Rel. Energy & Momentum.

$$\omega_x = \frac{v + \omega_x'}{1 + v\omega_x'}$$

$$\omega_y = \frac{1}{\gamma} \frac{\omega_y'}{1 + v\omega_x'}$$

$$\omega_z = \frac{1}{\gamma} \frac{\omega_z'}{1 + v\omega_x'}$$

$$\vec{p} = m\vec{\omega} f(\vec{\omega}^2)$$

↓ depends on the magnitude of velocity.

$$\gamma = \frac{1}{\sqrt{1-v^2}}$$

$$\left(\frac{1}{\gamma}\right)^2 = \left(\sqrt{1-v^2}\right)^2$$

Scenario

$$\vec{\omega_1}' = (\omega, 0, 0)$$

$$\vec{\omega_2}' = (-\omega, 0, 0)$$

$$\vec{\omega_3}' = (\omega\cos\theta, \omega\sin\theta, 0)$$

$$\vec{\omega_4}' = (-\omega\cos\theta, -\omega\sin\theta, 0)$$

Another coord system:

$$\vec{\omega_1} = \left(\frac{v+\omega}{1+v\omega}, 0, 0\right)$$

$$\vec{\omega_2} = \left(\frac{v-\omega}{1-v\omega}, 0, 0\right)$$

$$\vec{\omega_3} = \left(\frac{v+\omega\cos\theta}{1+v\omega\cos\theta}, \frac{1}{\gamma}\frac{\omega\sin\theta}{1+v\omega\cos\theta}, 0\right)$$

$$\vec{\omega_4} = \left(\frac{v-\omega\cos\theta}{1-v\omega\cos\theta}, \frac{1}{\gamma}\frac{-\omega\sin\theta}{1-v\omega\cos\theta}, 0\right)$$
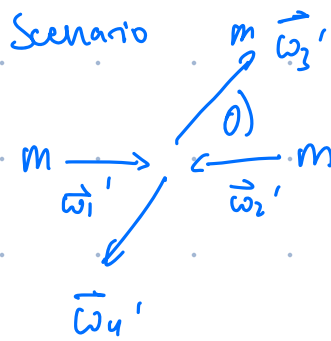
$$\vec{\omega}_{3,4}^2 = \frac{(v \pm \omega\cos\theta)^2 + \left(\frac{1}{\gamma}\right)^2(\pm\omega\sin\theta)^2}{(1 \pm v\omega\cos\theta)^2}$$

$$= \frac{v^2 + \omega^2\cos^2\theta \pm 2v\omega\cos\theta + \omega^2\sin^2\theta - \omega^2 v^2(1-\cos^2\theta)}{}$$

$$= \frac{v^2 + \omega^2 \pm 2v\omega\cos\theta - \omega^2 v^2 + v^2\omega^2\cos^2\theta}{}$$

$$\vec{\omega}_{3,4}^2 = \frac{(1 \pm v\omega\cos\theta)^2 - (1-v^2)(1-\omega^2)}{(1 \pm v\omega\cos\theta)^2}$$

$$= 1 - \frac{(1-v^2)(1-\omega^2)}{(1 \pm v\omega\cos\theta)^2}$$

$$f(17x \leftarrow)$$

$$\vec{P_{s \, before}} = \vec{P_{y \, after}}$$

$$0 = \frac{\omega \sin\theta/\gamma}{1 + v\omega \cos\theta} f\left(1 - \frac{(1-v^2)(1-\omega^2)}{(1 \neq v\omega \cos\theta^2)}\right)$$

$$- \frac{\omega \sin\theta/\gamma}{1 - v\omega \cos\theta} f(\cdots)$$

$$f(\cdots) \frac{1}{1 - v\omega \cos\theta} = f(\cdots) \frac{1}{1 + v\omega \cos\theta}$$

$$\frac{1}{f^2}\left(1 - \frac{(1-v^2)(1-\omega^2)}{(1 \neq v\omega \cos\theta)^2}\right)(1 - v\omega \cos\theta)^2$$

$$= \frac{1}{f^2}(\cdots)(1 + v\omega \cos\theta)^2$$

$$= \vec{\omega}_{3/4}^{\,2}$$

$$\frac{1}{f^2} = 1 - S \implies S = 1 - \frac{(1-v^2)(1-\omega^2)}{(1 - v\omega \cos\theta)^2}$$

