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Chapter 1

Introduction to Vectors

1.1 Vectors and Linear Combinations

1.2 Lengths and Dot Products

1.3 Matrices

1.3.1 Linear Equations

1.3.2 The Inverse Matrix

this part is so hard

1.3.3 Cyclic Differences

Chapter 2

Solving Linear Equations

2.1 Vectors and Linear Equations

2.2 The Idea of Elimination

Second lecture

Elimination

Back substitution

Here, we need to pay attention to the particularity of the matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix} \quad (2.1)$$

2.3 Elimination Using Matrices

2.4 Rules for Matrix Operations

2.5 Inverse Matrices

2.6 Elimination = Factorization: $A = LU$

2.7 Transposes and Permutations

Chapter 3

Vector Spaces and Subspaces

3.1 Spaces of Vectors

3.2 The Nullspace of A : Solving $Ax = 0$

3.3 The Rank and the Row Reduced Form

3.4 The Complete Solution to $Ax = b$

3.5 Independence, Basis and Dimension

3.6 Dimensions of the Four Subspaces

Chapter 4

Orthogonality

4.1 Orthogonality of the Four Subspaces

4.2 Projections

4.3 Least Squares Approximations

4.4 Orthogonal Bases and Gram-Schmidt

Chapter 5

Determinants

5.1 The Properties of Determinants

5.2 Permutations and Cofactors

5.3 Cramer's Rule, Inverse, and Volumes

Chapter 6

Eigenvalues and Eigenvectors

6.1 Introduction to Eigenvalues

6.2 Diagonalizing a Matrix

6.3 Applications to Differential Equations

6.4 Symmetric Matrices

6.5 Positive Definite Matrices

6.6 Similar Matrices

6.7 Singular Value Decomposition

Chapter 7

Linear Transformations

7.1 The idea of a Linear Transformation

7.2 The Matrix of a Linear Transformation

7.3 Diagonalization and the Pseudoinverse

Chapter 8

Applications

8.1 Matrices in Engineering

8.2 Graphs and Networks

8.3 Markov Matrices, Population, and Economics

8.4 Linear Programming

8.5 Fourier Series: Linear Algebra for Functions

8.6 Linear Algebra for Statistics and Probability

8.7 Computer Graphics

Chapter 9

Numerical Linear Algebra

9.1 Gaussian Elimination in Practice

9.2 Norms and Condition Numbers

9.3 Iterative Methods and Preconditioners

Chapter 10

Complex Vectors and Matrices

10.1 Complex Numbers

10.2 Hermitian and Unitary Matrices

10.3 The Fast Fourier Transform