# Lista 1.

Geometria Analítica (2024-1) Jeann Victor Batista.

1) a) $A+2B = \begin{pmatrix} 1 & 0 \\ 3 & 7 \end{pmatrix} + \begin{pmatrix} 0.2 & 2.5 \\ 2.2 & 2.-2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ 3 & 7 \end{pmatrix} + \begin{pmatrix} 0 & 10 \\ 4 & -4 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 10 \\ 7 & 3 \end{pmatrix}$

b)

$A.B = \begin{pmatrix} 0 & 5 \\ 14 & 1 \end{pmatrix}$ e $BA = \begin{pmatrix} 15 & 35 \\ -4 & -14 \end{pmatrix} \rightarrow AB - BA = \begin{pmatrix} -15 & -30 \\ 18 & 15 \end{pmatrix}$

c)

É impossível de se realizar por apresentar ordem diferente.

d)

$2.D^t - 3.E^t \quad \begin{pmatrix} -3 & 2 & 0 \\ 1 & 1 & 4 \\ -2 & 0 & 2 \end{pmatrix} .2 = \begin{pmatrix} -6 & 4 & 0 \\ 2 & 2 & 8 \\ -4 & 0 & 4 \end{pmatrix} \rightarrow \begin{pmatrix} -6 & 2 & -4 \\ 4 & 2 & 0 \\ 0 & 8 & 4 \end{pmatrix}$

$\begin{pmatrix} 2 & 4 & -3 \\ -1 & 0 & -4 \\ -6 & 0 & -1 \end{pmatrix} .3 \rightarrow \begin{pmatrix} 6 & 12 & -9 \\ -3 & 0 & -12 \\ -18 & 0 & -3 \end{pmatrix} \rightarrow \begin{pmatrix} 6 & -3 & -18 \\ 12 & 0 & 0 \\ -9 & -12 & -3 \end{pmatrix}$

$\begin{pmatrix} -6 & 2 & 4 \\ 4 & 2 & 0 \\ 0 & 8 & 4 \end{pmatrix} + \begin{pmatrix} -6 & 13 & 18 \\ -12 & 0 & 0 \\ 9 & 12 & 3 \end{pmatrix} \rightarrow \begin{pmatrix} -12 & 5 & 22 \\ -8 & 2 & 0 \\ 9 & 20 & 7 \end{pmatrix}$

e)

$\begin{pmatrix} -3 & 2 & 0 \\ 1 & 1 & 4 \\ -2 & 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} -3 & 2 & 0 \\ 1 & 1 & 4 \\ -2 & 0 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 9+2+0 & -6+2+0 & 0+8+0 \\ -3+1-8 & 2+1+0 & 0+4+8 \\ 6+0-4 & -4+0+0 & 0+0+4 \end{pmatrix} \begin{pmatrix} 11 & -4 \\ -10 & 3 \\ 2 & -4 \end{pmatrix}$

$\begin{pmatrix} -3 & 2 & 0 \\ 1 & 1 & 4 \\ -2 & 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} 2 & 4 & -3 \\ -1 & 0 & -4 \\ -6 & 0 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} -6-2+0 & -12+0+0 & 9-8\ 0 \\ 2-1-24 & 4+0+0 & -3-4-4 \\ -4+0-12 & -8\ 0\ 0 & +6+0-2 \end{pmatrix} \begin{pmatrix} -8 & -12 & 1 \\ -23 & 4 & -11 \\ -16 & -8 & 4 \end{pmatrix}$

$\begin{pmatrix} 11 & -4 & 8 \\ -10 & 3 & 12 \\ 2 & -4 & 4 \end{pmatrix} + \begin{pmatrix} -8 & -12 & 1 \\ -23 & 4 & -11 \\ -16 & -8 & 4 \end{pmatrix} \rightarrow \begin{pmatrix} 3 & -16 & 9 \\ -33 & 7 & 1 \\ -14 & -12 & 8 \end{pmatrix}$