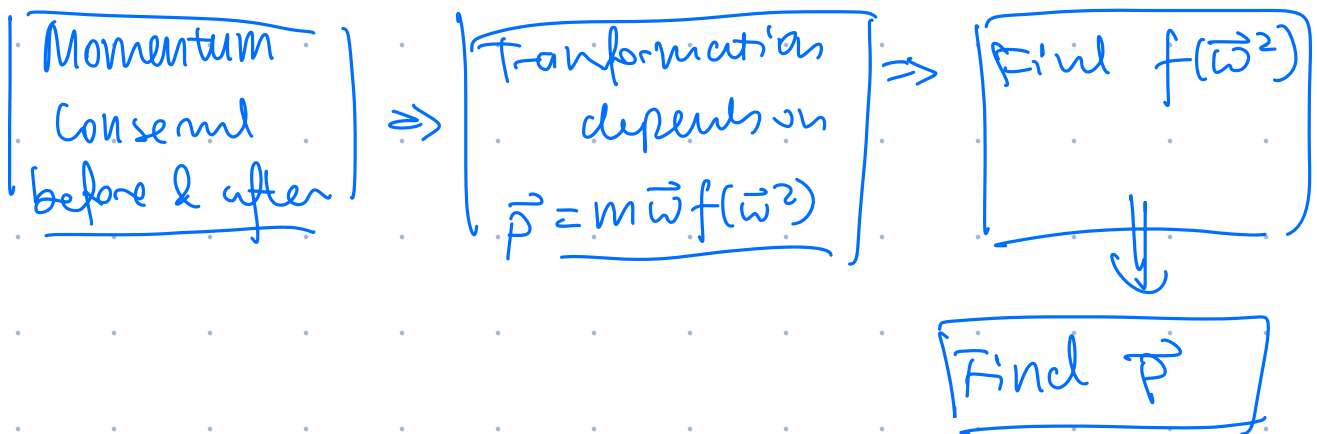$$\frac{1}{f^2}(\cdots)(1 - v\omega \cos\theta)^2 = (1 - S)(1 - v\omega \cos\theta)^2$$

$$= \left[1 - 1 + \frac{(1-v^2)(1-\omega^2)}{(1 - v\omega \cos\theta)^2}\right](1 - v\omega \cos\theta)^2$$

$$= (1-v^2)(1-\omega^2)$$

$$f^2 = \frac{1}{1-s} \Rightarrow f = \frac{1}{\sqrt{1-s}} = \frac{1}{\sqrt{1-\vec{\omega}_{1,4}^2}}$$

So $\qquad \vec{p}(\vec{\omega}) = \dfrac{m\vec{\omega}}{\sqrt{1-\vec{\omega}^2}}$

General Idea:

$\boxed{\begin{array}{c}\text{Momentum} \\ \text{Conserved} \\ \text{before \& after}\end{array}} \Rightarrow \boxed{\begin{array}{c}\text{Transformations} \\ \text{depends on} \\ \vec{p} = m\vec{\omega}\, f(\vec{\omega}^2)\end{array}} \Rightarrow \boxed{\text{Find } f(\vec{\omega}^2)}$

$$\Downarrow$$

$$\boxed{\text{Find } \vec{p}}$$

Energy Consv

$$E(\vec{\omega}) = mg(\vec{\omega}^2)$$

$$N.R. \quad g(s) = \tfrac{1}{2}s.$$

$$E_{before} = E_{after}.$$

→ from a homogeneous equation you can set whether constants you like it still satisfies the equation.

