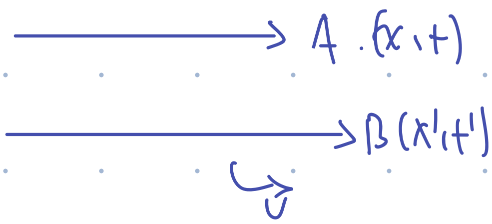
Relativity Lecture 1 Notes.

Suppose we have 2 coordinate systems:



B is moving with velocity v relative to A.

In B's perspective, A is moving with velocity $-v$ w.r.t B.

Assuming the (x, t) & (x', t') coordinate system is linearly related, we have the following.

$$\begin{cases} x' = \gamma(v)(x - vt) & ① \\ x = \gamma(v)(x' + vt') & ② \end{cases}$$

$$\frac{x}{\gamma} - vt' = x'$$

$$\frac{x}{\gamma} - vt' = \gamma(x - vt)$$

$$\frac{x}{\gamma} - \gamma x + \gamma v t = vt'$$

From ①: $\frac{x'}{\gamma} + vt = x$. ③

③ = ②

$$\frac{x'}{\gamma} + vt = \gamma(x' + vt')$$

$$\frac{x'}{\gamma^2} + \frac{v}{\gamma} t = x' + vt'$$

$$\frac{v}{\gamma} t = \left(1 - \frac{1}{\gamma^2}\right) x' + vt'$$

$$t = \frac{\gamma}{v}\left(1 - \frac{1}{\gamma^2}\right) x' + \frac{\gamma}{\cancel{v}} \cancel{v} t'$$

$$= \frac{\gamma}{v}\left(1 - \frac{1}{\gamma^2}\right) x' + \gamma t'$$

$$t = \gamma\left[\frac{1}{v}\left(1 - \frac{1}{\gamma^2}\right) x' + t'\right]$$

$$\frac{\gamma^2 - 1}{\gamma^2} \qquad \frac{\frac{1}{A^2} - 1}{\frac{1}{A^2}} = 1 - A^2$$

A

B ↳ v (w.r.t A)

C ↳ w' (w.r.t B)

① $x' = \omega' t'$ → where does that come from

② $$\omega = \frac{x}{t} = \frac{\cancel{\gamma}(x' + vt')}{\cancel{\gamma}\left[\frac{1}{v}\left(1 - \frac{1}{\gamma^2}\right)x' + t'\right]}$$

$$= \frac{\omega' t' + vt'}{\frac{1}{v}\left(1 - \frac{1}{\gamma^2}\right)\omega' t' + t'}$$

$$= \frac{\omega' + v}{\frac{1}{v}\left(1 - \frac{1}{\gamma^2}\right)\omega' + 1}$$

Now I am going to rewrite γ as $\gamma(v)$ to specify that it depends on v.

$$\omega = \frac{\omega' + v}{\frac{1}{v}\left(1 - \frac{1}{\gamma^2(v)}\right)\omega' + 1}$$

Now we want to see everything from observer C's perspective.

A

B ↳ v (w.r.t to A)

C ↳ w' (w.r.t. to B)

To observer B, C is moving with vel ω'

$$\begin{cases} x'' = \gamma(\omega')\left[x' - \omega' t'\right] & ① \\ x' = \gamma(\omega')\left[x'' + \omega' t''\right] & ② \end{cases}$$

③: $\frac{x'}{\gamma} - \omega' t'' = x''$

~~② = ③~~ ① = ③

also $x' = -vt'$

how did that come about?

