Nombre Diego Alejan... 10 24

Profesor Daniel Ascuntar — Materia Calculo Vectoral

UNIVERSIDAD DEL QUINDÍO

Programa: Ingeniería de Sistemas y Computación.
Espacio Académico: Cálculo Multivariado y Vectorial Nocturno.
Docente: Daniel Alfonso Ascuntar Rojas.
Fecha: 28 de octubre del 2024.
Parcial 3.

1. (Valor 1.0) Calcular el dominio y sus curvas de nivel de la función. Graficar las curvas de nivel.

$$f(x,y) = \sqrt{36 - 9x^2 - 4y^2}$$

2. (Valor 1.0) Demostrar que el siguiente límite **NO** existe

$$\lim_{(x,y)\to(0,0)} \frac{x^4 y}{x^4 + y^2}$$

$$\lim_{\substack{x\to 0\\ y\to 0}} \frac{x^2 y}{x^4 + y^2}$$

3. (Valor 1.0) Sea $z = \ln(e^x + e^y)$ verificar si cumple la identidad

$$\left(\frac{\partial^2 z}{\partial x^2}\right)\left(\frac{\partial^2 z}{\partial y^2}\right) - \left(\frac{\partial^2 z}{\partial x \partial y}\right)^2 = 0$$

4. (Valor 2.0) Suponga que en una función derivable $w = f(x,y)$ sustituimos las coordenadas polares $x = r\cos(\theta)$ y $y = r\sin(\theta)$. Demuestre que

(a)

$$\frac{\partial f}{\partial r} = f_x \cos(\theta) + f_y \operatorname{sen}(\theta)$$

(b)

$$\frac{1}{r}\frac{\partial w}{\partial \theta} = -f_x \operatorname{sen}(\theta) + f_y \cos(\theta)$$

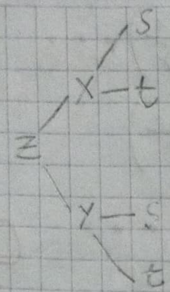
(c)

$$\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 = \left(\frac{\partial f}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial f}{\partial \theta}\right)^2$$

5. **Bonificación** Sea $z = f(x.y)$ con $x = g(s,t)$ y $y = h(s,t)$ calcular $\frac{\partial^2 z}{\partial s^2}$ y $\frac{\partial^2 z}{\partial t^2}$

Nombre	Diego Alejandro Flores Q.	Fecha	día 28 mes 10 año 24
Profesor	Daniel Ascuntar	Materia	Calculo Vectorial
Institución	Universidad del Quindio	Curso	Nota 5.0+0.5

⑤ Extra:



$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t}$$

$$\frac{\partial^2 z}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t}\right)$$

$$\frac{\partial^2 z}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial z}{\partial x}\right)\frac{\partial x}{\partial t} + \frac{\partial z}{\partial x}\cdot\frac{\partial}{\partial t}\left(\frac{\partial x}{\partial t}\right)\cdots$$

$$+\frac{\partial}{\partial t}\left(\frac{\partial z}{\partial y}\right)\frac{\partial y}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial}{\partial t}\left(\frac{\partial y}{\partial t}\right)$$

$$\frac{\partial^2 z}{\partial t^2} = \underbrace{\frac{\partial}{\partial t}\left(\frac{\partial z}{\partial x}\right)\frac{\partial x}{\partial t}}_{D_1} + \frac{\partial z}{\partial x}\cdot\frac{\partial^2 x}{\partial t^2}\cdots$$

$$+\underbrace{\frac{\partial}{\partial t}\left(\frac{\partial z}{\partial x}\right)\frac{\partial y}{\partial t}}_{D_2} + \frac{\partial z}{\partial y}\cdot\frac{\partial^2 y}{\partial t^2}$$

$$D_1 = \left[\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\frac{\partial x}{\partial t} + \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\frac{\partial y}{\partial t}\right]\frac{\partial x}{\partial t}$$

$$D_1 = \frac{\partial^2 z}{\partial x^2}\left(\frac{\partial x}{\partial t}\right)^2 + \frac{\partial^2 z}{\partial y\partial x}\cdot\frac{\partial x}{\partial t}\cdot\frac{\partial y}{\partial t}$$

$$D_2 = \left[\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\frac{\partial x}{\partial t} + \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\frac{\partial y}{\partial t}\right]\frac{\partial y}{\partial t}$$

$$D_2 = \frac{\partial^2 z}{\partial x\partial y}\cdot\frac{\partial x}{\partial t}\cdot\frac{\partial y}{\partial t} + \frac{\partial^2 z}{\partial y^2}\left(\frac{\partial y}{\partial t}\right)^2$$

$$\frac{\partial^2 z}{\partial t^2} = \frac{\partial^2 z}{\partial x^2}\left(\frac{\partial x}{\partial t}\right)^2 + \frac{\partial^2 z}{\partial y\partial x}\cdot\frac{\partial x}{\partial t}\cdot\frac{\partial y}{\partial t} + \frac{\partial z}{\partial x}\cdot\frac{\partial^2 x}{\partial t^2}\cdots$$

