Buckling Load Capacity of Composite Plates with and without Internal Flaws Using FEM

V.M.Sreehari, T.R. Jebieshia and Dipak Kumar Maiti

Abstract— The present study is an attempt to develop a finite element formulation for handling bending and buckling analysis of laminated composite plates, with and without damage. The effect of a damage is analysed for thin composite plates. Damage modelling is done using an anisotropic damage formulation, which is based on the concept of stiffness change. The primary aim of the buckling analysis of plates is to determine the critical buckling loads and the corresponding configuration of equilibrium. Mathematical formulation and Matlab coding using First Order Shear Deformation Theory have been done. Geometric nonlinearity has been included in the von Karman sense. The numerical results were obtained and compared with the available literature. The variations of critical buckling load for different lamination scheme, thicknesses, material properties, plate length to width ratios, the damaged area and intensity are presented.

Keywords--- Laminated Plates, Cross-Ply, Angle-Ply, Anisotropic Damage, Buckling Load, FEM

I. INTRODUCTION

AIRCRAFT and spacecraft structures consist of a large number of flat and curved panel type structural elements. The plates form an essential part of many aerospace, marine and automobile structures. In the past few decades, astonishing advances in the sciences and technology have motivated researchers to work on new structural materials. The development of composite materials has improved the performance and reliability of structural system. Increased use of composite laminated plates in primary structures necessitates the development of accurate theoretical models to predict their responses.

Damage or flaws are almost unavoidable in any structure at some stage of its operative life span. The damage may be due to projectile impact, fatigue and corrosion or caused by inclusions due to faulty manufacturing or fabrication procedures. The presence of damage significantly affects the reliable operation of the structure. The early detection of cracks, fatigue, corrosion, and structural failure is one of the major challenges in the aerospace, civil and mechanical industries. To ensure safety and to avoid any incidence of catastrophic failure a sound knowledge of the mechanism of

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failure and its influence on the response behaviour of the structure under load is inevitable.

Tamer Ozben [7] had reported the critical buckling load of fiber reinforced composite plate. S.S.Choudhary and V.B.Tungikar [10] presented Finite Element Analysis for geometrically nonlinear behaviour of laminated composite plates. G. Bao and W. Jiang [11] presented a critical review of bending and buckling of flat, rectangular, orthotropic thin plates. Jian wu Shang and W.B. Kratzig [12] presented a fournoded rectangular element of the Mindlin displacement model for thin plate bending and buckling analysis. S. Valliappan et al. [15] developed the elastic constitutive relationship for anisotropic damage mechanics. D.L.Prabhakara and P.K.Datta [16] studied the free vibration and static stability behaviour of a rectangular plate with localized zones of damage. The internal flaws present in the structural element altering the material behavior may affect the static and dynamic characteristics. Though the extensive study has been done on static and dynamic behavior of structural elements, no such literatures are available on the effects of region of damage in buckling of laminated composite plates.

In the present work, bending and buckling behaviour of laminated composite plates with and without localized regions of damage is studied using a finite element technique. The boundary conditions considered are simply supported on all the edges. Both the uniform uniaxial and biaxial loading conditions are taken into account for the buckling analysis. The effects of a region of damage are introduced by the use of an idealized model having a reduction in the elastic property in the zone of damage. The individual effects of parameters, such as size of damage, the extent of damage on the static stability behaviour are studied.

The advantage to this method is that this finite element damage formulation considers effects of transverse shear effects, geometric non linearity of composite plates while predicting buckling. The results and observations from this work will assist to address some important issues regarding structures where weight is a prime design factor like in aerospace applications.

II. MATHEMATICAL MODELING

For the finite element analysis total potential energy is given as,

$$U=\sum_{e=1}^{NE} U^{(e)}$$
 (1)

$$U = \sum_{e=1}^{NE} \frac{1}{2} \int \left[\left\{ \Delta \right\}^{(e)T} [L]^T [D] [L] \left\{ \Delta \right\}^{(e)} \right] dA$$

Where NE is the number of elements used for meshing the plate and [D] is elasticity matrix.

