Determining the tau decay constant for a guitar sound wave

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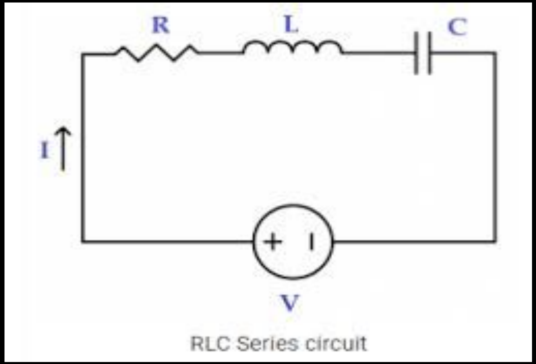
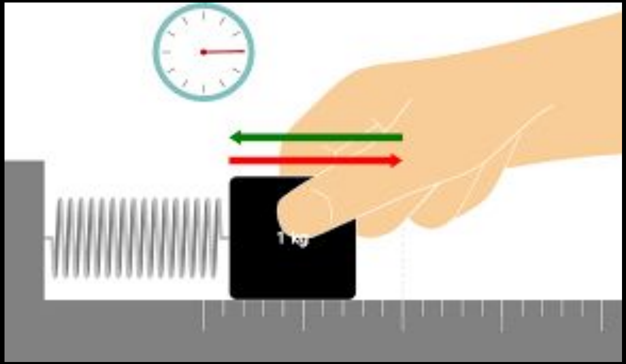
Damped Linear Oscillators

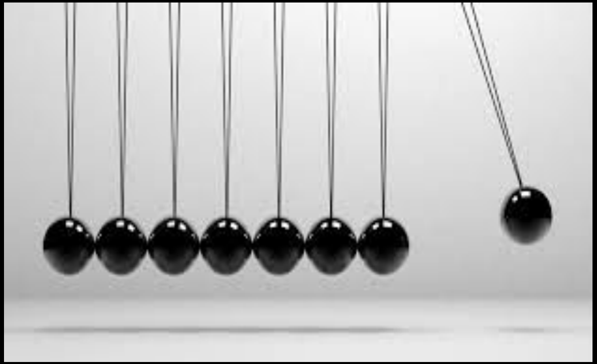
RLC Circuit:



Vibrating Spring:



Pendulum:



Mathematics of Damped Linear Oscillators

3. Damped Free Vibrations

$$mr^2 + cr + k = 0$$

Solutions to characteristic equation:

$$r_1, r_2 = \frac{-c \pm \sqrt{c^2 - 4km}}{2m} = \frac{c}{2m}\left(-1 \pm \sqrt{1 - \frac{4km}{c^2}}\right)$$

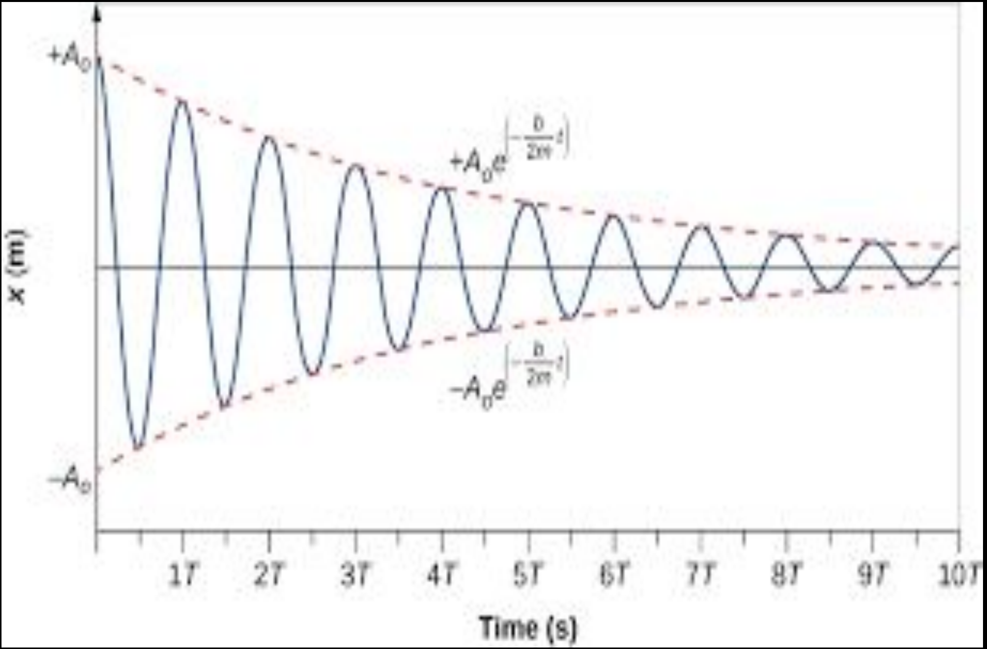
$c^2 - 4km > 0, \quad y = Ae^{r_1 t} + Be^{r_2 t}$ overdamped