$$+\frac{\partial^2 z}{\partial x\partial y}\cdot\frac{\partial x}{\partial t}\cdot\frac{\partial y}{\partial t} + \frac{\partial^2 z}{\partial y^2}\left(\frac{\partial y}{\partial t}\right)^2 + \frac{\partial z}{\partial y}\cdot\frac{\partial^2 y}{\partial t^2}$$

$$\frac{\partial^2 z}{\partial t^2} = \frac{\partial^2 z}{\partial x^2}\left(\frac{\partial x}{\partial t}\right)^2 + 2\cdot\frac{\partial^2 z}{\partial y\partial x}\cdot\frac{\partial x}{\partial t}\cdot\frac{\partial y}{\partial t}\cdots$$

$$+\frac{\partial^2 z}{\partial y^2}\left(\frac{\partial y}{\partial t}\right)^2 + \frac{\partial z}{\partial x}\left(\frac{\partial^2 x}{\partial t^2}\right) + \frac{\partial z}{\partial y}\left(\frac{\partial^2 y}{\partial t^2}\right)$$

$$\bullet\ \frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial s}$$

$$\frac{\partial^2 z}{\partial s^2} = \frac{\partial}{\partial s}\left(\frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial s}\right)$$

$$\frac{\partial^2 z}{\partial s^2} = \frac{\partial}{\partial s}\left(\frac{\partial z}{\partial x}\right)\frac{\partial x}{\partial s} + \frac{\partial z}{\partial x}\cdot\frac{\partial}{\partial s}\left(\frac{\partial x}{\partial s}\right)\cdots$$

$$+\frac{\partial}{\partial s}\left(\frac{\partial z}{\partial y}\right)\frac{\partial y}{\partial s} + \frac{\partial z}{\partial y}\cdot\frac{\partial}{\partial s}\left(\frac{\partial y}{\partial s}\right)$$

$$\frac{\partial^2 z}{\partial s^2} = \boxed{\frac{\partial}{\partial s}\left(\frac{\partial z}{\partial x}\right)\frac{\partial x}{\partial s} + \frac{\partial z}{\partial x}}\cdot\frac{\partial^2 x}{\partial s^2}\cdots$$

D_1

$$+\boxed{\frac{\partial}{\partial s}\left(\frac{\partial z}{\partial x}\right)\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}}\cdot\frac{\partial^2 y}{\partial s^2}$$

D_2

0.5

$$D_1 = \left[\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\frac{\partial x}{\partial s} + \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\frac{\partial y}{\partial s}\right]\frac{\partial x}{\partial s}$$

$$D_1 = \frac{\partial^2 z}{\partial x^2}\left(\frac{\partial x}{\partial s}\right)^2 + \frac{\partial^2 z}{\partial y\partial x}\cdot\frac{\partial x}{\partial s}\cdot\frac{\partial y}{\partial s}$$

$$D_2 = \left[\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\frac{\partial x}{\partial s} + \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\frac{\partial y}{\partial s}\right]\frac{\partial y}{\partial s}$$

$$D_2 = \frac{\partial^2 z}{\partial x\partial y}\cdot\frac{\partial x}{\partial s}\cdot\frac{\partial y}{\partial s} + \frac{\partial^2 z}{\partial y^2}\left(\frac{\partial y}{\partial s}\right)^2$$

$$\frac{\partial^2 z}{\partial s^2} = \frac{\partial^2 z}{\partial x^2}\left(\frac{\partial x}{\partial s}\right)^2 + \frac{\partial^2 z}{\partial y\partial x}\cdot\frac{\partial x}{\partial s}\cdot\frac{\partial y}{\partial s} + \frac{\partial z}{\partial x}\cdot\frac{\partial^2 x}{\partial s^2}\cdots$$

$$+\frac{\partial^2 z}{\partial x\partial y}\cdot\frac{\partial x}{\partial s}\cdot\frac{\partial y}{\partial s} + \frac{\partial^2 z}{\partial y^2}\left(\frac{\partial y}{\partial s}\right)^2 + \frac{\partial z}{\partial y}\cdot\frac{\partial^2 y}{\partial s^2}$$

$$\frac{\partial^2 z}{\partial s^2} = \frac{\partial^2 z}{\partial x^2}\left(\frac{\partial x}{\partial s}\right)^2 + 2\cdot\frac{\partial^2 z}{\partial y\partial x}\cdot\frac{\partial x}{\partial s}\cdot\frac{\partial y}{\partial s}\cdots$$

$$+\frac{\partial^2 z}{\partial y^2}\left(\frac{\partial y}{\partial s}\right)^2 + \frac{\partial z}{\partial x}\left(\frac{\partial^2 x}{\partial s^2}\right) + \frac{\partial z}{\partial y}\left(\frac{\partial^2 y}{\partial s^2}\right)$$

