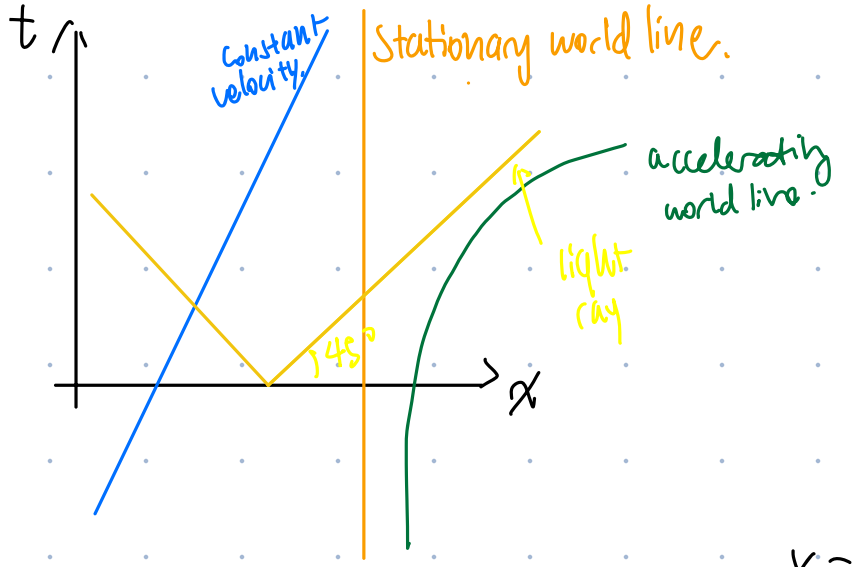
What we've known so far

$$\begin{cases} x = \gamma(x' + vt') \\ t = \gamma(t' + vx') \end{cases} \quad \begin{cases} x' = \gamma(x - vt) \\ t' = \gamma(t - vx) \end{cases} \quad \begin{cases} y = y' \\ z = z' \end{cases}$$

Tool: Space-Time Graph.



The line $t' = 0$ looks like:

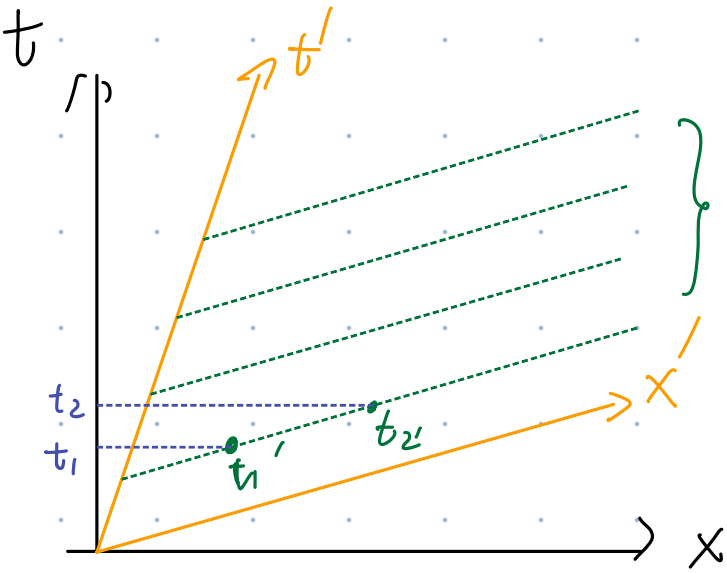
$$\begin{cases} x = \gamma x' \\ t = \gamma v x' \end{cases} \Rightarrow \frac{t}{x} = v$$

$t = vx$.

The line $x' = 0$ looks like:

$$\begin{cases} x = \gamma v t' \\ t = \gamma t' \end{cases} \Rightarrow \frac{x}{t} = v$$

$x = vt$



} lines of constant t.

times simultaneous in x-t are not simultaneous in x'-t'.

Suppose I have 2 events

$$(x_1, t_1) \quad (x_2, t_2) \Rightarrow \begin{cases} \Delta x = x_2 - x_1 \\ \Delta t = t_2 - t_1 \end{cases} \Rightarrow \begin{aligned} \Delta x' &= \gamma(\Delta x - v\Delta t) \\ \Delta t' &= \gamma(\Delta t - v\Delta x) \end{aligned}$$

$$(\Delta x')^2 - (\Delta t')^2 = \gamma^2 \left[\Delta x^2 + v^2 \Delta t^2 - 2\Delta x \cdot v\Delta t - \Delta t^2 - v^2 \Delta x^2 + 2\Delta x \cdot v\Delta t \right]$$

$$= \gamma^2 \left[(1 - v^2)\Delta x^2 + (v^2 - 1)\Delta t^2 \right].$$

$$= \frac{1 - v^2}{1 - v^2} (\Delta x^2 - \Delta t^2)$$

$$= \Delta x^2 - \Delta t^2.$$

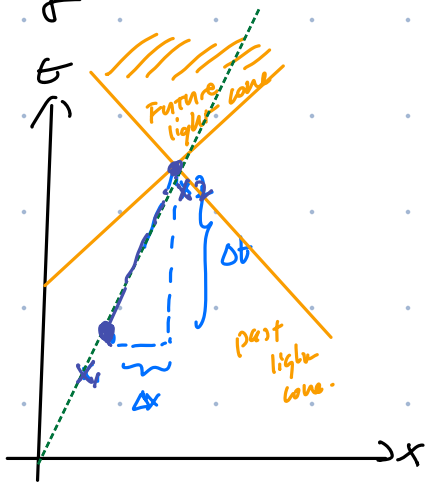
↳ the invariant

- The magnitude of a vector under rotation is invariant.

There are 3 classes of events

① $(\Delta t)^2 = (\Delta x)^2$ "light-like"

↳ there is a light ray connecting 2 events.



② $(\Delta t)^2 > (\Delta x)^2$ "time-like"

$\Delta t > 0$ by assumption.

$$\Delta t' = \gamma(\Delta t - v\Delta x) > 0$$

$$\downarrow$$

$|v| < 1 \quad |\Delta t| > |\Delta x|$

$\Delta x < \Delta t$

⇒causally connected.

event 2 is later than event 1.

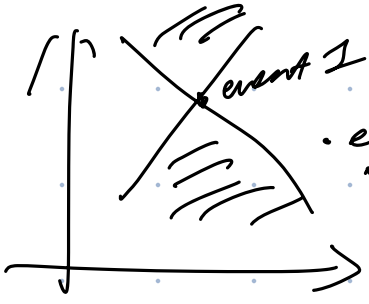
There exists a coordinate system such that these two events happen at the same place.

③ $(\Delta t)^2 < (\Delta x)^2$ "space-like"

$\Delta t' = \gamma(\Delta t - v\Delta x)$ → could be $> 0, = 0, < 0$.

