

La tasita

ICPC Notebook

Team: Turistas

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Parte I

Dynamic Programming

1.1. Longest Common Subsequence

```
1 // https://leetcode.com/problems/longest-common-subsequence/submissions
2 // 1815539365/
3 #include <bits/stdc++.h>
4 using namespace std;
5
6 string s1, s2;
7 const int n = 1000;
8 int dp[n][n];
9
10 int LCS(int i, int j) {
11     if (i < 0 || j < 0) return 0;
12     if (dp[i][j] != -1) return dp[i][j];
13
14     if (s1[i] == s2[j]) dp[i][j] = 1 + LCS(i - 1, j - 1);
15     else dp[i][j] = max(LCS(i - 1, j), LCS(i, j - 1));
16
17     return dp[i][j];
18 }
19
20 string LCSReconstruccion() {
21     int i = s1.size() - 1;
22     int j = s2.size() - 1;
23     string ans = "";
24
25     while (i >= 0 && j >= 0) {
26         if (s1[i] == s2[j]) {
27             ans += s1[i];
28             i--;
29             j--;
30         } else {
31             if (i > 0 && (j == 0 || dp[i - 1][j] >= dp[i][j - 1])) i--;
32             else j--;
33         }
34     }
35
36     reverse(ans.begin(), ans.end());
37     return ans;
38 }
39
40 int main() {
41     memset(dp, -1, sizeof(dp));
42     s1 = "abcde";
43     s2 = "ace";
44
45     cout << LCS(s1.size() - 1, s2.size() - 1) << endl;
46     cout << LCSReconstruccion() << endl;
47 }
```

1.2. Longest Increasing Subsequence

```
1 // https://leetcode.com/problems/longest-increasing-subsequence/
2 // description/
3 #include <bits/stdc++.h>
4 using namespace std;
5
6 const int max_n = 100000;
7 int dp[max_n + 1];
8
9 int LIS(vector<int> arr) {
10     int n = arr.size();
11     memset(dp, 0, sizeof(dp));
12
13     int res = 0;
14
15     for (int i = 0; i < n; i++) {
16         dp[i] = 1;
17         int valAc = arr[i];
18         for (int j = 0; j < i; j++) {
19             int valAn = arr[j];
20             if (valAn < valAc) dp[i] = max(dp[i], dp[j] + 1);
21         }
22         res = max(res, dp[i]);
23     }
24
25     return res;
26 }
27
28 vector<int> LISReconstruccion(vector<int> arr) {
29     int n = arr.size();
30     vector<int> padre(n, -1);
31
32     int res = 0;
33     int indiceMejorSecuencia = -1;
34
35     for (int i = 0; i < n; i++) {
36         dp[i] = 1;
37         int valAc = arr[i];
38         for (int j = 0; j < i; j++) {
39             int valAn = arr[j];
40             if (valAn < valAc && dp[j] + 1 > dp[i]) {
41                 dp[i] = dp[j] + 1;
42                 padre[i] = j;
43             }
44         }
45
46         if (dp[i] > res) {
47             res = dp[i];
48             indiceMejorSecuencia = i;
49         }
50     }
51
52     vector<int> secuencia;
53     for (int i = indiceMejorSecuencia; i != -1; i = padre[i]) {
54         secuencia.push_back(arr[i]);
55     }
56 }
```

```
56     reverse(secuencia.begin(), secuencia.end());
57
58     return secuencia;
59 }
60
61 int main() {
62     vector<int> nuevo = {1,3,6,7,9,4,10,5,6};
63     cout << LIS(nuevo) << endl;
64
65     vector<int> secuencia = LISReconstruccion(nuevo);
66     for (int& val : secuencia) cout << val << " ";
67 }
```

1.3. Longest Increasing Subsequence Fast

```
1 // https://leetcode.com/problems/longest-increasing-subsequence/
2 // description/
3 #include <bits/stdc++.h>
4 using namespace std;
5
6 int LISFast(vector<int> arr) {
7     int n = arr.size();
8     vector<int> res;
9     res.push_back(arr[0]);
10
11    for (int i = 1; i < n; i++) {
12        if (arr[i] > res.back()) {
13            res.push_back(arr[i]);
14        } else {
15            auto it = lower_bound(res.begin(), res.end(), arr[i]);
16            int busca = it - res.begin();
17            res[busca] = arr[i];
18        }
19    }
20
21    return res.size();
22}
23
24 vector<int> LISFastReconstruccion(vector<int> arr) {
25    int n = arr.size();
26    vector<int> res;
27    vector<int> resIndex;
28    vector<int> padre(n, -1);
29
30    for (int i = 0; i < n; i++) {
31        int x = arr[i];
32        auto it = lower_bound(res.begin(), res.end(), x);
33        int pos = it - res.begin();
34
35        if (it == res.end()) {
36            res.push_back(x);
37            resIndex.push_back(i);
38        } else {
39            *it = x;
40            resIndex[pos] = i;
41        }
42
43        if (pos > 0)
44            padre[i] = resIndex[pos - 1];
45    }
46
47    vector<int> lis;
48    int idx = resIndex.back();
49    while (idx != -1) {
50        lis.push_back(arr[idx]);
51        idx = padre[idx];
52    }
53    reverse(lis.begin(), lis.end());
54    return lis;
55}
```

```
56 int main() {
57     vector<int> nuevo = {1,3,6,7,9,4,10,5,6};
58     cout << LISFast(nuevo) << endl;
59
60     vector<int> reconstruccion = LISFastReconstruccion(nuevo);
61     for (int& val : reconstruccion) cout << val << " ";
62 }
```

1.4. Maximum Subarray

```
1 // https://leetcode.com/problems/maximum-subarray/submissions
2 /1815546613/
3 #include <bits/stdc++.h>
4 using namespace std;
5
6 const int max_n = 100001;
7 int dp[max_n];
8
9 int maximumSub(vector<int>& arr) {
10     memset(dp, 0, sizeof(dp));
11     int n = arr.size();
12
13     for (int i = 1; i <= n; i++) {
14         int& valor = arr[i - 1];
15         dp[i] = max(valor, dp[i - 1] + valor);
16     }
17
18     int maximo = -1000000000;
19     for (int i = 1; i <= n; i++) {
20         maximo = max(maximo, dp[i]);
21     }
22     return maximo;
23 }
24
25 int main() {
26     vector<int> valores = {-2, 1, -3, 4, -1, 2, 1, -5, 4};
27     cout << maximumSub(valores);
28 }
```

1.5. Knapsack

```
1 // Source: https://atcoder.jp/contests/dp/tasks/dp_d
2 #include <bits/stdc++.h>
3 using namespace std;
4
5 const int MAX_W = 100001;
6 long long dp[MAX_W];
7 int W = MAX_W;
8
9 long long knapsack(const vector<pair<int, long long>>& items) {
10     memset(dp, -1, sizeof(dp));
11     int n = items.size();
12
13     dp[0] = 0;
14     for (int i = 0; i < n; i++) {
15         int w = items[i].first;
16         long long v = items[i].second;
17
18         for (int j = W; j >= w; j--) {
19             if (dp[j - w] != -1) {
20                 dp[j] = max(dp[j], dp[j - w] + v);
21             }
22         }
23     }
24
25     long long max_v = 0;
26     for (int j = 0; j <= W; j++) {
27         max_v = max(max_v, dp[j]);
28     }
29     return max_v;
30 }
31
32 int main() {
33     vector<pair<int, long long>> items = {{2,3},{3,5},{6,5}};
34     cout << knapsack(items) << endl;
35     return 0;
36 }
```

1.6. Knapsack Binary

```
1 // https://cses.fi/problemset/task/1159/
2 #include <bits/stdc++.h>
3 using namespace std;
4
5 const int MAX_W = 100001;
6 int dp[MAX_W];
7 int W;
8
9 int knapsack(const vector<pair<int, int>>& items) {
10    memset(dp, -1, sizeof(dp));
11    int n = items.size();
12    dp[0] = 0;
13
14    for (int i = 0; i < n; i++) {
15        int w = items[i].first;
16        int v = items[i].second;
17
18        for (int j = W; j >= w; j--) {
19            if (dp[j - w] != -1) {
20                dp[j] = max(dp[j], dp[j - w] + v);
21            }
22        }
23    }
24
25    int max_v = 0;
26    for (int j = 0; j <= W; j++) {
27        max_v = max(max_v, dp[j]);
28    }
29    return max_v;
30 }
31
32 void res(const vector<int>& pesos, const vector<int>& valores, const
33           vector<int>& copias) {
34    int n = pesos.size();
35    vector<pair<int, int>> items;
36    for (int i = 0; i < n; i++) {
37        int k = copias[i];
38        int b = 1;
39        while (k > 0) {
40            int num = min(k, b);
41            items.push_back({pesos[i] * num, valores[i] * num});
42            k -= num;
43            b *= 2;
44        }
45    }
46    cout << knapsack(items) << endl;
47 }
48
49 int main() {
50    vector<int> pesos = {2, 6, 3};
51    vector<int> valores = {8, 5, 4};
52    vector<int> copias = {3, 5, 2};
53    res(pesos, valores, copias);
54 }
```

1.7. Knapsack Re

```
1 #include <bits/stdc++.h>
2 using namespace std;
3
4 const int MAX_N = 1001;
5 const int MAX_W = 1001;
6
7 int N, W;
8 long long dp[MAX_N][MAX_W];
9 vector<pair<int, long long>> items;
10
11 long long knapsack(int i, int w_rem) {
12     if (i == N || w_rem == 0) {
13         return 0;
14     }
15     if (dp[i][w_rem] != -1) {
16         return dp[i][w_rem];
17     }
18
19     int w = items[i].first;
20     long long v = items[i].second;
21
22     long long res = knapsack(i + 1, w_rem);
23     if (w_rem >= w) {
24         res = max(res, v + knapsack(i + 1, w_rem - w));
25     }
26
27     return dp[i][w_rem] = res;
28 }
29
30 vector<pair<int, long long>> knapsackReconstruccion() {
31     vector<pair<int, long long>> tomados;
32     int w_rem = W;
33
34     for (int i = 0; i < N; i++) {
35         int w = items[i].first;
36         long long v = items[i].second;
37
38         if (knapsack(i + 1, w_rem) == dp[i][w_rem]) {
39             continue;
40         }
41
42         else if (w_rem >= w && (v + knapsack(i + 1, w_rem - w) == dp[i][w_rem])) {
43             tomados.push_back(items[i]);
44             w_rem -= w;
45         }
46     }
47     return tomados;
48 }
49
50
51 int main() {
52     memset(dp, -1, sizeof(dp));
53     items = {{3, 30}, {4, 50}, {5, 60}};
54     W = 8;
55     N = items.size();
```

```
56     cout << knapsack(0, W) << endl;
57
58     vector<pair<int, long long>> res = knapsackReconstruccion();
59     cout << "Items tomados:" << endl;
60     for (auto& item : res) {
61         cout << "Peso: " << item.first << ", Valor: " << item.second <<
62         endl;
63     }
64 }
```