① $f(x,y) = \sqrt{36-9x^2-4y^2}$

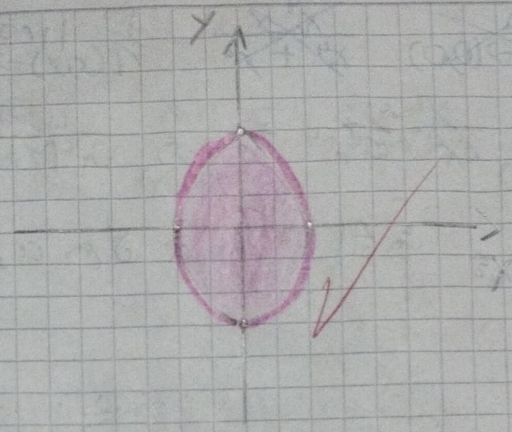
$36-9x^2-4y^2 \geq 0$

$(-1)\ -9x^2-4y^2 \geq -36\ (-1)$

$9x^2+4y^2 \leq 36$

$\frac{9x^2}{36}+\frac{4y^2}{36} \leq 1$

$\frac{x^2}{4}+\frac{y^2}{9} \leq 1$



$Dom f = \{(x,y) \in \mathbb{R}^2 \mid \frac{x^2}{4}+\frac{y^2}{9} \leq 1\}$

$K = \sqrt{36-9x^2-4y^2}$

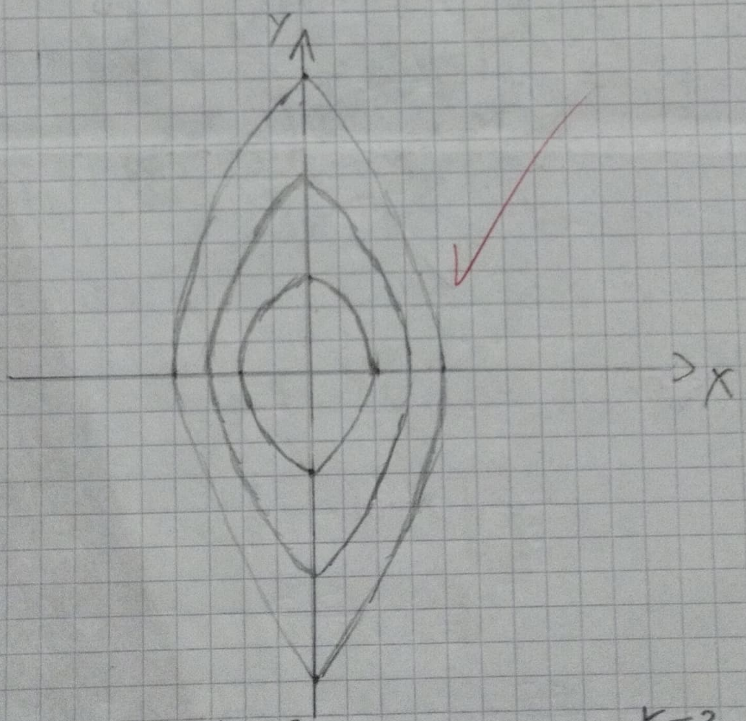
$K^2 = 36-9x^2-4y^2$

$-K^2+36 = 9x^2+4y^2$

$1 = \frac{9x^2}{-K^2+36} + \frac{4y^2}{-K+36}$

$K \neq 6$

10



$K=1$

$1 = \frac{9x^2}{35} + \frac{4y^2}{35}$

$K=2$

$1 = \frac{9x^2}{32} + \frac{y^2}{8}$

$K=3$

$1 = \frac{x^2}{3} + \frac{4y^2}{27}$

2) $\cancel{\lim_{(x,y)\to(0,0)} \frac{x^4 y}{x^4+x^2}}$ $\quad \lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4+y^2}$

$$\lim_{\substack{x\to 0 \\ y\to ax^2}} \frac{x^2 ax^2}{x^4+a^2x^4} \Rightarrow \lim_{\substack{x\to 0 \\ y\to ax^2}} \frac{a\cancel{x^4}}{\cancel{x^4}(1+a^2)} \Rightarrow \lim_{\substack{x\to 0 \\ y\to ax^2}} \frac{a}{1+a^2} \Rightarrow$$

$\frac{a}{1+a^2}$ ∴ Al depender de a, el límite no existe

10

Diego flores

(3) $z = \ln(e^x + e^y)$

$$\frac{\partial z}{\partial x} = \frac{e^x}{e^x + e^y}$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{(e^x + e^y)e^x - e^{2x}}{[e^x + e^y]^2} \Rightarrow \frac{\cancel{e^{2x}} + e^x e^y - \cancel{e^{2x}}}{[e^x + e^y]^2} \Rightarrow \frac{e^x e^y}{[e^x + e^y]^2}$$

$$\frac{\partial z}{\partial y} = \frac{e^y}{e^x + e^y}$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{(e^x + e^y)e^y - e^{2y}}{[e^x + e^y]^2} \Rightarrow \frac{e^x e^y + \cancel{e^{2y}} - \cancel{e^{2y}}}{[e^x + e^y]^2} \Rightarrow \frac{e^x e^y}{[e^x + e^y]^2}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{-e^y e^x}{[e^x + e^y]^2} \Rightarrow -\frac{e^x e^y}{[e^x + e^y]^2}$$

