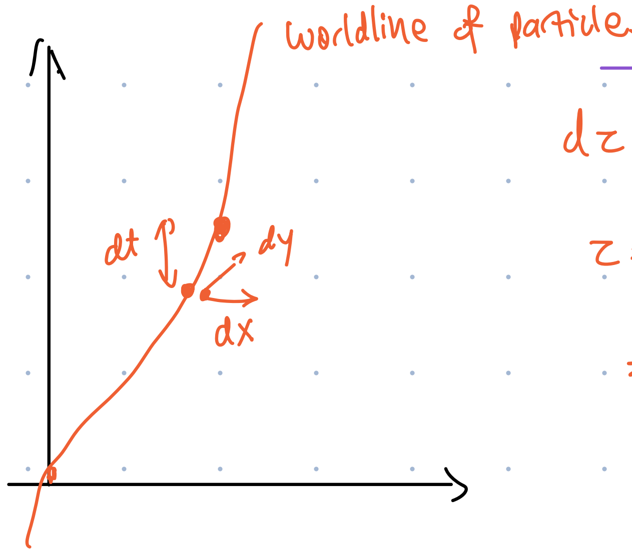
Relativistic Newton's laws.

invariant: same in every coordinate system.

conserved: does not evolve in time.



Proper time:

$$d\tau = \sqrt{dt^2 - d\vec{r}^{\,2}}$$

this is an invariant.

$$\tau = \int d\tau$$

$$= \int \sqrt{dt^2 - d\vec{r}^{\,2}}$$

$$= \int dt \sqrt{1 - \left(\frac{d\vec{r}}{dt}\right)^2}$$

the particle's velocity as seen in its own frame.

$$= \int dt \sqrt{1 - \vec{\omega}^{\,2}}$$

Proper velocity:

$$\vec{u} \equiv \frac{d\vec{r}}{d\tau}, \quad \vec{\omega} \equiv \frac{d\vec{r}}{dt}.$$

$$u^0 = \frac{dt}{d\tau} = \frac{1}{\sqrt{1-\vec{\omega}^{\,2}}}$$

zeroth component

In the object's frame:

$$\vec{\omega} = 0, \quad \tau = t$$

$$\vec{u} = \frac{d\vec{r}}{d\tau} = \frac{\frac{d\vec{r}}{dt}}{\frac{d\tau}{dt}} = \frac{\vec{\omega}}{\sqrt{1-\vec{\omega}^{\,2}}}$$

$$= \gamma \vec{\omega}.$$

$$E = \frac{m}{\sqrt{1-\vec{\omega}^{\,2}}}, \quad \vec{p} = \frac{m\vec{\omega}}{\sqrt{1-\vec{\omega}^{\,2}}} \implies E = mu^0, \quad \vec{p} = m\vec{u}$$

Transformation:

$$\begin{cases} u^{0\prime} = \gamma(u^0 - v u_x) \\ u_x' = \gamma(u_x - v u^0) \\ u_y' = u_y \\ u_z' = u_z \end{cases}$$

since $d\tau$ invariant
u^0 transforms like dt.
$\vec{u}$ transforms like $d\vec{r}$.

Similarly

$$\begin{cases} E' = \gamma(E - v p_x) \\ p_x' = \gamma(p_x - vE) \\ p_y' = p_y \\ p_z' = p_z \end{cases}$$

or

$$\begin{cases} E = \gamma(E' + v p_x') \\ p_x = \gamma(p_x' + vE') \\ p_y = p_y' \\ p_z = p_z' \end{cases}$$

Newton's Laws

$$p_{N.R} = m\vec{w}$$

$$F_{N.R} = \frac{dp_{N.R.}}{dt}, \quad \frac{dE}{dt} = \vec{F}\cdot\vec{w}$$

$$F = \frac{d\vec{p}}{dt}$$

(relativistic force)

or

$$f = \frac{d\vec{p}}{d\tau}$$

(4-force)

$$\begin{cases} t = \gamma(t' + v x') \\ x = \gamma(x' + v t') \end{cases}$$

$x^2 - t^2$ invar.

