

Rel. Energy & Momentum.

$$\omega_x = \frac{v + \omega_x'}{1 + v\omega_x'}$$

$$\omega_y = \frac{1}{\gamma} \frac{\omega_y'}{1 + v\omega_x'}$$

$$\omega_z = \frac{1}{\gamma} \frac{\omega_z'}{1 + v\omega_x'}$$

$$\vec{p} = m\vec{\omega} f(\vec{\omega})$$

↓
depends on
the magnitude of
velocity.

$$\gamma = \frac{1}{\sqrt{1-v^2}}$$

$$\left(\frac{1}{\gamma}\right)^2 = (1-v^2)^{-2}$$

$$\vec{\omega}_{3,4}^2 =$$

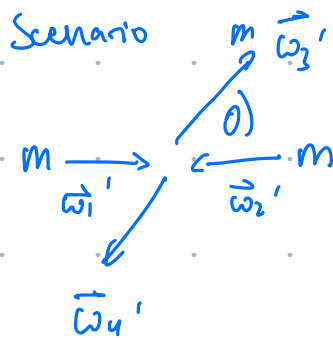
$$\frac{(v \pm \omega \cos \theta)^2 + \left(\frac{1}{\gamma}\right)^2 (\pm \omega \sin \theta)^2}{(1 \pm v\omega \cos \theta)^2}$$

$$= \frac{v^2 + \omega^2 \cos^2 \theta \pm 2v\omega \cos \theta + \omega^2 \sin^2 \theta - \omega^2 v^2 (1 - \omega^2)}{1 \pm 2v\omega \cos \theta + \omega^2 v^2}$$

$$= \frac{v^2 + \omega^2 \pm 2v\omega \cos \theta - \omega^2 v^2 + v^2 \omega^2 \cos^2 \theta}{1 \pm 2v\omega \cos \theta + \omega^2 v^2}$$

$$\vec{\omega}_{3,4}^2 = \frac{(1 \pm v\omega \cos \theta)^2 - (1-v^2)(1-\omega^2)}{(1 \pm v\omega \cos \theta)^2}$$

$$= 1 - \frac{(1-v^2)(1-\omega^2)}{(1 \pm v\omega \cos \theta)^2}$$



$$\vec{\omega}_1' = (\omega, 0, 0)$$

$$\vec{\omega}_2' = (-\omega, 0, 0)$$

$$\vec{\omega}_3' = (\omega \cos \theta, \omega \sin \theta, 0)$$

$$\vec{\omega}_4' = (-\omega \cos \theta, -\omega \sin \theta, 0)$$

Another coord system:

$$\vec{\omega}_1 = \left(\frac{v+\omega}{1+v\omega}, 0, 0 \right)$$

$$\vec{\omega}_2 = \left(\frac{v-\omega}{1-v\omega}, 0, 0 \right)$$

$$\vec{\omega}_3 = \left(\frac{v+\omega \cos \theta}{1+v\omega \cos \theta}, \frac{1}{\gamma} \frac{\omega \sin \theta}{1+v\omega \cos \theta}, 0 \right)$$

$$\vec{\omega}_4 = \left(\frac{v-\omega \cos \theta}{1-v\omega \cos \theta}, \frac{1}{\gamma} \frac{-\omega \sin \theta}{1-v\omega \cos \theta}, 0 \right)$$

f(17x ←)

$$\vec{p}_{\text{before}} = \vec{p}_{\text{after}}$$

$$0 = \frac{\cancel{\omega \sin \theta} / \gamma}{1 + v \omega \cos \theta} f \left(1 - \frac{(1-v^2)(1-\omega^2)}{(1 \mp v \omega \cos \theta)^2} \right)$$

$$- \frac{\cancel{\omega \sin \theta} / \gamma}{1 - v \omega \cos \theta} f \left(- \dots \right)$$

$$f \left(- \dots \right) \frac{1}{1 - v \omega \cos \theta} = f \left(\dots \right) \frac{1}{1 + v \omega \cos \theta}$$

$$\frac{1}{f^2} \left(1 - \frac{(1-v^2)(1-\omega^2)}{(1 \mp v \omega \cos \theta)^2} \right) (1 - v \omega \cos \theta)^2$$

$$= \frac{1}{f^2} \left(\dots \right) (1 + v \omega \cos \theta)^2$$

$$\Rightarrow \vec{\omega}_{\text{sin}}^2$$

$$\frac{1}{f^2} = 1 - S \Rightarrow S = 1 - \frac{(1-v^2)(1-\omega^2)}{(1 - v \omega \cos \theta)^2}$$

$$\frac{1}{f^2} (\dots) (1 - v \omega \cos \theta)^2 = (1 - S) (1 - v \omega \cos \theta)^2$$

