

Rel. Energy & Momentum:

$$\omega_x = \frac{v + \omega x'}{1 + v\omega x'}$$

$$\omega_y = \frac{1}{\gamma} \frac{\omega y'}{1 + v\omega x'}$$

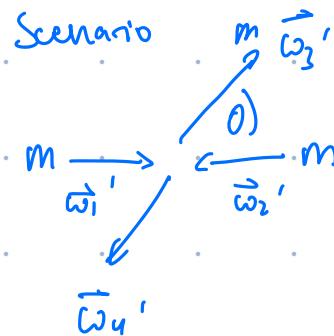
$$\omega_z = \frac{1}{\gamma} \frac{\omega z'}{1 + v\omega x'}$$

$$\vec{p} = m\vec{\omega}f(\vec{\omega})$$

↓
depends on
the magnitude of
velocity.

$$\gamma = \frac{1}{\sqrt{1-v^2}}$$

$$\left(\frac{1}{\gamma}\right)^2 = \left(\frac{1}{\sqrt{1-v^2}}\right)^2$$



$$\vec{\omega}_1' = (\omega_1, 0, 0)$$

$$\vec{\omega}_2' = (-\omega, 0, 0)$$

$$\vec{\omega}_3' = (\omega \cos \theta, \omega \sin \theta, 0)$$

$$\vec{\omega}_4' = (-\omega \cos \theta, -\omega \sin \theta, 0)$$

Another coord system:

$$\vec{\omega}_1 = \left(\frac{v + \omega}{1 + v\omega}, 0, 0 \right)$$

$$\vec{\omega}_2 = \left(\frac{v - \omega}{1 - v\omega}, 0, 0 \right)$$

$$\vec{\omega}_3 = \left(\frac{v + \omega \cos \theta}{1 + v\omega \cos \theta}, \frac{1}{\gamma} \frac{\omega \sin \theta}{1 + v\omega \cos \theta}, 0 \right)$$

$$\vec{\omega}_4 = \left(\frac{v - \omega \cos \theta}{1 - v\omega \cos \theta}, \frac{1}{\gamma} \frac{-\omega \sin \theta}{1 - v\omega \cos \theta}, 0 \right)$$

$$\vec{\omega}_{3,4}^2 = \frac{(v \pm \omega \cos \theta)^2 + \left(\frac{1}{\gamma}\right)^2 (\pm \omega \sin \theta)^2}{(1 \mp v\omega \cos \theta)^2}$$

$$= \frac{v^2 + \omega^2 \cos^2 \theta \mp 2v\omega \cos \theta + \omega^2 \sin^2 \theta - \omega^2 v^2 (1 - \omega^2 \theta)}{1 \mp v\omega \cos \theta}$$

$$= \frac{v^2 + \omega^2 \pm 2v\omega \cos \theta - \omega^2 v^2 + v^2 \omega^2 \cos^2 \theta}{1 \mp v\omega \cos \theta}$$

$$\vec{\omega}_{3,4}^2 = \frac{(1 \pm v\omega \cos \theta)^2 - (1 - v^2)(1 - \omega^2)}{(1 \mp v\omega \cos \theta)^2}$$

$$= 1 - \frac{(1 - v^2)(1 - \omega^2)}{(1 \pm v\omega \cos \theta)^2}$$

f(17x ←)

$$\vec{P}_3 \text{ before} = \vec{P}_3 \text{ after}$$

$$0 = \frac{\omega \sin \theta / \gamma}{1 + v \omega \cos \theta} f \left(1 - \frac{(1-v^2)(1-\omega^2)}{(1+v \omega \cos \theta)^2} \right)$$

$$- \frac{\omega \sin \theta / \gamma}{1 - v \omega \cos \theta} f \left(\dots \right)$$

$$f \left(\dots \right) \frac{1}{1 - v \omega \cos \theta} = f \left(\dots \right) \frac{1}{1 + v \omega \cos \theta}$$

$$\frac{1}{f^2} \left(1 - \frac{(1-v^2)(1-\omega^2)}{(1+v \omega \cos \theta)^2} \right) (1+v \omega \cos \theta)^2$$

$$= \frac{1}{f^2} \left(\dots \right) (1+v \omega \cos \theta)^2 = \overline{\omega_{\text{sin}}}^2$$

$$\frac{1}{f^2} = 1 - \zeta \Rightarrow \zeta = 1 - \frac{(1-v^2)(1-\omega^2)}{(1-v \omega \cos \theta)^2}$$

$$\frac{1}{f^2} \left(\dots \right) (1-v \omega \cos \theta)^2 = (1-\zeta) (1-v \omega \cos \theta)^2$$