F) $C_t \cdot A \rightarrow \begin{pmatrix} -2 & 3 & -7 \\ 7 & -3 & -2 \end{pmatrix} \rightarrow \begin{pmatrix} -2 & 7 \\ 3 & -3 \\ -7 & -2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 3 & 7 \end{pmatrix} \rightarrow \begin{pmatrix} -2+21 & 0+49 \\ 3-9 & 0-21 \\ -7-6 & 0-14 \end{pmatrix} \begin{pmatrix} 19 & 49 \\ -6 & -21 \\ & -14 \end{pmatrix}$

g) $E - AC \rightarrow \begin{pmatrix} 1 & 0 \\ 3 & 7 \end{pmatrix} \cdot \begin{pmatrix} -2 & 3 & -7 \\ 7 & -3 & -2 \end{pmatrix} \rightarrow \begin{pmatrix} -2+0 & 3+0 & -7+0 \\ -6+49 & 9-21 & -21-14 \end{pmatrix} \begin{pmatrix} -2 & 3 & -7 \\ 43 & -12 & -35 \end{pmatrix}$

Não dá pois na subtração de matriz a ordem precisa ser igual

h

$F^t \cdot E = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -2 & 0 \end{pmatrix} \cdot \rightarrow \begin{pmatrix} 2 & 4 & -3 \\ -1 & 0 & -4 \\ -6 & 0 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 2+2+12 \\ 4 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

i) $\begin{pmatrix} 0 & 5 \\ 2 & -2 \end{pmatrix} \cdot \begin{pmatrix} -2 & 3 & -7 \\ 7 & -3 & -2 \end{pmatrix} \rightarrow \begin{pmatrix} 0+35 & 0-15 & 0-10 \\ -4-14 & 6+6 & -14+4 \end{pmatrix} \rightarrow \begin{pmatrix} 35 & -15 & -10 \\ -18 & 12 & -10 \end{pmatrix}$

$\begin{pmatrix} 35 & -15 & -10 \\ -18 & 12 & -10 \end{pmatrix}_{2\times 3} \cdot \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}_{3\times 1} \rightarrow \begin{pmatrix} 35+30+0 \\ -18 & -24+0 \end{pmatrix} \rightarrow \begin{pmatrix} 65 \\ -42 \end{pmatrix}$

2) a) $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \cdot \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{pmatrix} \rightarrow \begin{pmatrix} xxxx & xxxx & xxxx & xxxx \\ xxxx & xxxx & xxxx & xxxx \end{pmatrix}$

→ Matriz de ordem → $C_{2\times 4}$ $\S$ $B \times A$ = Não está definido

b) $\begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \\ a_{41} \end{pmatrix} \cdot \begin{pmatrix} a_{11} & a_{12} \end{pmatrix} \rightarrow \begin{pmatrix} x & x \\ x & x \\ x & x \\ x & x \end{pmatrix}$

→ Matriz de ordem → $C = 4\times 2$

$B \times A$ não está definido

c)

$\begin{pmatrix} a_{11} & a_{12} \end{pmatrix} \cdot \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix} \rightarrow$ Não é possivel realizar.

BxA esta definido

d)

$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \\ a_{51} & a_{52} \end{pmatrix} \cdot \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \rightarrow \begin{pmatrix} x+x & x+x & x+x \\ x+x & x+x & x+x \\ x+x & x+x & x+x \\ x+x & x+x & x+x \\ x+x & x+x & x+x \end{pmatrix}$

→ Matriz de ordem 5x3 // BxA = não esta definido.

e)

$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \cdot \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{33} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$

BxA
→ Não esta definido

→ Não é possivel realizar multiplicação das matrizes.

