“Student Paper”

Paper Number: **1569**

Title: **Vibration Characteristics and Damage Detection of Composite Structures with Anisotropic Damage using Unified Particle Swarm Optimization Technique**

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ABSTRACT

Composite structures may undergo several types of damages such as delamination, fiber breakage, etc. due to the defects in manufacturing process or fatigue loading during service which in turn affects the effective performance of the structure. The damage in the composite is anisotropic in nature whose extent is distributed in several orthogonal directions and the extent of damage in any orthogonal direction is independent of the other. The anisotropic damage is introduced into the composite using concept of reduction in stiffness due to reduction in effective load bearing area which is represented by damage parameters or variables.

Vibration based damage detection is one of an effective scheme to detect damages due to its simplicity in implementation and ability of acquiring both the global and the local damage information of the structure. From the literature, it has been revealed that the frequency response function (FRF) based damage detection is more accurate compared to frequency based damage detection since it provide much more information on damage for a particular frequency range.

A numerical procedure is developed on MATLAB to detect and quantify damages in composite structures using vibration response parameters with the help of unified particle swarm optimization (UPSO) algorithm.

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From the numerical and experimental results it is inferred that the presence of damage in composite structure affect the vibration characteristics such as frequency, mode shapes and FRF curves as well. Moreover, the damage parameters show strong orthogonality on vibration characteristics. It has been observed that, damage in the direction of fibre results in steeper deterioration of natural frequencies. The outcome shows that as the intensity of damage or the area of damage increases, the natural frequency deteriorates irrespective of ply orientation. The proposed methodology is able to detect and quantify damages accurately for various single and multiple damage scenarios.

I. INTRODUCTION

When engineering structures are subjected to unfavorable situation such as cold and hot working processes, temperature variation, chemical action, radiation, mechanical loading or environmental condition, microscopic defects and cracks may build up inside the materials. This damage causes reduction in strength and that may lead to failure and thus shorten the operating life of the structures. This deterioration in mechanical properties of a material is known as a damage process.

The calculations involved in isotropic damage state are very much simplified due to the scalar nature of the damage variable. In composite structures, since the damage is anisotropic in nature, the stress variables are tensorial and hence the identification of the models is much more complicated. Santos et al. [1] developed a numerical technique for the identification of damage on laminated structures based on FSDT. Reduction of in-plane and bending stiffness and re-distribution of membrane stresses occurs due to the presence of damages in the system which inturn affect the static and dynamic response characteristics of the system. So understanding vibration characteristics is essential in fail safe design. Due to the complexities involved in an anisotropic damage, the use of numerical methods such as the FEM based damage mechanics theory has been proved to be very efficient.

Salawu [2] discussed the use of natural frequency as a diagnostic parameter in structural damage assessment. For obtaining the frequency response, the forward method is used and to determine the location and severity of damage inverse technique is used. The reverse technique is a numerical procedure to detect and quantify damage in a composite structure from changes natural frequencies and FRF using unified particle swarm optimization (UPSO) technique. Natural frequency based objective function is used for this optimization technique. PSO has found its application in several complex optimization problems in engineering including structural design optimization and structural damage detection because of its simplicity and increased convergence speed.

II. THEORETICAL FORMULATION

The governing differential equations for the present problem have been developed using first order shear deformation theory. For finite element analysis of problems involving complex geometry, it is difficult to use simple triangular and rectangular elements. However, these problems can easily be solved by using the concept of mapping. This concept is based on transformation of the parent elements such as regular triangular, rectangular or solid elements, in natural coordinate system to an arbitrary shape in the Cartesian coordinate system. A convenient way of expressing this transformation is by using a shape function of the rectilinear elements in their natural coordinate system and the nodal values of the coordinates. If the same shape functions are used for both the field variable and description of element geometry then it is called isoparametric mapping and the element defined by such a mapping is known as an isoparametric element. These isoparametric elements find their wide use in two and three dimensional stress analysis and analysis of plate and shells problem.