1.8. Knapsack Subset Sum

```
1 // https://leetcode.com/problems/tallest-billboard/description/
2 #include <bits/stdc++.h>
3 using namespace std;
4
5 const int MAX_N = 2501;
6 int diferencia[2][MAX_N];
7
8 int knapsackDiferencia(const vector<int>& arr) {
9     int n = arr.size();
10    memset(diferencia, -1, sizeof(diferencia));
11    diferencia[0][0] = 0;
12
13    for (int i = 0; i < n; i++) {
14        int ahora = arr[i];
15        int actual = i & 1;
16        int siguiente = actual ^ 1;
17
18        fill(diferencia[siguiente], diferencia[siguiente] + MAX_N, -1);
19
20        for (int valor = 0; valor < MAX_N; valor++) {
21            if (diferencia[actual][valor] == -1) continue;
22
23            diferencia[siguiente][valor] = max(
24                diferencia[siguiente][valor],
25                diferencia[actual][valor]
26            );
27
28            int nuevaDiferenciaMayor = valor + ahora;
29            if (nuevaDiferenciaMayor < MAX_N) {
30                diferencia[siguiente][nuevaDiferenciaMayor] = max(
31                    diferencia[siguiente][nuevaDiferenciaMayor],
32                    diferencia[actual][valor]
33                );
34            }
35
36            int nuevaDiferenciaMenor = abs(valor - ahora);
37            int nuevaAltura = diferencia[actual][valor] + min(valor,
38                ahora);
39            diferencia[siguiente][nuevaDiferenciaMenor] = max(
40                diferencia[siguiente][nuevaDiferenciaMenor],
41                nuevaAltura
42            );
43        }
44
45    return diferencia[n & 1][0];
46 }
47
48 int main() {
49     vector<int> roads = {1, 2, 3, 6};
50     cout << knapsackDiferencia(roads) << endl;
51 }
```

Parte II

Data Structures

2.1. Fenwick Tree

```
1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 #define fst first
6 #define snd second
7 #define all(c) ((c).begin()), ((c).end())
8 #define ll long long
9
10 const int INF = 1 << 30;
11 const int MAXN = 1e5+5;
12
13 struct FenwickTree {
14     vector<ll> bit;
15     ll n;
16
17     FenwickTree(ll n) {
18         this->n = n;
19         bit.assign(n, 0);
20     }
21
22     FenwickTree(vector<ll> const &a) : FenwickTree(a.size()) {
23         for (size_t i = 0; i < a.size(); i++)
24             add(i, a[i]);
25     }
26
27     ll sum(ll r) {
28         ll ret = 0;
29         for (; r >= 0; r = (r & (r + 1)) - 1)
30             ret += bit[r];
31         return ret;
32     }
33
34     ll sum(ll l, ll r) {
35         return sum(r) - sum(l - 1);
36     }
37
38     void add(ll idx, ll delta) {
39         for (; idx < n; idx = idx | (idx + 1))
40             bit[idx] += delta;
41     }
42 };
43
44
45 void solve() {
46     int n; cin >> n;
47     vector<ll> arr(n);
48     for (int i = 0; i < n; i++) {
49         cin >> arr[i];
50     }
51     FenwickTree ft(arr);
52
53     int m;
54     cin >> m;
55     for (int i = 1; i <= m; i++) {
56         int q,l,r;
```

```
57     cin >> q >> l >> r;
58     if (q == 's') {
59         cout << ft.sum(l, r) << " ";
60     }
61     else {
62         int delta = r - arr[l];
63         arr[l] = r;
64         ft.add(l, delta);
65     }
66 }
67 }
68
69 int main() {
70     ios::sync_with_stdio(false);
71     cin.tie(nullptr);
72     int t = 1;
73     //cin >> t;
74     while(t--)
75         solve();
76 }
```

2.2. Fenwick Tree 2d

```
1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 #define fst first
6 #define snd second
7 #define all(c) ((c).begin()), ((c).end())
8 #define ll long long
9
10 const int INF = 1 << 30;
11 const int MAXN = 1e3+5;
12
13 int n, m;
14 int qn;
15 char q[10];
16 int f[MAXN][MAXN];
17
18 void update(int x, int y, int delta) {
19     for (int i = x; i <= n; i = i | (i + 1))
20         for (int j = y; j <= m; j = j | (j + 1))
21             f[i][j] += delta;
22 }
23
24 int getSum(int x, int y) {
25     int res = 0;
26     for (int i = x; i > 0; i = (i & (i + 1)) - 1)
27         for (int j = y; j > 0; j = (j & (j + 1)) - 1)
28             res += f[i][j];
29     return res;
30 }
31
32 int getSum(int xFrom, int xTo, int yFrom, int yTo) {
33     return getSum(xTo, yTo) - getSum(xTo, yFrom - 1) - getSum(xFrom - 1,
34     yTo) + getSum(xFrom - 1, yFrom - 1);
35 }
36
37 void solve() {
38     cin >> n >> qn;
39     m = n;
40
41     for (int i = 1; i <= qn; i++) {
42         cin >> q;
43         if (q[0] == 'A') {
44             int x, y;
45             cin >> x >> y;
46             update(x, y, 1);
47         }
48         else {
49             int xFrom, xTo, yFrom, yTo;
50             cin >> xFrom >> yFrom >> xTo >> yTo;
51             if (xFrom > xTo)
52                 swap(xFrom, xTo);
53             if (yFrom > yTo)
54                 swap(yFrom, yTo);
55             cout << getSum(xFrom, xTo, yFrom, yTo);
56         }
57     }
58 }
```

```
56     }
57 }
58
59 int main() {
60     ios::sync_with_stdio(false);
61     cin.tie(nullptr);
62     int t = 1;
63     //cin >> t;
64     while(t--)
65         solve();
66 }
```

2.3. Treap

```
1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 #define fst first
6 #define snd second
7 #define all(c) ((c).begin()), ((c).end())
8 #define ll long long
9 #define endl '\n'
10
11 const int INF = 1 << 30;
12 const int MAXN = 1e5+5;
13
14 mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());
15
16 int random(int l, int r) {
17     return uniform_int_distribution<int>(l, r)(rng);
18 }
19
20 struct node {
21     int key;
22     int priority;
23     node* left;
24     node* right;
25     int cnt;
26     node(int v) : key(v) {
27         priority = random(0, 1e9);
28         left = right = nullptr;
29         cnt = 1;
30     }
31 };
32
33 int cnt(node* t) {
34     return t ? t->cnt : 0;
35 }
36
37 void update(node *t) {
38     if (t) t->cnt = 1 + cnt(t->left) + cnt(t->right);
39 }
40
41 node* merge(node* a, node* b) {
42     if (a == nullptr) return b;
43     if (b == nullptr) return a;
44     if (a->priority > b->priority) {
45         a->right = merge(a->right, b);
46         update(a);
47         return a;
48     } else {
49         b->left = merge(a, b->left);
50         update(b);
51         return b;
52     }
53 }
54
55 pair<node*, node*> split(node *T, int k) {
56     if (T == nullptr) return {nullptr, nullptr};
```

```

57     if(T->key <= k) {
58         auto p = split(T->right,k);
59         T->right = p.fst;
60         update(T);
61         return {T, p.snd};
62     } else {
63         auto p = split(T->left, k);
64         T->left = p.snd;
65         update(T);
66         return {p.fst, T};
67     }
68 }
69
70 node* insert(node* T, int x) {
71     auto p = split(T,x);
72     T = merge(p.fst, new node(x));
73     T = merge(T, p.snd);
74     return T;
75 }
76
77 node* remove(node* T, int x) {
78     auto p = split(T, x);           // p.fst:      x, p.snd: > x
79     auto p2 = split(p.fst, x-1);   // p2.fst: < x, p2.snd: == x
80
81     // p2.snd contains the node(s) with key = x      we discard it
82     return merge(p2.fst, p.snd);
83 }
84
85 bool find(node* T, int x) {
86     if (!T) return false;
87     if (T->key == x) return true;
88     if (x < T->key) return find(T->left, x);
89     return find(T->right, x);
90 }
91
92 node* kth(node* t, int k) {
93     int left_size = cnt(t->left);
94     if (k < left_size) return kth(t->left, k);
95     else if (k == left_size) return t;
96     else return kth(t->right, k - left_size - 1);
97 }
98
99 void inorder(node* T) {
100    if (!T) return;
101    inorder(T->left);
102    cout << T->key << " ";
103    inorder(T->right);
104 }
105
106 void solve() {
107     node* root = nullptr;
108
109     root = insert(root, 5);
110     root = insert(root, 2);
111     root = insert(root, 8);
112     root = insert(root, 1);
113     root = insert(root, 10);
114 }
```

```

115 cout << "Treap inorder (sorted): ";
116 inorder(root); // 1 2 5 8 10
117 cout << endl;
118
119 cout << "Find 8? " << (find(root, 8) ? "Yes" : "No") << endl;
120 cout << "Find 7? " << (find(root, 7) ? "Yes" : "No") << endl;
121
122 root = remove(root, 5);
123 cout << "After removing 5: ";
124 inorder(root); // 1 2 8 10
125 cout << endl;
126 }
127
128
129 int main() {
130 // ios::sync_with_stdio(false);
131 // cin.tie(nullptr);
132 int t = 1;
133 //cin >> t;
134 while(t--)
135 solve();
136 }
```

2.4. Implicit Treap

```
1 #include <bits/stdc++.h>
2 using namespace std;
3
4 #define endl '\n';
5
6 mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());
7
8 int random(int l, int r) {
9     return uniform_int_distribution<int>(l, r)(rng);
10 }
11
12 struct node {
13     int value;
14     int priority;
15     node* left;
16     node* right;
17     int cnt;
18
19     bool rev;
20     node(int v) : value(v) {
21         priority = random(0, 1e9);
22         left = right = nullptr;
23         cnt = 1; // FIX: initialize cnt
24     }
25 };
26
27 int cnt(node* t) {
28     return t ? t->cnt : 0;
29 }
30
31 // push for reverse or lazy propagation
32 void push(node* t) {
33     if (!t || !t->rev) return;
34     t->rev = 0;
35     swap(t->left, t->right);
36     if (t->left) t->left->rev ^= 1;
37     if (t->right) t->right->rev ^= 1;
38 }
39
40 void update(node* t) {
41     if (!t) return;
42     t->cnt = 1 + cnt(t->left) + cnt(t->right);
43 }
44
45 node* merge(node* a, node* b) {
46     if (!a) return b;
47     if (!b) return a;
48     if (a->priority > b->priority) {
49         push(a);
50         a->right = merge(a->right, b);
51         update(a);
52         return a;
53     } else {
54         push(b);
55         b->left = merge(a, b->left);
56         update(b);
57     }
58 }
```

```

57     return b;
58 }
59 }
60
61 // 0-index
62 pair<node*, node*> split(node* T, int k, int add = 0) {
63     if (!T) return {nullptr, nullptr};
64     push(T);
65     int cur_key = add + cnt(T->left);
66     if (k <= cur_key) {
67         auto p = split(T->left, k, add);
68         T->left = p.second;
69         update(T);
70         return {p.first, T};
71     } else {
72         auto p = split(T->right, k, add + 1 + cnt(T->left));
73         T->right = p.first;
74         update(T);
75         return {T, p.second};
76     }
77 }
78
79 node* insert(node* T, int pos, int value) {
80     auto p = split(T, pos);
81     T = merge(p.first, new node(value));
82     T = merge(T, p.second);
83     return T;
84 }
85
86 node* remove(node* T, int pos) {
87     auto p1 = split(T, pos);           // [0..pos-1], [pos..end]
88     auto p2 = split(p1.second, 1);    // [pos], [pos+1..end]
89                                         // delete p2.first; // optional, free
90                                         // memory
91     return merge(p1.first, p2.second);
92 }
93
94 void inorder(node* T) {
95     if (!T) return;
96     inorder(T->left);
97     cout << T->value << " ";
98     inorder(T->right);
99 }
100
101 node* kth(node* t, int k) {
102     push(t);
103     int left_size = cnt(t->left);
104     if (k < left_size) return kth(t->left, k);
105     else if (k == left_size) return t;
106     else return kth(t->right, k - left_size - 1);
107 }
108
109 node* update_range(node* t, int l, int r, int delta) {
110     auto p = split(t, r + 1);
111     auto p2 = split(p.first, l);
112
113     if (p2.second) {
114         p2.second->lazy += delta; // mark range for update