F)

$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix} \cdot \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{pmatrix} \rightarrow \begin{pmatrix} x+x & x+x & x+x & x+x \\ x+x & x+x & x+x & x+x \\ x+x & x+x & x+x & x+x \\ x+x & x+x & x+x & x+x \end{pmatrix}$

→ Matriz de ordem 4x4 // BxA esta definido.

g)

$\begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix} \cdot \begin{pmatrix} a_{11} & a_{12} & a_{13} \end{pmatrix} \qquad \begin{pmatrix} x & x & x \\ x & x & x \end{pmatrix}$

→ Matriz de ordem 2x3. // BxA = não esta definido

h) $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \cdot \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \rightarrow \begin{matrix} x+x & x+x \\ x+x & x+x \end{matrix}$

↳ A matriz $C = 2x2$ // BxA está definido.

③ a) $\begin{Bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{Bmatrix} \rightarrow \begin{pmatrix} 3.1-2.1= & 3.1-2.2 & 3.1-2.3 \\ 3.2-2.1= & 3.2-2.2 & 3.2-2.3 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & -3 \\ 4 & 2 & 0 \end{pmatrix}$

$a_{ij} = 3i - 2j$

b)

$\begin{Bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{Bmatrix} \rightarrow \begin{Bmatrix} 2.1+1=3 & 1^2-2=-1 & 1^2-3=-2 \\ 2^2-1=3 & 2.2+2=6 & 2^2-3=1 \\ 3^2-1=8 & 3^2-2=7 & 2.3+3=9 \end{Bmatrix} \rightarrow \begin{Bmatrix} 3 & -1 & -2 \\ 3 & 6 & 1 \\ 8 & 7 & 9 \end{Bmatrix}$

$\begin{cases} 2i+j, \text{se } i=j \\ i^2-j, \text{se } i \neq j \end{cases}$

c)

$\{a_{11} \ a_{12} \ a_{13} \ a_{14}\} \rightarrow \{1^1=1 \ \ 2^1=2 \ \ 3^1=3 \ \ 4^1=4\} \rightarrow \{1 \ 2 \ 3 \ 4\}$

$c_{ij} = j^i$

d)

$\begin{Bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{Bmatrix} \rightarrow \begin{Bmatrix} 1^2+1^2=2 & 2.1.2=4 & 2.1.3=6 & 2.1.4=8 \\ 2.2.1=4 & 2^2+2^2=8 & 2.2.3=12 & 2.2.4=16 \\ 2.3.1=6 & 2.3.2=12 & 3^2+3^2=18 & 2.3.4=24 \\ 2.4.1=8 & 2.4.2=16 & 2.4.3=24 & 4^2+4^2=32 \end{Bmatrix}$

$\begin{cases} i^2+j^2, \text{se } i=j \\ 2.i.j, \text{se } i \neq j \end{cases}$ ↳ $\begin{Bmatrix} 2 & 4 & 6 & 8 \\ 4 & 8 & 12 & 16 \\ 6 & 12 & 18 & 24 \\ 8 & 16 & 24 & 32 \end{Bmatrix}$

tilibra

4) a) $[BA]_{23} \rightarrow \begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 4 \\ -3 & -1 & -17 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 1 \\ -2 & -3 & 2 \\ 1 & 4 & 5 \end{pmatrix} \rightarrow \begin{pmatrix} 1+0+3 & 2+0+12 & 1+0+15 \\ 2+2+4 & 4+3+16 & 2-2+20 \\ -3+2-17 & -6+3-68 & -3-2-85 \end{pmatrix} \begin{pmatrix} 4 & 14 & 16 \\ 8 & 23 & 20 \\ -12 & -71 & -90 \end{pmatrix}$

↳ Posição 23 = 20

b) $[AB]_{23}$

$\begin{pmatrix} 1 & 2 & 1 \\ -2 & -3 & 2 \\ 1 & 4 & 5 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 4 \\ -3 & -1 & -17 \end{pmatrix} \rightarrow \begin{pmatrix} 1+4-3 & 0-2-1 & 3+8-17 \\ -2-6-6 & 0+3-2 & -6-12-34 \\ 1+8-15 & 0-4-5 & 3+16-85 \end{pmatrix} \begin{pmatrix} 2 & -3 & -6 \\ -14 & 1 & -52 \\ -6 & -9 & -67 \end{pmatrix}$

