homework 2

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1) Solution,

$$(N, M) = (13, 5)$$
$$answer = 39$$
$$(N, M) = (13, 7)$$
$$answer = 37$$
$$(N, M) = (19, 5)$$
$$answer = 73$$
$$(N, M) = (19, 7)$$
$$answer = 59$$
$$(N, M) = (23, 5)$$
$$answer = 101$$
$$(N, M) = (23, 7)$$
$$answer = 75$$

2) The minimum amount of Hanoi problem is $T_n = 2^n - 1$, and I think sub-minimum answer $T_n^{'} = T_n - 1$ is correct. I will give a possible way to implemnet it. Now we have 3 pegs (p_1, p_2, p_3), in the minimum solution, the last step is to move the smallest disk to p_3, instead of this step, we can move the last disk to another empty peg, then move the last disk to p_3.

3) if break Lucas rule once

Disks	Break	No breaks
1	1	1
2	3	3
3	5	7
4	9	15
5	17	31
...	...	...

if break Lucas rule once, the answer is $2^{n-1}+1$.
Suppose we have 3 pegs (p_1, p_2, p_3), firstly, move $n-2$ disks to p_2, using $2^{n-2}-1$ steps. Secondly, move the topest disk on p_1 to p_2 (breaking Lucas rule). then, move the last disk to p_3 and move the topest disk on p_2 to p_3, using 3 steps. Finally, move all the disks on p_2 to p_3, using $2^{n-2}-1$ steps. Totally, we'll use

$$(2^{n-2}-1)\times 2+3=2^{n-1}+1$$

steps.