The displacement vector Δ can be written in terms of shape functions N_i and nodal displacement vector, q for an element as:

$$\{\Delta\}^{(e)} = [N_i]^{(e)} \{q\}^{(e)}$$

On substituting, potential energy can be written as

$$U \!\! = \!\! \sum_{\scriptscriptstyle e=1}^{NE} \! \frac{1}{2} \! \int \! \! \left[\{q\}^{(e)T} [N]^{(e)T} [L]^T [D] [L] [N]^{(e)} \{q\}^{(e)} \right] \! dA$$

Element potential energy can be written as

$$U^{(e)} = \frac{1}{2} \int \left[\{q\}^{(e)T} [B]^{(e)T} [D] [B]^{(e)} \{q\}^{(e)} \right] dA$$

$$\begin{bmatrix} B \end{bmatrix}^{(e)} = \begin{bmatrix} L \end{bmatrix} \begin{bmatrix} N \end{bmatrix}^{(e)}$$

Where
$$[\mathbf{B}]^{(e)} = [\mathbf{B}_1 \quad \mathbf{B}_2 \quad \mathbf{B}_3 \quad \dots \quad \mathbf{B}_{NN}]$$

The element bending stiffness matrix is defined as

$$\left[K\right]^{(e)} = \int B^{(e)T} DB^{(e)} dA^{(e)}$$

Thus finally the elemental potential energy can be written as:

$$U^{(e)} = \frac{1}{2} q^{(e)T} K^{(e)} q^{(e)}$$

Now $K^{(e)}$, the elastic stiffness is computed numerically by transforming the existing coordinate system to natural coordinate system ξ and η as,

$$\left[\boldsymbol{K}_{ij}\right]^{(e)} = \int_{1}^{1} \int_{1}^{1} \boldsymbol{B}_{i}^{T} \boldsymbol{D} \boldsymbol{B}_{j} \ det \boldsymbol{J} \ d\xi d\eta$$

Where, J is the Jacobian Matrix and is given by

$$J = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix}$$

When numerical integration is adopted, the element matrix becomes:

$$K^{(e)} = \sum_{p=1}^{N} \sum_{q=1}^{N} W_p W_q B^T \overline{D} B det J$$
 (2)

Where \mathbf{w}_{p} \mathbf{w}_{q} are the weights used in the Gaussian quadrature numerical integration method.

The displacement w produces some additional extension in the x and y directions of the middle surface resulting in von Karman strain-displacement relationship as given below,

$$\left\{\epsilon\right\} = \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{bmatrix} + \begin{bmatrix} \frac{1}{2} \left(\frac{\partial w}{\partial x}\right)^2 \\ \frac{1}{2} \left(\frac{\partial w}{\partial y}\right)^2 \\ \left(\frac{\partial w}{\partial x}\right) \left(\frac{\partial w}{\partial y}\right) \end{bmatrix} = \left\{\epsilon_0^p\right\} + \left\{\epsilon_L^p\right\}$$

in which the first term is the linear expression and second gives the non-linear terms.

Similar as equation of linear stress, but here for non-linear components,

$$d\epsilon_L^p = B_L^b q$$

The non-linear strain components can be also written as,

$$\epsilon_{L}^{p} = \frac{1}{2} \begin{bmatrix} \frac{\partial w}{\partial x} & 0 \\ 0 & \frac{\partial w}{\partial y} \\ \frac{\partial w}{\partial y} & \frac{\partial w}{\partial x} \end{bmatrix} \begin{bmatrix} \frac{\partial w}{\partial x} \\ \frac{\partial w}{\partial y} \end{bmatrix} = \frac{1}{2} A \theta$$