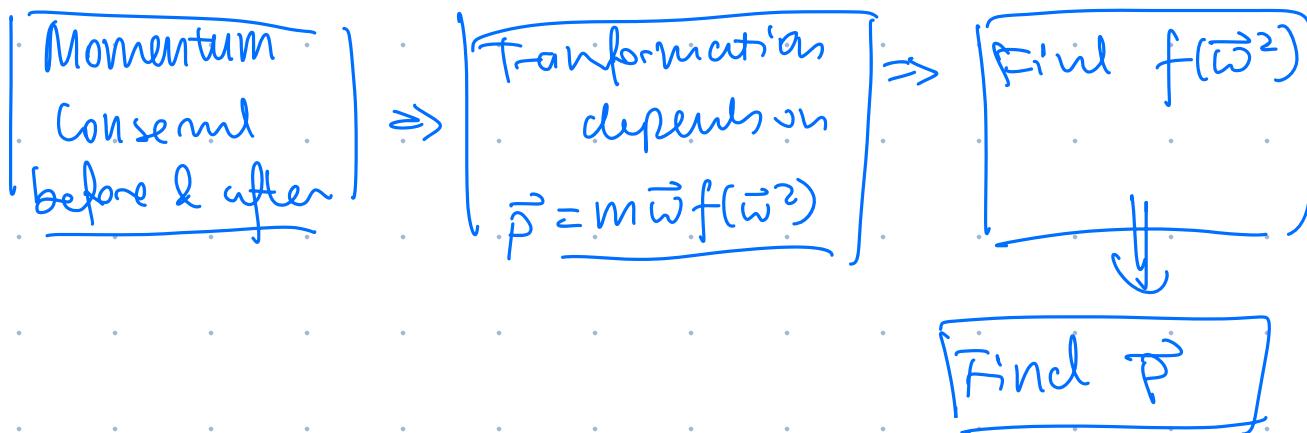
$$= \left[1 - 1 + \frac{(1-v^2)(1-\omega^2)}{(1-v \omega \cos \theta)^2} \right] (1-v \omega \cos \theta)^2$$

$$= (1-v^2)(1-\omega^2)$$

$$f^2 = \frac{1}{1-s} \Rightarrow f = \frac{1}{\sqrt{1-s}} = \frac{1}{\sqrt{1-\vec{\omega}_{i,j}^2}}$$

so $\vec{p}(\vec{\omega}) = \frac{m\vec{\omega}}{\sqrt{1-\vec{\omega}^2}}$

General idea:



Energy Conserv.

$$E(\vec{\omega}) = mg(\vec{\omega}^2)$$

$$\text{N.R. } g(s) = \frac{1}{2}s.$$

$$E_{\text{before}} = E_{\text{after}}$$

→ from a homogenous equation you can set whatever constants you like if it still satisfies the equation.

$$\cancel{mg \left(\left(\frac{v+w}{1+vw} \right)^2 \right)} + \cancel{mg \left(\left(\frac{v-w}{1-vw} \right)^2 \right)}$$

$$= \cancel{mg \left(1 - \frac{(1-v^2)(1-w^2)}{(1+vw\cos\theta)^2} \right)}$$

$$+ \cancel{mg \left(1 - \frac{(1-v^2)(1-w^2)}{(1-vw\cos\theta)^2} \right)}$$

?)

