PHYS 3450 lecture 4

Goals: reconcile continuous nature $|\psi\rangle$. with discrete measurement results.

Standard repre. $|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2} e^{i\varphi}|1\rangle$

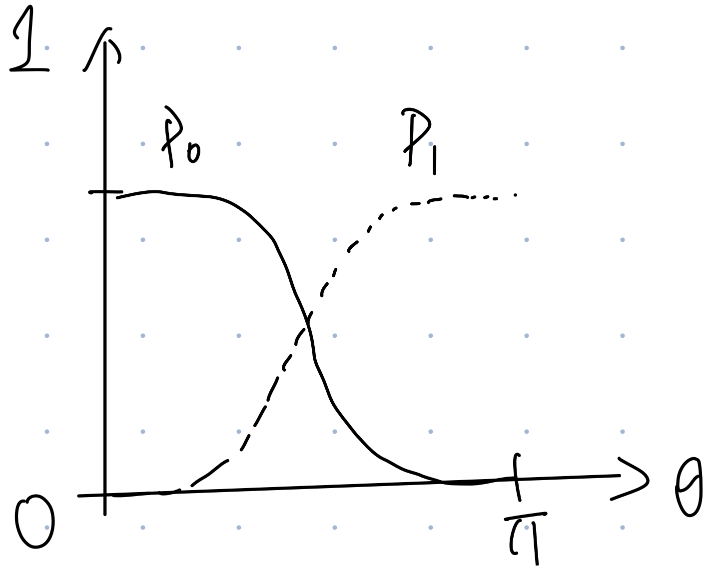
"ray": equivalent class in Hilbert Space.

Sunday. Prof might be there in person.

$\hat{n} = (\hat{n}_x, \hat{n}_y, \hat{n}_z) = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$

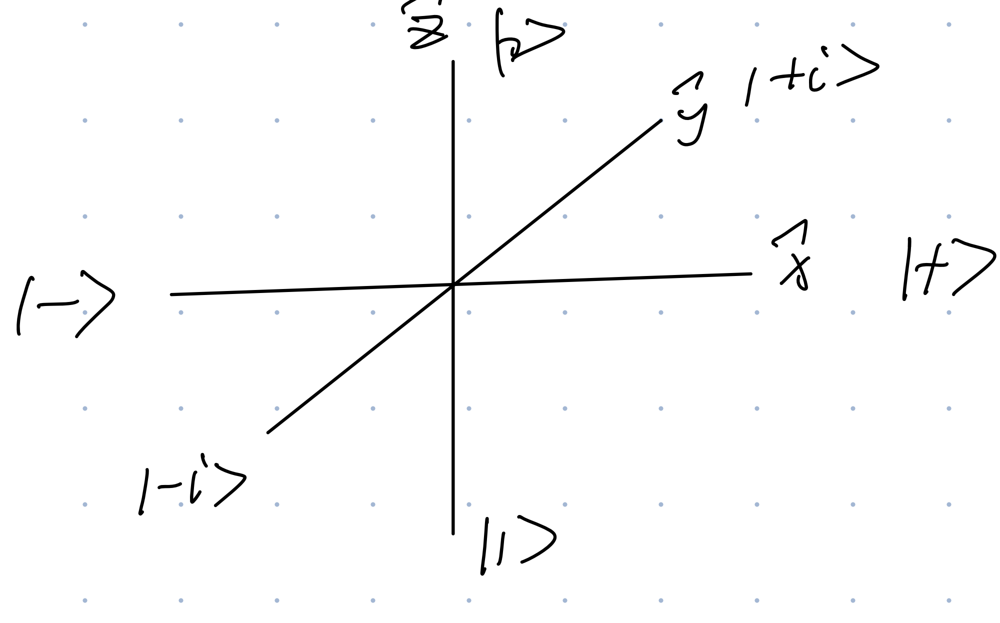
$|+\hat{n}\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2} e^{i\varphi}|1\rangle$

probability of $\begin{cases} \text{measuring } |0\rangle,\ P_0 = \left|\cos\frac{\theta}{2}\right|^2 = \frac{1-\cos\theta}{2} \\ \text{measuring } |1\rangle,\ P_1 = \left|\sin\frac{\theta}{2}\right|^2 = \frac{1+\cos\theta}{2} \end{cases}$



we can produce the same state every time, but our measurements random.

