Vibration based Damage Assessment of Laminated Composite Structures using UPSO

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Abstract

Structural damage identification and quantification has gained attention in the scientific and engineering fields because the unpredicted structural failure may cause economic and human life loss. So early detection of damage and its assessment is necessary to avoid the safety issues and to economize the costs of repair. The presence of damage in any structure alters its performance (reduction in stiffness and strength) and thus changes in the vibration response parameters such as frequency, mode shapes, damping ratio, frequency response functions (FRFs) etc. These response parameters can be used for the identification of fault in the composite structures. In the present study, the assessment of anisotropic damage in composite structure has been detected and quantified using first few natural frequencies and corresponding mode shapes with the help of Unified Particle Swarm Optimization (UPSO) technique.

1 Introduction

In the past few decades, many authors [1, 2] studied different types of vibration based damage identification techniques. Since frequency alone as a response may not be sufficient for the accurate detection or identification of damage, the authors used the combined frequency and mode shape (frequency and mode shape simultaneously) or Frequency Response Functions (FRFs) or combined frequency and FRF data to locate as well as to estimate the extent of damage in various laminated composite structures.

The damage identification process includes two steps – direct and inverse [3]. In the direct or forward method, it is determined whether the structure is healthy or damaged. And in the inverse technique, with the help of an optimization algorithm, the location and extent of damage is identified.

Since composite materials exhibits the anisotropic properties and the damages present in composite structure are of anisotropic in nature. And hence, it is necessary to use tensorial descriptions of damage [4]. The anisotropic damage is parametrically incorporated into the present finite element formulation by considering the principle damage parameter or variables which is a representation of reduction in effective load bearing area [5]. For the present damage assessment study, natural frequency as well as combination of both natural frequency and mode shape [6] is used as the objective function.

2 Theoretical Formulation

The governing differential equations for the present numerical analysis have been formulated using First order Shear Deformation Theory (FSDT). The free vibration and damage analysis of laminated composite structures is done using Finite Element Method (FEM), i.e., the governing differential equations have been developed using FEM and then implementing the problem into MATLAB environment to obtain the various results. An eight-noded serendipity quadratic element have been used to descretize the plates. The choice of an isoparametric element ensures that the same shape function can be used to represent the element geometry and the problem unknowns, which in structural mechanics are the displacements.

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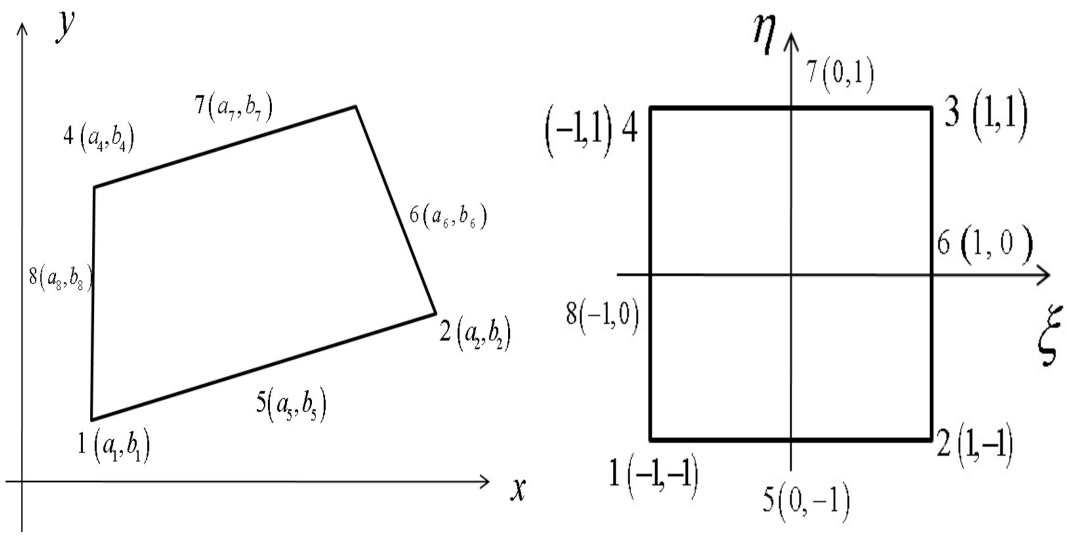
2.1 Finite Element Formulation

