Contents

Chapter 1

Introduction to Vectors

1.1 Vectors and Linear Combinations

REVIEW OF THE KEY IDEAS

1. A vector $\mathbf{v}$ in two-dimensional space has two components v_1 and v_2.
2. $\mathbf{v} + \mathbf{w} = (v_1 + w_1, v_2 + w_2)$ and $c\mathbf{v} = (cv_1, cv_2)$ are found a component at a time.
3. A linear combination of three vectors $\mathbf{u}$ and $\mathbf{v}$ and $\mathbf{w}$ is $c\mathbf{u} + d\mathbf{v} + e\mathbf{w}$.
4. Take all linear combination of $\mathbf{u}$, or $\mathbf{u}$ and $\mathbf{v}$, or $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$. In three dimensions, those combinations typically fill a line, then a plane, and the whole space $\mathbf{R}^3$.

1.2 Lengths and Dot Products

REVIEW OF THE KEY IDEAS

1. The dot product $\mathbf{v} \cdot \mathbf{w}$ multiplies each component v_i by w_i and adds all $v_i w_i$.
2. The length $\|\mathbf{v}\|$ of a vector is the square root of $\mathbf{v} \cdot \mathbf{v}$.
3. $\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$ is a **unit vector**, its length is 1.
4. The cosine product is $\mathbf{v} \cdot \mathbf{w} = 0$ when vector $\mathbf{v}$ and $\mathbf{w}$ are perpendicular.
5. The cosine of θ (the angle between any nonzero $\mathbf{v}$ and $\mathbf{w}$) never exceeds 1:

$$\cos\theta = \frac{\mathbf{v} \cdot \mathbf{w}}{\|\mathbf{v}\|\|\mathbf{w}\|} \quad \text{(Schwarz inequality)} \quad |\mathbf{v} \cdot \mathbf{w}| \le \|\mathbf{v}\|\|\mathbf{w}\|$$

Problem 21 will produce the **triangle inequality** $\|\mathbf{v} + \mathbf{w}\| \le \|\mathbf{v}\| + \|\mathbf{w}\|$.

1.3 Matrices

1.3.1 Linear Equations

1.3.2 The Inverse Matrix

this part is so hard

1.3.3 Cyclic Differences

REVIEW OF THE KEY IDEAS

1. **Matrix times vector:** $\mathbf{Ax} = combination of the columns of A.$
2. The solution to $Ax = b is x = A^{-1}b, when A is an invertible matrix.$
3. The difference matrix A is inverted by the sum matrix $\mathbf{S} = \mathbf{A^{-1}}$.
4. The cyclic matrix **C** has no inverse. Its three columns lie in the same plane
   Those dependent columns add to the zero vector. $\mathbf{Cx} = \mathbf{0}$ has many solutions.
5. This section is looking ahead to key ideas. not fully explained yet.

Chapter 2

Solving Linear Equations

2.1 Vectors and Linear Equations

REVIEW OF THE KEY IDEAS

1.

2.2 The Idea of Elimination

Second lecture

Elimination

Back substitution

Here, we need to pay attention to the particularity of the matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix} \tag{2.1}$$

REVIEW OF THE KEY IDEAS

1.

2.3 Elimination Using Matrices

REVIEW OF THE KEY IDEAS

1.

2.4 Rules for Matrix Operations

REVIEW OF THE KEY IDEAS

1.

2.5 Inverse Matrices

REVIEW OF THE KEY IDEAS

1.

2.6 Elimination = Factorization: A = LU

REVIEW OF THE KEY IDEAS

1.

2.7 Transposes and Permutations

REVIEW OF THE KEY IDEAS

1.

Chapter 3

Vector Spaces and Subspaces

Chapter 4

Orthogonality

Chapter 5

Determinants

Chapter 6

Elgenvalues and Elgenvectors

Chapter 7

Linear Transformations

Chapter 8

Applications

Chapter 9

Numerical Linear Algebra

Chapter 10

Complex Vectors and Matrices