Cardinal points on the Bloch sphere.



$|\hat{x}\rangle = |+\rangle$

$= \cos\left[\frac{\pi/2}{2}\right]|0\rangle + \sin\left[\frac{\pi/2}{2}\right]|1\rangle$

$= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$|-\rangle$

$= \cos\left[\frac{\pi/2}{2}\right]|0\rangle + \sin\left[\frac{\pi/2}{2}\right]e^{i\pi}|1\rangle$

$= \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

Colinear vec. on the bloch sphere are orthogonal.

$e^{i(\frac{\pi}{2})}$

$|\hat{y}\rangle = |+i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$

$|-\hat{y}\rangle = |-i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$

$\langle -i|+i\rangle = \frac{1}{\sqrt{2}}(\langle 0| + i\langle 1|)\frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$

$*$

$= \frac{1}{2}(\langle 0|0\rangle + i^2\langle 1|1\rangle + \cancel{\ldots})$

$= 0$

$\Gamma_{n|n\rangle} = (|+\hat{n}\rangle\langle +\hat{n}| + |-\hat{n}\rangle\langle -\hat{n}|)$ $[\langle n|x\rangle\sigma^x + \langle n|y\rangle\sigma^y + \langle n|z\rangle\sigma^z]$

$=$

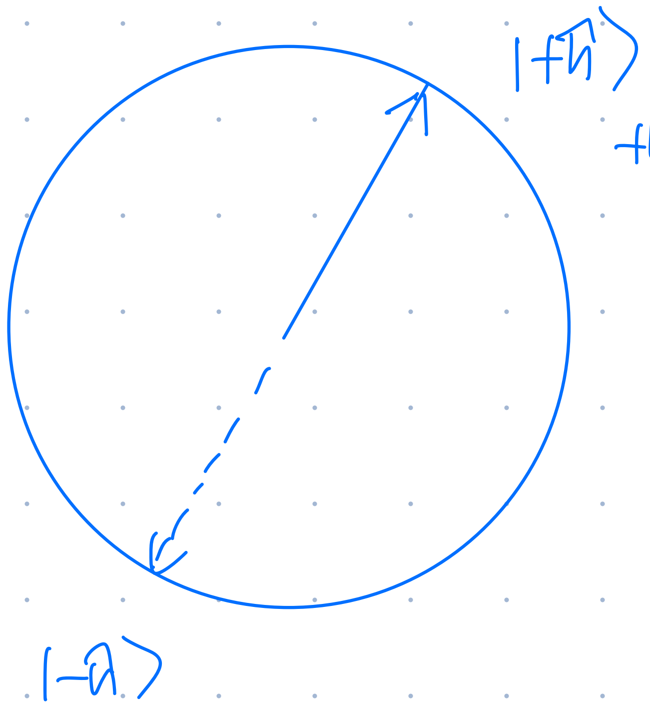
Next Big Questions

① Given a computational basis state, how do I transform/rotate it to $|+\hat{n}\rangle$?

② How do I change basis $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
$= \alpha'|+\hat{n}\rangle + \beta'|-\hat{n}\rangle$.

③ How do I make measurements in the basis $|+\hat{n}\rangle, |-\hat{n}\rangle$?

Do not confuse $|-\hat{n}\rangle$ with $-|+\hat{n}\rangle$



the minus sign is just a global phase of $|+\hat{n}\rangle$.

Goal #1: express in any basis for ordinary 2D cartesian coordinates

$\hat{x} = (1,0)$, $\hat{y} = (0,1)$

$\vec{V} = (x,y) = x\hat{x} + y\hat{y}$.

$\vec{V}\cdot\hat{x} = (x,y)\cdot(1,0) = x$

$\vec{V}\cdot\hat{y} = (x,y)(0,1) = y$

$\vec{V} = \hat{x}(\hat{x}\cdot\vec{V}) + \hat{y}(\hat{y}\cdot\vec{V})$

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

order? want to keep $|\psi\rangle$ the same.