The derivatives of w (i.e. slopes) can be related to the nodal parameters a^b as,

$$\theta = [G]\{q\}$$

$$\mathbf{W} \text{here,} \quad \mathbf{G}_{i} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \frac{\partial \mathbf{N}_{i}}{\partial \mathbf{x}} & \mathbf{0} & \mathbf{0} \\ & & & \\ \mathbf{0} & \mathbf{0} & \frac{\partial \mathbf{N}_{i}}{\partial \mathbf{y}} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

Now, the geometric stiffness matrix is obtained as in Eq. (3),

$$\mathbf{K}_{\sigma}^{b} = \int_{\mathbf{V}} \mathbf{G}^{\mathsf{T}} \begin{bmatrix} \mathbf{T}_{\mathsf{x}} & \mathbf{T}_{\mathsf{x}\mathsf{y}} \\ \mathbf{T}_{\mathsf{x}\mathsf{y}} & \mathbf{T}_{\mathsf{y}} \end{bmatrix} \mathbf{G} \, d\mathbf{V}$$
 (3)

Where T_x , T_y , and T_{xy} represents the in-plane loads.

As a result of buckling, an eigenvalue problem can exist as below.

$$[[K] + \lambda [K_{\sigma}]] \{\Delta\} = 0$$

Where λ is the inplane magnification factor.

- The eigenvalue problem is solved to extract minimum eigenvalues or critical buckling loads.
- The eigenvectors give the corresponding mode shapes.

III. INTRODUCING EFFECTS OF DAMAGE

By considering a parameter Γ_1 , the anisotropic damages can be parametrically incorporated into the formulation of structures like thin plate. This parameter is essentially a representation of reduction in effective area and is given by

$$\Gamma_{i} = \frac{A_{i} - A_{i}^{*}}{A_{i}}$$

Where A_i^* is the effective area (with unit normal) after damage and i denotes the three orthogonal directions. For a thin plate only Γ_1 and Γ_2 need to be considered. Γ_1 represents the damage in the direction of the fibre while Γ_2 refers to orthogonal damage. This method of parametrically modelling damage in any anisotropic material was proposed by Valliappan (1990). Using this formulation, a damaged stress-strain matrix for a two dimensional laminate is written as ,

$$\left\{\sigma^*\right\} = \left\lceil D^* \right\rceil \! \left\{\epsilon^*\right\}$$

Where,

$$\begin{bmatrix} \mathbf{D}^* \end{bmatrix} = \begin{bmatrix} \frac{\left(1 \cdot \Gamma_1\right)^2}{\left(1 \cdot \upsilon_{12}\upsilon_{21}\right)} E_1 & \frac{\left(1 \cdot \Gamma_1\right)\left(1 \cdot \Gamma_2\right)}{\left(1 \cdot \upsilon_{12}\upsilon_{21}\right)} E_2\upsilon_{12} & 0 \\ \\ \frac{\left(1 \cdot \Gamma_1\right)\left(1 \cdot \Gamma_2\right)}{\left(1 \cdot \upsilon_{12}\upsilon_{21}\right)} E_1\upsilon_{21} & \frac{\left(1 \cdot \Gamma_2\right)^2}{\left(1 \cdot \upsilon_{12}\upsilon_{21}\right)} E_2 & 0 \\ \\ 0 & 0 & 2 \frac{\frac{\left(1 \cdot \Gamma_1\right)^2\left(1 \cdot \Gamma_2\right)^2}{\left(1 \cdot \upsilon_{12}\upsilon_{21}\right)\left(1 \cdot \upsilon_{12}\upsilon_{21}\right)}}{\frac{\left(1 \cdot \Gamma_1\right)^2}{\left(1 \cdot \upsilon_{12}\upsilon_{21}\right)} + \frac{\left(1 \cdot \Gamma_2\right)^2}{\left(1 \cdot \upsilon_{12}\upsilon_{21}\right)}} G_{12} \end{bmatrix}$$

Proceeding as in undamaged case formulation, eigenvalue problem for a composite plate with damage can be obtained.