$$\frac{x'}{\gamma} - w't'' = \gamma\left(x' - w't'\right)$$

A moving at $-V$ w.r.t to B

a point x' moves

$$\frac{x'}{\gamma} - \gamma x' + \gamma w't' = w't''$$

$$\gamma\left[\left(\frac{1}{\gamma^2} - 1\right)x' + w't'\right] = w't''$$

$$\gamma\left[\frac{1}{w'}\left(\frac{1}{\gamma^2} - 1\right)x' + t'\right] = t''$$

$$-w = \frac{x''}{t''} = \frac{\cancel{\gamma}\left[x' - w't'\right]}{\cancel{\gamma}\left[\frac{1}{w'}\left(\frac{1}{\gamma^2} - 1\right)x' + t'\right]}$$

$$= \frac{-Vt' - w't'}{\frac{1}{w'}\left(\frac{1}{\gamma^2} - 1\right)\left(-Vt'\right) + t'}$$

$$= \frac{-\left(V + w'\right)}{\frac{1}{w'}\left(1 - \frac{1}{\gamma^2}\right)V + 1}$$

so

$$w = \frac{V + w'}{\frac{1}{w'}\left(1 - \frac{1}{\gamma^2}\right)V + 1}$$

Now, to specify that γ depends on w'.

I'll rewrite w as

$$w = \frac{v+w'}{\frac{1}{w'}\left(1-\frac{1}{\gamma^2(w')}\right)v+1}$$

Since $w = \dfrac{v+w'}{\frac{1}{w'}\left(1-\frac{1}{\gamma^2(w')}\right)v+1} = \dfrac{v+w'}{\frac{1}{v}\left(1-\frac{1}{\gamma^2(v)}\right)w'+1}$

that $\dfrac{\cancel{v}}{w'}\left(1-\frac{1}{\gamma^2(w')}\right) = \dfrac{w'}{v}\left(1-\frac{1}{\gamma^2(v)}\right)$

$$\frac{\cancel{vw'}}{w'^2}\left(1-\frac{1}{\gamma^2(w')}\right) = \frac{\cancel{vw'}}{v^2}\left(1-\frac{1}{\gamma^2(v)}\right)$$

$$\underbrace{\frac{1}{w'^2}\left(1-\frac{1}{\gamma^2(w')}\right)}_{1} = \underbrace{\frac{1}{v^2}\left(1-\frac{1}{\gamma^2(v)}\right)}_{2} = B.$$

That means the two can't depend on time, so we can set it to be a constant.

$$\omega = \frac{v+\omega'}{B\omega' v + 1}$$

we could have $\begin{cases} B = 0 : \text{galilean} \\ B > 1 : \text{relativistic} \end{cases}$

$$B = \frac{1}{v^2}\left(1 - \frac{1}{\gamma^2}\right)$$

$$v^2 B = 1 - \frac{1}{\gamma^2}$$

$$\gamma^2 v^2 B = \gamma^2 - 1$$

$$\gamma^2 \left[v^2 B - 1\right] = -1$$

$$\gamma^2 = \frac{1}{1 - v^2 B}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2 B}}$$

Fiteau's experiment measuring B yields $1.1 \times 10^{-17}\ s^2/m^2$

turns out that $B = \frac{1}{c^2}$

so $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

Hence we have the Lorentz transformation:

$$\begin{cases} x' = \dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}}(x - vt) \\[2ex] x = \dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}}(x' + vt') \end{cases}$$

$$1 - \frac{1}{1-\frac{v^2}{c^2}}$$

$$\frac{\cancel{1} - \frac{v^2}{c^2} - \cancel{1}}{1-\frac{v^2}{c^2}}$$

$$\begin{cases} t = \gamma\left[\frac{1}{v}\left(1-\frac{1}{\gamma^2}\right)x' + t'\right] \\ \quad = \dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}}\left[\frac{1}{v}\left[\cancel{1} - \cancel{1} + \frac{v^2}{c^2}\right]x' + t'\right] \\ \quad = \dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}}\left[\frac{v}{c^2}x' + t'\right]. \\[2ex] t' = \gamma\left[\frac{1}{v}\left(\frac{1}{\gamma^2} - 1\right)x + t\right] \\ \quad = \dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}}\left[\frac{1}{v}\left[\cancel{1} - \frac{v^2}{c^2} - \cancel{1}\right]x + t\right] \\ \quad = \dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}}\left(-\frac{v}{c^2}x + t\right). \end{cases}$$

$\square$ Lorentz Transformation