$$mg\left(\left(\frac{v+w}{1+vw}\right)^2\right) + mg\left(\left(\frac{v-w}{1-vw}\right)^2\right)$$

$$= mg\left(1 - \frac{(1-v^2)(1-w^2)}{(1+vw\cos\theta)^2}\right)$$

$$+ mg\left(1 - \frac{(1-v^2)(1-w^2)}{(1-vw\cos\theta)^2}\right)$$

(?)   ⋮ derivations skipped.

$$g(s) = \frac{1}{\sqrt{1-s}}$$

⋮

$$\overline{E}(\vec{\omega}) = \frac{m}{\sqrt{1-\vec{\omega}^2}}$$

An inelastic collision

$$\vec{w_1} \xrightarrow{\quad} \qquad \overleftarrow{\vec{w_2}}$$

$$\vec{w_1} = \left(\frac{v+w}{1+vw}, 0, 0 \right)$$

$$\vec{w_2} = \left(\frac{v-w}{1-vw}, 0, 0 \right)$$

$$\vec{w_3} = (v, 0, 0)$$

$$\uparrow \\ \text{blob}$$

$$E_{before} =$$

$$g(s) = \frac{1}{\sqrt{1-s}},$$

$$\frac{m}{\sqrt{1-\left[\frac{v+w}{1+vw}\right]^2}} + \frac{m}{\sqrt{1-\left(\frac{v-w}{1-vw}\right)^2}} = \boxed{?} \; \frac{1}{\sqrt{1-v^2}}$$

2m? something or else?

$$\|$$

$$\frac{[1+vw]m}{\sqrt{(1+vw)^2 - (v+w)^2}} + \frac{[1-vw]m}{\sqrt{(1-vw)^2 - (v-w)^2}}$$

$$\|$$

$$\frac{[\cdots -]}{\sqrt{1+v^2w^2 + 2vw - v^2 - w^2 - 2v \cdot w}} + \frac{(- + - -)}{\sqrt{1+v^2w^2 - 2vw - v^2 - w^2 + 2vw}}$$

$$\|$$

$$\frac{2m}{\sqrt{(1-v^2)(1-w^2)}} = \boxed{?} \; \frac{1}{\sqrt{1-v^2}}$$

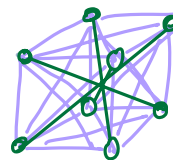!!

$$\boxed{\frac{2m}{\sqrt{1-w^2}}}$$

$$m_{blob}.$$

E here allows for — all energy

in the primed coordinate

$$E_{lab} = \frac{m}{\sqrt{1-w^2}} + \frac{m}{\sqrt{1-w^2}}$$

$$= \frac{2m}{\sqrt{1-w^2}}$$

Implications:

① $c=1 \Rightarrow [E]=[p]=[m]$

② $w \to 1$ $E = \frac{m}{\sqrt{1-v^2}} \Rightarrow \infty$

(2') only massless objects move with the speed of light.

$$|\vec{w}| = 1.$$

③ mass of an object = its energy at rest.

↳ heating smt up → making it heavier.

97% of proton mass comes from relativistic
effect of quark mass

↳ how do you measure the mass of a quark?

Proton = 3 quarks

quark = $\frac{1}{3}$ proton mass? ✗ NI.

M_{quark} = at most $\frac{1}{6}$ Mproton.