$$\swarrow \quad \searrow$$

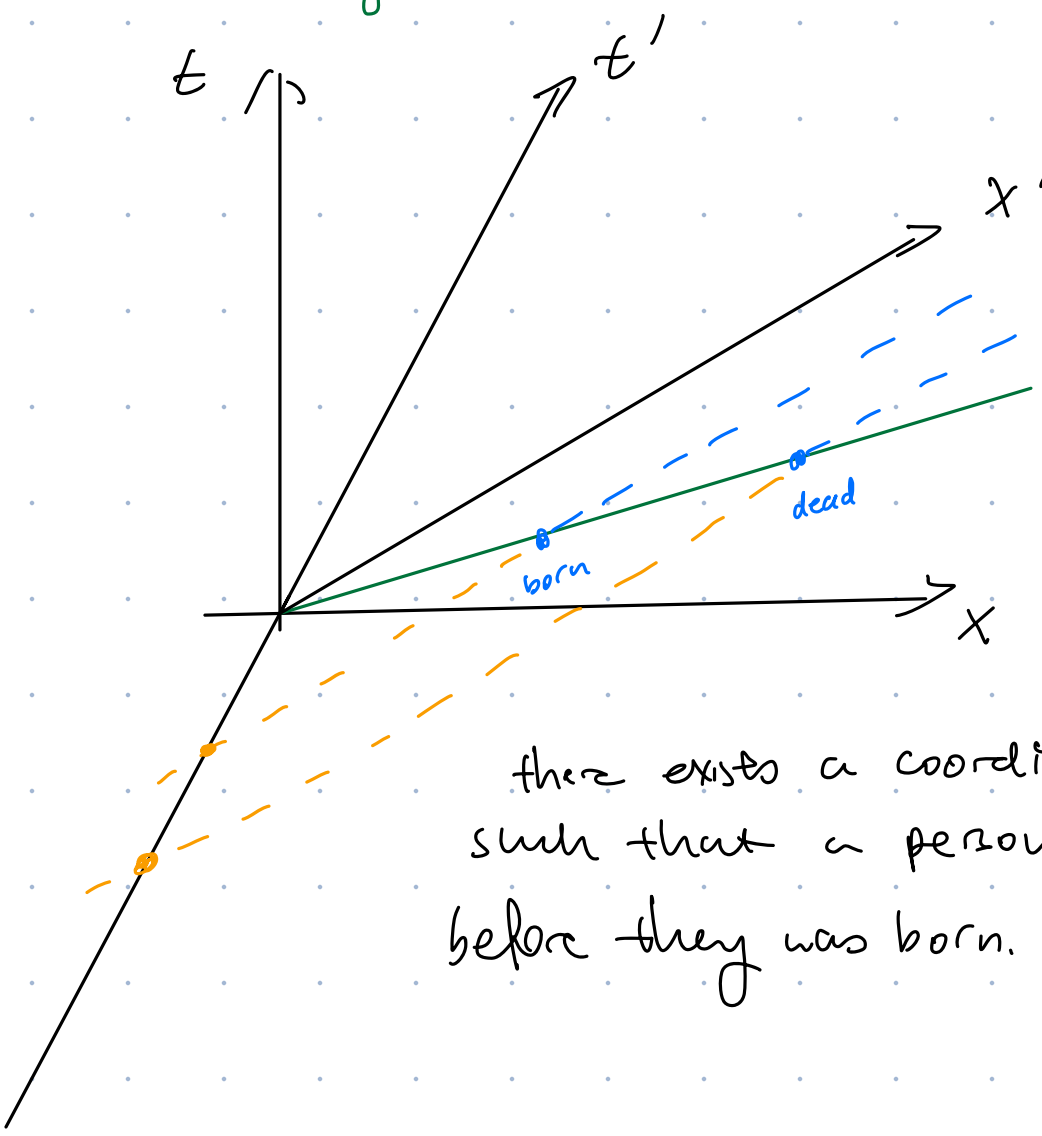
$|v| > 1 \quad |\Delta x| > |\Delta t|$



outside the light cone of 2

∃ a coordinate system where the events are simultaneous.

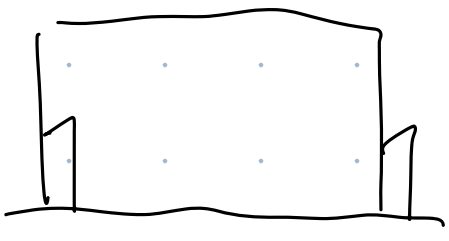
↳ An example of why spacelike events could not be causally correlated.



there exists a coordinate such that a person is deceased before they was born.

The Barn Paradox

stick with length L.



$L > L\sqrt{1-v^2} = B.$

- Conflict on the simultaneity of events.

$$\begin{cases} x_L = 0 \\ t_L = t \end{cases} \qquad \begin{cases} x_R = B \\ t_R = t. \end{cases}$$

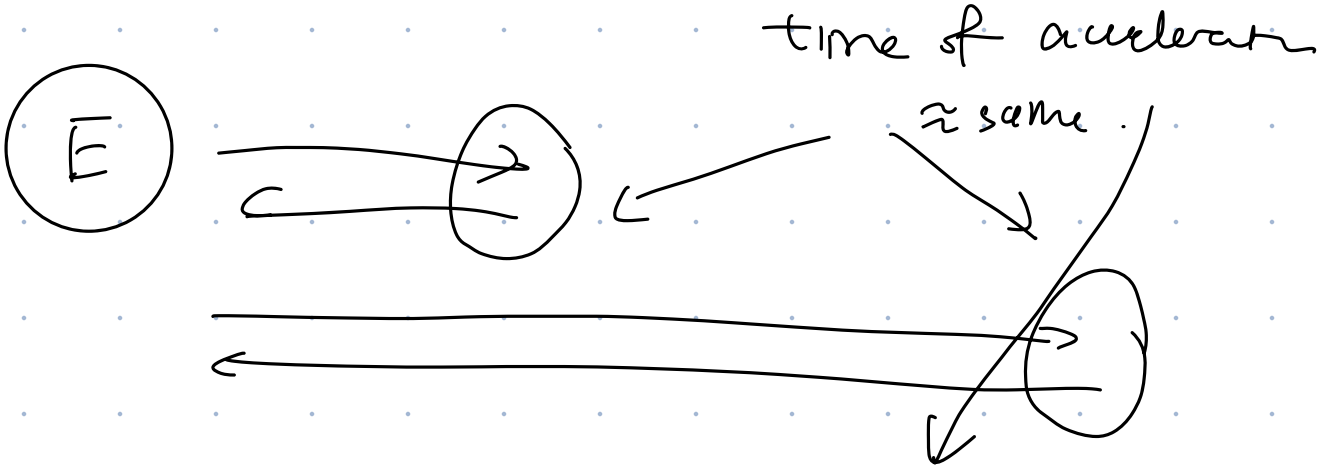
↓ Lorentz tranformation.

$$t'_L = \gamma t \quad , \quad t'_R = \gamma(t - VB)$$

Right door closed before the left door.
from the perspective of the pole.

symmetry is broken.

It is not the acceleration that makes the twin younger.



does not explain the difference in their age.