Lista 5 de G.A — Jeann Victor Batista

①

a) $\vec{AB}+\vec{BF}=\vec{AF} \rightarrow \vec{BF}=AF-AB.\ \vec{BF}=\vec{F}-\vec{b}$

b) $\vec{AG}=\vec{AC}+\vec{CG} \longrightarrow \vec{CG}=\vec{BF}\quad \vec{AG}=\vec{AC}+\vec{BF} \rightarrow \vec{AG}=\vec{AC}+\vec{F}-\vec{b}$

$\vec{AG}=\vec{C}+\vec{F}-\vec{b}$

c) $\vec{AE}=\vec{BF} \rightarrow \vec{AE}=\vec{AF}-\vec{AB}$

D) $\vec{BG}=\vec{BC}+\vec{CG}\quad \| \vec{BG}=\vec{BC}+\vec{F}-\vec{b}\ \| \vec{BG}=\vec{AC}-\vec{AB}+\vec{F}-b \rightarrow \vec{BG}=\vec{C}-\vec{B}+\vec{F}-\vec{b}$

$AB+BC=AC \rightarrow BC=AC-AB\qquad \vec{BG}=-2\vec{B}+\vec{C}+\vec{F}$

E) $\vec{HB}=\vec{HE}+\vec{EA}+\vec{AB}$

$\vec{HE}=\vec{DA}=\vec{CB}=-\vec{BC}=\quad AB+BC=AC \rightarrow BC=AC-AB \rightarrow -BC=-AC+AB$

$\vec{EA}=\vec{FB}=-\vec{BF}\quad \vec{AB}+\vec{BF}=\vec{AF}\quad BF=\vec{AF}-\vec{AB}\quad -BF=-\vec{AF}+\vec{AB}$

$-AC+AB-AF+AB+AB\quad \boxed{3AB-AC-AF}$

F) $\vec{AB}+\vec{FG} \rightarrow \vec{AB}+\vec{BC}=AC\qquad \vec{AB}+\vec{AC}-\vec{AB} \rightarrow \vec{AC}$

$BC\qquad BC=\vec{AC}-\vec{AB}$

G) $\vec{AD}+\vec{HG} \rightarrow \vec{BC}+\vec{AB} \rightarrow \vec{AC}-\vec{AB}+\vec{AB}$

$\vec{BC}+\vec{DC}\qquad \vec{AB}+\vec{BC}=AC\qquad \vec{AC}$

H) $\vec{HF}+\vec{AG}-\vec{EF} \rightarrow \cancel{\vec{AC}}-\cancel{\vec{AB}}+\cancel{\vec{AB}}+\vec{AC}+\vec{BF}+\vec{BC}-\cancel{\vec{AC}}=\|\ \vec{AC}+\vec{BF}+\vec{BC}\ \| \rightarrow \vec{AC}+\vec{AF}+\vec{AC}-\vec{AB}$

$\vec{AG}=\vec{C}+\vec{F}-\vec{b}\qquad \vec{C}+\vec{F}-\vec{b}+\vec{C}-\vec{b}$

$\vec{HF}=\vec{HE}+\vec{EF} \rightarrow \vec{DA}+\vec{EF} \rightarrow \vec{CB}+EF \rightarrow -\vec{CB}+EF \rightarrow \vec{BC}+\vec{EF}$

$\vec{AB}+\vec{BC}=\vec{AC} \rightarrow \vec{BC}=\vec{AC}-\vec{AB}\ \|\ \vec{AC}-\vec{AB}+\vec{AB}$

$-\vec{EF}=-\vec{AB} \rightarrow -\vec{AB}+\vec{AC}=\vec{BC}\quad -\vec{AB}=\vec{BC}+\vec{AC}$

$-2\vec{b}+2\vec{C}+\vec{F}$

1) $\vec{2AD} - \vec{FG} - \vec{BH} + \vec{GH}$

$2AD = 2BC = 2.(AB+BC = AF)$ $(2(AF-AB))$.

$-\vec{FG} = -BC = -AF+AB$

$-\vec{BH} = 3AB - AC - AF$

$\vec{GH} = -\vec{AB}$

$2\cancel{AF} - 2\cancel{AB} - \cancel{AF} + AB + 3\cancel{AB} - \vec{AC} - \cancel{\vec{AF}} - \cancel{\vec{AB}}$

$-\vec{AB} - \vec{AC}$

2 a) $\vec{DF}$

$\vec{DC} + \vec{CF} \rightarrow \vec{DC} + 2\vec{OF}$ $\vec{DC} + 2\vec{DE}$

b)

$\vec{DA}$

$\vec{DC} + \vec{CO} + \vec{OB} + \vec{BA} \rightarrow \vec{DC} + \vec{DE} + \vec{DC} + \vec{DE} \rightarrow \vec{2DC} + \vec{2DE}$

c)

$\vec{DB}$

$\vec{DC} + \vec{CO} + \vec{OB}$ $\vec{DC} + \vec{DE} + \vec{DC} \rightarrow \vec{2DC} + \vec{DE}$

d)

$\vec{DO}$

$\vec{DC} + \vec{CO} \rightarrow \vec{DC} + \vec{DE}$

e)

$\vec{EC}$

$-\vec{ED} + \vec{DC}$ $\vec{EO} + \vec{OC}$ $-\vec{DC} - \vec{DE}$

F)

$\vec{EB}$

$2\vec{EO} \rightarrow 2DC$

g)

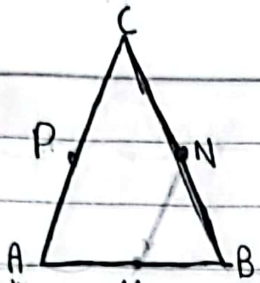
$\vec{OB}$

$\vec{OB} = \vec{DC}$

H)

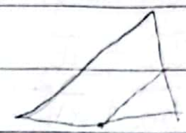
$\vec{AF} \rightarrow -\vec{DC}$

③



$\vec{BP} = \vec{BA} + AP$

$\vec{AB} + 1/2\,AC$



$\vec{AN} = \vec{AB} + \vec{BN}$

$\vec{AN} = \vec{AB} - \vec{NB}$

$\vec{AN} = \vec{AB} - (Nm + MP)$

$\vec{AN} = \vec{AB} - \vec{NM} + \vec{MB}$

$\vec{AN} = \vec{AB} - \vec{PA} + MB$

$\vec{AN} = \vec{AB} + 1/2\vec{AC} - 1/2\,AB$

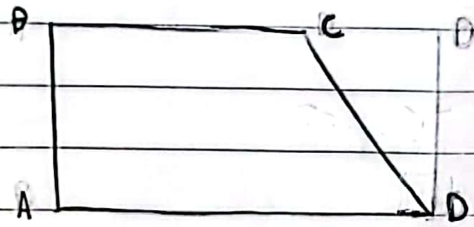
$\vec{AN} = 1/2\,AC + \vec{1/2AB}$

$CM = \vec{CA} + \vec{AM}$

$CM = -\vec{AC} + 1/2\,\vec{AB}$

④

a)



$\vec{AD} = 5\vec{v}$

$\vec{BC} = 3\vec{v}$

$\vec{AB}\ 2\vec{v}$

$\vec{CD} = \vec{AD} + \vec{DC} = \vec{AC} \longrightarrow \vec{AD} = \vec{AC} - \vec{DC}$ $\quad \vec{AD} = \vec{AB} + \vec{BC} - \vec{DC} \quad = -\vec{DC} = \vec{AD} - \vec{AB} - \vec{BC}$

$\vec{BD} = \vec{BA} + \vec{AD} = -\vec{AB} + \vec{AD} = 5v - 2\vec{v}$

$\vec{CA} = CB + BA \longrightarrow -\vec{CB} + (-\vec{BA}) = -3v - 2\vec{v}$

$\vec{CD} \longrightarrow -\vec{DC} = \vec{5v} - 2v - 3\vec{v} \qquad -\vec{DC} = 2v - 2\vec{v}$.

b) $\vec{AD}$ e $\vec{BC}$ são paralelos.