The finite element analysis is a numerical technique and one of the advantages of it is that a general purpose of computer program can be developed easily to analyze various kind of problems. In particular, any complex shape of the problem domain with prescribed conditions can be handled with ease using the finite element method. Because of its diversity and flexibility as an analysis tool, it is receiving much attention in engineering analysis. The finite element method requires division of the problem domain into many sub domains and each sub domain is called a finite element. For the present analysis, elements with five degrees of freedom (u, v, w, ϕ_x *and* ϕ_y), per node are used. The isoparametric element which is oriented in Cartesian co-ordinates can be transferred to the natural co-ordinates systems with the help of the Jacobian matrix.



(a) The element in Cartesian Co-ordinate system

(b) The element in Natural Co-ordinate system

Figure 1: Shape function transformation

The shape function for 8-noded isoparametric element can be defined as:

$$N_i = \left\{ \begin{array}{c} (1+\zeta\zeta_i)(1+\eta\eta_i)(\zeta\zeta_i+\eta\eta_i-1)/4, \;\; for\, i=1,2,3,4 \\ (1-\zeta^2)(1+\eta\eta_i)/2, \;\; for\, i=5,7 \\ (1+\zeta\zeta_i)(1-\eta^2)/2, \;\; for\, i=6,8 \end{array} \right\} \tag{1}$$

where, ζ and η are the local co-ordinates of the element, and ζ_i and η_i are the co-ordinates of i^{th} node.

2.1.1 Displacement field

A simple first order shear deformation theory in which transverse shear strains are assumed to be constant across the plate thickness are considered. The displacement components are assumed to be in the form:

$$\begin{bmatrix} u(x,y,z) \\ v(x,y,z) \\ w(x,y,z) \end{bmatrix} = \begin{bmatrix} u_0(x,y) \\ v_0(x,y) \\ w_0(x,y) \end{bmatrix} + z \begin{bmatrix} \phi_x(x,y) \\ \phi_y(x,y) \\ 0 \end{bmatrix} \tag{2}$$

where u, v, w are the displacement components in x, y, z directions respectively. u_0, v_0, w_0 are the displacements of a point on the mid plane $(x, y, 0)$ and ϕ_x, ϕ_y are the rotations of the cross-section perpendicular to x and y axes respectively.

2.2 Anisotropic Damage

Murti et.al [7] developed a finite element model for anisotropic damage based on the structural stiffness reduction factor and the elastic constitutive relationships for anisotropic damage mechanics. The reduction of in-plane and bending stiffness and re-distribution of membrane stresses are direct consequences of such damages being present in the system which in turn affect the static and dynamic response characteristics of the system. The initial model in damage mechanics was based on a scalar parameter to measure the damage variable under the assumption of isotropic damage within the material. For complex defects and their distribution, it is necessary to use tensorial descriptions of damage [4, 8, 9]. The anisotropic damage is parametrically incorporated into the present finite element formulation by considering the principle damage parameter or variables which is a representation of reduction in effective load bearing area and is given in Equation 3.

$$\Gamma_i = \frac{A_i - A_i^*}{A_i} \tag{3}$$

where A_i and A_i^* (i = 1,2), indicating the undamaged and damaged areas with unit normal n_i and n_i^* respectively, n_i is the orthotropic damage co-ordinate system.

It is assumed that the internal forces acting on any damaged section are the same as the one before damage. Therefore,

$$\sigma_{ij}\delta_{jk}A_k = \sigma_{ij}^*\delta_{jk}A_k^* \tag{4}$$

where σ_{ij} and σ_{ij}^* are the components of the Cauchy stress and effective (net) stress tensor respectively, and δ_{ij} is the Kronecker tensor.

