Relativity Lecture 2 Notes

- A consequence of the Lorentz transformations is that c is the speed limit.

  Suppose observer b is moving with velocity v w.r.t observer a.
  Observer b shoots a ball with velocity w' as measured from her p.o.v.

  Observer a would see the ball traveling at velocity w.

  Set $v=c$, then

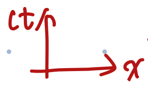
  $$w = \frac{v+w'}{1+\frac{vw'}{c^2}} = \frac{c+w'}{1+\frac{\cancel{c}w'}{c^{\cancel{2}}}} = \frac{c\,\cancel{(c+w')}}{\cancel{c+w'}} = c$$

  ★ a cannot observe a velocity faster than c.

- The Reason we want to unify units in time and space is because it makes our lives easier.

  Problem 1 in the HW shows that working with different units is a pain and ~~that~~ it also costs us some nice mathematical properties, such as $R^T = R^{-1}$ (or, inner product is no longer preserved under rotational transformation).

  prob: m & s are the same units?

  ⇒ a bit understable.

  We do this by setting $c=1$.

  So we can turn our original Lorentz transformation equations from

  please derive this again.

$$\begin{cases} x = \dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}}\left(x'+vt'\right) \\ t = \dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}}\left(t'+\frac{v}{c^2}x'\right) \end{cases} \Rightarrow \begin{cases} x = \dfrac{1}{\sqrt{1-v^2}}\left(x'+vt'\right) \\ t = \dfrac{1}{\sqrt{1-v^2}}\left(t'+vx'\right) \end{cases}$$

where v is now dimensionless

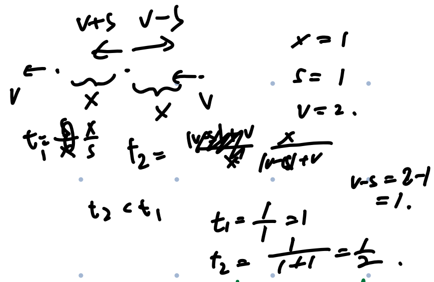
We abbreviate $\gamma = \frac{1}{\sqrt{1-v^2}}$, we also represent the Lorentz transformation with the matrix Λ.

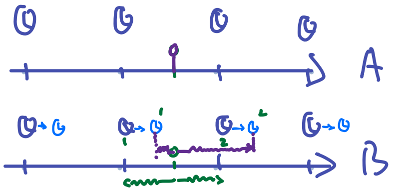
$$\begin{pmatrix} x \\ t \end{pmatrix} = \Lambda \begin{pmatrix} x' \\ t' \end{pmatrix} \quad \text{where } \Lambda = \begin{pmatrix} \gamma & \gamma v \\ \gamma v & \gamma \end{pmatrix} = \begin{pmatrix} \cosh\psi & \sinh\psi \\ \sinh\psi & \cosh\psi \end{pmatrix}$$

"It acts like a hyperbolic rotation in Minkowski spacetime"

there is some correlation with hyperbolic functions. why?

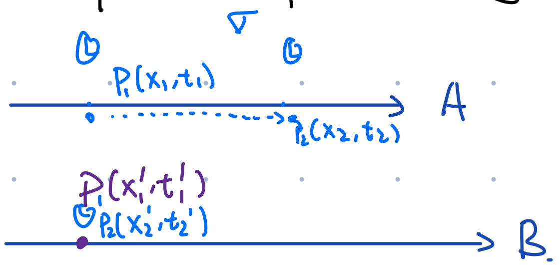
- Inertial Observers who are moving w.r.t each other cannot agree on time synchronization.





B believes light reached clocks 1 & 2 at the same time ⇒ 1 & 2 synchronized.
A believes that light reaches 1 first ⇒ clocks are not synchronized.

- A consequence of this disagreement is time dilation.



Suppose B is moving with velocity v w.r.t A,

Observer B sees a point that's located at $P_1(0,0)$. The point remains stationary and obtains the coordinate $P_2(0, T_{moving})$ after T_{moving} seconds have past.

Observer A sees the point initially at $P_1(0,0)$, then at $P_2(x, T_{stationary})$

By the Lorentz transformation, $t = \frac{1}{\sqrt{1-v^2}}(t' + vx')$

$$T_{stationary} = \frac{1}{\sqrt{1-v^2}} T_{moving}$$

Since $v < 1$, $T_{stationary} > T_{moving}$.

The time interval between these two events is large for the stationary observer than for the moving observer.

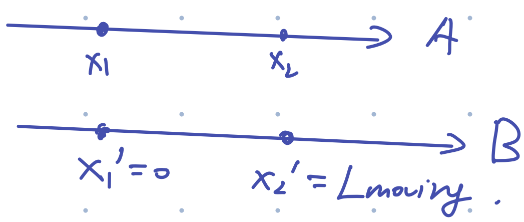
This is what's known as time dilation.

- Time Dilation explains why we can observe muons.
  - Cosmic rays hit the upper atmosphere, creating muons that rush towards Earth at almost the speed of light (0.99c).
  - A muon's rest lifetime is only ~2.2 μs. Classically, it could only travel 660m before decaying.
  - However, to a stationary observer, the decay time is:

$$t_{stat} = \frac{t_{moving}}{\sqrt{1-v^2}} = \frac{2.2\mu s}{\sqrt{1-0.998^2}} = 34.8\ \mu s$$

distance traveled: $34.8 \times 10^{-6} \times 3 \times 10^8 \times 0.99 = 10419$ m

- Another consequence of the Lorentz transformations is length contraction.



In the stationary observer's perspective:

$(x_1, t_1) \rightarrow$ left of the stick.

$(x_2, t_2) \rightarrow$ right of the stick.

For length measurement to make sense, we must measure the left and the right side at the same time. so $t_1 = t_2$.

$t_1 = t_2$

$$\gamma(t_1' + v x_1') = \gamma(t_2' + v x_2')$$

$$t_1' = t_2' + v L_{moving}$$

$$t_1' - t_2' = v L_{moving}$$

cannot agree on simultaneity.

$$L_{stationary} = x_2 - x_1$$

$$= \gamma[x_2' + v t_2'] - \gamma[x_1' + v t_1']$$

$$= \gamma[(x_2' - x_1') + v(t_2' - t_1')]$$

$$= \gamma[L_{moving} - v^2 L_{moving}]$$

$$= \frac{1-v^2}{\sqrt{1-v^2}} L_{moving} = \sqrt{1-v^2}\, L_{moving}$$

The stationary observer sees a length distance than the moving observer.

↳ reminded me of the ladder paradox.