$$\left(\frac{e^x e^y}{[e^x + e^y]^2}\right)\left(\frac{e^x e^y}{[e^x + e^y]^2}\right) - \left(-\frac{e^x e^y}{[e^x + e^y]^2}\right)^2 = 0$$

$$\cancel{\frac{e^{2x} e^{2y}}{[e^x + e^y]^4}} - \cancel{\frac{e^{2x} e^{2y}}{[e^x + e^y]^4}} = 0$$

$$0 = 0$$

∴ Si cumple la identidad

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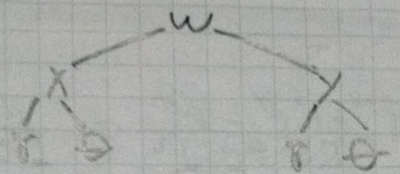
④ $w = f(x,y)$; $x = r\cos\theta$, $y = r\sin\theta$



$$\frac{\partial f}{\partial r} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial r}$$

$$\frac{\partial f}{\partial r} = \frac{\partial w}{\partial x}\cdot\cos\theta + \frac{\partial w}{\partial y}\cdot\text{sen}\,\theta$$

$$\frac{\partial f}{\partial \theta} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial \theta} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial \theta}$$

$$\frac{\partial f}{\partial \theta} = \frac{\partial w}{\partial x}(-r\,\text{sen}\,\theta) + \frac{\partial w}{\partial y}\cdot(r\cos\theta)$$

a) $\frac{\partial w}{\partial x}\cdot\cos\theta + \frac{\partial w}{\partial y}\,\text{sen}\,\theta = \frac{\partial w}{\partial x}\cos\theta + \frac{\partial w}{\partial y}\,\text{sen}\,\theta$

∴ Cumple la igualdad

20

b) $\frac{1}{\cancel{r}}\left(\frac{\partial w}{\partial x}(-\cancel{r}\,\text{sen}\,\theta) + \frac{\partial w}{\partial y}(\cancel{r}\cos\theta)\right) = -\frac{\partial w}{\partial x}\,\text{sen}\,\theta + \frac{\partial w}{\partial y}\cos\theta \Rightarrow$

$= -\frac{\partial w}{\partial x}\,\text{sen}\,\theta + \frac{\partial w}{\partial y}\cos\theta = -\frac{\partial w}{\partial x}\,\text{sen}\,\theta + \frac{\partial w}{\partial y}\cos\theta$

∴ Cumple la igualdad

c) $[\cdots] = \left(\frac{\partial w}{\partial x}\cdot\cos\theta + \frac{\partial w}{\partial y}\cdot\text{sen}\,\theta\right)^2 + \frac{1}{r^2}\left(\frac{\partial w}{\partial x}(-r\,\text{sen}\,\theta) + \frac{\partial w}{\partial y}(r\cos\theta)\right)^2 \Rightarrow$

$= [\cdots] = \left(\frac{\partial w}{\partial x}\right)^2\cos^2\theta + \cancel{2\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\cos\theta\cdot\text{sen}\,\theta} + \left(\frac{\partial w}{\partial y}\right)^2\text{sen}^2\theta \cdots$

$+\frac{1}{\cancel{r^2}}\left(\left(\frac{\partial w}{\partial x}\right)^2(\cancel{r^2}\,\text{sen}^2\theta) - \cancel{2r^2\frac{\partial w}{\partial x}\cdot\frac{\partial w}{\partial y}\,\text{sen}\,\theta\cos\theta} + \left(\frac{\partial w}{\partial y}\right)^2\cancel{r^2}\cos^2\theta\right) \Rightarrow$

$=[\cdots] = \left(\frac{\partial w}{\partial x}\right)^2\cos^2\theta + \left(\frac{\partial w}{\partial y}\right)^2\text{sen}^2\theta + \left(\frac{\partial w}{\partial x}\right)^2\text{sen}^2\theta + \left(\frac{\partial w}{\partial y}\right)^2\cos^2\theta \Rightarrow$

$=[\cdots] = \left(\frac{\partial w}{\partial x}\right)^2\left(\underbrace{\cos^2\theta + \text{sen}^2\theta}_{1}\right) + \left(\frac{\partial w}{\partial y}\right)^2\left(\underbrace{\text{sen}^2\theta + \cos^2\theta}_{1}\right) \Rightarrow$

$= \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 = \left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2$

$F = w$

∴ Cumple la igualdad

Diego Alejandro Flores Q

Programa: Ingeniería de Sistemas y Computación.
Espacio Académico: Cálculo Multivariado y Vectorial Nocturno.
Docente: Daniel Alfonso Ascuntar Rojas.
Fecha: 4 de Septiembre de 2024
Parcial 1