$c^2 - 4km = 0, \quad y = (A + Bt)e^{-ct/2m}$ critically damped

$c^2 - 4km < 0, \quad y = e^{-ct/2m}(A \cos \mu t + B \sin \mu t)$ underdamped

m, c, k are positive → The solution y decays as t goes to infinity *regardless* the values of ***A*** and ***B***

Damping gradually dissipates energy!



Guitar sound wave's behavior

Fundamental equations and assumptions:

1. $f = (\sqrt{(T/U)})/2L$

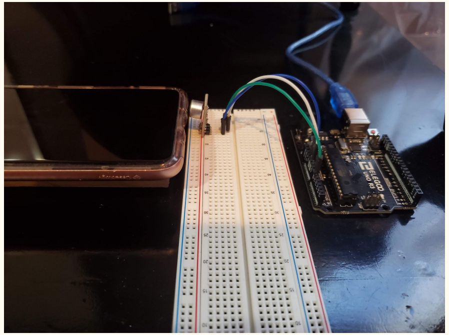
2. Amplitude decay= $A0e^{-t/tau}$

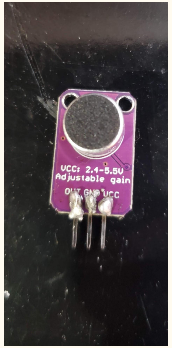
3. Tau=1/n*gamma

4. $A0=(a^2+b^2)^{1/2}$
    a. a and b are constant found after Solving the characteristic equation

Hypothesis

- Measure the intensity of a sound wave produced by guitars of varying linear mass density
- the initial amplitude and frequency for a steel guitar string should be higher than a brass guitar string
- the decay constant tau should be higher for each nth increase in harmonic
- because steel has a lower linear mass density than brass and an increase in harmonic correlates to a greater number of oscillations per second.





Method

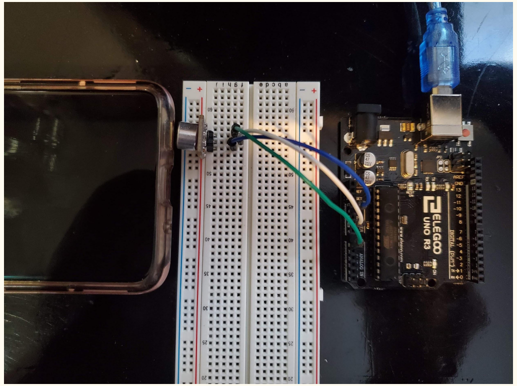
Acoustic/Brass

Electric/Steel

Equipment

- Arduino Uno R3 controller board
- Breadboard
- Jumper Wires
- Electret Microphone Amplifier MAX4466 Module Adjustable Gain Blue Breakout Board for Arduino
- Phone with audio recordings
- Phone stand
- Power source
- Laptop with Arduino software
- Acoustic and Electric guitars