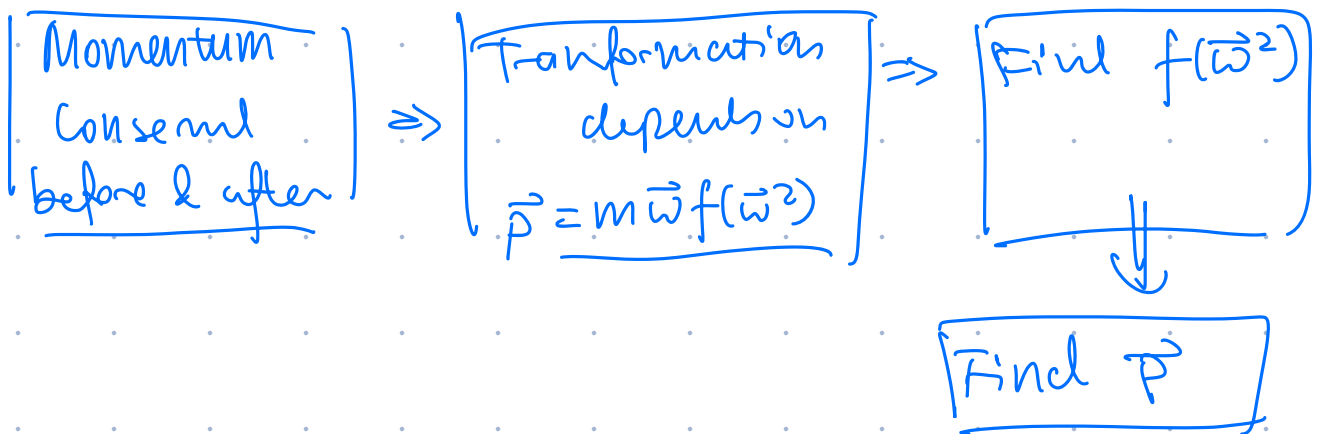
$$= \left[1 - 1 + \frac{(1-v^2)(1-\omega^2)}{(1 - v \omega \cos \theta)^2} \right] (1 - v \omega \cos \theta)^2$$

$$= (1-v^2)(1-\omega^2)$$

$$f^2 = \frac{1}{1-s} \Rightarrow f = \frac{1}{\sqrt{1-s}} = \frac{1}{\sqrt{1-\vec{\omega}_{1,4}^2}}$$

$$\text{so } \vec{p}(\vec{\omega}) = \frac{m\vec{\omega}}{\sqrt{1-\vec{\omega}^2}}$$

General idea:



Energy Conv

$$E(\vec{\omega}) = mg(\vec{\omega}^2)$$

$$\text{N.R. } g(s) = \frac{1}{2}s.$$

$$E_{\text{before}} = E_{\text{after}}$$

→ from a homogeneous equation you can set whatever constants you like if it still satisfies the equation.

$$\cancel{mg} \left(\left(\frac{v+w}{1+vw} \right)^2 \right) + \cancel{mg} \left(\left(\frac{v-w}{1-vw} \right)^2 \right)$$

$$= \cancel{mg} \left(1 - \frac{(1-v^2)(1-w^2)}{(1+vw \cos \theta)^2} \right)$$

$$+ \cancel{mg} \left(1 - \frac{(1-v^2)(1-w^2)}{(1-vw \cos \theta)^2} \right)$$

(?)

↓ derivations skipped.

$$g(s) = \frac{1}{\sqrt{1-s}}$$

↓

$$E(\vec{\omega}) = \frac{m}{\sqrt{1-\vec{\omega}^2}}$$

An inelastic collision

$$\vec{w}_1 \rightarrow \quad \leftarrow \vec{w}_2$$

$$\vec{w}_1 = \left(\frac{v+w}{1+vw}, 0, 0 \right)$$

↑
blob

$$\vec{w}_2 = \left(\frac{v-w}{1-vw}, 0, 0 \right)$$

$$\vec{w}_3 = (v, 0, 0)$$

Energy =

$$g(s) = \frac{1}{\sqrt{1-s}}$$

$$\frac{m}{\sqrt{1-\left(\frac{v+w}{1+vw}\right)^2}} + \frac{m}{\sqrt{1-\left(\frac{v-w}{1-vw}\right)^2}} = \boxed{?} \frac{1}{\sqrt{1-v^2}}$$

2m? or something else?

$$\frac{[1+vw]m}{\sqrt{(1+vw)^2 - (v+w)^2}} + \frac{[1-vw]m}{\sqrt{(1-vw)^2 - (v-w)^2}}$$

$$\frac{[\dots]}{\sqrt{1+v^2w^2+2vw-v^2-w^2-2vw}} + \frac{[\dots]}{\sqrt{1+v^2w^2-2vw-v^2-w^2+2vw}}$$

$$\frac{2m}{\sqrt{(1-v^2)(1-w^2)}} = \boxed{?} \frac{1}{\sqrt{1-v^2}}$$

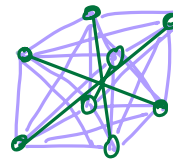
$$\frac{2m}{\sqrt{1-w^2}}$$

m_{blob}

Here
amounts
for an
energy

in the primed coordinate

$$E_{\text{tot}} = \frac{m}{\sqrt{1-v^2}} + \frac{m}{\sqrt{1-v^2}}$$
$$= \frac{2m}{\sqrt{1-v^2}}$$



Implications:

① $c=1 \Rightarrow [E]=[p]=[m]$

② $v \rightarrow 1 \quad E = \frac{m}{\sqrt{1-v^2}} \rightarrow \infty$

(2') only massless objects move with the speed of light.

$$|\vec{v}|=1.$$

③ mass of an object = its energy at rest.