For the present analysis an 8-node isoparametric quadrilateral shell element is employed in the present analysis with five degrees of freedom *i.e.*, u, v, w, θ_x and θ_y per node. The shape function N_i for the eight noded quadrilateral isoparametric element is given by:

$$N_i = \begin{cases} (1+\xi\xi_i)(1+\eta\eta_i)(\xi\xi_i+\eta\eta_i-1)/4 & \text{for i = 1, 2, 3, 4} \\ (1-\xi^2)(1+\eta\eta_i)/2 & \text{for i = 5, 7} \\ (1+\xi\xi_i)(1-\eta^2)/2 & \text{for i = 6, 8} \end{cases} \tag{1}$$

Where ξ and η are the local isoparametric coordinates of the element and ξ_i and η_i are the respective values at node i. In the isoparametric finite element formulation, the variation of the displacements $(u, v, w, \theta_x \text{ and } \theta_y)$ and the coordinates (x, y) at a point within the element are expressed in terms of shape functions and nodal degrees of freedom as follows:

$$\begin{gathered} x=\sum_{i=1}^{8} N_i x_i ; \quad x=\sum_{i=1}^{8} N_i y_i ; \quad u_0=\sum_{i=1}^{8} N_i u_{0i} ; \quad v_0=\sum_{i=1}^{8} N_i v_{0i} ; \\ w_0=\sum_{i=1}^{8} N_i w_{0i} ; \quad \theta_x=\sum_{i=1}^{8} N_i \theta_{xi} ; \quad \theta_y=\sum_{i=1}^{8} N_i \theta_{yi} ; \end{gathered} \tag{2}$$

Where, u_i, v_i and w_i denotes the displacements and θ_x and θ_y are the rotations at the i^{th} node. The derivatives of the shape functions with respect to global coordinate system are related to their derivatives in natural coordinate system by the following relation:

$$\begin{Bmatrix} \dfrac{\partial N_i}{\partial x} \\ \dfrac{\partial N_i}{\partial y} \end{Bmatrix} = \begin{bmatrix} \dfrac{\partial \xi}{\partial x} & \dfrac{\partial \eta}{\partial x} \\ \dfrac{\partial \xi}{\partial y} & \dfrac{\partial \eta}{\partial y} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial N_i}{\partial \xi} \\ \dfrac{\partial N_i}{\partial \eta} \end{Bmatrix} = [J]^{-1} \begin{Bmatrix} \dfrac{\partial N_i}{\partial \xi} \\ \dfrac{\partial N_i}{\partial \eta} \end{Bmatrix} \tag{3}$$

Where, $[J]$ is known as Jacobian matrix and is used to map natural coordinate system to Cartesian coordinate system.

Strain - displacement relationship

For the present structural analysis, Green-Lagrange's strain displacement relations are used. According to this the displacement field of any point at a distance *z* from the mid plane is given as,

$$\begin{Bmatrix} u(x,y,z) \\ v(x,y,z) \\ w(x,y,z) \end{Bmatrix} = \begin{Bmatrix} u_o(x,y) \\ v_o(x,y) \\ w_o(x,y) \end{Bmatrix} + z \begin{Bmatrix} \varphi_x(x,y) \\ \varphi_y(x,y) \\ 0 \end{Bmatrix} \tag{4}$$

Where, u, v and w denotes the displacement components in x, y and z directions respectively. u_0, v_0 and w_0 are the displacements of a point on the mid plane $(x, y, 0)$ and ϕ_x and ϕ_y are the rotations of the cross-section perpendicular to x and y axes respectively. The mid plane of the plate is considered as the reference plane and the mid plane strains of the laminates are given by:

$$\varepsilon_x = \frac{\partial u}{\partial x};\ \varepsilon_{yy} = \frac{\partial v}{\partial y};\ \gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y};\ \gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z};\ \gamma_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \tag{5}$$