↳ $M_{\bar{u}}$ has a ⓠ & ⓠ̄.

$$\boxed{M_{\bar{u}} = 0.135 M_p}$$

First nuclear bomb:

$$\Delta M \Big| = \frac{4 \cdot 10^{6 \text{ (Joules)}} \cdot 10^{4 \text{ (T) kg}}}{c^2} = \frac{4 \cdot 10^{10} \times 10^{3} \text{ kg}}{9 \cdot 10^{16}} \approx \frac{1}{2} 10^{-6} \text{ kg}$$

$\leftarrow 1\%T$

$10^{-6} \times 10^{3} = 10^{-3}$ g

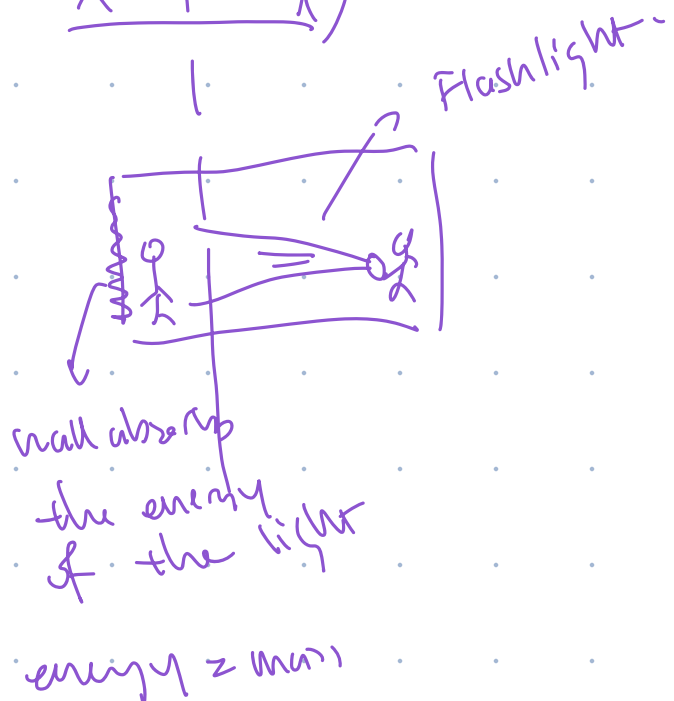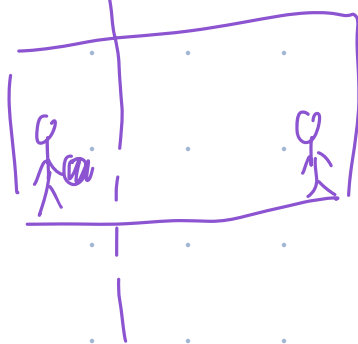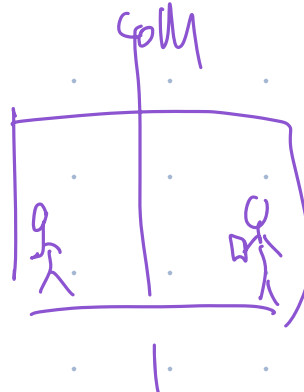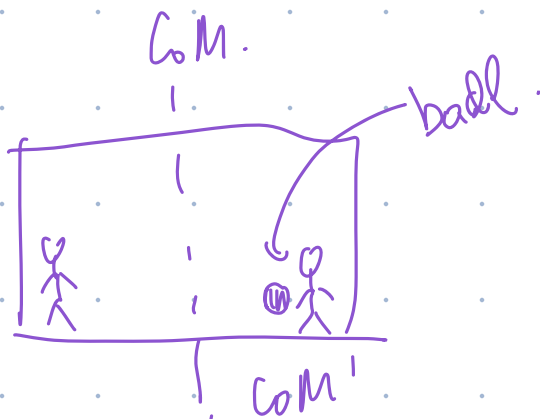$\frac{1}{2} \times 10^{-3}$ kg $= \frac{1}{2}$ g

Convert a of mass into energy

$$E(\vec{w}) = \frac{m}{\sqrt{1-\vec{w}^2}} \approx m\left(1 + \frac{\vec{w}^2}{2} + \frac{3}{8}\vec{w}^4 + - \right)$$

taylor expand

rest energy

nonrelativistic kinetic energy

relativistic corrections to kinetic energy.

CoM.

ball.

CoM'

CoM

Flashlight.

wall absorb the energy of the light

energy ≥ mass

It's not trivial that mass combine
all furrows of energy.