↳ Posição 23 = −52

c) $[B^2]_{23}$

$\begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 4 \\ -3 & -1 & -17 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 4 \\ -3 & -1 & -17 \end{pmatrix} \rightarrow \begin{pmatrix} 1+0-9 & 0+0-3 & 3+0-51 \\ 2-2-12 & 0+1-4 & 6-4-68 \\ -3-2+51 & 0+1+17 & -9-4+289 \end{pmatrix} \rightarrow \begin{pmatrix} -8 & -3 & -48 \\ -12 & -3 & -66 \\ 46 & +18 & 276 \end{pmatrix}$

↳ Posição 31 = 46

d) $tr(A) = a_{11} + a_{22} + a_{33}$

$\begin{pmatrix} 1 & 2 & 1 \\ -2 & -3 & 2 \\ 1 & 4 & 5 \end{pmatrix} \rightarrow 1 - 3 + 5 \rightarrow 3$

↳ $tr(A) = 3$

e) $tr(B^t) = [B^t]_{11} + [B^t]_{22} + [B^t]_{33}$

$B = \begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 4 \\ -3 & -1 & -17 \end{pmatrix} \quad B^t = \begin{pmatrix} 1 & 2 & -3 \\ 0 & -1 & -1 \\ 3 & 4 & -17 \end{pmatrix} \rightarrow 1 - 1 - 17 = -17$

↳ $tr(B^t) = -17$

F) $tr(A-B) = [A-B]_{11} + [A-B]_{22} + [A-B]_{33}$

$\begin{pmatrix} 1 & 2 & 1 \\ -2 & -3 & 2 \\ 1 & 4 & 5 \end{pmatrix} + \begin{pmatrix} -1 & 0 & -3 \\ -2 & 1 & 4 \\ 3 & 1 & 17 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 2 & -2 \\ 0 & -2 & 6 \\ 4 & 5 & 22 \end{pmatrix} \rightarrow 0 - 2 + 22 = 20$

↳ $tr(A-B) = 20$

g) $[AB]tr = [AB]_{11} + [AB]_{22} + [AB]_{33}$

Peguei da letra "B" a matriz $[A \times B]$ $\rightarrow \begin{pmatrix} 2 & -3 & -6 \\ -14 & 1 & -52 \\ -6 & -9 & -67 \end{pmatrix}$

↳ $2 + 1 - 67 = -64 = tr[AB]$.

5) a) $2X + A = 3B + C \rightarrow 2X = 3B + C - A$

$$\begin{bmatrix} 2r & 2s \\ 2t & 2v \end{bmatrix} + \begin{bmatrix} -1 & 7 \\ 2 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 2r-1 & 2s+7 \\ 2t+2 & 2v+6 \end{bmatrix}$$

$$3\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ 12 & 9 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 6 & 5 \\ 13 & 9 \end{bmatrix}$$

$$\begin{cases} 2r-1=6 \rightarrow 2r=7 \rightarrow r=7/2 \\ 2t+2=13 \rightarrow 2t=11 \rightarrow t=11/2 \\ 2s+7=5 \rightarrow 2s=-2 \rightarrow s=-1 \\ 2v+6=9 \rightarrow 2v=3 \rightarrow v=3/2 \end{cases} \rightarrow X = \begin{Bmatrix} 7/2 & -1 \\ 11/2 & 3/2 \end{Bmatrix}$$

b) $Y + A = 1/2\,(B-C)^t$

$$\begin{Bmatrix} r & s \\ t & v \end{Bmatrix} + \begin{Bmatrix} -1 & 7 \\ 2 & 6 \end{Bmatrix} \rightarrow \begin{Bmatrix} r-1 & s+7 \\ t+2 & v+6 \end{Bmatrix}$$

$$\begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix} - \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & -1 \\ 3 & 3 \end{pmatrix}^t \rightarrow \begin{pmatrix} 2 & 3 \\ -1 & 3 \end{pmatrix} \cdot \frac{1}{2} \rightarrow \begin{pmatrix} 1 & 3/2 \\ -1/2 & 3/2 \end{pmatrix}$$