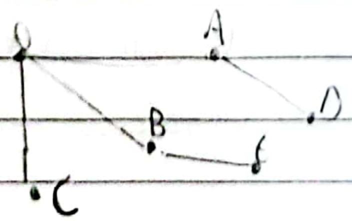
⑤ $\vec{a} = \vec{OA}$

$\vec{b} = \vec{OB}$

$\vec{c} = \vec{OC}$

$1/4\,\vec{c} = \vec{AD}$

$5/6\,\vec{a} = \vec{BE}$



$\vec{DA} + \vec{AO} + \vec{OB} + \vec{BE}$

$-\vec{AD} - \vec{OA} + \vec{OB} + \vec{BE}$

$-\frac{1}{4}\vec{c} - \vec{a} + \vec{b} + \frac{5}{6}\vec{a} \longrightarrow$

$5/6\,\vec{a} - \vec{a} + \vec{b} - 1/4\,\vec{c}$

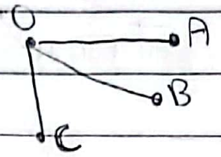
$-\frac{1}{6}\vec{a} + \vec{b} - \frac{1}{4}\vec{c}$

tilibra

6) $\vec{OA} = \vec{a} + 2\vec{b}$

$\vec{OB} = 3\vec{a} + 2\vec{b}$

$\vec{OC} = 5\vec{a} + x\vec{b}$



$\vec{AO} + \vec{OC} = \vec{AC}$

$\vec{BO} + \vec{OC} = \vec{BC}$

$-\vec{OA} + \vec{OC} = \vec{AC} \rightarrow -\vec{a} - 2\vec{b} + 5\vec{a} + x\vec{b} \rightarrow 4\vec{a} - 2\vec{b} + x\vec{b} = 0$

$-\vec{OB} + \vec{OC} = \vec{BC} \rightarrow -3\vec{a} - 2\vec{b} + 5\vec{a} + x\vec{b} \quad 2\vec{a} - 2\vec{b} + x\vec{b} = 0$

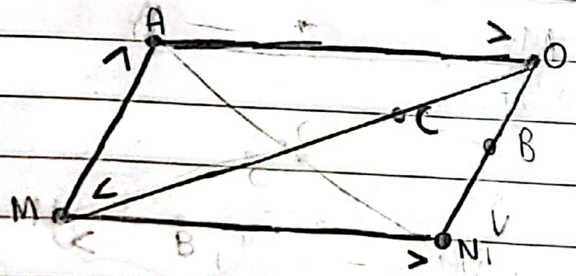
$4\vec{a} \ \overrightarrow{-2+xb} = 0$

$2\vec{a} \ \overrightarrow{-2+xb} = 0$

$\vec{AC} = K.\vec{BC}$

$\Rightarrow K = \dfrac{BC}{AC}$

7)



$\vec{OB} = 1/n \ \vec{ON}$

$\vec{OC} = \cancel{1/1+n} \ \vec{OM}$

$\vec{AC} = \vec{AO} + \vec{OC}$

$\vec{AC} = \vec{MN} + \vec{OC}$

$\vec{AC} = \vec{ON} - \vec{OM} + \vec{OC}$

$\vec{AC} = \vec{ON} - \vec{OM} + \dfrac{1}{1+n}\vec{OM}$

$\vec{AB} = \vec{AO} + \vec{OB}$

$\vec{AB} = \vec{MN} + \vec{OB}$

$\vec{AB} = \vec{ON} - \vec{OM} + \vec{OB}$

$\vec{AB} = \vec{ON} - \vec{OM} + \dfrac{1}{n}\vec{ON}$

$\vec{BC} = \vec{BO} + \vec{OC}$

$-\vec{OB} + \vec{OC}$

$-\dfrac{1}{n}\vec{ON} + \dfrac{1}{1+n}\vec{OM}$

$$\left\{\vec{ON} - \vec{OM}\left(\frac{1}{1+n} - 1\right), \vec{ON}\left(1 + \frac{1}{n}\right) - \vec{OM}\right\}$$ (8)