{ derivations skipped. }

$$g(s) = \frac{1}{\sqrt{1-s}}$$



$$E(\vec{\omega}) = \frac{m}{\sqrt{1-\vec{\omega}^2}}$$

An inelastic collision

$$\vec{\omega}_1 \rightarrow \vec{\omega}_2$$

$$\vec{\omega}_1 = \left(\frac{v+w}{1+vw}, 0, 0 \right)$$



$$\vec{\omega}_2 = \left(\frac{v-w}{1-vw}, 0, 0 \right)$$

$$\vec{\omega}_3 = (v, 0, 0)$$

E before =

$$g(s) = \frac{1}{\sqrt{1-s}}, \quad \frac{m}{\sqrt{1-\left(\frac{v+w}{1+vw}\right)^2}} + \frac{m}{\sqrt{1-\left(\frac{v-w}{1-vw}\right)^2}} = \boxed{?} \quad \frac{1}{\sqrt{1-v^2}}$$

$$\frac{[1+vw]m}{\sqrt{(1+vw)^2 - (v+w)^2}} + \frac{[1-vw]m}{\sqrt{(1-vw)^2 - (v-w)^2}}$$

$$\frac{[-\rightarrow]}{\sqrt{1+v^2w^2+2vw-v^2+w^2-2vw}} + \frac{[-\rightarrow]}{\sqrt{1+v^2w^2-2vw-v^2+w^2+2vw}}$$

$$\frac{2m}{\sqrt{(1-v^2)(1-w^2)}} = \boxed{?} \quad \frac{1}{\sqrt{1-v^2}}$$

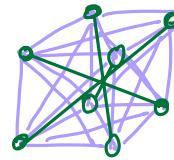
$$\boxed{\frac{2m}{\sqrt{1-w^2}}}$$

m_{blob}

E here
accounts
for all
energy

↓
in the primed coordinate

$$F_{\text{LdL}} = \frac{m}{\sqrt{1-\omega^2}} + \frac{m}{\sqrt{1-\omega^2}}$$
$$= \frac{2m}{\sqrt{1-\omega^2}}$$



(implications:

① $c=1 \Rightarrow [E]=[P]=[m]$

② $\omega \rightarrow 1 \quad E = \frac{m}{\sqrt{1-v^2}} \rightarrow \infty$

(2') only massive objects move with the speed of light.

$$|\vec{\omega}| = 1.$$

③ mass of an object = its energy at rest.

↪ heating isn't up → making it heavier.

97% of proton mass comes from relativistic effect of quark mass

↪ how do you measure the mass of a quark?



First nuclear bomb:

$$\Delta m = \frac{4 \cdot 10^6 \cdot 10^4}{c^2} = \frac{4 \cdot 10^{10}}{9 \cdot 10^{16}} \approx \frac{1}{2} \cdot 10^{-6} \text{ kg}$$

$10^{-6} \times 10^3 = 10^{-3} \text{ g}$

Convert a microgram
of mass into energy

$$E(\vec{\omega}) = \frac{m}{\sqrt{1-\vec{\omega}^2}} \approx m \left(1 + \frac{\vec{\omega}^2}{2} + \frac{3}{8} \vec{\omega}^4 + \dots \right)$$

↑
Taylor
expand

rest
energy

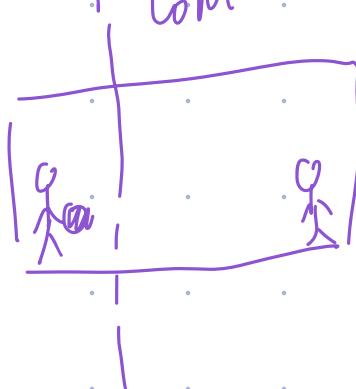
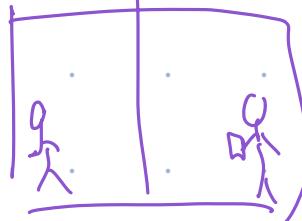
nonrelativistic
kinetic
energy

relativistic
corrections
to kinetic
energy

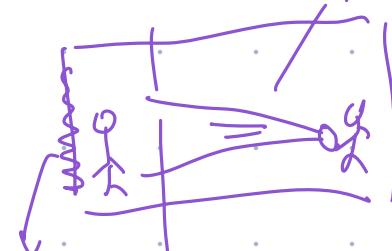
CoM.



CoM



Flashlight



wall absorbs
the energy
of the light

$\text{energy} \approx m c^2$

It's not trivial that such combination
all furrows of every: