

Binary Numbers

• Binary & Decimal Conversion

Method 1: Find largest power of 2, subtract & repeat

$$53_{10} = 32 \times 1 + 16 \times 1 + 4 \times 1 + 1 \times 1 \Rightarrow \frac{1}{2^5} \frac{1}{2^4} \frac{0}{2^3} \frac{1}{2^2} \frac{0}{2^1} \frac{1}{2^0}$$

Method 2: Repeatedly divide by 2

$$53/2 = 26 \text{ mod } 1$$

$$26/2 = 13 \text{ mod } 0$$

$$13/2 = 6 \text{ mod } 1$$

$$6/2 = 3 \text{ mod } 0$$

$$3/2 = 1 \text{ mod } 1$$

$$1/2 = 0 \text{ mod } 1$$

$$\Rightarrow 110101$$

• Binary Values & Range

N bit binary #

- ① 2^N values
- ② $[0, 2^N - 1]$

For example: 3-bit binary

$$\begin{cases} 2^3 = 8 \text{ possible val.} \\ \text{range } [0, 7] \end{cases}$$

Hexadecimal Numbers

Hex Digit	Decimal Equivalent	Binary Equivalent
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Conversions:

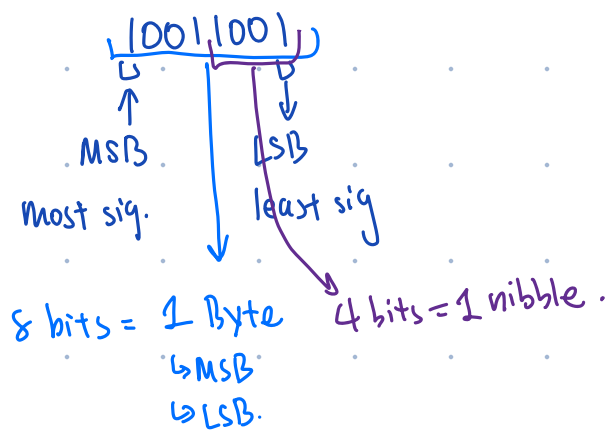
To binary

$$4AF_{16} \text{ (or } 0x4AF) \\ \downarrow \quad \downarrow \quad \downarrow \\ \underline{0100} \quad \underline{1010} \quad \underline{1111}_2$$

To decimal

$$\begin{aligned} &= 4 \times 16^2 + A \times 16^1 + F \times 16^0 \\ &= 4 \times 256 + 10 \times 16 + 15 \\ &= 1024 + 160 + 15 \\ &= 1199_{10} \end{aligned}$$

Bits, Bytes, and Nibbles



Large Powers of Two

$$\begin{aligned} 2^{10} &= 1 \text{ kilo} \approx 10^3 \\ 2^{20} &= 1 \text{ mega} \approx 10^6 \\ 2^{30} &= 1 \text{ giga} \approx 10^9 \\ 2^{40} &= 1 \text{ tera} \approx 10^{12} \\ 2^{50} &= 1 \text{ peta} \approx 10^{15} \\ 2^{60} &= 1 \text{ exa} \approx 10^{18} \end{aligned}$$

Overflow

$$\begin{array}{r} 11 \\ 1011 \\ + 0110 \\ \hline 10001 \\ \text{↑ overflow.} \end{array}$$

Two's Complement Numbers

$$\begin{array}{cccc} 1 & 0 & 1 & 1 \\ \hline -2^3 & 2^2 & 2^1 & 2^0 \\ \hline \end{array}$$

Value = $-2^3 + 2^1 + 2^0$
 $= -8 + 2 + 1$
 $= -5$

the -2^{N-1} place

range $[-2^{N-1}, (2^{N-1}-1)]$

When adding 2 numbers together ignore the overflow.

Wfs $-5 + (-4) = -9$

$$\begin{array}{r} 1 \\ 1011 \\ + 1100 \\ \hline 1111 \\ \hline \end{array}$$

$-2^3 \quad -2^2 \quad -2^1 \quad -2^0$
 $-8 \quad -4 \quad -2 \quad -1$
 $-8 \quad -4 \quad -2 \quad -1$
 $-8 \quad -4 \quad -2 \quad -1$

Try $-5 + (-1) = -6$.

$$\begin{array}{r} 1111 \\ 1011 \\ + 1111 \\ \hline 10010 \\ \hline \end{array}$$

$-8 + 4 + 2 + 1$
 $-8 + 2 = -6 \checkmark$

this is problematic because -9 is out of the range of 4-bits range.