↳ heating smt up → making it heavier.

97% of proton mass comes from relativistic effect of quark mass

↳ how do you measure the mass of a quark?



First nuclear bomb:

$$\Delta m = \frac{4 \cdot 10^6 \cdot 10^4}{c^2} = \frac{4 \cdot 10^{10}}{9 \cdot 10^{16}} \approx \frac{1}{2} 10^{-6} \text{ kg}$$

Joules 1 kT

$$10^{-6} \times 10^3 = 10^{-3} \text{ g}$$

Convert a microgram
of mass into energy

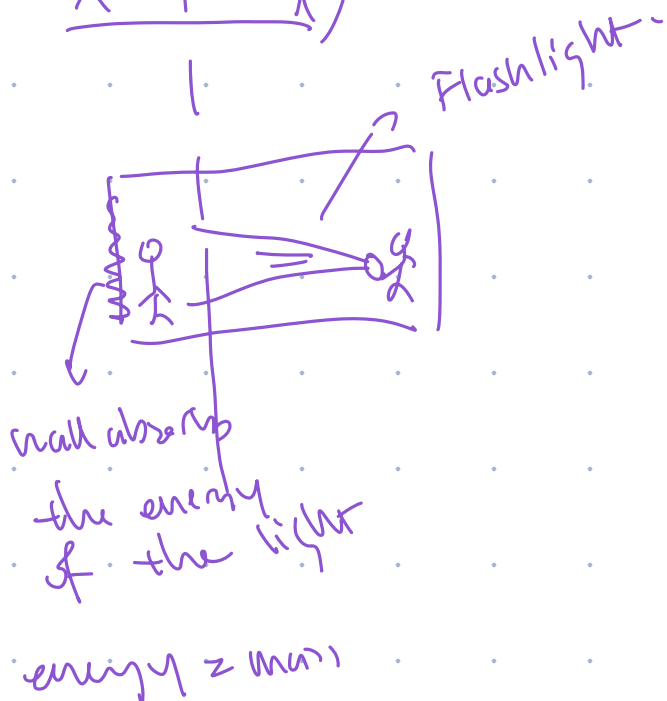
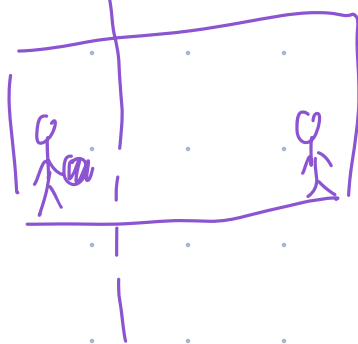
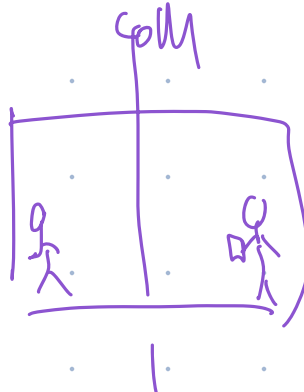
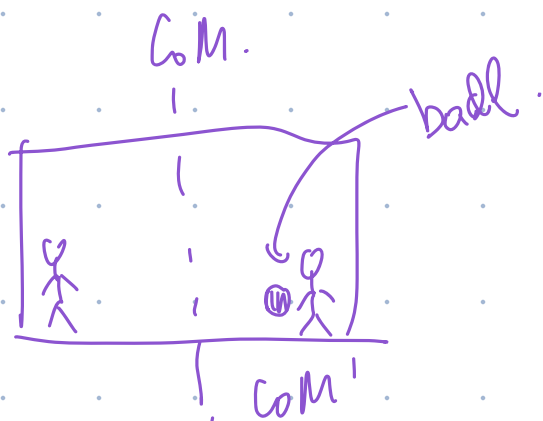
$$E(\vec{v}) = \frac{m}{\sqrt{1-\vec{v}^2}} \approx m \left(1 + \frac{\vec{v}^2}{2} + \frac{3}{8} \vec{v}^4 + \dots \right)$$

taylor
expand

rest
energy

nonrelativistic
kinetic
energy

relativistic
corrections
to kinetic
energy.



It's not trivial that mass combines
all forms of energy: