Project 1 Report

Martingale

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***Abstract*—**In this project, Martingale Theory is performed in American Roulette, and Monte Carlo Method is utilized to evaluate its performance. The results show that with an infinite bankroll, there is a 100% probability of achieving the $80 winning goal within 1000 successive bets, while with a -$256 bankroll, the probability reduces to 64%. Meanwhile, estimated expected values of winnings from the two outcomes are $80 and -$40 respectively.

1 EXPERIMENT DESIGN

With the initial bets of $1, double up the bets after a loss and reset it to the initial amount ($1) after a win. The goal is to achieve $80 cumulative winnings within 1000 sequential trials while keep betting on black numbers. When the cumulative winnings hit $80 within an episode (i.e., a series of 1000 successive bets), stop betting and the cumulative winnings stay at $80 for the rest of the episode. Repeat the whole process 1000 times.

In experiment 1, the bankroll is infinite. In experiment 2, the bankroll is limited to $256, so once the cumulative winnings hit -$256, stop betting and the cumulative winnings stay at -$256.

2 RESULTS *&* ANALYSES

2.1 Experiment 1

2.1.1 *Estimated probability (Question 1)*

In experiment 1, the estimated probability of winning $80 within 1000 sequential bets is 100%.

Bets are doubled after a loss and reset to $1 after a win. Accordingly, a win after a loss gains $1 as a net profit and offsets the previous loss. Therefore, a win ultimately adds $1 to the cumulative winnings. In other words, the cumulative winnings hit $80 by 80 wins.

Besides, there are 18 black numbers in American Roulette, which has 38 numbers in total, so the winning odds of each bet is 18/38, i.e., 47.4%. Theoretically, 1000 rounds of bets yield 474 wins, and 169 rounds yield 80 wins – the cumulative winnings always hit $80 by the first 169 bets, hence a 100% probability to reach the winning goal.

Meanwhile, outputs from the 1000-episode simulation support the reasoning. For the entire simulation, the cumulative winnings always reach $80 by the first 215 bets (169 bets on average).

The 100% probability to reach the winning goal is the result of several factors working together: 1) an infinite bankroll makes bets keep going until a win offsets the previous loss and gains a net profit; 2) a considerable number of betting rounds works together with the infinite bankroll to allow the ongoing bets to yield enough net profits to reach the winning goal.

2.1.2 ***Estimated expected value (Question 2)***

According to 2.1.1, the winning odds of $80 is 100%, thus the estimated expected value of winnings after 1000 sequential bets is $80 × 100% = $80.

Additionally, it can also be calculated in this way: The odds to gain a $1 net profit is 47.4%, and there are an average of **169 active bets** (a bet is wagered and a win/loss outcome follows) per episode, so the estimated expected value of winnings after 1000 sequential bets is $1 × 47.4% × 169 = $80.

Also, outputs from the 1000-episode simulation support the claim. None of them fails the winning goal.

Noticeably, an infinite bankroll guarantees that any possible huge loss will be offset and yield a win. On the other hand, a limited bankroll will make the case different. Extreme cases can be found in Fig. 1: The cumulative winnings of

Episode 1 once drop below -$10,000. If there is a limited bankroll, which might limit the financial ability to double a bet after such a huge loss, the expected value will be different.

2.1.3 *Upper/lower standard deviation (Question 3)*

Fig. 2 shows that the upper standard deviation line does not reach a maximum value and then stabilize, and the lower standard deviation line does not reach a minimum value and then stabilize. Both lines stabilize after the cumulative winnings reach $80 by the first 215 bets and converge afterward.

It is because the cumulative winnings are set at $80 after hitting the winning goal; simultaneously, the standard deviation is fixed at $0.

Besides, the lower standard deviation line reaches a maximum value ($80) and then stabilizes. The reason is that the standard deviation is the square root of the variance, and the lower standard deviation line is based on the subtraction of the standard deviation from the mean value. As the mean value has a steady trend of increase before it hits $80, the lower standard deviation line reaches its maximum when the mean value stays at $80 and the standard deviation is $0.

Both lines change sharply at certain points, and their patterns are random. This is because of the sharp drops of the cumulative winnings, such as the examples of Fig.1 presented in 2.1.2 – random huge losses lead to random sharp changes in standard deviation.

2.2 Experiment 2

2.2.1 *Estimated probability (Question 4)*

In the 1000-episode simulation, 638 episodes hit $80, while 359 episodes hit - $256; the rest 3 episodes hit $64, $40, and $22 respectively. Therefore, the estimated probability of winning $80 within 1000 sequential bets is 63.8%.

Experiment 2 introduces a $256 bankroll, which plays a significant role in the outcome.

2.2.2 ***Estimated expected value (Question 5)***

According to 2.2.1, the winning odds of \$80 in one episode are 63.8%, while the losing odds of -\$256 in one episode are 35.9%; there is also a 1% chance to gain \$64, \$40, and \$22. The estimated expected value is:

$$\$80 \times 63.8\% - \$256 \times 35.9\% + (\$64 + \$40 + \$22) \times 1\% = - \$40$$

The estimated expected value is -\$40, which means the average outcome of all possible outputs is a \$40 loss after 1000 consecutive bets. It can also be verified by the mean value of cumulative winnings, which ultimately settles down at -\$40.

2.2.3 ***Upper/lower standard deviation (Question 6)***

In Fig. 4, the upper standard deviation line reaches a maximum of \$120, the lower standard deviation line reaches a minimum of -\$200; then they stabilize afterward. They do not converge as the number of sequential bets increases.

It is because 638 episodes reach \$80 winning goals and 328 episodes reach -\$256 within 1000 sequential bets, thus they ultimately set the cumulative winnings at \$80 and -\$256 respectively. As the number of sequential bets increases, the mean value stabilizes at -\$40, and the standard deviation stabilizes at \$160, the upper/lower standard deviation hence stabilizes at \$120/-\$200.

Compared with Fig. 2, the lines in Fig. 4 are smoother because the limited bankroll "filters" sharp drops. In other words, it prevents huge losses.

3 BENEFITS OF EXPECTED VALUES (QUESTION 7)

The expected value is a generalization of "a weighted average of all possible outcomes."[1] It considers each possible scenario, including benefits and risks. The expected value in Experiment 2 is negative, which is a telling sign of the risk, i.e., the possibility to hit negative cumulative winnings at the end. Moreover, the expected value is also an easy-to-calculate and easy-to-understand index: It is not bound to complicate mathematical calculations and presents a quick, brief, and direct result of the average outcome.

[1] https://en.wikipedia.org/wiki/Expected_value

On the other hand, one specific random episode cannot fully comprehend all the possible scenarios, especially a rare and damaging situation. Take Experiment 2 for instance, there is a 63.8% chance per episode to win $80 and a 35.9% chance to lose -$256. If only one episode is considered, the result will be either a 100% chance to win or a 100% chance to lose, and none of them are close to the fact. It is the reason why Monte Carlo Method is introduced in this project.

4 CONCLUSION

This project is based on Martingale Theory and Monte Carlo Method. The key of Martingale Strategy is doubling the losing bets to regain the previous loss. It seems a loss-averse strategy; however, Martingale also increases the possibility of quick and damaging financial losses. The key of Monte Carlo is repeating the simulations of random variables and using the simulation results/statistics to predict the probability of random outcomes. Estimated expected values play a positive role in predicting the likely outcomes; nonetheless, as a weighted average, it cannot indicate some rare and severe losses if the losses can be offset by an ideal condition (such as an infinite bankroll).

5 CHARTS

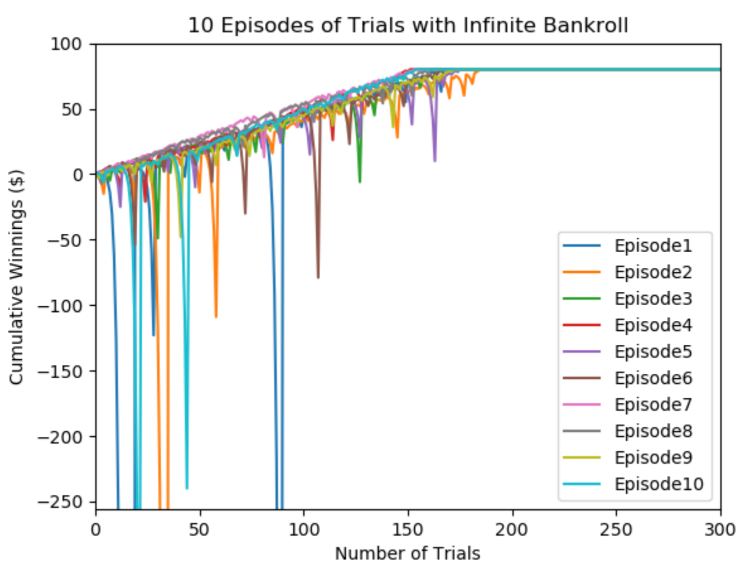

Figure 1—Cumulative winnings for 10-episode trials with infinite bankroll

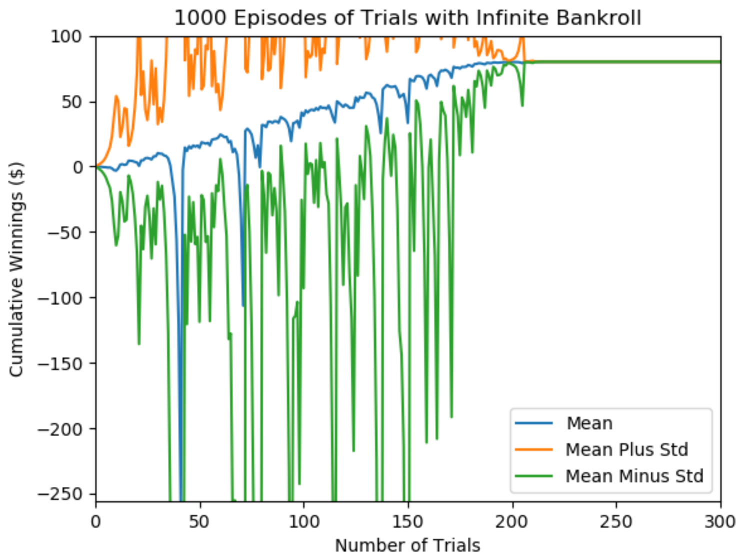

Figure 2—Mean values of cumulative winnings for 1000-episode trials with infinite bankroll