The damage model should not assume the damage tensor to be symmetric in order to define the damage effectively in composites materials. Therefore the effective non-symmetric stress tensor relation, which is the compatibility relation of the effective shear stress components can be given as:

$$\sigma_{21}^* = \frac{1-\Gamma_2}{1-\Gamma_1}\sigma_{12}^* \tag{5}$$

Therefore,

$$\left\{\begin{array}{c} \sigma_{11}^* \\ \sigma_{22}^* \\ \sigma_{12}^* \\ \sigma_{21}^* \end{array}\right\} = \left[\begin{array}{ccc} \frac{1}{1-\Gamma_1} & 0 & 0 \\ 0 & \frac{1}{1-\Gamma_2} & 0 \\ 0 & 0 & \frac{1}{1-\Gamma_1} \\ 0 & 0 & \frac{1}{1-\Gamma_2} \end{array}\right] \left\{\begin{array}{c} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{array}\right\} \tag{6}$$

which can also be written as: $\{\sigma^*\} = [\psi]\{\sigma\}$

In order to obtain the constitutive relation for the anisotropic damage model, the complementary elastic energy of the damage state is assumed to be equal to that of the undamaged state therefore,

$$[Q^*]^{-1} = [\psi]^T [Q]^{-1} [\psi] \tag{7}$$

where. $[Q]$ is the elastic stress-strain matrix of the undamaged state.

$$[Q^*] = \left[\begin{array}{ccc} \frac{E_{11}(1-\Gamma_1)^2}{1-\nu_{12}\nu_{21}} & \frac{E_{22}\nu_{12}(1-\Gamma_1)(1-\Gamma_2)}{1-\nu_{12}\nu_{21}} & 0 \\ \frac{E_{11}\nu_{21}(1-\Gamma_1)(1-\Gamma_2)}{1-\nu_{12}\nu_{21}} & \frac{E_{11}(1-\Gamma_2)^2}{1-\nu_{12}\nu_{21}} & 0 \\ 0 & 0 & \frac{2G_{12}(1-\Gamma_1)^2(1-\Gamma_2)^2}{(1-\Gamma_1)^2+(1-\Gamma_2)^2} \end{array}\right] \tag{8}$$

The constitutive relations for the anisotropic damage model can be given as:

$$\{\sigma_d\} = \left[\overline{Q}_d\right]\{\varepsilon_d\} \tag{9}$$

where, $\overline{Q}_d = T^T [Q^*] T$ and T is the transformation matrix which transform the damage co-ordinate system to general or Cartesian co-ordinate system and is given as

$$[T] = \begin{bmatrix} cos^2\theta & sin^2\theta & sin2\theta \\ sin^2\theta & cos^2\theta & -sin2\theta \\ -0.5sin2\theta & 0.5sin2\theta & cos2\theta \end{bmatrix}$$

Elemental elastic stiffness matrix can be given as:

$$K^{(e)}_{d\,or\,ud} = \int B^{(e)^T} D_{d\,or\,ud}\ B^{(e)} dA^{(e)} dz \tag{10}$$

where $K^{(e)}_{d\,or\,ud}$ is the elastic stiffness matrix for damaged or undamaged element, $D_{d\,or\,ud}$ is the stress-strain matrix for damaged or undamaged element, $[B]^{(e)} = [B_1\,B_2\,B_3\,......B_{NN}]$, the strain-displacement matrix; NN is the number of nodes per element.

$$[B_i] = \begin{bmatrix} N_{i,x} & 0 & 0 & 0 & 0 \\ 0 & N_{i,y} & 0 & 0 & 0 \\ N_{i,y} & N_{i,x} & 0 & 0 & 0 \\ 0 & 0 & 0 & N_{i,x} & 0 \\ 0 & 0 & 0 & 0 & N_{i,y} \\ N_{i,y} & -N_{i,x} & 0 & N_{i,y} & N_{i,x} \\ 0 & 0 & N_{i,x} & N_i & 0 \\ 0 & 0 & N_{i,y} & 0 & N_i \end{bmatrix} i = 1, 2 \ldots NN$$