$$y\left(\vec{ON} - \vec{OM}\left(\frac{1}{1+n} - 1\right)\right) = \vec{ON}\left(1 + \frac{1}{n}\right) - \vec{ON}$$

$$y.\vec{ON} - y\,\vec{OM}\left(\frac{1}{1+n} - 1\right) = \vec{ON}.\left(1 + \frac{1}{n}\right) - \vec{OM}$$

$$\begin{cases} y = 1 + \frac{1}{n} \rightarrow y = \frac{n+1}{n} \\ y.\left(\frac{1}{1+n} - 1\right) = 1 \rightarrow y = \frac{1}{\frac{1}{1+n} - 1} \rightarrow y = \frac{1}{\frac{1}{1} \frac{-1+n}{1+n}} \end{cases}$$ (10)

$$y = \frac{1}{\frac{n}{1+n}} \rightarrow y = 1 . \frac{1+n}{n} \rightarrow y = \frac{1+n}{n}$$

(d)

Portanto é possivel achar um Sistema LD quando $n \neq \{0, -1\}$
Pois se forem 0 ou -1 é LI

(8) $x(\vec{2u}+\vec{v})+y(\vec{u}-\vec{2v})=0$

$\vec{2xu}+\vec{xv}+\vec{yu}-\vec{2yv}=\vec{0}$

$\vec{u}(\vec{2x}+\vec{y})+\vec{v}(\vec{x}+\vec{2y})=0$

$\begin{cases} 2x+y=0 \\ x-2y=0 \end{cases} \times 2 \rightarrow \begin{cases} 4x+2y=0 \\ x-2y=0 \end{cases} \rightarrow \begin{matrix} 5x=0 \\ x=0 \end{matrix}$

$0-2y=0$ $\quad y=0$ $\quad \begin{cases} x=0 \\ y=0 \end{cases}$

$-2y=0$

(9) a) $\vec{u}+\vec{v}$

$\vec{u}-\vec{v}+\vec{w}$

$\vec{u}+\vec{v}+\vec{w}$

$x(\vec{u}+\vec{v})+y(\vec{u}-\vec{v}+\vec{w})+z(\vec{u}+\vec{v}+\vec{w})=0$

$\vec{xu}+\boxed{\vec{xv}}+\vec{yu}-\vec{yv}+\vec{yw}+\vec{zu}+\vec{zv}+\vec{zw}=0$

$\vec{u}(x+y+z)+\vec{v}(x-y+z)+\vec{w}(y+z)=0$

$\begin{cases} x+y+z=0 \\ x-y+z=0 \\ y+z=0 \end{cases} \rightarrow \begin{Bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & 1 & 1 \end{Bmatrix} \begin{matrix} 1 & 1 \\ 1 & -1 \\ 0 & 1 \end{matrix} \rightarrow \{-1+0+1\}-\{0+1+1\}$

$0-2 \neq 2$

É L.I pois Det $\neq 0$

b) $t=a\vec{u}+b\vec{v}+c\vec{w}$

$(\vec{u}+a\vec{u}+\boxed{b\vec{v}}+c\vec{w})+(\vec{v}+a\vec{u}+\boxed{b\vec{v}}+c\vec{w})+(\vec{w}+a\vec{u}+\boxed{b\vec{v}}+c\vec{w})=0$