```

```

114     push(p2.second);           // optionally apply now (or can defer)
115     update(p2.second);        // recompute cnt/sum
116 }
117
118 return merge(merge(p2.first, p2.second), p.second);
119 }
120
121 void solve() {
122     node* root = nullptr;
123
124     root = insert(root, 0, 10);
125     root = insert(root, 1, 20);
126     root = insert(root, 2, 30);
127     cout << "Initial array: ";
128     inorder(root); cout << "\n";
129
130     root = insert(root, 1, 15);
131     cout << "After inserting 15 at pos 1: ";
132     inorder(root); cout << endl;
133
134     root = remove(root, 2);
135     cout << "After removing element at pos 2: ";
136     inorder(root); cout << endl;
137 }
138
139 int main() {
140     solve();
141 }
```

2.5. Ordered Set

```
1 #include <bits/stdc++.h>
2 #include <ext/pb_ds/assoc_container.hpp>
3 #include <ext/pb_ds/tree_policy.hpp>
4 using namespace std;
5 using namespace __gnu_pbds;
6
7 template <class T>
8 using ordered_set = tree<
9 T,
10 null_type,
11 less<T>, // use less_equal<T> if you want to allow
12 duplicates (see note below)
13 rb_tree_tag,
14 tree_order_statistics_node_update>;
15
16 // --- Example usage ---
17 int main() {
18     ios::sync_with_stdio(false);
19     cin.tie(nullptr);
20
21     ordered_set<int> s;
22
23     s.insert(10);
24     s.insert(20);
25     s.insert(30);
26     s.insert(40);
27
28     // 1      Get element by index (0-based)
29     cout << "Element at index 2: " << *s.find_by_order(2) << "\n"; // ->
30     30
31
32     // 2      Get index of an element
33     cout << "Index of 30: " << s.order_of_key(30) << "\n"; // -> 2
34
35     // 3      Lower bound behavior
36     cout << "Index where 25 would be inserted: " << s.order_of_key(25) <<
37     "\n"; // -> 2
38
39     // 4      Iterate through all elements
40     for (auto x : s) cout << x << " ";
41     cout << "\n";
42
43     // 5      Check if an element exists
44     if (s.find(20) != s.end())
45         cout << "20 exists in set\n";
46 }
```

2.6. Segment Tree

```
1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 #define fst first
6 #define snd second
7 #define all(c) ((c).begin()), ((c).end())
8 #define ll long long
9
10 const int INF = 1 << 30;
11 const int MAXN = 1e5+5;
12
13 int n, t[4*MAXN];
14
15 void build(vector<int> a, int v, int tl, int tr) {
16     if (tl == tr) {
17         t[v] = a[tl];
18     } else {
19         int tm = tl + (tr - tl) / 2;
20         build(a, v*2, tl, tm);
21         build(a, v*2+1, tm+1, tr);
22         t[v] = t[v*2] + t[v*2+1];
23     }
24 }
25
26 void update(int v, int tl, int tr, int pos, int new_val) {
27     if (tl == tr) {
28         t[v] = new_val;
29     } else {
30         int tm = tl + (tr - tl) / 2;
31         if (pos <= tm)
32             update(v*2, tl, tm, pos, new_val);
33         else
34             update(v*2+1, tm+1, tr, pos, new_val);
35         t[v] = t[v*2] + t[v*2+1];
36     }
37 }
38
39 int sum(int v, int tl, int tr, int l, int r) {
40     if (l > r)
41         return 0;
42     if (l == tl && r == tr) {
43         return t[v];
44     }
45     int tm = (tl + tr) / 2;
46     return sum(v*2, tl, tm, l, min(r, tm))
47         + sum(v*2+1, tm+1, tr, max(l, tm+1), r);
48 }
49
50
51 int main(){
52     vector<int> arr = {1,1,1,1,1};
53     int n = arr.size();
54     build(arr, 1, 0, n-1);
55     cout << sum(1,0,n-1,0,4) << endl; // 5
56     cout << sum(1,0,n-1,0,1) << endl; // 2
```

```
57 cout << sum(1,0,n-1,1,4) << endl; // 4
58 update(1, 0, n-1, 4, 0);
59
60 cout << sum(1,0,n-1,0,4) << endl; // 4
61 cout << sum(1,0,n-1,0,1) << endl; // 2
62 cout << sum(1,0,n-1,1,4) << endl; // 3
63
64 return 0;
65 }
```

2.7. Segment Tree Summary

```
1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 #define fst first
6 #define snd second
7 #define all(c) ((c).begin()), ((c).end())
8 #define ll long long
9
10 const int INF = 1 << 30;
11 const int MAXN = 1e5+5;
12
13 struct Summary {
14     ll gcd;
15     ll cnt;
16     Summary(Summary left, Summary right){
17         gcd = __gcd(left.gcd,right.gcd);
18         cnt = (gcd==left.gcd?left.cnt:0) + (gcd==right.gcd?right.cnt:0);
19     }
20     Summary(ll gcd, ll cnt) : gcd(gcd), cnt(cnt) {}
21     Summary() : gcd(0), cnt(0) {} //elemento neutro
22 };
23
24 struct Node {
25     Summary value;
26     int start, end;
27     Node* left;
28     Node* right;
29     ll lazy;
30
31     Node(int start, int end) :
32         start(start), end(end), value(0,0), lazy(0), left(nullptr), right(
33             nullptr) {}
34 };
35 Node* build(vector<ll> &arr, int l, int r){
36     Node* node = new Node(l,r);
37     if(l == r) {
38         node->value = Summary(arr[l],1);
39         return node;
40     }
41
42     int m = (l + r) / 2;
43     node->left = build(arr, l, m);
44     node->right = build(arr, m+1, r);
45     node->value = Summary(node->left->value, node->right->value);
46     return node;
47 }
48
49 Summary query(Node* root, int start, int end) {
50     if(start > end) return Summary();
51     if(root->start == start && root->end == end)
52         return root->value;
53
54     int m = (root->start+root->end)/2;
55     Summary ans(query(root->left,start,min(m,end)), query(root->right,max(
```

```

        m+1,start),end));
56     return ans;
57 }
58
59 void solve() {
60     int n; cin >> n;
61     vector<ll> arr(n);
62     for(int i = 0; i < n; i++) {
63         cin >> arr[i];
64     }
65     Node* root = build(arr, 0, n-1);
66     int m; cin >> m;
67     while(m--) {
68         int l,r;
69         cin >> l >> r;
70         l--,r--;
71         Summary ans = query(root, l, r);
72         cout << (r - l + 1) - (ans.cnt) << endl;
73     }
74 }
75
76 int main() {
77     ios::sync_with_stdio(false);
78     cin.tie(nullptr);
79     int t = 1;
80     //cin >> t;
81     while(t--)
82         solve();
83 }
```

2.8. Persistent Segment Tree

```
1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 #define fst first
6 #define snd second
7 #define all(c) ((c).begin()), ((c).end())
8 #define ll long long
9
10 const int INF = 1 << 30;
11 const int MAXN = 1e5+5;
12
13 struct Node {
14     ll value;
15     int start, end;
16     Node* left;
17     Node* right;
18     ll lazy;
19
20     Node(int start, int end) :
21         start(start), end(end), value(0), lazy(0), left(nullptr), right(
22             nullptr) {}
23 };
24
25 ll aggregate_function(ll left, ll right){
26     return left + right;
27 }
28
29 Node* build(vector<ll> &arr, int l, int r){
30     Node* node = new Node(l,r);
31     if(l == r) {
32         node->value = arr[l];
33         return node;
34     }
35     int m = (l + r) / 2;
36     node->left = build(arr, l, m);
37     node->right = build(arr, m+1, r);
38     node->value = aggregate_function(node->left->value, node->right->value
39     );
40     return node;
41 }
42
43 void pushDown(Node* node) {
44     if(node->lazy != 0) {
45         if(node->left) {
46             int leftRange = node->left->end - node->left->start + 1;
47             node->left->value += node->lazy * leftRange;
48             node->left->lazy += node->lazy;
49         }
50         if(node->right) {
51             int rightRange = node->right->end - node->right->start + 1;
52             node->right->value += node->lazy * rightRange;
53             node->right->lazy += node->lazy;
54         }
55     }
56     node->lazy = 0;
57 }
```

```

55     }
56 }
57
58 ll query(Node* root, int start, int end) {
59     if(start > end) return 0;
60     if(root->start == start && root->end == end)
61         return root->value;
62
63     pushDown(root);
64     int m = (root->start+root->end)/2;
65     ll ans = aggregate_function(query(root->left,start,min(m,end)), query(
66         root->right,max(m+1,start),end));
67     return ans;
68 }
69
70 Node* persistent_update(Node* root, int pos, ll value) {
71     Node* node = new Node(root->start, root->end);
72     if(root->start == root->end) {
73         node->value = value;
74         return node;
75     }
76     int m = (root->start + root->end) / 2;
77     if(pos <= m) {
78         node->left = persistent_update(root->left, pos, value);
79         node->right = root->right;
80     } else {
81         node->right = persistent_update(root->right, pos, value);
82         node->left = root->left;
83     }
84     node->value = aggregate_function(node->left->value, node->right->value
85     );
86     return node;
87 }
88
89 void update(Node* root, int pos, ll value) {
90     if(root->start == root->end) {
91         root->value = value;
92         return;
93     }
94     int m = (root->start + root->end) / 2;
95     if(pos <= m) {
96         update(root->left, pos, value);
97     } else {
98         update(root->right, pos, value);
99     }
100    root->value = aggregate_function(root->left->value, root->right->value
101 );
102
103 void update_range(Node* node, int l, int r, ll delta) {
104     if(l > r) return;
105     if(l <= node->start && node->end <= r) {
106         int rangeLength = node->end - node->start + 1;
107         node->value += delta * rangeLength;
108         node->lazy += delta;
109         return;
110     }
111     pushDown(node);

```

```

110 int m = (node->start + node->end) / 2;
111 if(l <= m)
112     update_range(node->left, l, min(r, m), delta);
113 if(r > m)
114     update_range(node->right, max(l, m+1), r, delta);
115 node->value = aggregate_function(node->left->value, node->right->value
116 );
117
118 void solve() {
119     vector<ll> arr = {1,1,1,1,1,1};
120     int n = arr.size();
121     Node* root = build(arr,0,n-1);
122     cout << query(root,0,n-1) << endl;
123     Node* mod1 = persistent_update(root,0,2);
124     cout << query(mod1,0,n-1) << endl;
125     Node* mod2 = persistent_update(mod1,0,10);
126     cout << query(mod2,0,n-1) << endl;
127
128     cout << endl;
129     update_range(root, 0, 2, 1);
130     cout << query(root,0,n-1) << endl;
131 }
132
133 int main() {
134     ios::sync_with_stdio(false);
135     cin.tie(nullptr);
136     int t = 1;
137     //cin >> t;
138     while(t--)
139         solve();
140 }
```

2.9. Sparse Table

```
1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 #define fst first
6 #define snd second
7 #define all(c) ((c).begin()), ((c).end())
8 #define ll long long
9
10 const int INF = 1 << 30;
11 const int MAXN = 1e5+5;
12
13 int MAXL;
14 int n;
15
16 int arr[MAXN];
17 vector<vector<int>> st; // sparse table
18
19 void preprocess() {
20     MAXL = ceil(log2(n));
21
22     st.assign(n, vector<int>(MAXL + 1, INF));
23
24     for(int i = 0; i < n; i++)
25         st[i][0] = arr[i];
26
27     for(int j = 1; j <= MAXL; j++) {
28         for (int i = 0; i + (1 << j) - 1 < n; i++) {
29             st[i][j] = min(st[i][j - 1], st[i + (1 << (j - 1))][j - 1]);
30         }
31     }
32 }
33
34
35 void solve() {
36     cin >> n;
37     for(int i = 0; i < n; i++) {
38         cin >> arr[i];
39     }
40
41     preprocess();
42
43     int q; cin >> q;
44     while(q--){
45         int l,r;
46         cin >> l >> r;
47         int k = floor(log2(r - l + 1));
48         cout << min(st[l][k], st[r - (1 << k) + 1][k]) << endl;
49     }
50 }
51
52 int main() {
53     ios::sync_with_stdio(false);
54     cin.tie(nullptr);
55     int t = 1;
56     //cin >> t;
```

```
57     while(t--)
58         solve();
59 }
```

Parte III

Geometry

3.1. Convex Hull

```
1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 struct point {
6     double x, y;
7     point() { x = y = 0; }
8     point(double _x, double _y) : x(_x), y(_y) {}
9     bool operator == (point const& t) const {
10         return x == t.x && y == t.y;
11     }
12 };
13
14 int orientation(point a, point b, point c) {
15     double v = a.x*(b.y-c.y)+b.x*(c.y-a.y)+c.x*(a.y-b.y);
16     if (v < 0) return -1; // clockwise
17     if (v > 0) return +1; // counter-clockwise
18     return 0;
19 }
20
21 bool cw(point a, point b, point c, bool include_collinear) {
22     int o = orientation(a, b, c);
23     return o < 0 || (include_collinear && o == 0);
24 }
25 bool collinear(point a, point b, point c) { return orientation(a, b, c)
26     == 0; }
27
28 void convex_hull(vector<point>& a, bool include_collinear = false) {
29     point p0 = *min_element(a.begin(), a.end(), [](point a, point b) {
30         return make_pair(a.y, a.x) < make_pair(b.y, b.x);
31     });
32     sort(a.begin(), a.end(), [&p0](const point& a, const point& b) {
33         int o = orientation(p0, a, b);
34         if (o == 0)
35             return (p0.x-a.x)*(p0.x-a.x) + (p0.y-a.y)*(p0.y-a.y)
36             < (p0.x-b.x)*(p0.x-b.x) + (p0.y-b.y)*(p0.y-b.y);
37         return o < 0;
38     });
39     if (include_collinear) {
40         int i = (int)a.size()-1;
41         while (i >= 0 && collinear(p0, a[i], a.back())) i--;
42         reverse(a.begin()+i+1, a.end());
43     }
44     vector<point> st;
45     for (int i = 0; i < (int)a.size(); i++) {
46         while (st.size() > 1 && !cw(st[st.size()-2], st.back(), a[i],
47             include_collinear))
48             st.pop_back();
49         st.push_back(a[i]);
50     }
51     if (include_collinear == false && st.size() == 2 && st[0] == st[1])
52         st.pop_back();
53
54     a = st;
```

```

55 }
56
57 double area(const vector<point>& fig) {
58     double res = 0;
59     for (unsigned i = 0; i < fig.size(); i++) {
60         point p = i ? fig[i - 1] : fig.back();
61         point q = fig[i];
62         res += (p.x - q.x) * (p.y + q.y);
63     }
64     return fabs(res) / 2;
65 }
66
67
68 // example use case
69 void solve() {
70     double n,r;
71     cin >> n >> r;
72
73     vector<point> coords(n);
74     for(int i = 0; i < n; i++) {
75         cin >> coords[i].x >> coords[i].y;
76     }
77
78     auto dist = [] (point &a, point&b) {
79         return sqrt(pow(a.x-b.x,2)+pow(a.y-b.y,2));
80     };
81
82     const double PI = 3.141516;
83     double cir = 2 * PI * r;
84     convex_hull(coords);
85
86     double per = 0;
87     coords.push_back(coords[0]);
88     for(int i = 1; i < coords.size(); i++) {
89         per += dist(coords[i-1],coords[i]);
90     }
91
92     int ans = round(per+cir);
93     cout << ans << endl;
94 }
95
96 int main() {
97     ios::sync_with_stdio(false);
98     cin.tie(0);
99     int t = 1;
100    //cin >> t;
101    while(t--)
102        solve();
103 }
```

Parte IV

Graphs

4.1. 2 Sat

```
1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 #define fst first
6 #define snd second
7 #define all(c) ((c).begin()), ((c).end())
8 #define ll long long
9 #define endl '\n'
10
11 const int INF = 1 << 30;
12 const int MAXN = 1e5+5;
13
14 struct TwoSatSolver {
15     int n_vars;
16     int n_vertices;
17     vector<vector<int>> adj, adj_t;
18     vector<bool> used;
19     vector<int> order, comp;
20     vector<bool> assignment;
21
22     TwoSatSolver(int _n_vars) : n_vars(_n_vars), n_vertices(2 * n_vars),
23         adj(n_vertices), adj_t(n_vertices), used(n_vertices), order(), comp(
24             n_vertices, -1), assignment(n_vars) {
25         order.reserve(n_vertices);
26     }
27     void dfs1(int v) {
28         used[v] = true;
29         for (int u : adj[v]) {
30             if (!used[u])
31                 dfs1(u);
32         }
33         order.push_back(v);
34     }
35     void dfs2(int v, int cl) {
36         comp[v] = cl;
37         for (int u : adj_t[v]) {
38             if (comp[u] == -1)
39                 dfs2(u, cl);
40         }
41     }
42     bool solve_2SAT() {
43         order.clear();
44         used.assign(n_vertices, false);
45         for (int i = 0; i < n_vertices; ++i) {
46             if (!used[i])
47                 dfs1(i);
48         }
49
50         comp.assign(n_vertices, -1);
51         for (int i = 0, j = 0; i < n_vertices; ++i) {
52             int v = order[n_vertices - i - 1];
53             if (comp[v] == -1)
54                 dfs2(v, j++);
```

```

55     }
56
57     assignment.assign(n_vars, false);
58     for (int i = 0; i < n_vertices; i += 2) {
59         if (comp[i] == comp[i + 1])
60             return false;
61         assignment[i / 2] = comp[i] > comp[i + 1];
62     }
63     return true;
64 }
65
66 void add_disjunction(int a, bool na, int b, bool nb) {
67     // na and nb signify whether a and b are to be negated
68     a = 2 * a ^ na;
69     b = 2 * b ^ nb;
70     int neg_a = a ^ 1;
71     int neg_b = b ^ 1;
72     adj[neg_a].push_back(b);
73     adj[neg_b].push_back(a);
74     adj_t[b].push_back(neg_a);
75     adj_t[a].push_back(neg_b);
76 }
77
78 };
79
80 void solve() {
81     int n,m;
82     cin >> n >> m;
83
84     TwoSatSolver solver(m);
85     for(int i = 0; i < n; i++) {
86         int topping1, topping2;
87         char op1, op2;
88         cin >> op1 >> topping1 >> op2 >> topping2;
89         bool no_good1 = op1 == '_';
90         bool no_good2 = op2 == '_';
91         topping1--, topping2--;
92         solver.add_disjunction(topping1,no_good1, topping2, no_good2);
93     }
94
95     bool poss = solver.solve_2SAT();
96     if(!poss) {
97         cout << "IMPOSSIBLE" << endl;
98         return;
99     }
100
101    for(int i = 0; i < m; i++) {
102        if(solver.assignment[i]) {
103            cout << "+ ";
104        } else {
105            cout << "- ";
106        }
107    }
108 }
109
110 int main() {
111     ios::sync_with_stdio(false);
112     cin.tie(nullptr);

```

```
113     int t = 1;  
114     //cin >> t;  
115     while(t--)  
116         solve();  
117 }
```

4.2. Bellman Ford

```
1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 #define fst first
6 #define snd second
7 #define all(c) ((c).begin()), ((c).end())
8 #define ll long long
9
10 const ll INF = LONG_LONG_MAX;
11 const int MAXN = 1e5+5;
12
13 struct Edge {
14     int u, v;
15     ll weight;
16 };
17
18 int n, m;
19 vector<Edge> edges;
20
21 // Bellman-Ford algorithm to detect negative cycles and find shortest
22 // paths
23 bool bellman(int src, vector<ll> &dist) {
24     dist.assign(n + 1, INF);
25     dist[src] = 0;
26     for (int i = 2; i <= n; i++)
27     {
28         for (int j = 0; j < m; j++)
29         {
30             int u = edges[j].u;
31             int v = edges[j].v;
32             ll weight = edges[j].weight;
33             if (dist[u] != INF && dist[u] + weight < dist[v])
34                 dist[v] = dist[u] + weight;
35         }
36     }
37     for (int i = 0; i < m; i++)
38     {
39         int u = edges[i].u;
40         int v = edges[i].v;
41         ll weight = edges[i].weight;
42         // True if neg-cylce exists
43         if (dist[u] != INF && dist[u] + weight < dist[v])
44             return true;
45     }
46     return false;
47 }
48 int main() {
49     cin >> n >> m;
50     edges.resize(m);
51
52     for (int i = 0; i < m; i++) {
53         int a, b;
54         ll w;
55         cin >> a >> b >> w;
```

```

56     edges[i] = {a, b, w};
57 }
58
59 vector<ll> dist;
60
61 bool hasNegativeCycle = bellman(1, dist);
62
63 if (hasNegativeCycle) {
64     cout << "negative-weight cycle\n";
65 } else {
66     if (dist[n] == INF) {
67         cout << "no path from 1 to " << n << "\n";
68     } else {
69         cout << "shortest path from 1 to " << n << " is: " << dist[n] << "
70             "\n";
71     }
72 }
```

