$$\begin{cases} t' = \gamma(t - vx) \\ x' = \gamma(x - vt) \\ y' = y \\ z' = z \end{cases} \qquad \begin{cases} u^{0\prime} = \gamma(u^0 - v u^x) \\ u^{x\prime} = \gamma(u^x - v u^0) \\ u^{y\prime} = u_y \\ u_z' = u_z \end{cases} \qquad u_0 = \frac{dt}{d\tau}$$

$$E = m u^0$$

$$\vec{p} = m\vec{u}$$

$$\rho = \rho_0 u^0$$

$$\vec{j} = \rho_0 \vec{u}$$

$$\rho' = \gamma(\rho - v j_x)$$

$$E' = \gamma(E - v p_x)$$

$$p_x' = \gamma(p_x - vE)$$

$$p_y' = p_y$$

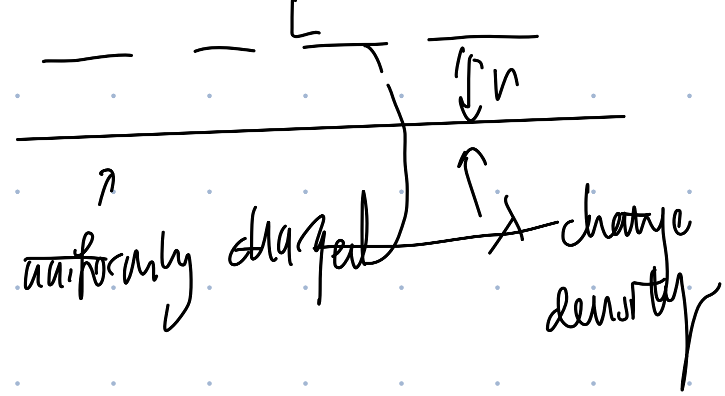
$$p_z' = p_z$$

$$\begin{cases} F_x' = \dfrac{F_x - v \vec{w} \cdot \vec{F}}{1 - v w_x} \\ F_y' = \dfrac{F_y / \gamma}{1 - v w_x} \\ F_z' = \dfrac{F_z / \gamma}{1 - v w_x} \end{cases} \qquad \begin{cases} w_x' = \dfrac{v - w_x}{1 - v w_x} \\ w_y' = \dfrac{w_y / \gamma}{1 - v w_x} \\ w_z' = \dfrac{w_z / \gamma}{1 - v w_x} \end{cases}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}, \quad \vec{\nabla} \times \vec{B} = \mu_0 \vec{j} \qquad \frac{1}{\epsilon_0 \mu_0} = c^2$$

$$\int_{\text{boundary}} \vec{E} \cdot d\vec{s} = \frac{\int_{\text{volume}} \rho}{\epsilon_0} = \frac{Q}{\epsilon_0}$$

(1)



$$\bar{E}\,(2\pi r \cancel{L}) = \frac{\lambda \cancel{L}}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi \epsilon_0 r}$$

if we are moving along $\vec{x}$.

(b)

from length contraction, density increase

$$\bar{E}'_\perp = \gamma \bar{E}_\perp .$$

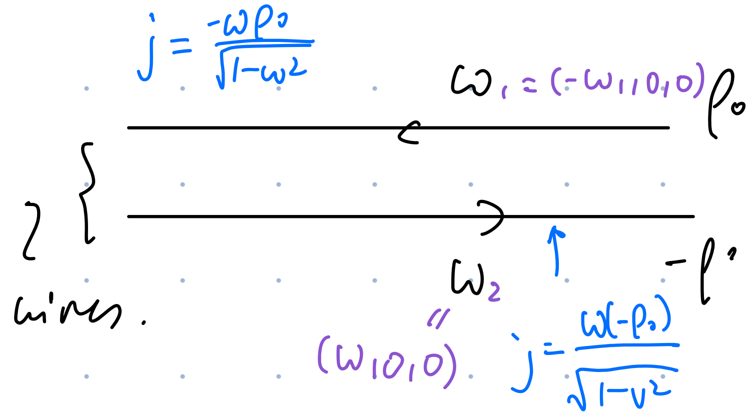
perpendicular to velocity

$$\bar{E}_\perp = \gamma \bar{E}'_\perp$$

changing dir doesn't

$$\underline{\bar{E}_\perp = \gamma \bar{E}_\perp' = \gamma^2 \bar{E}_\perp .}$$

inconsistent



no electric fields
(not net charges)

In the primed coordinate system,

$$w_1' = \left(\frac{v-w}{1-vw}, 0, 0\right) \qquad \rho_1' = \frac{\rho_0}{\sqrt{1-\left(\frac{v-w}{1-vw}\right)^2}}$$

$$w_2' = \left(\frac{v+w}{1+vw}, 0, 0\right) \qquad \rho_2' = \frac{-\rho_0}{\sqrt{1-\left(\frac{v+w}{1+vw}\right)^2}}$$

$\rho_1' + \rho_2' \neq 0 \Rightarrow \exists$ electric fields

$$\vec{F} = q(\vec{E} + \vec{w} \times \vec{B})$$

$$\vec{w} \times \vec{B} = \hat{x}(w_y B_z - w_z B_y) + \hat{y}(w_z B_x - w_x B_z) + \hat{z}(w_x B_y - w_y B_x)$$

① $\vec{E} = (E_x, 0, 0)$, $\vec{w} = (0, 0, 0)$ | $E_x' = E_x$

$F_x = qE_x$

assume particle not moving

$$F_x' = \frac{F_x - v\vec{w}\vec{F}}{1 - vw_x} = F_x = qE_x$$

$\parallel$

qE_x'

② E $\quad \vec{w} = (0, w_y, 0)$

$\vec{B} = (B_x, 0, 0)$

$\vec{F} = -qw_yB_x$

$$F_z' = \frac{F_z/\gamma}{1 - v\cancel{w_x}} = \frac{F_z}{\gamma} = -q\frac{w_y}{\gamma}B_x$$

$= -qw_y'B_x$

$\parallel$

$q\left(-w_y'B_x' + \cancel{w_x'B_y'}\right)$ $\quad w_x' = vB_y'$

↳ since there's no v on the RHS, $B_y' = 0$

Hence you have $B_x' = B_x$.

③ $\vec{E} = (0, E_y, 0)$, $\vec{\omega} = (v, 0, 0)$ $\vec{\omega}' = (0, 0, 0)$

$\vec{B} = (0, 0, B_z)$, $F_y = q(E_y - vB_z)$

$$F_y' = \frac{F_y/\gamma}{1-v^2} = \gamma F_y = \gamma q(E_y - vB_z)$$

$\parallel$

$$q(E_y' + \vec{\omega}' \times \vec{B}') = q(E_y')$$

$\uparrow$

this is 0

$$\boxed{E_y' = \gamma(E_y - vB_z)}$$

④ $\vec{E} = (0, 0, E_z)$, $\vec{\omega} = (v, 0, 0)$, $\vec{\omega}' = (0, 0, 0)$

$\vec{B} = (0, B_y, 0)$,

$($

$$\bar{F}_z = q(\bar{E}_z + vB_y)$$

$$F_z' = \frac{\bar{F}_z/\gamma}{1-v^2} = \gamma \bar{F}_z = \gamma q(\bar{E}_z + vB_y)$$

$$\|$$

$$q(E_z' + \cancel{\underbrace{\vec{w}'}_{0} \times \vec{B}'})$$

$$\boxed{E_z' = \gamma(\bar{E}_z + B_y)}$$

More General Case.

$$\vec{E} = (0, \bar{E}_y, 0), \quad \vec{\omega} = (\omega_x, 0, 0)$$

$$\vec{B} = (0, 0, \bar{B}_z), \quad \bar{F}_y = q(\bar{E}_y - \omega_x \bar{B}_z)$$

$$F_y' = \frac{\bar{F}_y/\gamma}{1 - vW_x} = \frac{1}{\gamma(1 - vW_x)} q(\bar{E}_y - W_x \bar{B}_z)$$

$$\|$$

$$q[E_y' + \cancel{\vec{w}' \times \vec{B}}],$$

$$\underset{\|}{} (w_z' B_x' - W_x' B_z')$$

$$W_x' = \frac{\omega_x - v}{1 - W_x v}$$

$$E_y' = \frac{1}{\gamma\left(1 - v\,\frac{w_x' + v}{1 + v w_x'}\right)}\left(E_y - \frac{v + w_x'}{1 + v w_x'} B_z\right)$$

$$= \frac{(1 + v w_x')E_y - (1 + v w_x')(v + w_x')B_z}{\gamma\left(1 + v \cancel{w_x'} - v(\cancel{w_x'} + v)\right)}$$

$$= \gamma\left[\underbrace{(E_y - vB_z)}_{E_y'} - w_x'(B_z - vE_y)\right]$$

E_y', which confirms what we've derived previously.

$B_z' = \gamma(B_z - vE_y)$

By staring @ stuff, we can derive

^ spotting patterns

$$B_y' = \gamma(B_y + vE_z)$$

Hence you'll have

$E_x' = E_x$ as is. $\qquad B_x' = B_x$

$E_y' = \gamma(E_y - vB_z)$ $\qquad B_y' = \gamma(B_y + vE_z)$

$E_z' = \gamma(E_z + vB_y)$ $\qquad B_z' = \gamma(B_z - vE_y)$

$B_y' = \gamma(B_y + \frac{v}{c^2}E_z)$

Check consistency:

$\mu_0 = \frac{1}{\epsilon_0 c^2}$

$$E_y' = \gamma(E_y - vB_z)$$
$$= \gamma(\gamma(E_y' + vB_z') - v\gamma(B_z' + vE_y'))$$
$$= \gamma^2[E_y'(1-v^2) + vB_z' - vB_z']$$
$$= E_y' \Rightarrow \text{consistent.}$$

imagine you can change c.

then as $c \to \infty$

$\mu_0 \to 0.$

then you have no magnetic fields.

Magnetism is a relativistic phenomenon.

Restoring dimension