Table II: Convergence of FEM solution of deflection for different mesh size for anti-symmetric angle-ply laminate

		Mesh size				
Lamination Scheme	a/h	2x2	4x4	6x6	8x8	Percentage change in last two values
[-45/45]	4	0.768	0.829	0.829	0.829	0.0602
	30	0.617	0.676	0.675	0.674	0.0740
	100	0.493	0.655	0.656	0.656	0.0152
[-45/45] ₄	4	0.402	0.419	0.420	0.420	0.0238
	30	0.281	0.266	0.265	0.265	0.0752
	100	0.233	0.248	0.248	0.247	0.0403

Table III: Non dimensionalized Buckling load for varied lamination scheme and thicknesses

Lamination scheme	a/h	Ref [18]	Present (From program)
$(0/90)_4$	10	21.631	25.176
$(0/90)_4$	20	29.965	31.708
$(0/90)_4$	100	34.381	34.578
(0/90)	10	10.381	11.352
(0/90)	20	11.980	12.515
(0/90)	100	12.601	12.939

IV. RESULTS AND DISCUSSIONS

Finite element methods results for laminated composite square (aspect ratio = 1) plates are obtained by analyzing the formulation explained in previous section and programming in MATLAB. An eight noded C_0 , isoparametric element has been employed for discretization of the laminated plate. For the FSDT, a shear correction factor 5/6 has been used. Non dimensional results are presented in figures and tabular form. The non dimensionality used for transverse deflection and critical buckling load are given respectively as,

$$\overline{\mathbf{w}} = \mathbf{w} \left(\frac{100 h^3 \mathbf{E}_2}{q_0 a^4} \right) \quad \text{and} \quad \mathbf{P}_{nd} = \mathbf{P}_{cr} \left(\frac{\mathbf{a}^2 \mathbf{E}_2}{h^3} \right)$$

Analysis is carried out using different material models but the result using material model 1 is produced generally in this work unless specified and material model 2 also in damaged cases. Since the result for this material model is accurate, the implementation of the program with different material model is also accurate. These two material models are presented in Table I.

Table I: Material Types

Material model	E ₁ (in N/m ²)	E ₁ //E ₂	ν_{12}	G ₁₂ =G ₁₃	G_{23}
Material 1	25×10 ¹⁰	25	0.25	0.5 E ₂	0.2 E ₂
Material 2	40×10 ¹⁰	40	0.25	0.6 E ₂	0.5 E ₂

A. Convergence Study

As the number of mesh increases the convergence of the results is found to be fairly accurate. The Table II shows the non-dimensional central transverse deflection for antisymmetric angle ply plates subjected to transverse sinusoidal load (SSL). Table III results also validates the present mathematical formulation and Matlab programming. Table IV shows the comparison of the non dimensional buckling loads of obtained by the present analysis with the exact values for square, laminated composite plate for inplane uniaxial loading condition, which were given in Reference [8]. The material properties taken were E_1 = 130 GPa, E_2 = 9 GPa, E_3 = 0.28. Analysis is also performed for both antisymmetric and symmetric cross ply plates.

Table IV: Comparison of Non Dimensional Buckling Load Results with Varying Number of Layers

Number of layers	Anti symme cross- pl		Symmetric cross–ply		
	Exact (Ref 8)	Present	Exact (Ref 8)	Present	
4	13.106	13.086	14.989	14.955	
6	14.152	14.129	14.989	14.960	
8	14.518	14.494	14.989	14.961	
10	14.688	14.663	14.989	14.962	
20	14.914	14.888	14.9895	14.963	

B. The effect of number of layers on buckling load

Here a composite plate with uniaxial loading and simply supported is considered and results are shown in Fig.1. Antisymmetric cross ply plates with 2 and 8 layers are used. As number of layers are high, buckling loads also are high. (a/b = 1; Material 1). The eight-layer antisymmetric cross-ply plate behaves much like an orthotropic plate.