UNIVERSIDAD DEL QUINDÍO

1. (Valor 1.0) Describa el conjunto de puntos en el espacio cuyas coordenadas satisfacen las siguientes condiciones.
   (a) $x^2 + z^2 \leq 1$ y $0 \leq y$
   (b) $1 \leq x^2 + y^2 + z^2 \leq 4$
2. (Valor 1.0) Encontrar la ecuación de la recta que pasa por $A = (2, 4, 5)$ y es perpendicular al plano $3x + 7y - 5z = 21$
3. (Valor 1.0) Obtenga el punto de intersección de las rectas $x = 2t + 1$, $y = 3t + 2$, $z = 4t + 3$ y $x = s + 2$, $y = 2s + 4$, $z = -4s - 1$. Luego encuentre el plano determinado por estas rectas.
4. (Valor 2.0) Clasificar que tipo de superficie son las siguientes ecuaciones y graficar.
   (a) $x^2 + z^2 - 2x + 4y - y^2 + 2z = 0$
   (b) $4y^2 + z^2 - x - 16y - 4z + 20 = 0$

piedad :c

Nombre: Diego Alejandro Flores Q

Fecha: 04 09 24

Profesor: Daniel Ascuntar

Materia: calculo vectorial

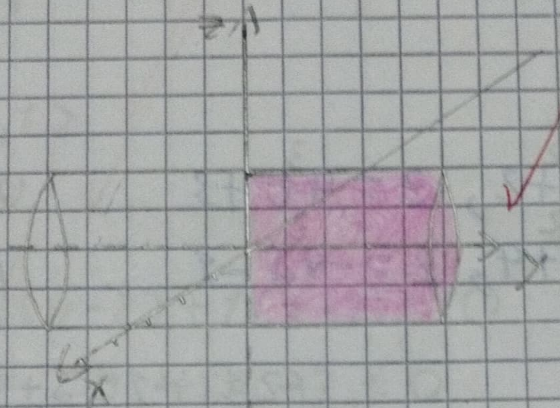
Institución: Universidad del Quindio

Curso:

Nota: 5.0

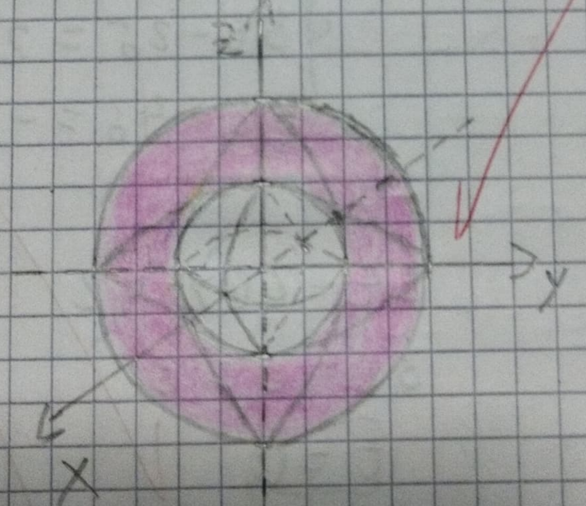
1

a) $x^2+z^2\le 1$ y $0\le y$



1.0

b) $1\le x^2+y^2+z^2\le 4$



② $A = (2,4,5)$; $3x + 7y - 5z = 21$

$A = (2,4,5)$

$\vec{\delta} = \langle 3, 7, -5 \rangle$

$x = 2 + 3t$

$y = 4 + 7t$

$z = 5 - 5t$

10

③ $x = 2t + 1$, $y = 3t + 2$, $z = 4t + 3$

$x = \delta + 2$, $y = 2\delta + 4$, $z = -4\delta - 1$

$2t + 1 = \delta + 2$

$t = \dfrac{\delta + 1}{2}$

$4\left(\dfrac{\delta + 1}{2}\right) + 4\delta = -4$

$2(\delta + 1) + 4\delta = -4$

$2\delta + 2 + 4\delta = -4$

$6\delta + 2 = -4$

$6\delta = -6$

$\delta = -1$

$$\begin{cases} 2t + 1 = \delta + 2 \\ 3t + 2 = 2\delta + 4 \\ 4t + 3 = -4\delta - 1 \end{cases}$$

$$\begin{cases} 2t - \delta = 1 \\ 3t - 2\delta = 2 \\ 4t + 4\delta = -4 \end{cases}$$

$3t - 2(-1) = 2$

$3t + 2 = 2$

$t = 0$

$P = (1, 2, 3)$

$$\left.\begin{array}{l} x = 2(0) + 1 \Rightarrow 1 \\ x = (-1) + 2 \Rightarrow 1 \end{array}\right\} 1$$

$$\left.\begin{array}{l} y = 3(0) + 2 \Rightarrow 2 \\ y = 2(-1) + 4 \Rightarrow 2 \end{array}\right\} 2$$

$$\left.\begin{array}{l} z = 4(0) + 3 \Rightarrow 3 \\ z = -4(-1) - 1 \Rightarrow 3 \end{array}\right\} 3$$

$\vec{u} = \langle 2, 3, 4 \rangle$

$\vec{v} = \langle 1, 2, -4 \rangle$

$\vec{n} = \overrightarrow{u \times v}$

$$\vec{n} = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 1 & 2 & -4 \end{vmatrix} = (-20)i - (-12)j + (1)k \Rightarrow = -20i + 12j + 1k$$