$\langle 0|\psi\rangle = \langle 0|\alpha|0\rangle + \langle 0|\beta|1\rangle$

$= \alpha$

$\langle 1|\psi\rangle = \beta$

Thus $|\psi\rangle = |0\rangle\langle 0|\psi\rangle + |1\rangle\langle 1|\psi\rangle$

$= |+\hat{n}\rangle\langle +\hat{n}|\psi\rangle + |-\hat{n}\rangle\langle -\hat{n}|\psi\rangle$

$$|\psi\rangle = |+\hat{n}\rangle\langle+\hat{n}|\,(\alpha|0\rangle + \beta|1\rangle) + |-\hat{n}\rangle\langle-\hat{n}|\,(\alpha|0\rangle+\beta|1\rangle)$$

$$= (\alpha|+\hat{n}\rangle\langle+\hat{n}|0\rangle + \beta|+\hat{n}\rangle\langle+\hat{n}|1\rangle) +$$

$$(\alpha|-\hat{n}\rangle\langle-\hat{n}|0\rangle + \beta|-\hat{n}\rangle\langle-\hat{n}|1\rangle)$$

$$= (\underbrace{\alpha\langle+\hat{n}|0\rangle + \beta\langle+\hat{n}|1\rangle}_{\alpha'})|+\hat{n}\rangle +$$

$$(\underbrace{\alpha\langle-\hat{n}|0\rangle + \beta\langle-\hat{n}|1\rangle)|-\hat{n}\rangle}_{\beta'}$$

$$\begin{pmatrix}\alpha' \\ \beta'\end{pmatrix} = \underbrace{\begin{pmatrix}\langle+\hat{n}|0\rangle & \langle+\hat{n}|1\rangle \\ \langle-\hat{n}|0\rangle & \langle-\hat{n}|1\rangle\end{pmatrix}}_{U_y\text{; rotation matrix}}\begin{pmatrix}\alpha \\ \beta\end{pmatrix}$$

$$|+\hat{n}\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

$$|-\hat{n}\rangle = \sin\frac{\theta}{2}|0\rangle - e^{i\varphi}\cos\frac{\theta}{2}|1\rangle$$

Plug into the equation:

$$\langle +\hat{n}|0\rangle = \cos\frac{\theta}{2}$$

$$\langle +\hat{n}|1\rangle = e^{-i\varphi}\sin\frac{\theta}{2}$$

$$\langle -\hat{n}|0\rangle = \sin\frac{\theta}{2}$$

$$\langle -\hat{n}|1\rangle = -e^{-i\varphi}\cos\frac{\theta}{2}$$

$$\begin{pmatrix} \cos\frac{\theta}{2} & e^{-i\varphi}\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & -e^{-i\varphi}\cos\frac{\theta}{2} \end{pmatrix}$$

$U^\dagger U = I \Rightarrow$ isometry

$UU^\dagger = I \Rightarrow$ co-isometry

U is square $\rightarrow U$ is unitary.

Why should the basis be orthogonal?

example: $+\hat{n} = +\hat{x}$.

$\theta = \frac{\pi}{2}, \ \phi = 0.$

$P_{+x} = |\alpha'|^2 \qquad |\alpha'|^2 = |\cos\frac{\pi}{4}\alpha + \sin\frac{\pi}{4}\beta|^2$

$= \frac{1}{2}|\underbrace{\alpha+\beta}|^2$ → constructive interferen

$P_{-x} = |\beta'|^2 = \frac{1}{2}|\underbrace{\alpha-\beta}|^2$

destructive interference

example: $|\psi\rangle = \underbrace{\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)}$

$|+x\rangle$. 100% measuring $|+x\rangle$.

$P_{+\hat{n}} = \langle\psi|+\hat{n}\rangle\langle+\hat{n}|\psi\rangle.$

$P_{-\hat{n}} = \langle\psi|\underbrace{-\hat{n}\rangle\langle-\hat{n}}_{\text{operator}}|\psi\rangle$

expiration f