C. Effect of a/h ratio on critical buckling loads in uniaxial and biaxial loading condition

To understand the effect of shear deformation in uniaxial and biaxial loading conditions, analysis is done with side to thickness ratio varying from 10 to 100. For this we have taken a $(0/90)_s$ laminated composite plate with simply supported at all ends and material-1. It is found that critical loads are higher for uniaxial loading. And as thickness decreases, the non dimensional buckling load increases. The variation is very less above a/h=40 for both cases, as seen clearly in Fig.2.

D. Effect of ply orientation on buckling load

Fig.3 presents the results for even number of layers of layers of square antisymmetric angleply laminated composite plates under uniaxial compression.

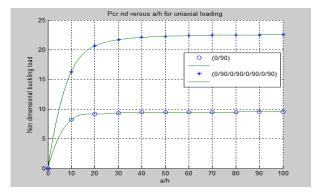


Figure 1: Variation of buckling load as thickness decreases for a 2-layered and 8-layered anti symmetric cross ply plates.

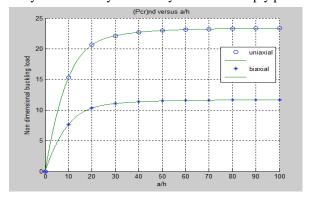


Figure.2: The effect of transverse shear deformation on critical buckling loads of symmetric (0/90/90/0) laminates under uniaxial and biaxial compression, a/b = 1; Material 1)

Percentage differences between the exact solution and the present solution for even-layered, square antisymmetric, angle-ply laminated composite plates subjected to uniaxial compression are greatest in the range of 5°-15 ° ply angle and the difference becomes insignificant with increasing number of layers. SS2 boundary conditions are used and the material properties taken was E_1 =130 GPa, E_2 = 9 GPa, G_{12} = G_{23} =4. 8 GPa and V_{12} =0. 28. From θ =0 to 90 degrees, the buckling load first increases and then decreases. The values were compared with the values of the plot given in Ref 8. For a variation of angle from θ =0 to 90 degrees in an angle ply plate, the maximum buckling load occurs at 45 degrees.

E. Effect of material parameters on buckling load

Critical buckling loads of two-layer antisymmetric crossply laminated plates under uniaxial and biaxial loading are presented in Fig.4 and Fig.5 respectively for modulus ratios $E_1/E_2=10$, 15, 20, 25, 30, 35 and 40. Here a two layered cross ply plate (0/90) is taken and is simply supported. In Fig.4 E_1/E_2 variations from 10 to 40 is plotted against the corresponding buckling loads for 3 cases viz a/h=10, 20 and 100. As the modulus of elasticity in 1-direction increases, the buckling load increases. Similarly can explain from Fig.5 too, but in biaxial cases, the buckling loads are less than the corresponding uniaxial cases at each E_1/E_2 ratio analysed.

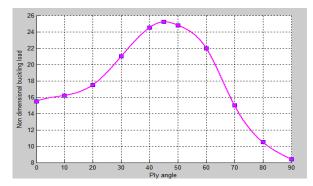


Figure 3: Buckling load of square antisymmetric laminates (20 layers) subjected to uniaxial compression

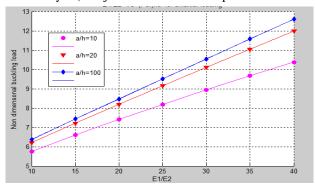


Figure 4: Critical buckling loads of two-layer antisymmetric cross-ply laminated plates under uniaxial loading conditions for modulus ratio variation

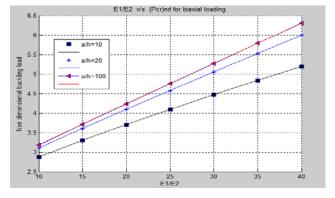


Figure 5: Critical buckling loads of two-layer antisymmetric cross-ply laminated plates under biaxial loading conditions for modulus ratio variation

F. Effect of damage in bending analysis.

A laminated composite plate of 4 layered $(0/90)_s$ with damage in two layers with $\Gamma_1/\Gamma_2=9$ is analysed and central transverse deflection and buckling loads are found. The results obtained are shown as a plot in Fig.6. As the percentage of damaged area increases, the central deflection gets increased. For plates having higher E_1/E_2 , the deflection is found less.