$\vec{n} = \langle -20, 12, 1 \rangle$

$P = (1, 2, 3)$

$-20(x-1) + 12(y-2) + 1(z-3) = 0$

$-20x + 20 + 12y - 24 + 1z - 3 = 0$

$-20x + 12y + z = 7$

:c

(4)

a) $x^2+z^2-2x+4y-y^2+2z=0$

$x^2-2x-y^2+4y+z^2+2z=0$

$x^2-2x-(y^2-4y)+z^2+2z=0$

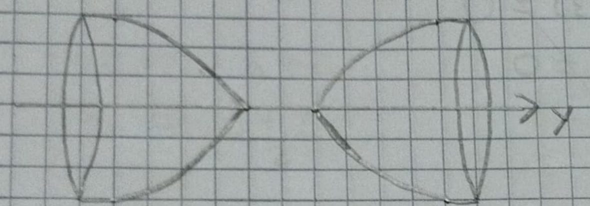
$(x-1)^2-1-[(y-2)^2-4]+(z+1)^2-1=0$

$(x-1)^2-1-(y-2)^2+4+(z+1)^2-1=0$

$(x-1)^2-(y-2)^2+(z+1)^2=-2$

$-\frac{(x-1)^2}{2}+\frac{(y-2)^2}{2}-\frac{(z+1)^2}{2}=1$ → hiperboloide de dos hojas

1,0



b) $4y^2+z^2-x-16y-4z+20=0$

$-x+4y^2-16y+z^2-4z+20=0$

$-x+4(y^2-4y)+z^2-4z+20=0$

$-x+4[(y-2)^2-4]+(z-2)^2-4+20=0$

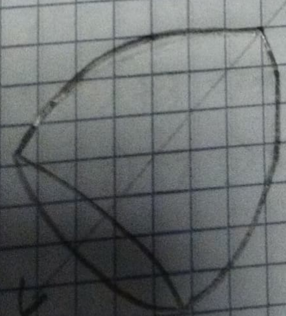
$-x+4(y-2)^2-16+(z-2)^2-4+20=0$

$-x+4(y-2)^2+(z-2)^2=0$

$4(y-2)^2+(z-2)^2=x$

$\frac{(y-2)^2}{\frac{1}{4}}+\frac{(z-2)^2}{1}=\frac{x}{1}$ → Paraboloide

1,0



UNIVERSIDAD DEL QUINDÍO

Programa: Ingeniería de Sistemas y Computación.
Espacio Académico: Cálculo Multivariado y Vectorial Nocturno.
Docente: Daniel Alfonso Ascuntar Rojas.
Fecha: 27 de septiembre de 2024
Parcial 2

1. (Valor 1.0) En qué puntos cruza la curva $\mathbf{r}(t) = t\mathbf{i} + (2t - t^2)\mathbf{k}$ y el paraboloide $x = z^2 + y^2$
2. (Valor 1.0) Identifique y dibuje la curva representadas por $x = t\cos t$, $y = t$, $z = t\sin t$ con $\cancel{0 \leq}\, t$ escriba además la ecuación rectangular correspondiente.
3. (Valor 1.0) Calcule la longitud de la curva de $\mathbf{r}(t) = \left\langle e^t\cos(t), e^t\mathrm{sen}(t), e^t \right\rangle$ con $-\ln 4 \leq t \leq 0$
4. (Valor 1.0) Calcule la curvatura de $\mathbf{r}(t) = t^2\mathbf{i} + \ln t\mathbf{j} + t\ln t\mathbf{k}$ en el punto $(1,0,0)$. $r' \times r''$
5. (Valor 1.0) Encuentre el vector de posición de una partícula que tiene como función de aceleración $\mathbf{a}(t) = t\mathbf{i} + e^t\mathbf{j} + e^{-t}\mathbf{k}$ con condiciones iniciales de velocidad $\mathbf{v}(0) = \mathbf{k}$ y posición $\mathbf{r}(0) = \mathbf{j} + \mathbf{k}$.

Nombre Diego Alejandro Flores Q. Fecha 27 9 24

Profesor Materia

Institución Curso Nota 50

① $r(t) = ti + (2t - t^2)K$ curva

$x = z^2 + y^2$ paraboloide

$x = t$

$y = 0$

$z = 2t - t^2$

$t = (2t - t^2)^2$

$t = 4t^2 - 4t^3 + t^4$

$0 = t^4 - 4t^3 + 4t^2 - t$

$0 = t(t^3 - 4t^2 + 4t - 1)$

$0 = t$

$0 = t^3 - 4t^2 + 4t - 1$

1	-4	4	-1	1
	1	-3	1	
1	-3	1	$\boxed{0}$	

$t^2 - 3t + 1 = 0$

$\frac{3 \pm \sqrt{9 - 4(1)(1)}}{2}$

$\frac{3 \pm \sqrt{5}}{2}$

$\frac{3 + \sqrt{5}}{2}, \frac{3 - \sqrt{5}}{2}$

$t = 0$

$t = 1$

$t = \frac{3 + \sqrt{5}}{2}$

$t = \frac{3 - \sqrt{5}}{2}$

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$t = 0$: $(0, 0, 0)$

$t = 1$: $(1, 0, 1)$

$t = \frac{3 + \sqrt{5}}{2}$: $(2.62, 0, -1.62)$

$t = \frac{3 - \sqrt{5}}{2}$: $(0.38, 0, 0.62)$

$\frac{45}{64}$

③ $r(t) = \langle e^t \cos(t), e^t \operatorname{sen}(t), e^t \rangle; \; -\ln 4 \leq t \leq 0$