$\vec{u}(1+a+a+a)+\vec{v}(1+b+b+b)+\vec{w}(1+c+c+c)=0$

$\begin{cases} 3a+1=0 \\ 3b+1=0 \\ 3c+1=0 \end{cases} \rightarrow \begin{cases} 3a=1 \\ 3b=1 \\ 3c=1 \end{cases} \rightarrow \begin{cases} a=-1/3 \\ b=-1/3 \\ c=-1/3 \end{cases}$

tilibra

10) a) $\vec{AB} = \vec{B} - \vec{A} \rightarrow (1-1, 0-3, -1-2) \rightarrow (0, -3, -3)$

$\vec{BC} \rightarrow \vec{C} - \vec{B} \rightarrow (1-1, 1-0, 0-(-1)) \rightarrow (0, 1, 1)$

$\vec{CA} \rightarrow \vec{A} - \vec{C} \rightarrow (1-1, 3-1, 2-0) \rightarrow (0, 2, 2)$

b) $\vec{AB} \rightarrow \vec{B} - \vec{A} \quad (1-1, 0-3, -1-2) \rightarrow (0, -3, -3)$

$\vec{BC} \rightarrow \vec{C} - \vec{B} \rightarrow (1-1, 1-0, 0-(-1)) \rightarrow (0, 1, +1)$

$2/3 \cdot (0, 1, +1) \rightarrow \left(0, \frac{2}{3}, \frac{2}{3}\right)$

$\left(0+0 \,/\, -3+\frac{2}{3} \,/\, -3+\frac{2}{3}\right) \rightarrow \left(0, -7/3, -7/3\right)$

C) $\vec{C} + \frac{1}{2}\vec{AB} \rightarrow \vec{C} = (1, 1, 0)$

$\vec{B} - \vec{A} = (1-1, 0-3, -1-2) \rightarrow (0, -3, -3)$

$1/2 \cdot (0, -3, -3) = \rightarrow (0, -3/2, -3/2)$

$(1, 1, 0) + (0, -3/2, -3/2) \rightarrow (1+0, 1-3/2, 0-3/2)$

$\rightarrow (1, -1/2, -3/2)$

D) $\vec{A} - 2\vec{BC} \rightarrow \vec{A} = (1, 3, 2)$

$2 \cdot \vec{BC} = \vec{C} - \vec{B} \rightarrow (1-1, 1-0, 0-(-1)) \rightarrow 2 \cdot (0, 1, 1) \rightarrow (0, 2, 2)$

$\vec{A} - 2\vec{BC}$

$\rightarrow (1-0, 3-2, 2-2) \rightarrow (1, 1, 0)$

11) a) $\{(2,3),(0,2)\}$

$$\begin{Bmatrix} 2 & 3 \\ 0 & 2 \end{Bmatrix} \quad 4-0 \longrightarrow Det \neq 0$$

Logo é L.I

b) $\{(3,0),(-2,0)\}$

$$\begin{Bmatrix} 3 & 0 \\ -2 & 0 \end{Bmatrix} \quad 0-0 \rightarrow Det = 0$$

Logo é L.D.

C) $\{(2,3,4),(0,3,3)\}$

$2,3,4 = X(0,3,3) \rightarrow 2X=0 \quad 3X=4 \quad | \; X=0 \quad X = 3|4$

$2,3,4 = 0X, 3X, 3X \quad\quad 3X=3 \quad\quad | \; X=1$

Como X não tem valor definido

É L.I

d) $\{(1,-1,2),(1,1,0),(1,-1,1)\}$

$\quad\quad A \quad\quad B \quad\quad C$

$$\begin{Bmatrix} 1 & -1 & 2 \\ 1 & 1 & 0 \\ 1 & -1 & 1 \end{Bmatrix} \begin{matrix} 1 & -1 \\ 1 & 1 \\ 1 & -1 \end{matrix} \quad \{1+0-2\} - \{2+0-1\}$$

$$-1 \quad -1$$

$$-2 \quad\quad Det \neq 0$$

Logo é L.I

e)