Assuming small deformations, the generalized linear in-plane strains of the laminate at a distance z from the mid surface are expressed as,

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \partial u_0/\partial x \\ \partial v_0/\partial y \\ \partial u_0/\partial y + \partial v_0/\partial x \\ \phi_y + \partial w_0/\partial y \\ \phi_x + \partial w_0/\partial x \end{Bmatrix} + z \begin{Bmatrix} \partial \phi_x/\partial x \\ \partial \phi_y/\partial y \\ \partial \phi_x/\partial y + \partial \phi_y/\partial x \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} + z \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \\ 0 \\ 0 \end{Bmatrix} \tag{6}$$

Where $\varepsilon_x^0,\ \varepsilon_y^0,\ \gamma_{xy}^0, \gamma_{yz}^0$ and γ_{xz}^0 are the mid plane strains and k_x, k_y and k_{xy} are the curvatures of laminated plate.

Stress - strain relationship

The force and moment resultants can be obtained by integrating the stresses and their moments through the laminate thickness as expressed bellow:

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \sum_{k=1}^{n} \left[Q_{ij} \right] \left\{ \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{xy}^0 \end{Bmatrix} dz + \int_{z_{k-1}}^{z_k} \begin{Bmatrix} k_{xx} \\ k_{yy} \\ k_{xy} \end{Bmatrix} zdz \right\} \tag{7}$$

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \sum_{k=1}^{n} \left[Q_{ij} \right] \left\{ \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{xy}^0 \end{Bmatrix} zdz + \int_{z_{k-1}}^{z_k} \begin{Bmatrix} k_{xx} \\ k_{yy} \\ k_{xy} \end{Bmatrix} z^2 dz \right\} \tag{8}$$

$$\begin{Bmatrix} S_{xz} \\ S_{yz} \end{Bmatrix} = \sum_{k=1}^{n} \left[Q_{ij} \right] \left\{ \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix} dz \right\}, \qquad i, j = 4, 5 \tag{9}$$

In matrix form, the above three equations can be expressed as:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ S_x \\ S_y \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{21} & A_{31} & B_{11} & B_{21} & B_{31} & 0 & 0 \\ A_{12} & A_{22} & A_{32} & B_{12} & B_{22} & B_{32} & 0 & 0 \\ A_{13} & A_{23} & A_{33} & B_{13} & B_{23} & B_{33} & 0 & 0 \\ B_{11} & B_{21} & B_{31} & D_{11} & D_{21} & D_{31} & 0 & 0 \\ B_{12} & B_{22} & B_{32} & D_{12} & D_{22} & D_{32} & 0 & 0 \\ B_{13} & B_{23} & B_{33} & D_{13} & D_{23} & D_{33} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{44} & S_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{45} & S_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \\ k_x \\ k_y \\ k_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{10}$$

In simplified matrix form,

$$\begin{Bmatrix} N \\ M \\ S \end{Bmatrix} = \begin{bmatrix} A & B & 0 \\ B & D & 0 \\ 0 & 0 & S \end{bmatrix} \begin{Bmatrix} \varepsilon \\ k \\ \gamma \end{Bmatrix} \tag{11}$$

Or, $$\{F\} = [D]\{\varepsilon\} \tag{12}$$

In Eq.11, A, B and D denotes the extension, bending - stretching coupling, and bending stiffness respectively. S denotes the transverse shear stiffness of the laminate. These are mathematically calculated by:

$$A_{ij} = \sum_{k=1}^{n} \left(Q_{ij}\right)_k \left(z_k - z_{k-1}\right) \tag{13}$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} \left(Q_{ij}\right)_k \left(z_k^2 - z_{k-1}^2\right) \tag{14}$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} \left(Q_{ij}\right)_k \left(z_k^3 - z_{k-1}^3\right) \qquad For\, i, j = 1,\ 2, 6 \tag{15}$$

$$S_{ij} = \kappa \sum_{k=1}^{n} \left(Q_{ij}\right)_k \left(z_k - z_{k-1}\right) \qquad For\, i, j = 4,\ 5 \tag{16}$$

The term κ denotes the shear correction factor $= (5/6)$ and z_k and z_{k-1} denotes the top and the bottom distance of the lamina from the mid plane.