$\frac{\partial x}{\partial t} = e^t \cos t - e^t \operatorname{sen} t$

$\frac{\partial y}{\partial t} = e^t \operatorname{sen} t + e^t \cos t$

$\frac{\partial z}{\partial t} = e^t$

$L = \int_{-\ln 4}^{0} \sqrt{(e^t \cos t - e^t \operatorname{sen} t)^2 + (e^t \operatorname{sen} t + e^t \cos t)^2 + (e^t)^2}\, \partial t$

$\sqrt{(e^{2t}\cos^2 t \cancel{- 2e^{2t}\cos t \operatorname{sen} t} + e^{2t}\operatorname{sen}^2 t) + (e^{2t}\operatorname{sen}^2 t \cancel{+ 2e^{2t}\operatorname{sen} t \cos t} + e^{2t}\cos^2 t) \ldots}$

$\ldots + e^{2t} \Rightarrow$

$= e^{2t}\cos^2 t + e^{2t}\operatorname{sen}^2 t + e^{2t}\operatorname{sen}^{2t} + e^{2t}\cos^2 t + e^{2t} \Rightarrow$

$= 2e^{2t}\cos^2 t + 2e^{2t}\operatorname{sen}^2 t + e^{2t} \Rightarrow$

$= e^{2t}(2\cos^2 t + 2\operatorname{sen}^{2t} + 1) \Rightarrow e^{2t}(2[\cos^2 t + \operatorname{sen}^2 t] + 1) \Rightarrow$

$= e^{2t}(2[1] + 1) \Rightarrow e^{2t}(2 + 1) \Rightarrow e^{2t}(3) \Rightarrow 3e^{2t}$

$\int_{-\ln 4}^{0} \sqrt{3e^{2t}}\, \partial t$

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$\sqrt{3}\int e^t \, \partial t \Rightarrow \sqrt{3}\, e^t + C$

$\sqrt{3}\left(e^t\right)_{-\ln 4}^{0} \Rightarrow \sqrt{3}\, e^0 - \sqrt{3}\, e^{-\ln 4} \Rightarrow \frac{3\sqrt{3}}{4} \approx 1{,}299$

$\frac{d}{dx}\ln x = \frac{1}{x}$

④ $r(t) = t^2\hat{\imath} + \ln t\,\hat{\jmath} + t\ln t\,\hat{k}$; $P = (1, 0, 0)$

$t^2 = 1$ | $x(1) = 1^2$ | $y(1) = \ln(1)$ | $z(1) = (1)\ln(1)$

$t = \sqrt{1}$ | $x(1) = 1$ ✓ | $y(1) = 0$ ✓ | $z(1) = 0$ ✓

$t = \pm 1$ ∴ $t = 1$ //

$t = 1$

$$\frac{|r'(t) \times r''(t)|}{|r'(t)|^3}$$

$\vec{r}(t) = \langle t^2, \ln t, t\ln t \rangle$

$\vec{r}'(t) = \langle 2t, \frac{1}{t}, 1 + \ln t \rangle$

$\vec{r}''(t) = \langle 2, -\frac{1}{t^2}, \frac{1}{t} \rangle$

1,0

$r'(1) = \langle 2, 1, 1 \rangle$

$r''(1) = \langle 2, -1, 1 \rangle$

$$r'(1) \times r''(1) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & 1 \\ 2 & -1 & 1 \end{vmatrix} = (1+1)\hat{\imath} - (2-2)\hat{\jmath} + (-2-2)\hat{k} \Rightarrow$$

$= 2\hat{\imath} - 0\hat{\jmath} - 4\hat{k}$

$\langle 2, 0, -4 \rangle$

$|r'(1) \times r''(1)| = \sqrt{4+16} \Rightarrow \sqrt{20}$

$|r'(1)| = \sqrt{4+1+1} \Rightarrow \sqrt{6}$

$|r'(1)|^3 = (\sqrt{6})^3 \Rightarrow 6\sqrt{6}$

$K = \frac{\sqrt{20}}{6\sqrt{6}} \Rightarrow \frac{2\sqrt{5}}{6\sqrt{6}} \Rightarrow \frac{\sqrt{5}}{3\sqrt{6}}$ // $\approx 0,304$

