Relativity Lecture 4.

- Relativistic doppler effect

there is a preferred.

$V_r, V_e > 0$

$e \to$ emitter
$r \to$ receiver

$$f_r = \frac{C_s - V_r}{C_s + V_e} f_e$$

speed of sound

$$t = \gamma(t' + vx)$$
$$x = \gamma(x' + vx)$$

2 pings $\quad p_1'(l, t)$
$\quad p_2'(l, t + T_e)$

$$p_1 = (\gamma(l + vt'),\ \gamma(t' + vl))$$
$$p_2 = (\gamma(l + v(t' + T_e)),\ \gamma(t' + T_e + vl))$$

time it takes for light to travel, $c = 1$

p_1 received at

$$\gamma(t + vl) + \underbrace{\gamma(l + vt)/c}$$

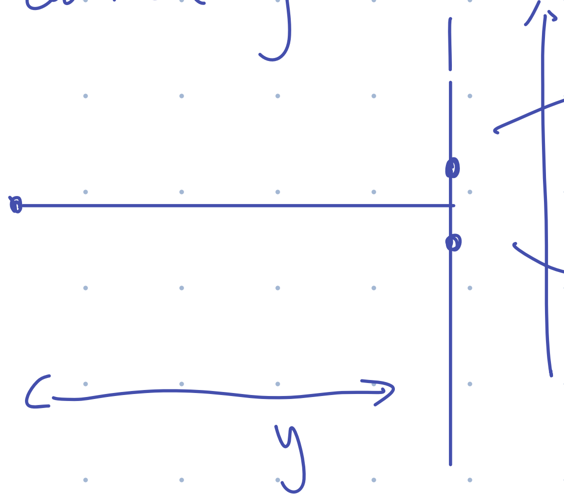
$$p_2 \quad \gamma(t + T_e + vl) + \gamma(l + v(t + T_e))/c$$

$$T_r = P_2 - P_1$$
$$= \gamma T_e + \gamma v T_e$$
$$= \gamma(1 + v) T_e = \frac{1 + v}{\sqrt{1 - v^2}} T_e$$

$$= \frac{\cancel{\sqrt{1+v}}\,\sqrt{1+v}}{\cancel{\sqrt{1+v}}\,\sqrt{1-v}}\, T_e.$$

$$f_r = \frac{\sqrt{1-v}}{\sqrt{1+v}}\, f_e.$$ $\to$ longitudinal doppler effect.

In the perpendicular direction, the doppler effect is caused by time dilation



x-dir

$P_2' = (l, y, t+T_e)$

x

x

$P_1' = (l, y, t)$

$$\begin{cases} P_2 = (\overset{\text{smt.}}{\cancel{l}},\ y',\ \gamma(t+T_e+vl)) \\ P_1 = (\overset{\text{smt.}}{\cancel{l}},\ y',\ \gamma(t+vl)) \end{cases}$$

$T_r = \gamma T_e$

$$\boxed{f_r = \sqrt{1-v^2}\, f_e}$$

$$\cos(\vec{k}\cdot\vec{r} + \omega t)$$

$$r = \frac{\vec{k}\cdot\vec{r}}{|\vec{k}|} \qquad \cos(kr + \omega t)$$

$$= \cos k(r + \underbrace{\tfrac{\omega}{k}}_{\text{wave velocity}} t)$$

moves to the left in the $\frac{\vec{k}}{|\vec{k}|}$ direction.

$$\boxed{\vec{k} = \omega(\cos\theta_e, \sin\theta_e, 0)}$$

For light $\frac{\omega}{k} = c = 1$

so $|\vec{k}| = \omega$.