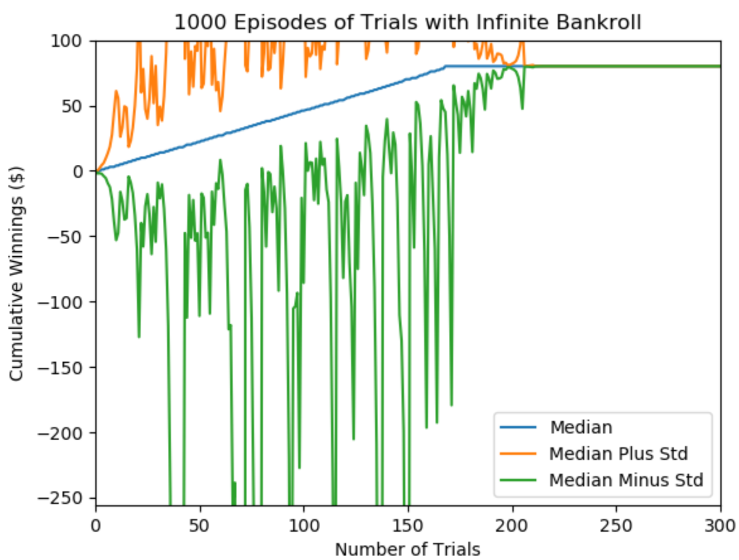

Figure 3—Median of cumulative winnings for 1000-episode trials with infinite bankroll

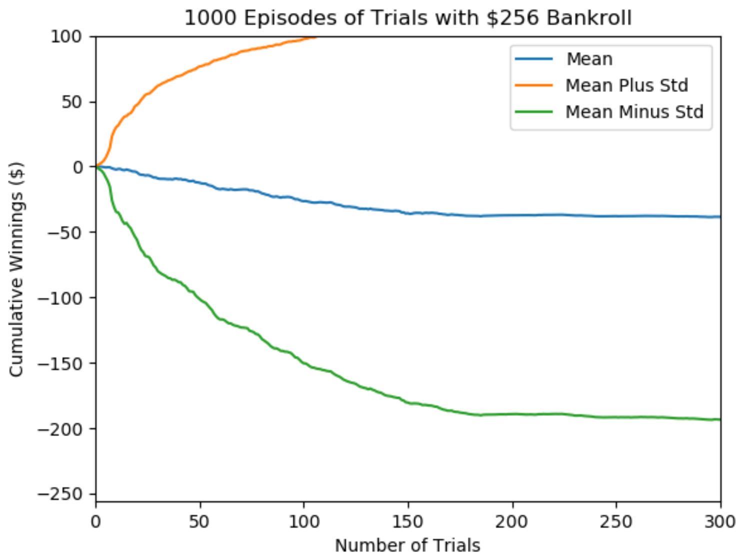

Figure 4—Mean values of cumulative winnings for 1000-episode trials with $256 bankroll

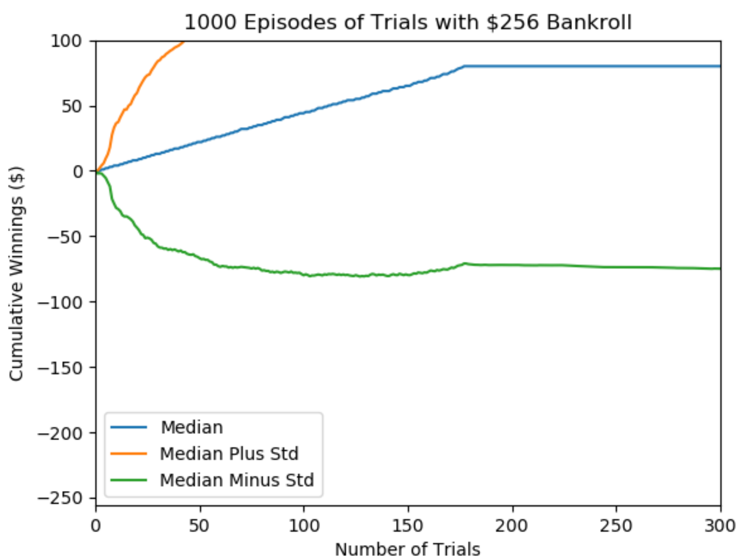

Figure 5—Median of cumulative winnings for 1000-episode trials with $256 bankroll