4.3. Dijkstra

```
1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 #define fst first
6 #define snd second
7 #define all(c) ((c).begin()), ((c).end())
8 #define ll long long
9
10 const int INF = 1 << 30;
11 const int MAXN = 1e5+5;
12
13 int n,m;
14 vector<pair<int, int>> adj[MAXN];
15
16 void dijkstra(int s, vector<int> & d, vector<int> & p) {
17     d.assign(n+1, INF);
18     p.assign(n+1, -1);
19     vector<bool> visited(n+1, 0);
20
21     d[s] = 0;
22     using pii = pair<int, int>;
23     priority_queue<pii, vector<pii>, greater<pii>> q;
24     q.push({0, s});
25     while (!q.empty()) {
26         int v = q.top().second;
27         q.pop();
28         if(visited[v]) continue;
29         visited[v] = 1;
30
31         for (auto edge : adj[v]) {
32             int to = edge.first;
33             int len = edge.second;
34
35             if (d[v] + len < d[to]) {
36                 d[to] = d[v] + len;
37                 p[to] = v;
38                 q.push({d[to], to});
39             }
40         }
41     }
42 }
43
44 vector<int> restore_path(int s, int t, vector<int> const& p) {
45     vector<int> path;
46
47     for (int v = t; v != s; v = p[v])
48         path.push_back(v);
49     path.push_back(s);
50
51     reverse(path.begin(), path.end());
52     return path;
53 }
54
55
56 int main()
```

```
57 {
58     cin >> n >> m;
59     // adj.resize(n+1);
60     while(m--) {
61         int a,b,w;
62         cin >> a >> b >> w;
63         adj[a].push_back({b,w});
64     }
65
66     int str=1, target=n;
67
68     vector<int> distance, path;
69
70     dijkstra(str, distance, path);
71     cout << distance[n] << '\n';
72 }
```

4.4. Floyd Warshall

```
1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 #define fst first
6 #define snd second
7 #define all(c) ((c).begin()), ((c).end())
8 #define ll long long
9
10 const int INF = 1 << 30;
11 const int MAXN = 1e3+5;
12
13 int n;
14 ll d[MAXN][MAXN];
15
16 void floyd_marshall() {
17     for (int k = 1; k <= n; k++)
18         for (int i = 1; i <= n; i++)
19             for (int j = 1; j <= n; j++)
20                 if (d[i][k] + d[k][j] < d[i][j])
21                     d[i][j] = d[i][k] + d[k][j];
22 }
23
24 void solve() {
25     int s, t;
26     cin >> n;
27     cin >> s >> t;
28     for(int i = 1; i <= n; i++) {
29         for(int j = 1; j <= n; j++) {
30             cin >> d[i][j];
31             if(d[i][j] == -1) d[i][j] = INF;
32         }
33     }
34
35     if(d[s][t] == INF)
36         cout << "-1\n";
37     else
38         cout << d[s][t] << '\n';
39 }
40
41 int main() {
42     ios::sync_with_stdio(false);
43     cin.tie(nullptr);
44     int t = 1;
45     //cin >> t;
46     while(t--)
47         solve();
48 }
```

4.5. Ford Fulkerson

```
1 // max flow algorithm
2
3 #include <bits/stdc++.h>
4
5 using namespace std;
6
7 #define fst first
8 #define snd second
9 #define all(c) ((c).begin()), ((c).end())
10
11 const int INF = 1e9;
12
13 struct graph {
14     typedef long long flow_type;
15     struct edge {
16         int src, dst;
17         flow_type capacity, flow;
18         size_t rev;
19     };
20     int n;
21     vector<vector<edge>> adj;
22
23     graph(int n) : n(n), adj(n) { }
24
25     void add_edge(int src, int dst, flow_type capacity) {
26         adj[src].push_back({src, dst, capacity, 0, adj[dst].size()});
27         adj[dst].push_back({dst, src, 0, 0, adj[src].size() - 1});
28     }
29
30     int max_flow(int s, int t) {
31         vector<bool> visited(n);
32         function<flow_type(int, flow_type)> augment = [&](int u, flow_type cur) {
33             if (u == t) return cur;
34             visited[u] = true;
35             for (auto &e : adj[u]) {
36                 if (!visited[e.dst] && e.capacity > e.flow) {
37                     flow_type f = augment(e.dst, min(e.capacity - e.flow, cur));
38                     if (f > 0) {
39                         e.flow += f;
40                         adj[e.dst][e.rev].flow -= f;
41                         return f;
42                     }
43                 }
44             }
45             return flow_type(0);
46         };
47         for (int u = 0; u < n; ++u)
48             for (auto &e : adj[u]) e.flow = 0;
49
50         flow_type flow = 0;
51         while (1) {
52             fill(all(visited), false);
53             flow_type f = augment(s, INF);
54             if (f == 0) break;
55             flow += f;
```

```

56     }
57     return flow;
58 }
59
60 // test function to understand the code
61 // Function to print the residual graph
62 void print_residual_graph() {
63     cout << "Residual Graph:" << endl;
64     for (int u = 0; u < n; ++u) {
65         for (const auto &e : adj[u]) {
66             if (e.capacity > 0) { // Ignore reverse edges with 0 original
67                 cout << "Edge from " << e.src << " to " << e.dst
68                 << " | Capacity: " << e.capacity
69                 << " | Flow: " << e.flow
70                 << " | Residual Capacity: " << (e.capacity - e.flow) << endl
71             ;
72         }
73     }
74     cout << "-----" << endl;
75 }
76
77 void print_mincut_edges() {
78     // Step 1: Find reachable nodes from the source in the residual
79     // graph
80     vector<bool> visited(n, false);
81     queue<int> q;
82     q.push(0); // Assuming 0 is the source node
83     visited[0] = true;
84
85     while (!q.empty()) {
86         int u = q.front();
87         q.pop();
88         for (const auto &e : adj[u]) {
89             if (!visited[e.dst] && e.capacity > e.flow) { // Residual
90                 capacity > 0
91                 visited[e.dst] = true;
92                 q.push(e.dst);
93             }
94         }
95     }
96
97     // Step 2: Print edges that cross the cut
98     for (int u = 0; u < n; ++u) {
99         if (!visited[u]) continue;
100        for (const auto &e : adj[u]) {
101            if (!visited[e.dst] && e.capacity > 0) {
102                cout << u + 1 << " " << e.dst + 1 << '\n'; // Output edge in
103                1-based index
104            }
105        }
106    }
107 }
108 };

```

```
109
110 int main() {
111     int n,m;
112     cin >> n >> m;
113     graph g(n);
114     for (int i = 0; i < m; ++i) {
115         int u, v, w;
116         cin >> u >> v >> w;
117         g.add_edge(u, v, w);
118     }
119     cout << g.max_flow(0,n-1) << endl;
120     g.print_residual_graph();
121 }
```

4.6. Functional Graph

```
1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 #define endl '\n'
6
7 const int MAXN = 2e5+5;
8 const int LOG = 22;
9
10 int up[MAXN][LOG];
11 bool visited[MAXN];
12 bool on_stack[MAXN];
13 int next_node[MAXN];
14 int cycle[MAXN];
15 int cycle_length[MAXN];
16 int depth[MAXN];
17
18 int cycle_idx[MAXN];
19
20 void mark_cycle(int u, int id) {
21     cycle[u] = id;
22     depth[u] = 0;
23     cycle_length[id]++;
24     cycle_idx[u] = cycle_length[id];
25     if(!cycle[next_node[u]])
26         mark_cycle(next_node[u], id);
27 }
28
29 void dfs(int u) {
30     visited[u] = 1;
31     on_stack[u] = 1;
32
33     int v = next_node[u];
34     if(on_stack[v]) {
35         mark_cycle(v, v);
36     } else if(!visited[v]){
37         dfs(v);
38     }
39
40     if(!cycle[u])
41         depth[u] = depth[v] + 1;
42
43     on_stack[u] = 0;
44 }
45
46 int jumpto(int u, int k) {
47     int v = u;
48     for(int i = 0; i < LOG; i++) {
49         if((1 << i) & k) {
50             v = up[v][i];
51         }
52     }
53     return v;
54 }
```

```

57 int distcycle(int u, int v) {
58     if(cycle_idx[u] < cycle_idx[v]) {
59         return cycle_idx[v] - cycle_idx[u];
60     }
61     int turn = (cycle_length[cycle[u]] - cycle_idx[u] + 1) % cycle_length[
62         cycle[u]];
63     return turn + cycle_idx[v] - 1;
64 }
65 void solve() {
66     int n,q;
67     cin >> n >> q;
68
69     for(int i = 1; i <= n; i++) {
70         cin >> next_node[i];
71         up[i][0] = next_node[i];
72     }
73
74     for(int j = 1; j < LOG; j++) {
75         for(int i = 1; i <= n; i++) {
76             up[i][j] = up[up[i][j-1]][j-1];
77         }
78     }
79
80     for(int i = 1; i <= n; i++) {
81         if(!visited[i]) dfs(i);
82     }
83
84     while(q--) {
85         int u,v;
86         cin >> u >> v;
87
88         int cyu = cycle[u];
89         int cyv = cycle[v];
90
91         if(cyu == 0) {
92             cyu = cycle[jumpto(u,depth[u])];
93         }
94         if(cyv == 0) {
95             cyv = cycle[jumpto(v,depth[v])];
96         }
97
98         int a = u;
99         int b = v;
100
101        if(cyu != cyv) {
102            cout << -1 << endl;
103            continue;
104        }
105
106        if(cycle[u] == 0 && cycle[v] == 0) {
107            if(depth[u] < depth[v]) {
108                cout << -1 << endl;
109                continue;
110            }
111            int diff = depth[u] - depth[v];
112            if(jumpto(u,diff) != v) {
113                cout << -1 << endl;

```

```

114     } else {
115         cout << diff << endl;
116     }
117     continue;
118 }
119
120 if(cycle[u] && cycle[v]) {
121     cout << distcycle(u,v) << endl;
122     continue;
123 }
124
125 if(cycle[u] == 0) {
126     int steps = depth[u];
127     steps += distcycle(jumpto(u,depth[u]), v);
128     cout << steps << endl;
129     continue;
130 }
131
132 if(cycle[v] == 0) {
133     cout << -1 << endl;
134     continue;
135 }
136 }
137 }
138
139 int main() {
140     ios::sync_with_stdio(false);
141     cin.tie(nullptr);
142     int t = 1;
143     //cin >> t;
144     while(t--)
145         solve();
146 }
```