emitted wave $\quad r = (x, y, 0)$

$$\cos[\omega_e[\cos\theta_e x' + \sin\theta_e y' + t']]$$

$$= \cos[\omega_e[\gamma(x - vt)\cos\theta_e + y\sin\theta_e + \gamma(t - vx)]]$$

$$= \cos[\omega_e[\gamma(\cos\theta_e - v)x + y\sin\theta_e + \gamma(1 - v\cos\theta_e)t]]$$

$$\omega_r = \gamma(1 - v\cos\theta_e)t\,\omega_e.$$

$$f_r = f_e\,\gamma(1 - v\underbrace{\cos\theta_e}_{})t.$$

Knowing the angle is not very useful.

we want to trade for $\theta_{received}$.

$$\cos\theta_r = \frac{\gamma(\cos\theta_e - v)}{\sqrt{\gamma^2(\cos\theta_e - v)^2 + \sin^2\theta_e}}$$

$$= \frac{\frac{1}{\sqrt{1-v^2}}(\cos\theta_e - v)}{\sqrt{\frac{1}{1-v^2}(\cos\theta_e - v)^2 + \sin^2\theta_e}}$$

$$= \frac{\cos\theta_e - v}{\sqrt{(\cos\theta_e - v)^2 + (1-v^2)\sin^2\theta_e}}$$

$$= \frac{\cos\theta_e - v}{\sqrt{\cos^2\theta_2 + v^2 - 2v\cos\theta_e + \sin^2\theta_e - v^2\sin^2\theta_e}}$$

$$= \frac{\cos\theta_e - v}{\sqrt{v^2+1-v^2\sin^2\theta_e - 2v\cos\theta_e}}$$

$$= \frac{\cos\theta_e - v}{\sqrt{1+v^2\cos^2\theta_e - 2v\cos\theta_2}}$$

$$= \frac{\cos\theta_e - v}{\sqrt{(1-v\cos\theta_e)^2}}$$

solve for $\cos\theta_e$

$$= \frac{\cos\theta_e - v}{1 - v\cos\theta_e} = \cos\theta_r$$

$$(1-v\cos\theta_e)(1+v\cos\theta_r)$$

$= \text{— — —}$ plug in

$= 1-v^2$

$$1 - v\cos\theta_e = \frac{1-v^2}{1+v\cos\theta_r}$$

plug into

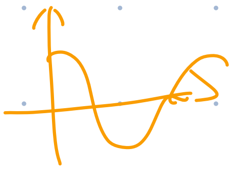
$$f_r = f_e \gamma (1 - v\cos\theta_e)$$

$$= f_e \frac{1}{\sqrt{1-v^2}} \frac{1-v^2}{1+v\cos\theta_r}$$

$$= f_e \sqrt{1-v^2} \frac{1}{1+v\cos\theta_r}$$

if $\cos\theta_r = 1 \Rightarrow f_r = f_e \sqrt{1-v^2} \frac{1}{\sqrt{1+v}\sqrt{1+v}}$