$\{(1,-1,1),(-1,2,1),(-1,2,2)\}$

$$\begin{Bmatrix} 1 & -1 & 1 \\ -1 & 2 & 1 \\ -1 & 2 & 2 \end{Bmatrix} \begin{matrix} 1 & -1 \\ -1 & 2 \\ -1 & 2 \end{matrix} \quad \{4+1-2\} - \{2+2+2\}$$

$$3 - 6$$

$$-3 \quad \neq 0$$

Logo L.I

f) $\{(1,0,1),(0,0,1),(2,0,5)\}$

$$\begin{Bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 2 & 0 & 5 \end{Bmatrix} \begin{matrix} 1 & 0 \\ 0 & 0 \\ 2 & 0 \end{matrix}$$

$0\,0\,0 - 0{+}0{+}0$

0 Logo Det $= 0$

Logo é L.D.

12) a) $a\vec{u} + b\vec{v} = w$

$a(2,-1) + b(1,-1) = (1,1)$

$(2a,-a) + (b,-b) = (1,1)$

$2a + b = 1$

$-a - b = 1 \rightarrow a = 2$

$-2 - b = 1 \rightarrow b = -3.$

$-b = 3$

$a = 2$

$b = -3.$

$W = 2\vec{u} + (-3)\vec{v}$

$(1,1) = (4,-2) + (-3,+3)$

b)

$z = x\vec{a} + y\vec{b} + z\vec{c}$

$(1,2,3) = x(1,1,1) + y(0,1,1) + z(1,1,0)$

$(1,2,3) = (x|x|x) + (0|y|y) + (z|z|0)$

$$\begin{cases} x + 0 + z = 1 \\ x + y + z = 2 \\ x + y + 0 = 3 \end{cases} \rightarrow \begin{matrix} 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{matrix}$$

$1 - 2$

$-1 \rightarrow$ Det 1 $= -1$

Det 2 $= \begin{Bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 3 & 1 & 0 \end{Bmatrix} \begin{matrix} 1 & 0 \\ 2 & 1 \\ 3 & 1 \end{matrix} \rightarrow$

$0{+}0{+}2 - 3{+}1{+}0$

$2 - 4$

Det 2 $= -2$

Det 3 $= \begin{Bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 3 & 0 \end{Bmatrix} \begin{matrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{matrix}$

$1{+}3 - 2{+}3$

$4 - 5$

Det 3 $= -1$

$$\text{Det } 4 = \begin{matrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & 3 \end{matrix} \begin{matrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{matrix}$$

$3+1 - 1+2$

$4 - 3$

$\text{Det } 4 = 1$

$X = -2|-1 = \quad X = 2$

$Y \quad -1|-1 = \quad Y = 1$

$Z \quad 1|-1 = \quad Z = -1$

13) $\vec{U} = x \cdot \vec{V}$ $\{ (1, m-1, m) = (Xm, X2n, 4X)$

$1 = Xm$ $\quad m = 1/X$

$m-1 = 2Xn \longrightarrow \frac{1}{X} = 4X \longrightarrow 1 = 4X \cdot X \longrightarrow 1 = 4X^2 \longrightarrow \frac{1}{4} = X^2$

$m = 4X$

$X^2 = -\frac{1}{2}$ ou $\frac{1}{2}$ $\quad m = 4 \cdot \frac{1}{2} \quad m = \frac{4}{2} \quad m = \pm 2$