Governing differential equation for the present problem is:

$$[M]\left\{\ddot{d}\right\} + [K_{d\,or\,ud}]\{d\} = 0 \tag{11}$$

where, the mass matrix [M] is:

$$[M] = \int\limits_v [N]^T [\rho] [N] dAdz$$

$$[M] = \int\limits_A [N]^T [\overline{\rho}] [N] dA \tag{12}$$

where,

$$[N]\ is\ the\ shape\ function\ matrix [\overline{\rho}] = \int [\rho] dz\ and\ [\rho] = \rho \begin{bmatrix} 1 & 0 & 0 & z & 0 \\ 0 & 1 & 0 & 0 & z \\ 0 & 0 & 1 & 0 & 0 \\ z & 0 & 0 & z^2 & 0 \\ 0 & z & 0 & 0 & z^2 \end{bmatrix}$$

3 Optimization Technique

Optimization is the process of maximizing or minimizing the desired objective function while satisfying the prevailing constraints. For the present damage assessment method, Particle Swarm Optimization has been used as an algorithm for the inverse method. Particle Swarm Optimization (PSO) is a population based algorithm, first proposed by Kennedy and Eberhart [10] inspired by the collective motion of insects and birds trying to reach an unknown destination, known as “swarm behavior”. This process involves both social interaction and intelligence so that birds, insects, fish, etc., learn from their own experience (self experience) and also from the experience of others around them (social experience). PSO algorithm has advantages lies in its simplicity and architecture (requires updating of two simple equations) and fast rate of convergence to an optimal solution. In spite of these advantages, the main drawback associated with standard PSO is that the algorithm may be trapped into some local optima. This may cause the prediction of wrong results. Further, to improve its efficiency, numerous alternations and variations are proposed to the standard PSO algorithm.

3.1 Simplified Particle Swarm Optimization (SPSO)

The two simplified equation to update the velocity and position of the particles for the Simplified PSO algorithm is given as:

$$v(t+1) = v(t) + c_1(P_l - x(t)) + c_2(P_g - x(t)) \tag{13a}$$
$$x(t+1) = x(t) + v(t+1) \tag{13b}$$

where, P_l is the local best position and P_g is the global best position. The parameters c_1 and c_2 are called acceleration coefficients which influence the maximum size of the step that a particle can take in a single iteration. In the SPSO method the accelerations coefficients are kept constant, $c_1 = c_2 = 2$.

3.2 Unified Particle Swarm Optimization (UPSO)

Unified Particle Swarm Optimization (UPSO) is one of the PSO algorithm that has the ability to harness both exploration and exploitation capacity simultaneously [11] by balancing the influence of both global and local search directions simultaneously. During the search process, the particle moves to new positions considering two factors: the best previously position visited by itself denoted by *pbest*, and the best position so far found by any individual of the population denoted by *gbest* (for global best neighborhood approach) or the best position so far found by any of its neighborhood denoted by *lbest* (for local best neighborhood approach). Mathematically, for a swarm size of P number of particles, in an S-dimensional search space, each particle occupies a position $X_i = \{x_{i1}, x_{i2}, ..., x_{id}, ..., x_{iS}\}$, with a velocity $V_i = \{v_{i1}, v_{i2}, ..., v_{id}, ..., v_{iS}\}$, where $i = 1, 2, .., P$ and $d = 1, 2, ..., S$. In each iteration of the PSO algorithm, each particle moves towards its best position and the best particle *pbest* in the swarm. The change of position of each particle can be computed according to the distance between the current position and its previous best position and the distance between the current position and the best position of swarm. Suppose the best previously visited position of the ith particle gives the best fitness value as $Pbest = \{pbest_{i1}, pbest_{i2}, ..., pbest_{id}, ..., pbest_{iS}\}$ and the best previously visited position of the swarm gives best fitness as $Gbest = \{gbest_1, gbest_2, ..., gbest_d, ..., gbest_S\}$. The best position so far found by any of its neighborhood is $lbest = \{lbest_{i1}, lbest_{i2}, ..., lbest_{id}, ..., lbest_{iS}\}$. Let G_{ij}^{t+1} and L_{ij}^{t+1} denotes the velocity update of i^{th} particle in global and local variants of PSO respectively for the $(t+1)^{th}$ iteration in the d^{th} dimensional search space as given by,

