More Gates

- AND gate (irreversible)

$x \wedge y = 1$ iff $x=1$ and $y=1$.

Universal Gate Set

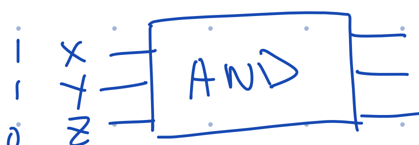
↳ Defined to be any set of gates that can be used to execute all Z ($Z = 2^{m(2^n)}$) programs.

↳ Fundamental building blocks of logic gates.

↳ You can use them to make other gates.

* Example: A Toffoli gate.

- Reversible AND gate / Toffoli / CCNOT.

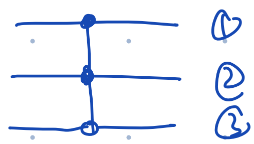


$q = z \oplus (x \wedge y)$

$0 \oplus 1 \Rightarrow 1.$

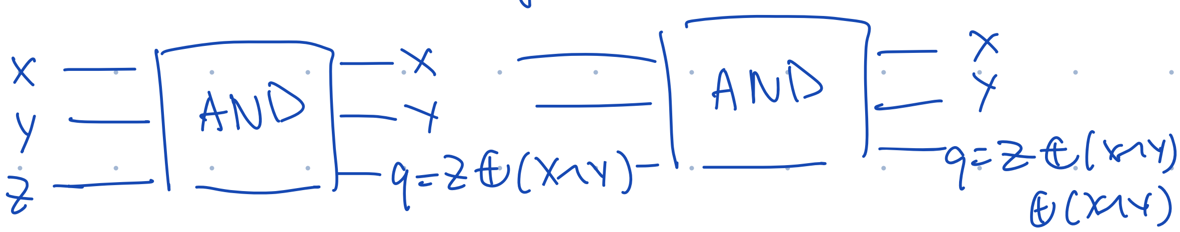
if z flips, then we know that $x \wedge y = 1$.

This is also called Toffoli / CCNOT.



flip this only when both ① & ② are true.

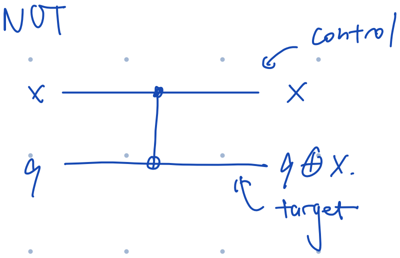
You can use 2 Toffoli to make an "identity gate"



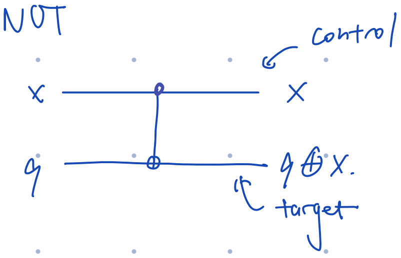
This is because $b \oplus b = 0$ $\begin{cases} 1 \oplus 1 = 0 \\ 0 \oplus 0 = 0 \end{cases}$

*Peculiar quantum effects allow the Toffoli gate to be synthesized from 2 qubit gates.

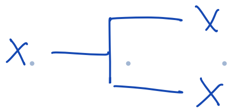
• CNOT



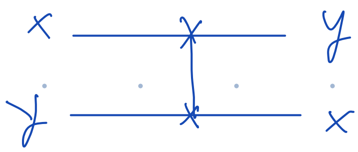
• $\overline{\text{CNOT}}$

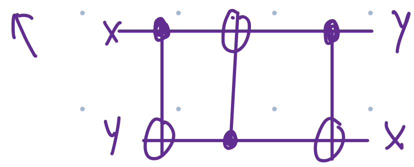


• FANOUT



• SWAP



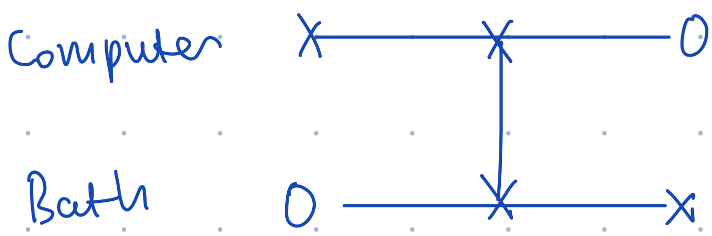


→ can be synthesized from 3 CNOT gates.

FANOUT is a classically ok gate.

It is forbidden in Q.M. due to no-cloning theorem.

Making "Reset Gates" Reversible.



Information is not erased.

The computer forgets, but the bath knows.

Circuit Depth

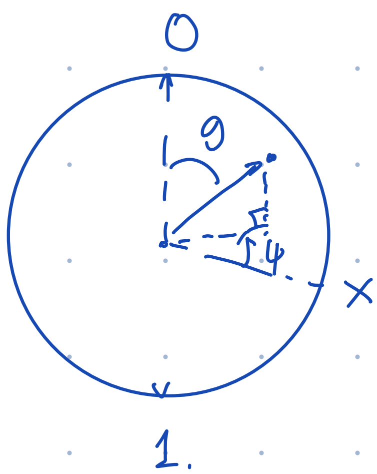
↳ the # of timesteps it takes to complete the circuit.

Phenomenology of Quantum Bits

① Energy level is quantized & controllable. Measurement of the energy always yields ε_0 or ε_1

② There exists a continuum of superposition states intermediate between 0 and 1.

③ It turns out that this continuum of states can be represented by points on the Bloch sphere.

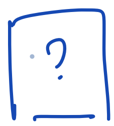


θ: the polar angle.

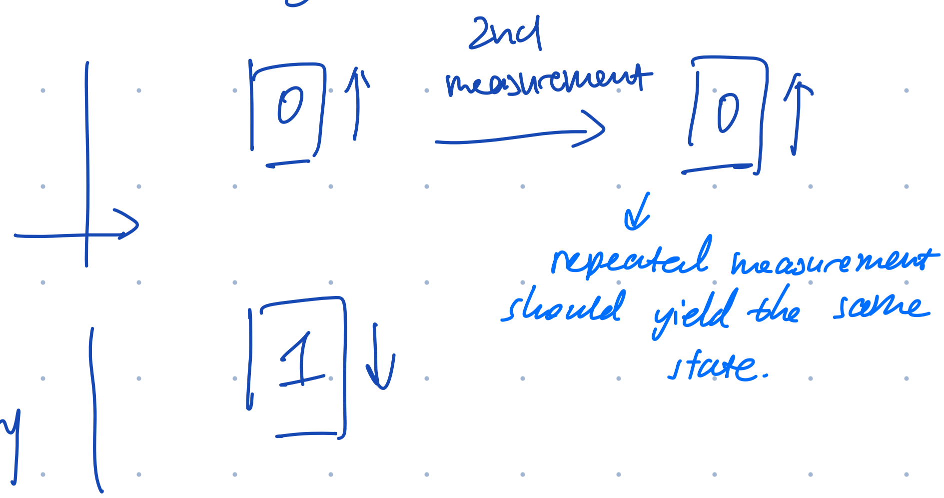
φ: the azimuthal angle.

$$(x, y, z) = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$$

① The act of measurement changes the state.



Before measurement, our knowledge of the qubit state is denoted by a probability vector.



$$\begin{bmatrix} A \\ B \end{bmatrix} \begin{matrix} \to 0 \text{ state} \\ \to 1 \text{ state} \end{matrix}$$

After measurement, our knowledge of the state changes.

We are certain that it is 1 or 0.

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$