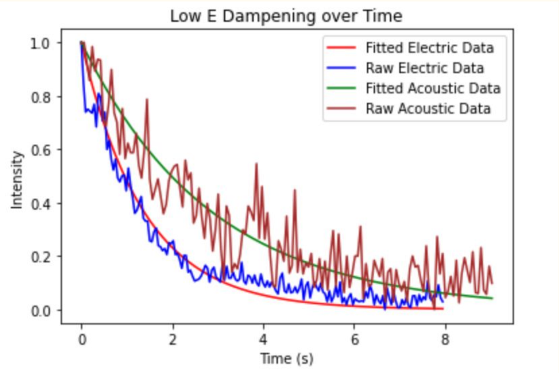
Experimental Setup and Procedure

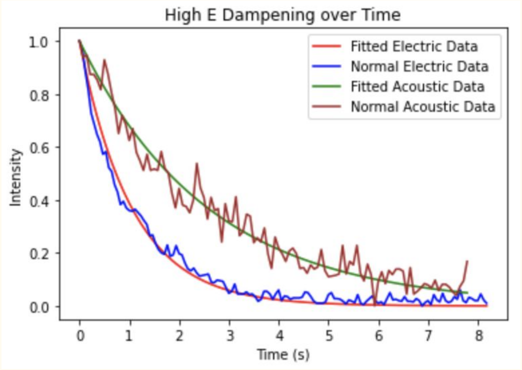


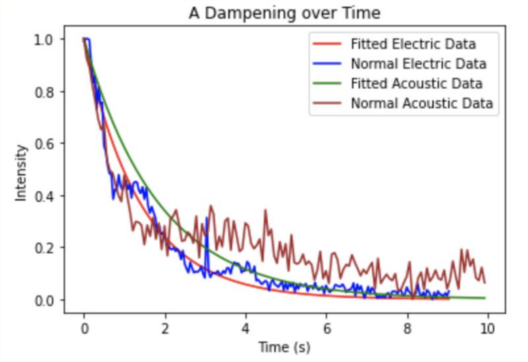
- Soldering
- Wiring
- Arduino Code
- Experimental Setup
- Data collecting
- Analysis

```
const int sampleWindow = 50; // Sample window width in mS (50 mS = 20Hz)
unsigned int sample;

void setup()
{
   Serial.begin(9600);
}


void loop()
{
   unsigned long startMillis= millis();  // Start of sample window
   unsigned int peakToPeak = 0;   // peak-to-peak level

   unsigned int signalMax = 0;
   unsigned int signalMin = 1024;

   // collect data for 50 mS
   while (millis() - startMillis < sampleWindow)
   {
      sample = analogRead(0);
      if (sample < 1024)  // toss out spurious readings
      {
         if (sample > signalMax)
         {
            signalMax = sample;  // save just the max levels
         }
         else if (sample < signalMin)
         {
            signalMin = sample;  // save just the min levels
         }
      }
   }
   peakToPeak = signalMax - signalMin;  // max - min = peak-peak amplitude
   double volts = (peakToPeak * 5.0) / 1024;  // convert to volts

   Serial.println(volts);
}
```

Analysis
[Code]



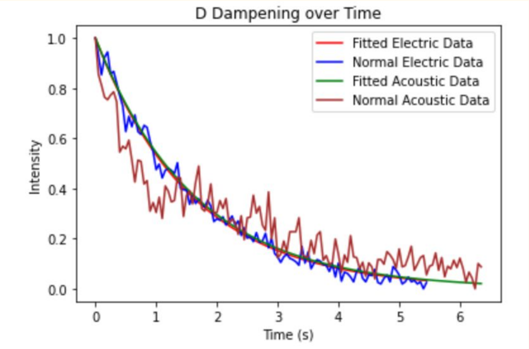


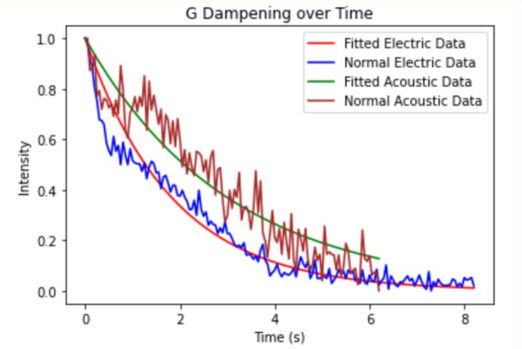


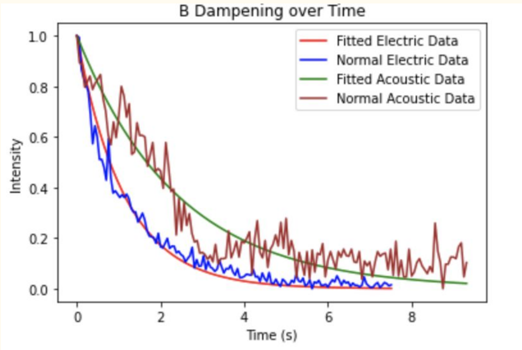
Guitar Type	Tao - Low E	Tao - High E	Tao - A
Electric/Steel	1.37	1.05	1.40
Acoustic/Brass	2.85	2.59	1.85

Plots with Normalized, and Fitted Curves for Acoustic and Electric Guitars, Intensity on Y-axis with respect to time on X-axis.