$$G_{id}^{t+1} = \chi\left[v_{id}^t + c_1 r_1\left(pbest_{id} - x_{id}^t\right) + c_2 r_2\left(gbest_d - x_{id}^t\right)\right] \tag{14a}$$
$$L_{id}^{t+1} = \chi\left[v_{id}^t + c_1 r_3\left(pbest_{id} - x_{id}^t\right) + c_2 r_4\left(lbest_{id} - x_{id}^t\right)\right] \tag{14b}$$

where *pbest*, *gbest and lbest* respectively denotes the best position explored by individual particle, any particle in the swarm and in the neighborhood of individual swarm. χ denotes the constriction factor which equals to 0.72984. c_1 and c_2 are two acceleration coefficients and is considered as 2.05 each in present study. Finally, all r terms denote random numbers between [0, 1] and independent of each other. Combining Equations (14a) and (14b), the aggregate updating velocity of the particles in the search directions is defined as,

$$V_{id}^{t+1} = uG_{id}^{t+1} + (1-u)L_{id}^{t+1}, \;\; u\,\varepsilon\,[0, 1] \tag{15}$$

The new position of the particles for $(t+1)^{th}$ iteration is,

$$x_{id}^{t+1} = x_{id}^t + V_{id}^{t+1}, \; \forall\, i\,\varepsilon\, P \; and \; \forall\, d\,\varepsilon\, S \tag{16}$$

The parameter u in Equation 15 is called the unification factor and its value is modified throughout the iteration according to the equation,

$$u(t) = exp\left(\frac{t\,log(2.0)}{t_{max}}\right) - 1.0 \tag{17}$$

where t_{max} is the maximum number of iteration.

3.3 Objective Function

Natural frequencies and mode shapes obtained from the damaged structure are used for constructing the objective function of the present problem and is given as:

$$F=\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\left(\frac{f_i^m}{f_i^c}\right)-1\right)^2}+\sum_{i=1}^{n}(1-MAC_{ii}) \tag{18}$$

where Modal Assurance Criterion, $MAC_{ii}=\frac{[\{\phi_i^m\}^T\{\phi_i^c\}]^2}{\{\phi_i^m\}^T\{\phi_i^m\}\{\phi_i^c\}^T\{\phi_i^c\}}$

The terms f_i^m are the measured natural frequencies for damaged structure and f_i^c are the natural frequency computed from numerical analysis (finite element simulation) for damaged structure. n is the number of input response parameters, ϕ_i denotes the i^{th} mode shape. The superscripts m denote the mode shape measured (mode shape of actual damaged structure) and c denote the mode shape of the damaged structure updated numerically through the optimization algorithm.

4 Results and Discussions

Identification of damage and estimating its severity has been done using finite element formulation based on vibration techniques with the help of UPSO algorithm. Real world measurements which are obtained from experiments are always contaminated by noise from different sources in the working environment moreover, the theoretical fixity conditions cannot be attained experimentally and it can add up noise to a significant level. So a standard noise is added to the numerical modal data to simulate the experimental condition.

Since natural frequency alone may not be sufficient to detect and quantify multiple damages in any structure, a numerical procedure is carried out to detect and quantify the damages in a composite structures based on changes in natural frequency and the corresponding mode shape data using UPSO technique. In this section, a thorough comparative study has been done with natural frequency alone and combined natural frequency and corresponding mode shapes as objective functions.

4.1 Damage assessment using UPSO considering different objective functions

To detect and quantify the damage in a structure effectively, combined frequency and mode shape data has been used as objective function in the present UPSO technique. In order to demonstrate the developed algorithm, a composite beam of 500 mm length, 30 mm breadth and 2.8 mm thickness having material properties given in Table 1, is considered. The algorithm is verified for both single and multiple element damage cases. For finite element simulation the beam is modeled with 10 orthotropic elements as shown in Figure 2.

