Contents

Chapter 1

Introduction to Vectors

1.1 Vectors and Linear Combinations

REVIEW OF THE KEY IDEAS

1. A vector $\mathbf{v}$ in two-dimensional space has two components v_1 and v_2.
2. $\mathbf{v} + \mathbf{w} = (v_1 + w_1, v_2 + w_2)$ and $c\mathbf{v} = (cv_1, cv_2)$ are found a component at a time.
3. A linear combination of three vectors $\mathbf{u}$ and $\mathbf{v}$ and $\mathbf{w}$ is $c\mathbf{u} + d\mathbf{v} + e\mathbf{w}$.
4. Take all linear combination of $\mathbf{u}$, or $\mathbf{u}$ and $\mathbf{v}$, or $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$. In three dimensions, those combinations typically fill a line, then a plane, and the whole space $\mathbf{R}^3$.

1.2 Lengths and Dot Products

REVIEW OF THE KEY IDEAS

1. The dot product $\mathbf{v} \cdot \mathbf{w}$ multiplies each component v_i by w_i and adds all $v_i w_i$.
2. The length $\|\mathbf{v}\|$ of a vector is the square root of $\mathbf{v} \cdot \mathbf{v}$.
3. $\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$ is a **unit vector**, its length is 1.
4. The cosine product is $\mathbf{v} \cdot \mathbf{w} = 0$ when vector $\mathbf{v}$ and $\mathbf{w}$ are perpendicular.
5. The cosine of θ (the angle between any nonzero $\mathbf{v}$ and $\mathbf{w}$) never exceeds 1:

$$\cos\theta = \frac{\mathbf{v} \cdot \mathbf{w}}{\|\mathbf{v}\|\|\mathbf{w}\|} \quad \text{(Schwarz inequality)} \quad |\mathbf{v} \cdot \mathbf{w}| \le \|\mathbf{v}\|\|\mathbf{w}\|$$

   Problem 21 will produce the **triangle inequality** $\|\mathbf{v} + \mathbf{w}\| \le \|\mathbf{v}\| + \|\mathbf{w}\|$.

1.3 Matrices

1.3.1 Linear Equations

1.3.2 The Inverse Matrix

this part is so hard

1.3.3 Cyclic Differences

REVIEW OF THE KEY IDEAS

1. **Matrix times vector:** $\mathbf{Ax} = combination of the columns of A.$

2. The solution to $Ax = bisx = A^{-1}b, when A is an invertible matrix.$

3. The difference matrix A is inverted by the sum matrix $\mathbf{S} = \mathbf{A^{-1}}$.

4. The cyclic matrix **C** has no inverse. Its three columns lie in the same plane
   Those dependent columns add to the zero vector. $\mathbf{Cx} = \mathbf{0}$ has many solutions.

5. This section is looking ahead to key ideas. not fully explained yet.

Chapter 2

Solving Linear Equations

2.1 Vectors and Linear Equations

REVIEW OF THE KEY IDEAS

1.

2.2 The Idea of Elimination

Second lecture

Elimination

Back substitution

Here, we need to pay attention to the particularity of the matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix} \tag{2.1}$$

REVIEW OF THE KEY IDEAS

1.

2.3 Elimination Using Matrices

REVIEW OF THE KEY IDEAS

1.

2.4 Rules for Matrix Operations

REVIEW OF THE KEY IDEAS

1.

2.5 Inverse Matrices

REVIEW OF THE KEY IDEAS

1.

Lecture 3

Matrix multiplication

Four different way to slove matrices

1. $C_{34} = (row_3 \, of \, A)(col_4 \, of \, B)$. we can use a row of A times another column of B equal to a special product of a value of matrix c
2. As before, we can use matrix A times each columns of B equal to matrix c
3. We can use matrix A times each rows of matrix B equal to matrix c
4. We can see $AB = sumof(cols \, of \, A) \times (rows \, of \, B)$
5. Addition, we should know the textbfBlock multiplication law

Inverse of A, AB, A^T

Inverses (square matrix)
In this case, we should know if $A^{-1}A = I = AA^{-1}$ exists, then A is invertible and nonsingular. If we were in a singular case, we can find $Ax = 0$ exist

$\mathbf{Gauss-Jodan \, find \, A^{-1}}$

Gauss-Jodan we can find $E[AI] = [IA^{-1}]$

2.6 Elimination = Factorization: A = LU

REVIEW OF THE KEY IDEAS

1.

2.7 Transposes and Permutations

REVIEW OF THE KEY IDEAS

1.

Chapter 3

Vector Spaces and Subspaces

3.1 Spaces of Vectors

3.2 The Nullspace of A: Solving Ax = 0

3.3 The Rank and the Row Reduced Form

3.4 The Complete Solution to Ax = b

3.5 Independence, Basis and Dimension

3.6 Dimensions of the Four Subspaces

Chapter 4

Orthogonality

4.1 Orthogonality of the Four Subspaces

4.2 Projections

4.3 Least Squares Approximations

4.4 Orthogonal Bases and Gram-Schmidt

Chapter 5

Determinants

5.1 The Properties of Determinants

5.2 Permutations and Cofactors

5.3 Cramer's Rule, Inverse, and Volumes

Chapter 6

Elgenvalues and Elgenvectors

Chapter 7

Linear Transformations

7.1 The idea of a Linear Transformation

7.2 The Matrix of a Linear Transformation

7.3 Diagonalization and the Pseudoinverse

Chapter 8

Applications

8.1 Matrices in Engineering

8.2 Graphs and Networks

8.3 Markov Matrices, Population, and Economics

8.4 Linear Programming

8.5 Fourier Series: Linear Algebra for Functions

8.6 Linear Algebra for Statistics and Probability

8.7 Computer Graphics

Chapter 9

Numerical Linear Algebra

9.1 Gaussian Elimination in Practice

9.2 Norms and Condition Numbers

9.3 Iterative Methods and Preconditioners

Chapter 10

Complex Vectors and Matrices

10.1 Complex Numbers

10.2 Hermitian and Unitary Matrices

10.3 The Fast Fourier Transform