Analysis







Guitar Type	Tao - D	Tao - G	Tao - B
Electric/Steel	1.59	1.83	1.13
Acoustic/Brass	1.63	3.01	2.4

Plots with Normalized, and Fitted Curves for Acoustic and Electric Guitars, Intensity on Y-axis with respect to time on X-axis.

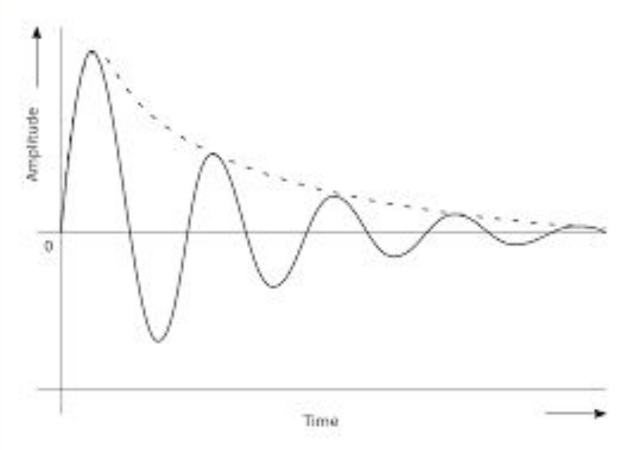
Analysis of Amplitudes/Frequencies

Please Note Initial Amplitude Based on Amplitude Normalized to 1 Cm, intended frequencies from MTU.edu

Note	Electric Initial Amplitude (Cm)	Acoustic Initial Amplitude (Cm)	Electric Error from Intended Frequency (%)	Acoustic Error from Intended Frequency (%)
Low E	.877	.92	1.1%	.7%
A	0.94	0.65	.9%	.9%
D	.99	0.74	1.9%	.1%
G	0.86	0.98	1.5%	.3%
B	0.92	0.94	1.2%	.4%
High E	0.92	0.96	.6%	.2%

Conclusion

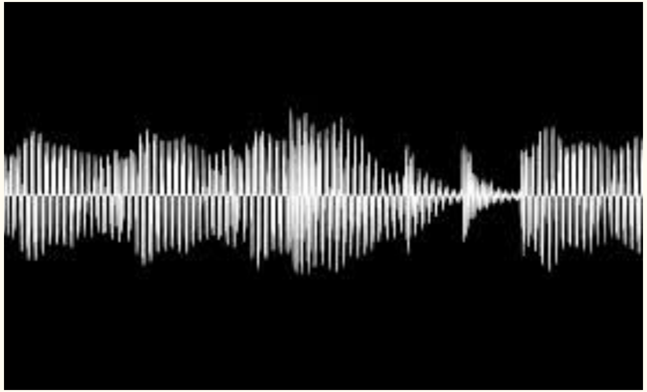
The experimentally collected data supports our hypothesis that the linear mass density and harmonic number are proportional to a guitar sound waves decay constant and oscillation frequency.



Sources of error

Systematic errors:

1. Playing a pre recorded audio file
2. Modeling the string-guitar system without any internal resistance
3. Assuming gamma is a constant
4. Acoustic vs Electric Build (ie. Acoustic is Hollow-Bodied)



Random errors

1. The Arduino's connection to the computer
2. The position/strength of the guitar string player's finger
3. The efficacy of the Arduino microphone
4. Qualities of the medium (ie. temperature, humidity)

Implications and magnitude of results

Our results illustrate that a guitar sound wave can be accurately modeled as a damped linear oscillator (one variable), and has a tau decay constant proportional to intrinsic properties of the sound wave!

- There is a higher average Tau for the acoustic guitar

- Guitar/String design

Future research

1. Conduct the same experiment but vary the temperature in the environment by an integer number of kelvin

2. Conduct the experiment in fluids of varying viscosities-water, oil, etc..

Thank you for listening to our presentation!

Questions?

References?