G. Effect of damage in buckling analysis.

For Material 1, with simply supported boundary conditions on all sides is analysed for uniaxial and biaxial loads. As damage area increases the buckling load gets decreases and is plotted in Fig.7. The variation of nondimensional buckling load for centrally located damage patches having damage intensities of Γ_1/Γ_2 =3 and Γ_1/Γ_2 =7 is presented in Table.V.

The damage ratio Γ_1/Γ_2 takes values between 0.0 and 9.0. A mild damage may be represented with a damage ratio $0.0 \le \Gamma_1/\Gamma_2 \le 3.0$ while a heavy damage may be denoted by the range of values, $7.0 \le \Gamma_1/\Gamma_2 \le 9.0$.

TABLE V: Variation of Nondimensional Buckling Load of Damaged Composite Plate as Damage Intensity Varies

Damage area in percentage	$\Gamma_1/\Gamma_2 = 3$	$\Gamma_1/\Gamma_2 = 7$
35	21.8243	17.2371
45	20.2866	15.4235

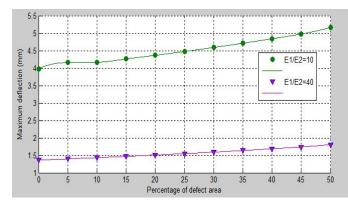


Figure 6: Effect of damage area on the transverse central deflection (at $E_1/E_2=10$, 40)

V. CONCLUSION

Buckling analysis of a laminated composite plate with and without damage has been studied. A finite element formulation based on FSDT have been done. MATLAB codes for the analysis has done.

The codes are providing satisfactory results when compared with references. The convergence is obtained in plate bending and buckling analysis. The results of critical buckling loads at various conditions are calculated and compared with several published results and analytical solutions available in the literature. A very good agreement of the results obtained by the present method with reference solutions, shows that the method is robust, effective and highly accurate. The observations are summarized in the following points,

- As the a/h ratio is increased, the non-dimensional transverse deflection decreases. And it is observed that for a/h ratio greater than 40, the deflection becomes almost constant.
- As the a/h ratio is increased, the non-dimensional buckling load increases. It is true in both uniaxial and biaxial loading cases. The variation is very less above a/h=40 for both cases.
- As the number of layers increases, buckling load too increases.
- It is observed that uniaxial loading cases have higher buckling loads than corresponding biaxial loading cases.
- From θ=0 to 90 degrees, the buckling load first increases and then decreases. For a variation of angle from θ=0 to 90 degrees in an angle ply plate, the maximum buckling load occurs at 45 degrees.

- As the modulus of elasticity ratio E_1 / E_2 increases, the buckling load increases, but in biaxial cases, the buckling loads are less than the corresponding uniaxial cases at each E_1 / E_2 ratio analysed.
- As the percentage of damaged area increases, the transverse central deflection increases.

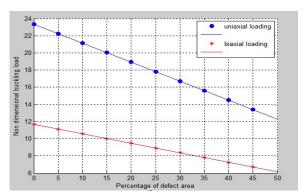


Figure 7: Effect of damage area on the non dimensional buckling load

- As the damage intensity increases from $\Gamma_1/\Gamma_2 = 1$ (mild) to $\Gamma_1/\Gamma_2 = 9$ (heavy), the buckling load decreases.
- The damage in top and bottom layers decreases the stiffness of plate more than damage inside the plate. Therefore, the displacement is increased and the buckling load is decreased when the damage occurs in top and bottom layers.
- As the percentage of damaged area increases, the buckling load decreases.

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