Table 1: Material properties considered for numerical composite beam

E_1 (GPa)	E_2 (GPa)	G_{12}(GPa)	ν_{12}	$\rho\,(kg/m^3)$
9.5	6.8	1.4	0.14	1761

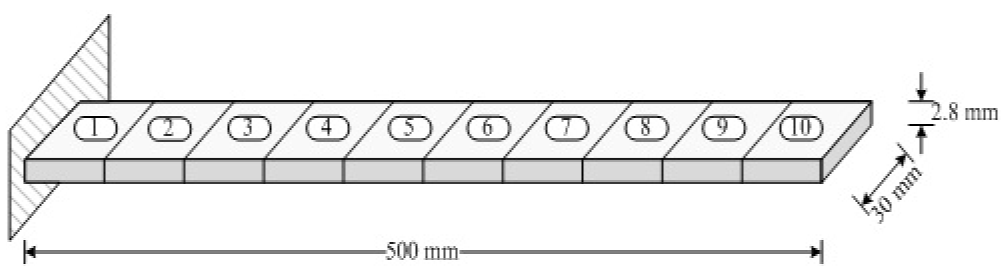


Figure 2: Beam discretization

First six numerically evaluated natural frequencies and the corresponding mode shapes of the numerically developed cantilever beam are used for constructing the objective function. The single element damage and multiple element damage conditions are considered for the study and the results of damage assessment for various noise levels (up to 1% noise in natural frequency is added to simulate the real life conditions) of cantilever beams are evaluated. It is inferred from the results (Figures 5 and 8) that for noiseless conditions, the present algorithm is able to detect the damage accurately. But the addition of noise in frequency, alters the results (Figures 4, 5, 7 and 8) considerably.

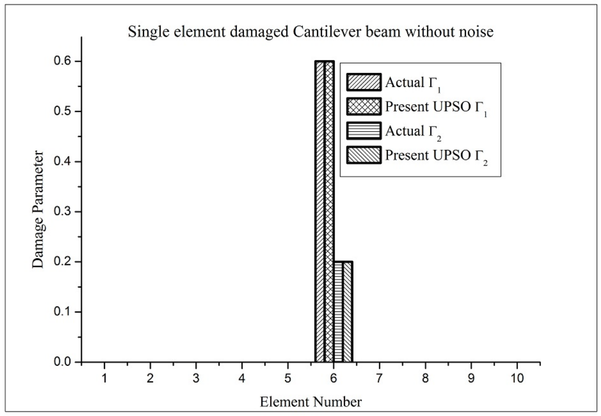


Figure 3: Single element damaged cantilever beam without noise

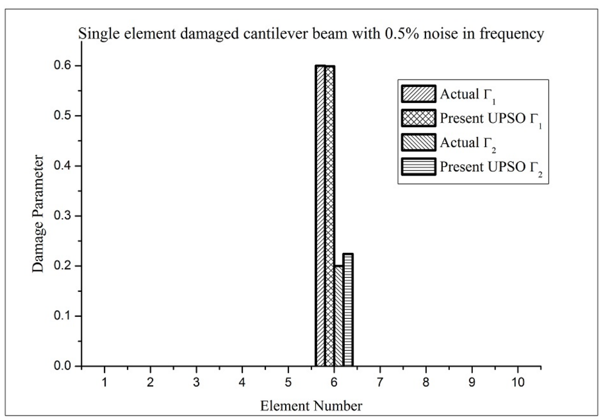


Figure 4: Single element damaged cantilever beam with 0.5% noise in natural frequency

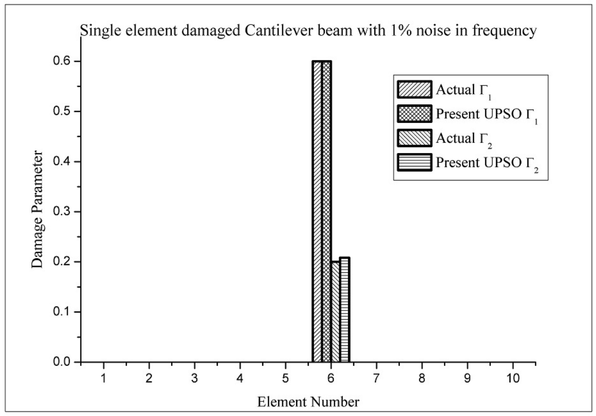


Figure 5: Single element damaged cantilever beam with 1% noise in natural frequency