⑤ $a(t) = t\hat{\imath} + e^{t}\hat{\jmath} + e^{-t}\hat{k}$; $v(0) = \hat{k}$; $r(0) = \hat{\jmath} + \hat{k}$

$$\begin{cases} a(t) = \langle t, e^{t}, e^{-t} \rangle \\ v(0) = \langle 0, 0, 1 \rangle \\ r(0) = \langle 0, 1, 1 \rangle \end{cases}$$

$v(t) = \int a(t)\, dt$:

- $\int t\, dt \Rightarrow \frac{t^2}{2} + C_1$
- $\int e^{t}\, dt \Rightarrow e^{t} + C_2$
- $\int e^{-t}\, dt \Rightarrow -e^{-t} + C_3$

$v(t) = \langle \frac{t^2}{2} + C_1, e^{t} + C_2, -e^{-t} + C_3 \rangle = \langle 0, 0, 1 \rangle$

$0 = \frac{(0)^2}{2} + C_1$ | $0 = e^{0} + C_2$ | $1 = -e^{-0} + C_3$

$0 = C_1$ | $0 = 1 + C_2$ | $1 = -1 + C_3$

$-1 = C_2$ | $2 = C_3$

$v(t) = \langle \frac{t^2}{2}, e^{t} - 1, -e^{-t} + 2 \rangle$

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$r(t) = \int v(t)\, dt$:

- $\frac{1}{2}\int t^2\, dt \Rightarrow \frac{t^3}{6} + C_1$
- $\int e^{t}\, dt - \int 1\, dt \Rightarrow e^{t} - t + C_2$
- $-\int e^{-t} + 2\int 1\, dt \Rightarrow e^{-t} + 2t + C_3$

$r(t) = \langle \frac{t^3}{6} + C_1, e^{t} - t + C_2, e^{-t} + 2t + C_3 \rangle = \langle 0, 1, 1 \rangle$

$0 = \frac{(0)^3}{6} + C_1$ | $1 = 1 - 0 + C_2$ | $1 = 1 + 0 + C_3$

$0 = C_1$ | $1 = 1 + C_2$ | $1 = 1 + C_3$

$0 = C_2$ | $0 = C_3$

$r(t) = \langle \frac{t^3}{6}, e^{t} - t, e^{-t} + 2t \rangle$

go Flores

② $x = t\cos t$, $y = t$, $z = t\sin t$

$y = t$

$x = y\cos y$

$z = y\sin y$

$\frac{x}{y} = \cos y$

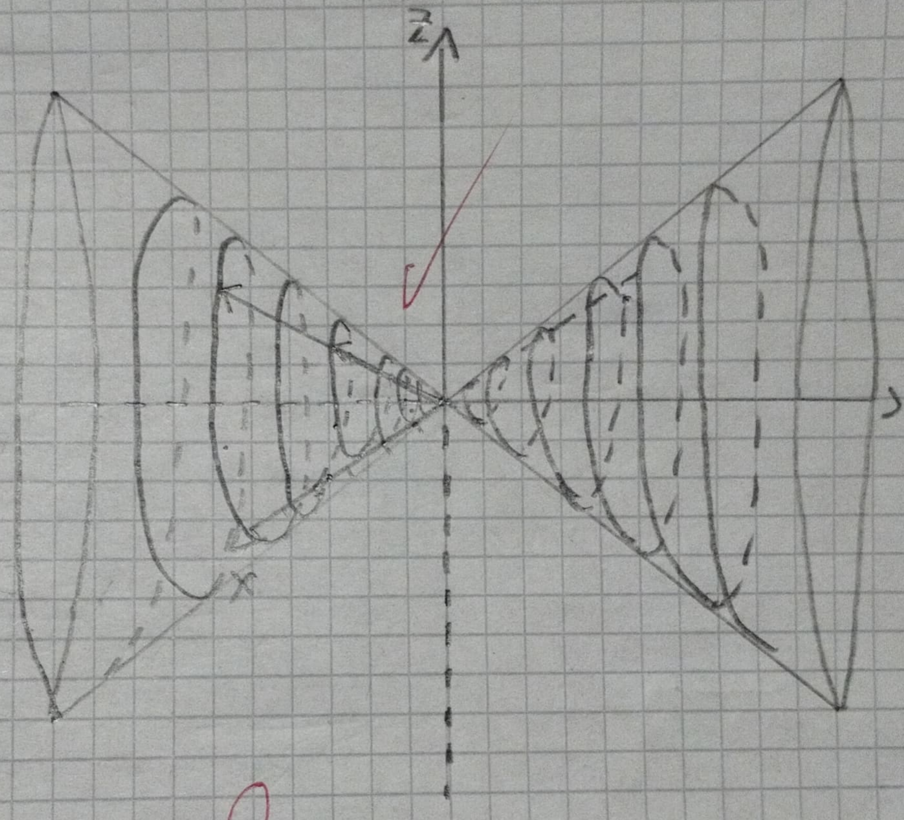
$\frac{z}{y} = \sin y$

$\cos^2 y + \sin^2 y = 1$

$\frac{x^2}{y^2} + \frac{z^2}{y^2} = 1$

$x^2 + z^2 = y^2$

$x^2 + z^2 - y^2 = 0$



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