Para $= m = 2 \quad X = \frac{1}{2}$ e $n = 2-1 = 2 \cdot \frac{1}{2} \cdot n$

$1 = 1 \cdot n \longrightarrow n = 1$ | $m = 2 \quad X = 1/2$ $\quad n = 1$

Para $m = -2 \quad X = -\frac{1}{2} \quad -2-1 = 2 \cdot -\frac{1}{2} \cdot n$

$-3 = -1 \cdot n \longrightarrow n = 3$

Para $m = 2 \quad X = 1/2$

b) $\vec{u} = x.\vec{v}$ $(1, m, n+1) = x.(m, n+1, 8)$

$1 = mx$ $1 = mx$ $1 = mx$

$m = (n+1)x \longrightarrow m = (nx + x).x$ $m = (8x - 1 + x).x$

$n+1 = 8x$ $n = 8x - 1$ $n = 8x - 1$

$8x^2 = m$ $1 = m.x$ $1 = 8x^2.x$ $1 = 8.x^3$ $\frac{1}{8} = x^3$ $x = \frac{1}{2}$ ou $-\frac{1}{2}$

$1 = m.\frac{1}{2} \rightarrow \frac{1}{\frac{1}{2}} = m$ $\frac{1.2}{1} = m$ $m = 2$

$n+1 = 8.\frac{1}{2} \rightarrow n+1 = 4$ $n = 3$

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$$\begin{array}{ccc|cc} m & -1 & m^2+1 & m & -1 \\ m^2+1 & m & 0 & m^2+1 & m \\ m & 1 & 1 & m & 1 \end{array}$$

$m^2 + 0 + (m^2+1).(m^2+1) \rightarrow m^2 + m^4 + m^2 + m^2 + 1 \rightarrow m^4 + 3m^2 + 1$

$(m^2+1).m^2 + 0\ (-m^2-1) \rightarrow m^4 + \cancel{m^2} - \cancel{m^2} - 1$ $m^4 - 1$

$m^4 + 3m^2 + 1 + -m^4 + 1$

$3m^2 + 2 \neq 0$

São LI

/ /

15) a) 1 1 0 1 1 $\quad 0+1+0-0+1-1$

1 0 1 1 0 $\quad$ Det = 1

1 1 -1 1 1 $\quad$ Det ≠ 0 Logo com isso provo que é L.I

b)

$\vec{U}(x,y,z) = x f_1 + y f_2 + z f_3$

$\vec{U}(x,y,z) = 2 f_1 + 3 f_2 + 7 f_3$

$\vec{U} = 2(1,1,0) + 3(1,0,1) + 7(1,1,-1)$

$\vec{U} = (2,2,0) + (3,0,3) + (7,7,-7)$

$\vec{U} = (12,9,-4)_B$

c) $\vec{U}(2,3,7) \longrightarrow C + (f_1, f_2, f_3)$

$C(l_1,l_2,l_3) = \begin{cases} 1\,1\,0 = f_1 \\ 1\,0\,1 = f_2 \\ 1\,1\,-1 = f_3 \end{cases}$ $\quad l_2 \leftarrow l_2 - l_1$, $\quad l_3 \leftarrow l_3 - l_1$ $\quad \begin{cases} 1\,1\,0 = f_1 \\ 0\,-1\,1 = f_2 - f_1 \\ 0\,0\,-1 = f_3 - f_1 \end{cases}$

$\begin{cases} 1\,1\,0 = f_1 \\ 0\,-1\,+1 \quad f_2 - f_1 \\ 0\,0\,-1 = f_3 - f_1 \end{cases}$ $\quad l_1 \leftarrow l_1 + l_2 \longrightarrow$ $\quad \begin{matrix} 1\,0\,1 & f_1 + f_2 \\ 0\,-1\,1 & f_2 - f_1 \\ 0\,0\,-1 & f_3 - f_1 \end{matrix}$ $\quad l_1 \leftarrow l_1 + l_3$, $\quad l_2 \leftarrow l_2 + l_3$

$\begin{matrix} 1\,0\,0 & f_1 + f_2 + f_3 \\ 0\,-1\,0 & f_2 - f_1 + f_3 \\ 0\,0\,-1 & f_3 - f_1 \end{matrix} \longrightarrow \begin{matrix} 1\,0\,0 & f_1 + f_2 + f_3 \\ 0\,1\,0 & f_1 - f_2 - f_3 \\ 0\,0\,1 & f_1 - F_3 \end{matrix}$