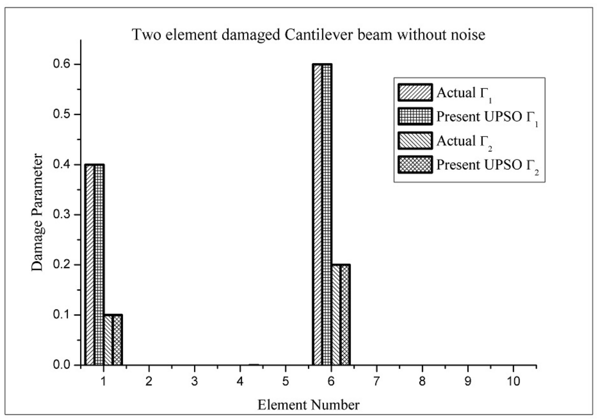


Figure 6: Two element damaged cantilever beam without noise

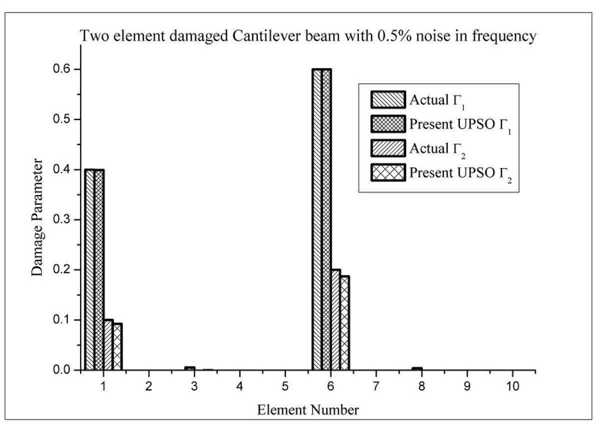


Figure 7: Two element damaged cantilever beam with 0.5% noise in frequency

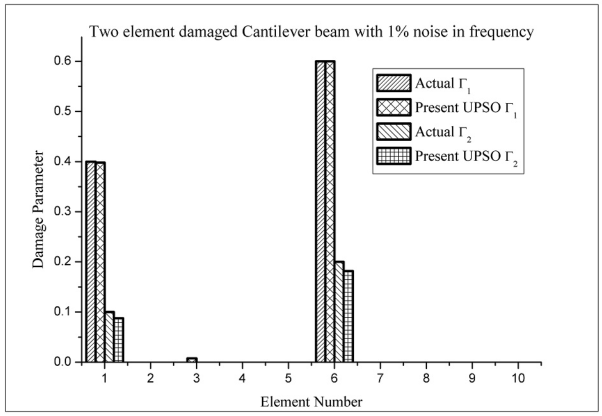


Figure 8: Two element damaged cantilever beam with 1% noise in frequency

4.1.1 Comparison of frequency based and combined frequency and mode shape based objective function

From Figures 5 and 8, it can be seen that without noise both frequency alone and combined frequency and mode shape based objective function gives the accurate results. As the noise level increases, combined frequency and mode shape based objective function is more efficient to detect and quantify the damage compared to frequency alone as objective function. Table 3 shows the damage assessment results of cantilever beam with multiple damages.

Table 2: Comparison with different objective functions for cantilever beam with multiple damages

Actual Damage	Frequency alone as objective function		
Element No. (Γ_1, Γ_2)	Without noise	0.5 % noise in frequency	1 % noise in frequency
2 (0.25, 0.1)	2 (0.25, 0.1)	2 (0.256, 0.088)	2 (0.281, 0.161)
5 (0.5, 0.3)	5 (0.5, 0.3)	5 (0.49, 0.297)	5 (0.477, 0.314)
7 (0.4, 0.2)	7 (0.4, 0.2)	7 (0.405, 21)	7 (0.408, 0)
Actual Damage	Frequency + Mode shape as objective function		
Element No. (Γ_1, Γ_2)	Without noise	0.5 % noise in frequency	1 % noise in frequency
2 (0.25, 0.1)	2 (0.25, 0.1)	2 (0.249, 0.113)	2 (0.25, 0.107)
5 (0.5, 0.3)	5 (0.5, 0.3)	5 (0.501, 0.289)	5 (0.503, 0.251)
7 (0.4, 0.2)	7 (0.4, 0.2)	7 (0.4, 0.193)	7 (0.396, 0.213)

Simply supported composite plate discretized as in Figure 9 has also been considered for damage detection. Table 3 shows the damage detection of simply supported composite plate with single and multiple damages. From Tables 3 and 3, it is clear that damages can be effectively identified and quantified with the help of combined natural frequency and corresponding mode shapes as objective function.