$$\begin{cases} r-1=1 \rightarrow r=2 \\ s+7=3/2 \rightarrow s=-14/2+3/2 \rightarrow s=-11/2 \\ t+2=-1/2 \rightarrow t=-2-1/2 \rightarrow t=-5/2 \\ v+6=3/2 \rightarrow v=3/2+6 \rightarrow v=3/2+12/2 \rightarrow v=-9/2 \end{cases}$$

$$\begin{Bmatrix} 2 & -11/2 \\ -5/2 & -9/2 \end{Bmatrix}$$

tilibra

c) $3X + A = B - X \rightarrow 3 \cdot \begin{pmatrix} r & s \\ t & u \end{pmatrix} = \begin{pmatrix} 3r & 3s \\ 3t & 3u \end{pmatrix} + \begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix} \rightarrow \begin{pmatrix} 3r-1 & 3s+7 \\ 3t+2 & 3u+6 \end{pmatrix}$

$\begin{pmatrix} 3r-1 & 3s+7 \\ 3t+2 & 3u+6 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix} - \begin{pmatrix} r & s \\ t & u \end{pmatrix} \rightarrow \begin{pmatrix} 2-r & 1-s \\ 4-t & 3-u \end{pmatrix}$

$3r - 1 = 2 - r \rightarrow 4r = 3 \rightarrow r = 3/4.$

$3s + 7 = 1 - s \rightarrow 4s = -6 \rightarrow s = -6/4 \rightarrow s = -3/2.$

$3t + 2 = 4 - t \rightarrow 4t = 2 \rightarrow t = 2/4 \rightarrow t = 1/2.$

$3u + 6 = 3 - u \rightarrow 4u = -3 \rightarrow u = -3/4.$

$\begin{pmatrix} 3/4 & -3/2 \\ 1/2 & -3/4 \end{pmatrix}$

Resul-tado na próxima pág.

d) $\begin{cases} X + Y = 3A \\ X - Y = 2B + C \end{cases} \rightarrow 2X = 3A + 2B + C$

$3A = \begin{pmatrix} -3 & 21 \\ 6 & 18 \end{pmatrix} \; || \; 2B = \begin{pmatrix} 4 & 2 \\ 8 & 6 \end{pmatrix} + C = \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} +1 & 25 \\ 15 & 24 \end{pmatrix} \rightarrow \begin{pmatrix} 1/2 & 25/2 \\ 15/2 & 12 \end{pmatrix}$

$X + Y = 3A \rightarrow Y = \begin{pmatrix} -3 & 21 \\ 6 & 18 \end{pmatrix} - \begin{pmatrix} 1/2 & 25/2 \\ 15/2 & 12 \end{pmatrix} \rightarrow Y = \begin{pmatrix} -7/2 & 17/2 \\ -3/2 & 6 \end{pmatrix}$

$Y = 3A - X$

$$X = \begin{Bmatrix} 1/2 & 25/2 \\ 15/2 & 1/2 \end{Bmatrix}, \quad Y = \begin{pmatrix} -7/2 & 17/2 \\ -3/2 & 6 \end{pmatrix}$$

**6)** x sendo 2 → $A = \begin{pmatrix} 1 & 1/2 \\ 2 & 1 \end{pmatrix}$ Logo $A^2 = \begin{pmatrix} 1 & 1/2 \\ 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1/2 \\ 2 & 1 \end{pmatrix}$ $\begin{pmatrix} 1+1 & 1/2+1/2 \\ 2+2 & 1+1 \end{pmatrix}$

$A^2 = \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$, e $2.A = \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$.

$A^3 = X.A \rightarrow \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1/2 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 2+2 & 1+1 \\ 4+4 & 2+2 \end{pmatrix} \rightarrow A^3 = \begin{pmatrix} 4 & 2 \\ 8 & 4 \end{pmatrix}$

Basicamente dobrou o valor dos elementos quando comparada a $A^2$.