4.7. Hungarian

```
1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 #define fst first
6 #define snd second
7 #define all(c) ((c).begin()), ((c).end())
8 #define ll long long
9 #define ms(a, b) memset(a, b, sizeof(a))
10
11 const int INF = 1 << 30;
12 const int MAXN = 25;
13 const int MAXM = 25;
14
15 // hungarian weighted matching algo
16 // finds the MAX cost of MAX matching, to find mincost, add edges as
17 // negatives
18 // Nodes are indexed from 1 on both sides
19 template<typename T>
20 struct KuhnMunkras { // n for left, m for right
21     int n, m, match[MAXM];
22     T g[MAXN][MAXM], lx[MAXN], ly[MAXM], slack[MAXM];
23     bool vx[MAXN], vy[MAXM];
24
25     void init(int n_, int m_) {
26         ms(g, 0); n = n_, m = m_;
27     }
28
29     void add(int u, int v, T w) {
30         g[u][v] = w;
31     }
32
33     bool find(int x) {
34         vx[x] = true;
35         for (int y = 1; y <= m; ++y) {
36             if (!vy[y]) {
37                 T delta = lx[x] + ly[y] - g[x][y];
38                 if (delta==0) {
39                     vy[y] = true;
40                     if (match[y] == 0 || find(match[y])) {
41                         match[y] = x;
42                         return true;
43                     }
44                 } else slack[y] = min(slack[y], delta);
45             }
46         }
47         return false;
48     }
49
50     T matching() { // maximum weight matching
51         fill(lx + 1, lx + 1 + n, numeric_limits<T>::lowest());
52         ms(ly, 0);
53         ms(match, 0);
54         for (int i = 1; i <= n; ++i) {
55             for (int j = 1; j <= m; ++j) lx[i] = max(lx[i], g[i][j]);
56         }
57     }
58 }
```

```

56     for (int k = 1; k <= n; ++k) {
57         fill(slack + 1, slack + 1 + m, numeric_limits<T>::max());
58         while (true) {
59             ms(vx, 0);
60             ms(vy, 0);
61             if (find(k)) break;
62             else {
63                 T delta = numeric_limits<T>::max();
64                 for (int i = 1; i <= m; ++i) {
65                     if (!vy[i]) delta = min(delta, slack[i]);
66                 }
67                 for (int i = 1; i <= n; ++i) {
68                     if (vx[i]) lx[i] -= delta;
69                 }
70                 for (int i = 1; i <= m; ++i) {
71                     if (vy[i]) ly[i] += delta;
72                     if (!vy[i]) slack[i] -= delta;
73                 }
74             }
75         }
76     }
77     T result = 0;
78     for (int i = 1; i <= n; ++i) result += lx[i];
79     for (int i = 1; i <= m; ++i) result += ly[i];
80     return result;
81 }
82 };
83
84 void solve() {
85     int n, m;
86     cin >> n >> m;
87
88     KuhnMunkras<double> km;
89     km.init(n, m);
90
91     for (int i = 1; i <= n; ++i) {
92         for (int j = 1; j <= m; ++j) {
93             double c;
94             cin >> c;
95             km.add(i, j, -c);
96         }
97     }
98
99     double result = km.matching();
100    double total_cost = -result;
101    cout << total_cost << '\n';
102 }
103
104
105 int main() {
106     ios::sync_with_stdio(false);
107     cin.tie(nullptr);
108     int t = 1;
109     //cin >> t;
110     while(t--)
111         solve();
112 }
```

4.8. Kosaraju

```
1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 #define fst first
6 #define snd second
7 #define all(c) ((c).begin()), ((c).end())
8 #define ll long long
9 #define ms(a, b) memset(a, b, sizeof(a))
10
11 const int INF = 1 << 30;
12 const int MAXN = 25;
13 const int MAXM = 25;
14
15 vector<bool> visited; // keeps track of which vertices are already
16 // visited
17 // runs depth first search starting at vertex v.
18 // each visited vertex is appended to the output vector when dfs leaves
19 void dfs(int v, vector<vector<int>> const& adj, vector<int> &output) {
20     visited[v] = true;
21     for (auto u : adj[v])
22         if (!visited[u])
23             dfs(u, adj, output);
24     output.push_back(v);
25 }
26
27 // input: adj -- adjacency list of G
28 // output: components -- the strongly connected components in G
29 // output: adj_cond -- adjacency list of G^SCC (by root vertices)
30 void strongly_connected_components(vector<vector<int>> const& adj,
31     vector<vector<int>> &components,
32     vector<vector<int>> &adj_cond) {
33     int n = (int)adj.size();
34     components.clear();
35     adj_cond.clear();
36
37     // 1) Order the vertices by finish time in a first DFS.
38     vector<int> order;
39     visited.assign(n, false);
40     for (int i = 0; i < n; i++) {
41         if (!visited[i])
42             dfs(i, adj, order);
43     }
44 }
45
46 // 2) Reverse all edges to get the transpose graph.
47 vector<vector<int>> adj_rev(n);
48 for (int v = 0; v < n; v++) {
49     for (int u : adj[v])
50         adj_rev[u].push_back(v);
51 }
52
53 // 3) Do a second DFS in descending order of finish times (from step
```

```

    1).
55     visited.assign(n, false);
56     reverse(order.begin(), order.end());
57
58     // this array will store the "root" (representative) of each SCC
59     vector<int> roots(n, -1);
60
61     for (auto v : order) {
62         if (!visited[v]) {
63             vector<int> component;
64             dfs(v, adj_rev, component);
65             // store the newly found component
66             components.push_back(component);
67             // the root is the minimum vertex in that component (arbitrary
68             choice)
69             int root = *min_element(begin(component), end(component));
70             for (auto u : component) {
71                 roots[u] = root;
72             }
73         }
74     }
75
76     // 4) Build the condensation graph (SCC DAG).
77     adj_cond.assign(n, {});
78     for (int v = 0; v < n; v++) {
79         for (auto u : adj[v]) {
80             if (roots[v] != roots[u]) {
81                 adj_cond[roots[v]].push_back(roots[u]);
82             }
83         }
84     }
85
86     void solve() {
87
88         int n = 5, m = 5;
89         vector<vector<int>> adj(n);
90
91         vector<pair<int, int>> edges = {
92             {0,1},
93             {1,2},
94             {2,0},
95             {1,3},
96             {3,4}
97         };
98
99         for (auto &e : edges) {
100             adj[e.first].push_back(e.second);
101         }
102
103         vector<vector<int>> components, cond_graph;
104         strongly_connected_components(adj, components, cond_graph);
105
106         for (auto &comp : components) {
107             sort(comp.begin(), comp.end());
108         }
109
110         cout << "Number of SCCs: " << (int)components.size() << "\n";

```

```

111 for (int i = 0; i < (int)components.size(); i++) {
112     cout << "SCC " << i << ":" ;
113     for (auto &v : components[i]) {
114         cout << v << " ";
115     }
116     cout << "\n";
117 }
118
119 cout << "Condensation graph adjacency list:\n";
120 // We'll print only non-empty adjacency to see edges
121 for (int i = 0; i < (int)cond_graph.size(); i++) {
122     if (!cond_graph[i].empty()) {
123         cout << "root " << i << ":" ;
124         for (auto &v : cond_graph[i]) {
125             cout << v << " ";
126         }
127         cout << "\n";
128     }
129 }
130 }
131
132 int main() {
133     ios::sync_with_stdio(false);
134     cin.tie(nullptr);
135     int t = 1;
136     //cin >> t;
137     while(t--)
138         solve();
139 }
```

4.9. Kruskal

```
1 #include <bits/stdc++.h>
2 using namespace std;
3
4 #define ll long long
5 #define endl '\n'
6
7 struct Edge {
8     ll u, v, weight;
9     bool operator<(Edge const& other) const {
10         return weight < other.weight;
11     }
12 };
13
14 struct DSU {
15     vector<int> parent, size;
16
17     DSU(int n) {
18         parent.resize(n);
19         size.assign(n, 1);
20         iota(parent.begin(), parent.end(), 0);
21     }
22
23     int find(int v) {
24         if (v == parent[v]) return v;
25         return parent[v] = find(parent[v]);
26     }
27
28     bool unite(int a, int b) {
29         a = find(a);
30         b = find(b);
31         if (a == b) return false;
32         if (size[a] < size[b]) swap(a, b);
33         parent[b] = a;
34         size[a] += size[b];
35         return true;
36     }
37 };
38
39 ll kruskal(int n, vector<Edge>& edges, vector<Edge>& result) {
40     sort(edges.begin(), edges.end());
41     DSU dsu(n);
42     ll cost = 0;
43
44     for (auto& e : edges) {
45         if (dsu.unite(e.u, e.v)) {
46             cost += e.weight;
47             result.push_back(e);
48         }
49     }
50     return cost;
51 }
52
53 int main() {
54     ios::sync_with_stdio(false);
55     cin.tie(nullptr);
56 }
```

```

57 int n, m;
58 cin >> n >> m;
59
60 vector<Edge> edges;
61 edges.reserve(m);
62
63 for (int i = 0; i < m; i++) {
64     ll u, v, w;
65     cin >> u >> v >> w;
66     u--;
67     v--;
68     edges.push_back({u, v, w});
69 }
70
71 vector<Edge> result;
72 ll total_cost = kruskal(n, edges, result);
73
74 if(result.size() == n-1) {
75     cout << total_cost << endl;
76 } else {
77     cout << "IMPOSSIBLE" << endl;
78 }
79 // Optional: print MST edges
80 // for (auto &e : result)
81 //     cout << e.u + 1 << " " << e.v + 1 << " " << e.weight << endl;
82 }
```

4.10. Heavy Light Descomposition

```

1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5
6 template <class T> class MaxSegmentTree {
7 private:
8 // const T DEFAULT = std::numeric_limits<T>().max();
9 const T DEFAULT = 0;
10 int len;
11 vector<T> segtree;
12 public:
13 MaxSegmentTree(int len) : len(len), segtree(len * 2, DEFAULT) {}
14 void set(int ind, T val) {
15     ind += len;
16     segtree[ind] = val;
17     for (; ind > 1; ind /= 2) {
18         segtree[ind / 2] = std::max(segtree[ind], segtree[ind ^ 1]);
19     }
20 }
21
22 T range_max(int start, int end) {
23     T max = DEFAULT;
24     for (start += len, end += len; start < end; start /= 2, end /= 2) {
25         if (start % 2 == 1) { max = std::max(max, segtree[start++]); }
26         if (end % 2 == 1) { max = std::max(max, segtree[--end]); }
27     }
28     return max;
29 }
30
31 };
32
33
34 template <class T, bool VALS_IN_EDGES> class HLD {
35 private:
36     int N, R, tim = 0; // n, root node, time
37     vector<vector<int>> adj;
38     vector<int> par, siz, depth, rt, pos; // parent, size, depth, root,
39     // position arrays
40     MaxSegmentTree<T> segtree; // Modify as needed
41     /** Compute the size of each subtree and set parent-child relationship
42      * Subtree of node v corresponds to segment [pos[v], pos[v] + sz[v])
43      */
44
45     void dfs_sz(int v) {
46         if (par[v] != -1) adj[v].erase(find(adj[v].begin(), adj[v].end(), par[v]));
47         for (int &u : adj[v]) {
48             par[u] = v, depth[u] = depth[v] + 1;
49             dfs_sz(u);
50             siz[v] += siz[u];
51             if (siz[u] > siz[adj[v][0]]) swap(u, adj[v][0]);
52         }
53     }
54     /** Assign positions for nodes

```

```

54 * Path from v to the last vertex in ascending heavy path
55     corresponds to [ pos[rt[v]], pos[v] ] */
56
57 void dfs_hld(int v) {
58     pos[v] = tim++;
59     for (int u : adj[v]) {
60         rt[u] = (u == adj[v][0] ? rt[v] : u);
61         dfs_hld(u);
62     }
63 }
64 /** process all heavy path and combine their results */
65 template <class B> void process(int u, int v, B op) {
66     for (; rt[u] != rt[v]; v = par[rt[v]]) {
67         if (depth[rt[u]] > depth[rt[v]]) swap(u, v);
68         op(pos[rt[v]], pos[v]);
69     }
70     if (depth[u] > depth[v]) swap(u, v);
71     op(pos[u] + VALS_IN_EDGES, pos[v]);
72 }
73
74 public:
75 HLD(vector<vector<int>> adj_, int _R)
76     : N(adj_.size()), R(_R), adj(adj_), par(N, -1), siz(N, 1), depth(N),
77     rt(N),
78     pos(N), segtree(N) // modify as needed
79 {
80     rt[R] = R;
81     dfs_sz(R);
82     dfs_hld(R);
83 }
84 T query_path(int u, int v) {
85     T res = 0; // default value, modify depending on problem
86     process(u, v, [&](int l, int r) {
87         res = max(res, segtree.range_max(l, r + 1)); // modify depending
88         on problem
89     });
90     return res;
91 }
92 void modify_node(int u, T val) { segtree.set(pos[u], val); }
93 };
94
95 int main() {
96     ios_base::sync_with_stdio(false);
97     cin.tie(0);
98
99     int n, q;
100    cin >> n >> q;
101    vector<int> v(n);
102    vector<vector<int>> adj(n);
103    for (int i = 0; i < n; i++) { cin >> v[i]; }
104    for (int i = 0; i < n - 1; i++) {
105        int a, b;
106        cin >> a >> b;
107        --a, --b;
108        adj[a].push_back(b);
109        adj[b].push_back(a);