$$E^2 - \vec{p}^{\,2} = E'^2 - \vec{p}'^{\,2} = m^2\left(\frac{1}{1-\vec{w}^{\,2}} - \frac{\vec{w}^{\,2}}{1-\vec{w}^{\,2}}\right) = m^2.$$

$$m^2(u^0)^2 - m^2(\vec{u})^2 = m^2$$

$$\boxed{(u^0)^2 - (\vec{u})^2 = 1}$$

Finding the relativistic force:

$$F_x' = \frac{dp_x'}{dt'} = \frac{d(\gamma p_x - vE)}{dt'} = \frac{\frac{d(\gamma p_x - vE)}{dt}}{\frac{dt'}{dt}}$$

$t' = \gamma(t - v\vec{x})$

$dt' = \gamma(dt - v\vec{w})$

$$= \frac{\gamma\left[\left(\frac{dp_x}{dt}\right) - v\frac{dE}{dt}\right]}{\gamma[1 - vw_x]} = \frac{F_x - v\frac{dE}{dt}}{1 - vw_x}$$

$$\vec{E}^2 - \vec{p}^2 = m^2.$$

$$2E\frac{dE}{dt} - 2\vec{p}\frac{d\vec{p}}{dt} = 0$$

$$E\frac{dE}{dt} - \vec{p}\,\cancel{\frac{d\vec{p}}{dt}}\,F_x = 0$$

$$E\frac{dE}{dt} = \vec{p}\frac{d\vec{p}}{dt}$$

$$\frac{dE}{dt} = \frac{\vec{p}}{E}\vec{F}_x$$

$$\frac{dE}{dt} = \vec{\omega}\cdot\vec{F}_x$$

$$E = \frac{m}{\sqrt{1-\omega^2}}$$

$$\vec{p} = \frac{m\vec{\omega}}{\sqrt{1-\vec{\omega}^2}}$$

$$\frac{\vec{p}}{E} = \vec{\omega}$$

$$F_x' = \frac{F_x - v\vec{\omega}\cdot\vec{F}}{1 - v\omega_x}$$

$$F_y' = \frac{dp_y'}{dt'} = \frac{\frac{dp}{dt}}{\frac{dt'}{dt}} = \frac{F_y}{\gamma(1 - v\omega_x)}$$

$$F_z' = \frac{F_z}{\gamma(1 - v\omega_x)}$$

Understanding what that is by setting ω to 0.

$$\vec{F}_x' = \frac{F_x - v\vec{F}\vec{\omega}}{1 - vW_x} \quad \& \quad \boxed{\vec{\omega} = 0}$$

$$\Downarrow$$

$$\begin{cases} \bar{F}_x = F_{NR,x} \\ F_y = \dfrac{F_{NR,y}}{\gamma} \\ F_z = \dfrac{\bar{F}_{NR,z}}{\gamma}. \end{cases}$$

$$\vec{f} = \gamma \vec{F}$$

$$f^0 = m \frac{du^0}{d\tau}$$

$$\vec{f} = m \frac{d\vec{u}}{d\tau}.$$

$$(u^0)^2 - (\vec{u})^2 = 1$$

$$2u^0 \frac{du^0}{d\tau} - 2\vec{u}\frac{d\vec{u}}{d\tau} = 0$$

$$u^0 f^0 - \vec{u}\vec{f} = 0$$

$\downarrow$

they are orthogonal.

$$w_x' = \frac{w_x + v}{1 + v w_x}$$

$$u_x' = \gamma(u_x - \cancel{v} u^0)$$

$\rightarrow$ a certain density ρ at rest.

$$\vec{j} = \rho \cdot \vec{w}$$

$\xrightarrow{v}$ see: $\rho \uparrow$. # particles are const.

length contraction

↳ space shrinks.

$$\rho = \frac{\rho_0}{\sqrt{1 - \vec{w}^2}}$$

$$\rho \cdot \vec{w} : \quad \vec{j} \equiv \frac{\rho_0 \vec{w}}{\sqrt{1 - \vec{w}^2}}$$

$$\rho = \rho_0 u^0$$

$$\vec{\rho} = \rho_0 \vec{u}$$