Logo se isso for padrão → $\boxed{A^n = 2^{n-1}.A}$

**7) a)** $A(B+C) \rightarrow A.B + A.C \rightarrow X+Y$

**b)** De acordo com a propiedade de matrizes, $(XY)^T = Y^t.X^t$

Logo → $B^t A^t \rightarrow (AB)^t$

→ $AB = X$ logo $(X)^t$.

**c)** De acordo com a propiedade de matrizes $(XY)^t = Y^t X^t$

Logo → $C^t A^t \rightarrow (AC)^t$

→ $AC = Y$ logo $(Y)^t$

**D)**

$(ABA).C \rightarrow (XA)C \rightarrow X(AC) \rightarrow (X.Y)$

8) $A = \begin{pmatrix} 4 & X+2 \\ 2X-3 & X+1 \end{pmatrix} \Rightarrow A^t = \begin{pmatrix} 4 & 2X-3 \\ X+2 & X+1 \end{pmatrix} \rightarrow$ $4=4$ $X+2=2X-3$ $2X-3=X+2$ $X+1=X+1$

$X+2=2X-3 \rightarrow 5=X$ // $A = \begin{pmatrix} 4 & 7 \\ 7 & 6 \end{pmatrix}$

b) $B^t = \begin{pmatrix} 0 & X & Y \\ -4 & 0 & 2Z \\ 2 & 1-Z & 0 \end{pmatrix}$ e $-B \begin{pmatrix} 0 & 4 & -2 \\ -X & 0 & -1+Z \\ -Y & -2Z & 0 \end{pmatrix} \rightarrow \boxed{X=4}$ $\boxed{Y=-2}$ $\boxed{Z=-1}$ $B \begin{pmatrix} 0 & -4 & 2 \\ 4 & 0 & 2 \\ -2 & -2 & 0 \end{pmatrix}$

$1-Z = -2Z \rightarrow 1 = -2Z+Z$ $1=-Z \rightarrow Z=-1$

9) $\begin{pmatrix} 3X & 3Y \\ 3Z & 3t \end{pmatrix} = \begin{pmatrix} 4+X & 6+X+Y \\ Z+t-1 & 2t+3 \end{pmatrix}$

$3X = 4+X \rightarrow 2X=4 \rightarrow \boxed{X=2}$

$3Y = 6+X+Y \rightarrow 2Y = 6+2 \rightarrow 2Y=8 \rightarrow \boxed{Y=4}$

$3Z = Z+t-1 \rightarrow 2Z+3-1 \rightarrow 2Z=2$ $\boxed{Z=1}$

$3t = 2t+3 \rightarrow \boxed{t=3}$

10) a)

$\begin{pmatrix} \cos\theta & \operatorname{sen}\theta \\ -\operatorname{sen}\theta & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} \cos\theta & -\operatorname{sen}\theta \\ \operatorname{sen}\theta & \cos\theta \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\begin{pmatrix} \cos\theta^2 + \operatorname{sen}^2 & \cos\cdot\operatorname{sen} + \operatorname{sen}\cdot\cos \\ -\operatorname{sen}\cdot\cos + \cos\cdot\operatorname{sen} & -\operatorname{sen}^2 + \cos^2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\rightarrow \cos^2 + \operatorname{sen}^2 = 1$

b)

$\begin{pmatrix} 1 & 0 & X \\ 0 & 1/\sqrt{2} & Y \\ 0 & 1/\sqrt{2} & Z \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ X & Y & Z \end{pmatrix}$

$X = 0$
$Y = -1/\sqrt{2}$
$Z = 1/\sqrt{2}$

$\begin{pmatrix} 1+0+X^2 & 0+0+X.Y & 0+0+X.Z \\ 0+0+X.Y & 0+(1/\sqrt{2})^2+Y^2 & 0+(1/\sqrt{2})^2+Y.Z \\ 0+0+Z.X & 0+(1/\sqrt{2})^2+Z.Y & 0+(1/\sqrt{2})^2+Z^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$