```

```
110 }
111 HLD<int, 0> H(adj, 0);
112 for (int i = 0; i < n; i++) { H.modify_node(i, v[i]); }
113 while (q--) {
114     int type, s, a, b, x;
115     cin >> type;
116     if (type == 1) {
117         cin >> s >> x;
118         --s;
119         H.modify_node(s, x);
120     } else if (type == 2) {
121         cin >> a >> b;
122         --a, --b;
123         cout << H.query_path(a, b) << " ";
124     }
125 }
126 }
127
128 }
```

4.11. LCA

```
1 #include <bits/stdc++.h>
2 using namespace std;
3
4 #define endl '\n'
5 using ll = long long;
6
7 int n, l;
8 vector<vector<pair<int,int>>> adj;
9 int timer;
10 vector<int> tin, tout;
11 vector<vector<int>> up;
12 vector<ll> dist; // distance from root to node
13
14 void dfs(int v, int p)
15 {
16     tin[v] = ++timer;
17     up[v][0] = p;
18
19     for (int i = 1; i <= l; ++i)
20         up[v][i] = up[up[v][i-1]][i-1];
21
22     for (auto [u, w] : adj[v]) {
23         if (u != p) {
24             dist[u] = dist[v] + w;
25             dfs(u, v);
26         }
27     }
28
29     tout[v] = ++timer;
30 }
31
32 bool is_ancestor(int u, int v) {
33     return tin[u] <= tin[v] && tout[u] >= tout[v];
34 }
35
36 int lca(int u, int v)
37 {
38     if (is_ancestor(u, v)) return u;
39     if (is_ancestor(v, u)) return v;
40     for (int i = l; i >= 0; --i)
41         if (!is_ancestor(up[u][i], v))
42             u = up[u][i];
43     return up[u][0];
44 }
45
46 ll distance(int u, int v)
47 {
48     int ancestor = lca(u, v);
49     return dist[u] + dist[v] - 2 * dist[ancestor];
50 }
51
52 void preprocess(int root)
53 {
54     tin.resize(n);
55     tout.resize(n);
56     dist.assign(n, 0);
```

```

57     timer = 0;
58     l = ceil(log2(n));
59     up.assign(n, vector<int>(l + 1));
60     dfs(root, root);
61 }
62
63 int main() {
64     ios::sync_with_stdio(false);
65     cin.tie(nullptr);
66
67     cin >> n;
68     adj.assign(n, {});
69
70     for (int i = 0; i < n - 1; i++) {
71         int u, v, w;
72         cin >> u >> v >> w;
73         --u; --v;
74         adj[u].push_back({v, w});
75         adj[v].push_back({u, w});
76     }
77
78     preprocess(0); // root = 0
79
80     int q; cin >> q;
81     while (q--) {
82         int u, v;
83         cin >> u >> v;
84         --u; --v;
85         cout << distance(u, v) << endl;
86     }
87 }
```

4.12. Topological Sort

```
1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 #define fst first
6 #define snd second
7 #define all(c) ((c).begin()), ((c).end())
8 #define ll long long
9
10 const int INF = 1 << 30;
11 const int MAXN = 1e5+5;
12
13 vector<int> adj[MAXN];
14 vector<int> topo;
15
16 int n,m;
17
18 void dfs(int u, vector<int> &visited) {
19     visited[u] = 1;
20     for (int &v : adj[u]) {
21         if (!visited[v])
22             dfs(v, visited);
23         else if(visited[v] == 1){
24             cout << "IMPOSSIBLE\n";
25             exit(0);
26         }
27     }
28     visited[u] = 2;
29     topo.push_back(u);
30 }
31
32 void topological_sort() {
33     topo.clear();
34     vector<int> visited(n+1,0);
35
36     for (int i = 1; i <= n; i++)
37         if (!visited[i])
38             dfs(i, visited);
39
40     reverse(topo.begin(), topo.end());
41 }
42
43 void solve() {
44     cin >> n >> m;
45     for(int i = 0; i <= n; i++) adj[i].clear();
46     while(m--) {
47         int a,b;
48         cin >> a >> b;
49         adj[a].push_back(b);
50     }
51     topological_sort();
52     for (int x : topo) cout << x << " ";
53     cout << "\n";
54 }
55
56 int main() {
```

```
57     int t = 1;  
58     // cin >> t;  
59     while(t--)  
60         solve();  
61 }
```

4.13. Centroid Descomposition

```
1 #include "bits/stdc++.h"
2 using namespace std;
3
4 using ll = long long;
5 using ld = long double;
6 #define sz(v) ((int)((v).size()))
7
8 const int MAXN = 100005;
9
10 int n;
11 vector<int> adj[MAXN];
12 int sub[MAXN];
13 bool removed[MAXN];
14 int parCentroid[MAXN];
15 vector<vector<pair<int,int>>> elements(MAXN);
16
17 vector<vector<pair<int,int>>> sufix(MAXN);
18 #define linea() cerr << "-----" << '\n',
19 int dfs_size(int u, int p) {
20     sub[u] = 1;
21     for (int v : adj[u]) {
22         if (v == p || removed[v]) continue;
23         sub[u] += dfs_size(v, u);
24     }
25     return sub[u];
26 }
27
28 int find_centroid(int u, int p, int total) {
29     for (int v : adj[u]) {
30         if (v == p || removed[v]) continue;
31         if (sub[v] > total / 2)
32             return find_centroid(v, u, total);
33     }
34     return u;
35 }
36 void bfs(int node) {
37     map<int, int> dist;
38     queue<int> q;
39     q.push(node);
40     elements[node].push_back({node, 0});
41     while (!q.empty()) {
42         auto l = q.front();
43         q.pop();
44         for (auto& x : adj[l]) {
45             if (dist[x] == 0 && !removed[x] && x != node) {
46                 dist[x] = dist[l] + 1;
47                 q.push(x);
48                 elements[node].push_back({x, dist[x]});
49             }
50         }
51     }
52     sort(elements[node].begin(), elements[node].end());
53     sufix[node].resize(sz(elements[node]) + 1);
54
55 }
```

```

57 pair<int, int> otro {1e9, 1e9};
58 for (int i = sz(elements[node]) - 1; i >= 0; i--) {
59     if (otro.second > elements[node][i].second) {
60         otro = elements[node][i];
61     }
62     else if (otro.second == elements[node][i].second) {
63         if (otro.first > elements[node][i].first) otro = elements[
64             node][i];
65     }
66     sufix[node][i] = otro;
67 }
68 void decompose(int u, int p = -1) {
69     int total = dfs_size(u, -1);
70     int c = find_centroid(u, -1, total);
71     bfs(c);
72     removed[c] = true;
73
74     parCentroid[c] = (p == -1 ? c : p); // root centroid points to
75     // itself
76
77     // Example: hook custom logic here (before recursing)
78     // e.g., preprocess distances from c to other nodes in its component
79
80     for (int v : adj[c]) {
81         if (!removed[v])
82             decompose(v, c);
83     }
84     int bs(vector<pair<int, int>>& v, int target) {
85         int low = 0, high = sz(v) - 1;
86         int ans = -1;
87         while (low <= high) {
88             int mid = low + (high - low) / 2;
89             if (v[mid].first > target) {
90                 ans = mid;
91                 high = mid - 1;
92             }
93             else low = mid + 1;
94         }
95         return ans;
96     }
97     int dist(vector<pair<int, int>>& v, int target) {
98         int low = 0, high = sz(v) - 1;
99         int ans = -1;
100        while (low <= high) {
101            int mid = low + (high - low) / 2;
102            if (v[mid].first == target) {
103                return mid;
104            }
105            else if (v[mid].first < target) low = mid + 1;
106            else high = mid - 1;
107        }
108        return -1;
109    }
110    int32_t main() {
111        ios::sync_with_stdio(false);
112        cin.tie(nullptr);

```

```

113     cin >> n;
114     for (int i = 0; i < n - 1; i++) {
115         int u, v;
116         cin >> u >> v;
117         adj[u].push_back(v);
118         adj[v].push_back(u);
119     }
120
121     decompose(1);
122     for (int i = 1; i < n; i++) {
123         int aux = i;
124         int ori = aux;
125         pair<int, int> res = {1e9, 1e9};
126         bool ya = false;
127         do {
128             int it = bs(elements[aux], ori);
129             if (it == -1) {
130                 aux = parCentroid[aux];
131                 linea();
132                 continue;
133             }
134             pair<int, int> ans = sufix[aux][it];
135             int it2 = dist(elements[aux], ori);
136             ans.second += elements[aux][it2].second;
137             if (ans.second < res.second) {
138                 res = ans;
139             }
140             else if (ans.second == res.second) {
141                 if (ans.first < res.first) {
142                     res = ans;
143                 }
144                 aux = parCentroid[aux];
145                 linea();
146                 if (ya) break;
147                 if (parCentroid[aux] == aux) ya = true;
148             } while (true);
149             cout << res.first << ' ';
150         }
151     }
152     cout << n << '\n';
153
154     return 0;
155 }
```

Parte V

Math

5.1. FFT

```
1 #include <bits/stdc++.h>
2 using namespace std;
3
4 #define ll long long
5 const double PI = acos(-1);
6
7 namespace FFT {
8     static vector<int> rev;
9     static vector<complex<double>> raices{{0,0},{1,0}};
10
11    void fft(vector<complex<double>>& a, bool invertido) {
12        int n = a.size();
13
14        if (rev.size() != n) {
15            int k = __builtin_ctz(n); // log2(n)
16            rev.assign(n, 0);
17            for (int i = 0; i < n; i++) {
18                rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (k - 1));
19            }
20        }
21
22        for (int i = 0; i < n; i++) {
23            if (i < rev[i]) swap(a[i], a[rev[i]]);
24        }
25
26        if (raices.size() < n) {
27            int k = __builtin_ctz(raices.size());
28            raices.resize(n);
29            while ((1 << k) < n) {
30                double angulo = 2 * PI / (1 << (k + 1));
31                for (int i = 1 << (k - 1); i < (1 << k); i++) {
32                    raices[2 * i] = raices[i];
33                    double ang = angulo * (2 * i + 1 - (1 << k));
34                    raices[2 * i + 1] = complex<double>(cos(ang), sin(
ang));
35                }
36                k++;
37            }
38        }
39
40        for (int len = 1; len < n; len = len << 1) {
41            for (int i = 0; i < n; i += 2 * len) {
42                for (int j = 0; j < len; j++) {
43                    complex<double> u = a[i + j];
44                    complex<double> v = a[i + j + len] * raices[len + j
];
45                    a[i + j] = u + v;
46                    a[i + j + len] = u - v;
47                }
48            }
49        }
50
51        if (invertido) {
52            reverse(a.begin() + 1, a.end());
53            for (auto &x : a) x /= n;
54        }
55    }
56}
```

```

55     }
56 } // namespace FFT
57
58 vector<ll> multiplicacion_polinomio(const vector<ll>& A, const vector<ll>& B) {
59     int n1 = A.size(), n2 = B.size();
60     if (n1 == 0 || n2 == 0) return vector<ll>();
61
62     int n = 1;
63     while (n < n1 + n2 - 1) n = n << 1;
64     vector<complex<double>> fa(n), fb(n);
65     for (int i = 0; i < n1; i++) fa[i] = complex<double>(A[i], 0);
66     for (int i = 0; i < n2; i++) fb[i] = complex<double>(B[i], 0);
67
68     FFT::fft(fa, false);
69     FFT::fft(fb, false);
70     for (int i = 0; i < n; i++) fa[i] *= fb[i];
71     FFT::fft(fa, true);
72
73     vector<ll> res(n1 + n2 - 1);
74     for (int i = 0; i < res.size(); i++) {
75         res[i] = llround(fa[i].real()); // Redondeo
76     }
77     return res;
78 }
79
80
81 int main() {
82     vector<ll> A = {1, 2, 3}; // 1 + 2x + 3x^2
83     vector<ll> B = {2, 0, 1}; // 2 + 0x + 1x^2
84     vector<ll> C = multiplicacion_polinomio(A, B);
85     return 0;
86 }
```