$\Downarrow$

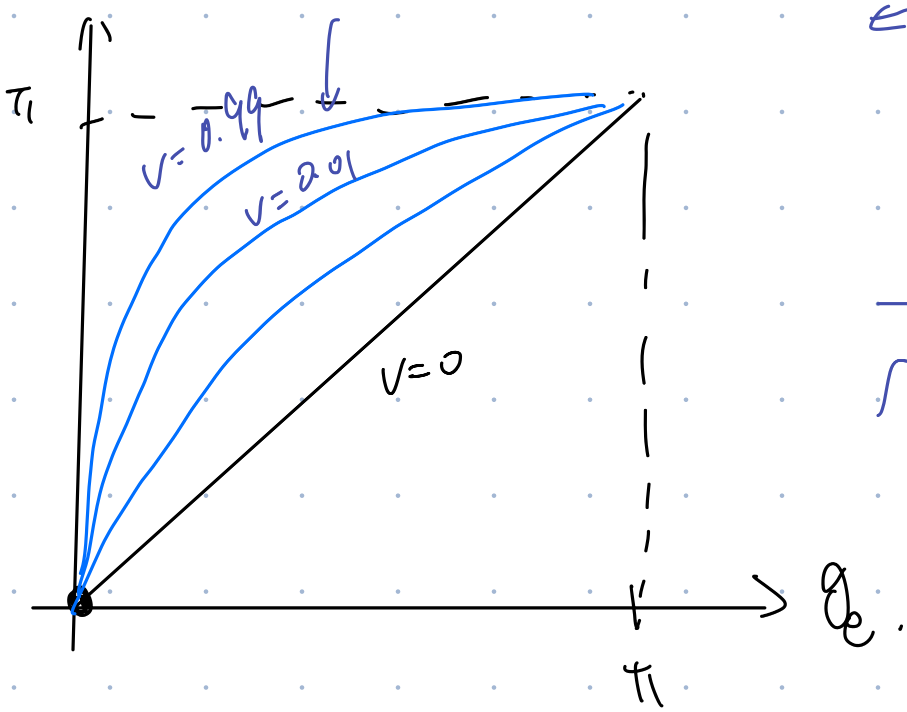


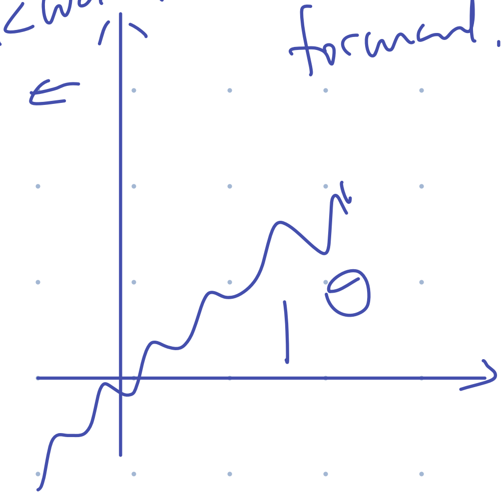
$\theta = 0$

$$= f_e \frac{\sqrt{1-v}\,\cancel{\sqrt{1+v}}}{\sqrt{1+v}\,\cancel{\sqrt{1+v}}}$$

$$= \frac{\sqrt{1-v}}{\sqrt{1+v}} f_e$$

corresponds to the longitudinal effect.

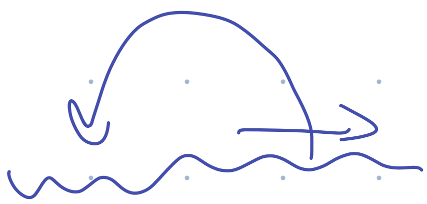




$\theta = 0$: backward

$\theta = \pi$: forward

$$\frac{\cos\theta_e - V}{1 - V\cos\theta_e}$$



Relativistic Momentum

$$\vec{p} = m\vec{v}\, f(\vec{v}^{\,2})$$

depend on $\vec{v}^{\,2}$ because of rotational symmetry.

$$\vec{r}\,' = \vec{r}_0\,' + (\omega_x', \omega_y', \omega_z')t'$$

$$\frac{d\vec{r}\,'}{dt'} = \vec{\omega}\,' = (\omega_x', \omega_y', \omega_z')$$

$$\omega_x = \frac{dx}{dt} = \frac{d(\gamma(x'+vt'))}{dt'} \frac{dt'}{dt} = \frac{1}{\frac{dt}{dt'}} \frac{d\gamma(x'+vt')}{dt}$$

$$= \frac{\gamma(\omega_x'+v)}{\gamma(1+v\omega x')} = \frac{v+\omega_x'}{1+v\omega x'}$$

$$\omega_y = \frac{dy}{dt} = \frac{\frac{dy'}{dt'}}{\frac{dt}{dt'}} = \frac{\omega_y'}{\gamma(1+v\omega_x')}$$

$$\omega_z = \frac{\omega_z'}{\gamma(1+v\omega_z')}$$