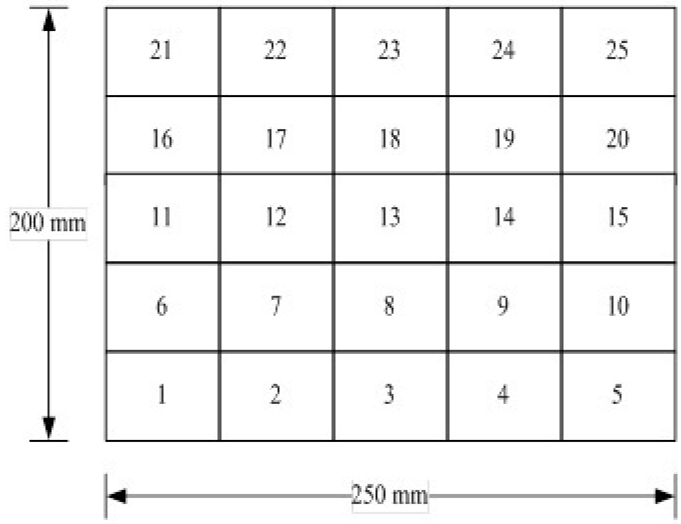


Figure 9: Simply supported plate discretization

Table 3: Comparison with different objective functions for simply supported plate

Actual Damage	Frequency as objective function		
Element No, Layer No., (Γ_1, Γ_2)	Without noise	0.5 % noise in frequency	1 % noise in frequency
5, 1 (0.5, 0.3)	1,1 (0.498, 0.282)	1,1 (0.4984, 0.5)	1,1 (0.249, 0)
11, 1 (0.3, 0.1)	15,1 (0.298, 0.272)	15,2 (0.317, 0)	15,2 (0.267, 0)
15, 1 (0.4, 0.2)	11,1 (0.402, 0)	11,1 (0.463, 0.194)	11,1 (0.554, 0)
20, 1 (0.25, 0.1)	16,1 (0.253,0.119)	16, 1 (0.205, 0.05)	6,2 (0.44, 0.275)
22, 2 (0.2, 0.2)	24,2 (0.203, 0.198)	24,1 (0.259, 0.209)	24,2 (0.438,0.169)
Actual Damage	Frequency + Mode shape as objective function		
Element No, Layer No., (Γ_1, Γ_2)	Without noise	0.5 % noise in frequency	1 % noise in frequency
5, 1 (0.5, 0.3)	5,1 (0.5, 0.3)	5,1 (0.514, 0.242)	5, 1 (0.5, 0.3)
11, 1 (0.3, 0.1)	11,1 (0.3, 0.1)	11,1 (0.289,0.113)	11, 1 (0.3, 0.1)
15, 1 (0.4, 0.2)	15,1 (0.4, 0.2)	15,1 (0.387, 0.225)	15, 1 (0.4, 0.2)
20, 1 (0.25, 0.1)	20,1 (0.25,0.1)	20,1 (0.248, 0.081)	20, 1 (0.25,0.1)
22, 2 (0.2, 0.2)	22,2 (0.2,0.2)	22,2 (0.173,0.204)	22, 2 (0.2,0.2)

Without noise both frequency alone and combined frequency and mode shape based objective function gives the accurate results. As the noise level increases, combined frequency and mode shape is more efficient compared to frequency alone as objective function.

5 Conclusion

A numerical procedure is presented to detect and quantify single and multiple damages in a composite beam based on changes in natural frequency and combined frequency and mode shape data using UPSO technique. The proposed methodology is demonstrated using a numerically simulated composite structures containing single and multiple damages. As indicated by the simulation results, the present method is able to detect and quantify the damage accurately using first few natural frequencies and the corresponding mode shapes for considered damage cases for noisy and noise free conditions. As the noise level increases, combined frequency and mode shape based objective function is more efficient to detect and quantify the damage compared to frequency alone as objective function.

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