5.2. NTT

```
1 #include <bits/stdc++.h>
2 using namespace std;
3
4 #define ll long long
5
6 namespace NTT {
7     // MOD debe ser un primo de la forma c * 2^k + 1
8     // 998244353 = 119 * 2^23 + 1. Raíz primitiva G = 3.
9     const ll MOD = 998244353;
10    const ll G = 3; // Raíz primitiva para MOD
11
12    ll fast_pow(ll a, ll b) {
13        ll res = 1;
14        a %= MOD;
15        while (b > 0) {
16            if (b & 1) res = (res * a) % MOD;
17            a = (a * a) % MOD;
18            b >>= 1;
19        }
20        return res;
21    }
22
23    ll inv(ll a) {
24        return fast_pow(a, MOD - 2);
25    }
26
27    static vector<int> rev;
28    static vector<ll> w;
29
30    void precompute(int n) {
31        if (rev.size() == n) return;
32        int k = __builtin_ctz(n);
33        rev.assign(n, 0);
34        for (int i = 0; i < n; i++) {
35            rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (k - 1));
36        }
37
38        w.resize(n);
39        w[0] = 1;
40        ll g_n = fast_pow(G, (MOD - 1) / n);
41        for (int i = 1; i < n; i++) {
42            w[i] = (w[i - 1] * g_n) % MOD;
43        }
44    }
45
46    void fft(vector<ll>& a, bool invertido) {
47        int n = a.size();
48        precompute(n);
49
50        for (int i = 0; i < n; i++) {
51            if (i < rev[i]) swap(a[i], a[rev[i]]);
52        }
53
54        for (int len = 1; len < n; len <= 1) {
55            int step = n / (2 * len);
56            for (int i = 0; i < n; i += 2 * len) {
```

```

57         for (int j = 0; j < len; j++) {
58             ll u = a[i + j];
59             ll v = (a[i + j + len] * w[j * step]) % MOD;
60
61             a[i + j] = (u + v) % MOD;
62             a[i + j + len] = (u - v + MOD) % MOD;
63         }
64     }
65
66     if (invertido) {
67         reverse(a.begin() + 1, a.end());
68         ll n_inv = inv(n);
69         for (auto &x : a) {
70             x = (x * n_inv) % MOD;
71         }
72     }
73 }
74 }
75 } // namespace NTT
76
77 vector<ll> multiplicacion_polinomio_mod(const vector<ll>& A, const
78 vector<ll>& B) {
79     int n1 = A.size(), n2 = B.size();
80     if (n1 == 0 || n2 == 0) return vector<ll>();
81
82     int n = 1;
83     while (n < n1 + n2 - 1) n = n << 1;
84
85     vector<ll> fa(n), fb(n);
86     for (int i = 0; i < n1; i++) fa[i] = (A[i] % NTT::MOD + NTT::MOD) %
87 NTT::MOD;
88     for (int i = 0; i < n2; i++) fb[i] = (B[i] % NTT::MOD + NTT::MOD) %
89 NTT::MOD;
90
91     NTT::fft(fa, false);
92     NTT::fft(fb, false);
93
94     for (int i = 0; i < n; i++) {
95         fa[i] = (fa[i] * fb[i]) % NTT::MOD;
96     }
97
98     NTT::fft(fa, true);
99
100    fa.resize(n1 + n2 - 1);
101    return fa;
102}
103
104int main() {
105    vector<ll> A = {1, 2, 3}; // 1 + 2x + 3x^2
106    vector<ll> B = {2, 0, 1}; // 2 + 0x + 1x^2
107    vector<ll> C = multiplicacion_polinomio_mod(A, B);
108    return 0;
109 }
```

5.3. Binary Exponentiation

```
1 #include <bits/stdc++.h>
2
3 #define ll long long
4
5 ll binpow(ll a, ll b) {
6     if (b == 0)
7         return 1;
8     ll res = binpow(a, b / 2);
9     if (b % 2)
10        return res * res * a;
11    else
12        return res * res;
13 }
14
15 ll binpowIt(ll a, ll b) {
16     ll res = 1;
17     while (b > 0) {
18         if (b & 1)
19             res = res * a;
20         a = a * a;
21         b >>= 1;
22     }
23     return res;
24 }
25
26 int power(ll x, ll y, ll M)
27 {
28     if (y == 0)
29         return 1;
30
31     int p = power(x, y / 2, M) % M;
32     p = (p * p) % M;
33
34     return (y % 2 == 0) ? p : (x * p) % M;
35 }
36 // inverse modular power(A, M - 2, M)
```

5.4. Matrix Exponentiation

```
1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 #define ll long long
6 const int MOD = 1e9+7;
7
8 struct Matrix {
9     int n;
10    vector<vector<ll>> mat;
11
12    Matrix(int size) : n(size), mat(size, vector<ll>(size)) {}
13
14    // Identity matrix
15    static Matrix identity(int size) {
16        Matrix I(size);
17        for (int i = 0; i < size; i++)
18            I.mat[i][i] = 1;
19        return I;
20    }
21
22    // Matrix multiplication
23    Matrix operator*(const Matrix& other) const {
24        Matrix product(n);
25        for (int i = 0; i < n; ++i)
26            for (int k = 0; k < n; ++k)
27                for (int j = 0; j < n; ++j) {
28                    product.mat[i][j] += mat[i][k] * other.mat[k][j];
29                    product.mat[i][j] %= MOD;
30                }
31        return product;
32    }
33
34    // Matrix exponentiation
35    Matrix pow(ll power) const {
36        Matrix result = identity(n);
37        Matrix base = *this;
38
39        while (power > 0) {
40            if (power & 1)
41                result = result * base;
42            base = base * base;
43            power >>= 1;
44        }
45        return result;
46    }
47 };
48
49 ll fibonacci(int n) {
50     if (n == 0) return 0;
51     Matrix base(2);
52     base.mat = {{1, 1}, {1, 0}};
53     base = base.pow(n - 1);
54     return base.mat[0][0]; // F(n)
55 }
```

```
57 int main() {
58     for(int i = 0; i < 25; i++)
59         cout << fibonacci(i) << " ";
60 }
```

Parte VI

Strings

6.1. KMP Automaton

```
1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 #define ll long long
6 #define endl '\n'
7
8 vector<int> prefix_function(string s) {
9     int n = (int)s.length();
10    vector<int> pi(n);
11    for (int i = 1; i < n; i++) {
12        int j = pi[i-1];
13        while (j > 0 && s[i] != s[j])
14            j = pi[j-1];
15        if (s[i] == s[j])
16            j++;
17        pi[i] = j;
18    }
19    return pi;
20}
21
22 void compute_automaton(string s, vector<vector<int>>& aut) {
23     s += '#';
24     int n = s.size();
25     vector<int> pi = prefix_function(s);
26     aut.assign(n, vector<int>(26));
27     for (int i = 0; i < n; i++) {
28         for (int c = 0; c < 26; c++) {
29             if (i > 0 && 'a' + c != s[i])
30                 aut[i][c] = aut[pi[i-1]][c];
31             else
32                 aut[i][c] = i + ('a' + c == s[i]);
33         }
34     }
35 }
36
37 const int MAXN = 1e4+5;
38 const int MAXK = 1e3+5;
39 const int MOD = 1e9+7;
40
41
42 int n,k;
43 string pattern;
44 vector<vector<int>> aut;
45
46 long long binpow(long long a, long long b)
47 {
48     long long result = 1;
49     while (b) {
50         if (b & 1)
51             result = (result * a) % MOD;
52         a = (a * a) % MOD;
53         b >>= 1;
54     }
55     return result;
56 }
```

```

57
58 ll dp[MAXN][MAXK];
59 ll fdp(int pos, int state) {
60     bool found = state == pattern.size();
61     if(pos == n) {
62         return found;
63     }
64
65     &val = dp[pos][state];
66     if(val != -1) return val;
67
68     val = 0;
69
70     if(found) {
71         ll l = n-pos;
72         return val = binpow(k, l);
73     }
74
75     for(int i = 0; i < k; i++) {
76         val = (val + fdp(pos+1, aut[state][i])) % MOD;
77     }
78
79     return val;
80 }
81
82 void solve() {
83     memset(dp, -1, sizeof dp);
84     cin >> n >> k >> pattern;
85
86     compute_automaton(pattern, aut);
87
88     cout << fdp(0,0) << endl;
89 }
90
91 int main() {
92     solve();
93 }
```

6.2. Hashing

```
1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 #define ll long long
6
7 const int MAXN = 1e5+5;
8 const int BASE = 31;
9 const int M = 1e9 + 7;
10
11 int pot[MAXN];
12
13 void compute_powers(int n) {
14     int p_pow = 1;
15     for (int i = 0; i < n; i++) {
16         pot[i] = p_pow;
17         p_pow = (p_pow * BASE) % M;
18     }
19 }
20
21 vector<ll> compute_hash(string const& s) {
22     int n = s.size();
23     vector<ll> hash_values(n + 5);
24     hash_values[0] = 0;
25     for (int i = 1; i <= n; i++) {
26         hash_values[i] = (hash_values[i - 1] * BASE + s[i - 1]) % M;
27     }
28     return hash_values;
29 }
30
31 ll get_hash(vector<ll> &hash_values, int a, int b) {
32     return (hash_values[b] - (hash_values[a - 1] * pot[b - a + 1])) % M +
33     M) % M;
34 }
35
36 int main() {
37     string word = "ALLEY";
38     compute_powers(word.size());
39     vector<ll> hash = compute_hash(word);
40
41     for (int i = 1; i <= word.size(); i++) {
42         cout << hash[i] << ',';
43     }
44     cout << endl;
45     for (int i = 1; i <= word.size(); i++) {
46         cout << word[i - 1] << ":" << get_hash(hash, i, i) << endl;
47     }
48
49     return 0;
50 }
```