Stiffness matrix

The linear strain is described in terms of nodal displacement as follows:

$$\{\varepsilon\} = [B]\{d_e\} \tag{17}$$

Where, $\{d_e\} = \{u_1 v_1 w_1 \theta_{x1} \theta_{y1} u_2 v_2\ \ldots w_8 \theta_{x8} \theta_{y8}\}^T$ and $[B] = [[B_1][B_2] \ldots [B_8]]$

$$[B_i]=\begin{bmatrix} \partial N_i/\partial x & 0 & 0 & 0 & 0 \\ 0 & \partial N_i/\partial y & 0 & 0 & 0 \\ \partial N_i/\partial y & \partial N_i/\partial x & 0 & 0 & 0 \\ 0 & 0 & 0 & \partial N_i/\partial x & 0 \\ 0 & 0 & 0 & 0 & \partial N_i/\partial y \\ 0 & 0 & 0 & \partial N_i/\partial y & \partial N_i/\partial x \\ 0 & 0 & \partial N_i/\partial y & N_i & 0 \\ 0 & 0 & \partial N_i/\partial x & 0 & N_i \end{bmatrix}_{i=1,2,\dots,8} \tag{18}$$

The potential energy of deformation for the element is given by

$$U_e=\frac{1}{2}\iint\{d_e\}^T[B]^T[D][B]\{d_e\}\;=\frac{1}{2}\{d_e\}^T[K_e]\{d_e\} \tag{19}$$

Where the matrix $[K_e]$ denotes the elemental stiffness matrix and it is given mathematically as:

$$[K_e]=\int_{-1}^{1}\int_{-1}^{1}[B]^T[D][B]|J|\,d\xi d\eta \tag{20}$$

$|J|$, denotes the determinant of the Jacobian matrix. The element stiffness matrix can be expressed in local natural coordinates of the element. The integration of above equation is carried out using Gauss quadrature method.

Consistent mass matrix

The consistent element mass matrix $[M_e]$ is expresses as

$$[M_e]=\int_{-1}^{1}\int_{-1}^{1}[N]^T[P][N]|J|\,d\xi d\eta \tag{21}$$

Where $[N]$ is the shape function matrix and $[P]$ is the inertia matrix given bellow.

$$[N_i]=\begin{bmatrix} N_i & 0 & 0 & 0 & 0 \\ 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & N_i & 0 & 0 \\ 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & N_i \end{bmatrix}_{i=1,2,\dots,8} \tag{22}$$

$$[P]=\begin{bmatrix} P_1 & 0 & 0 & P_2 & 0 \\ 0 & P_1 & 0 & 0 & P_2 \\ 0 & 0 & P_1 & 0 & 0 \\ P_2 & 0 & 0 & P_3 & 0 \\ 0 & P_2 & 0 & 0 & P_3 \end{bmatrix}_{i=1,2,\dots,8} \tag{23}$$

Where, $\left(P_1,\, P_2,\, P_3\right) = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \left(\rho\right)_k \left(1, z, z^2\right) dz$

Anisotropic damage

In a thin plate, anisotropic damage is parametrically incorporated into the finite element formulation by considering the damage parameter which is a representation of reduction in effective load bearing area [3] and is given by:

$$\Gamma_i = \frac{A_i - A_i^*}{A_i} \tag{24}$$

Where A_i^* is the effective area (with unit normal) after damage

A_i is the area of damaged material with unit normal n_i

$i \in \{1, 2, 3\}$ are the three orthogonal directions

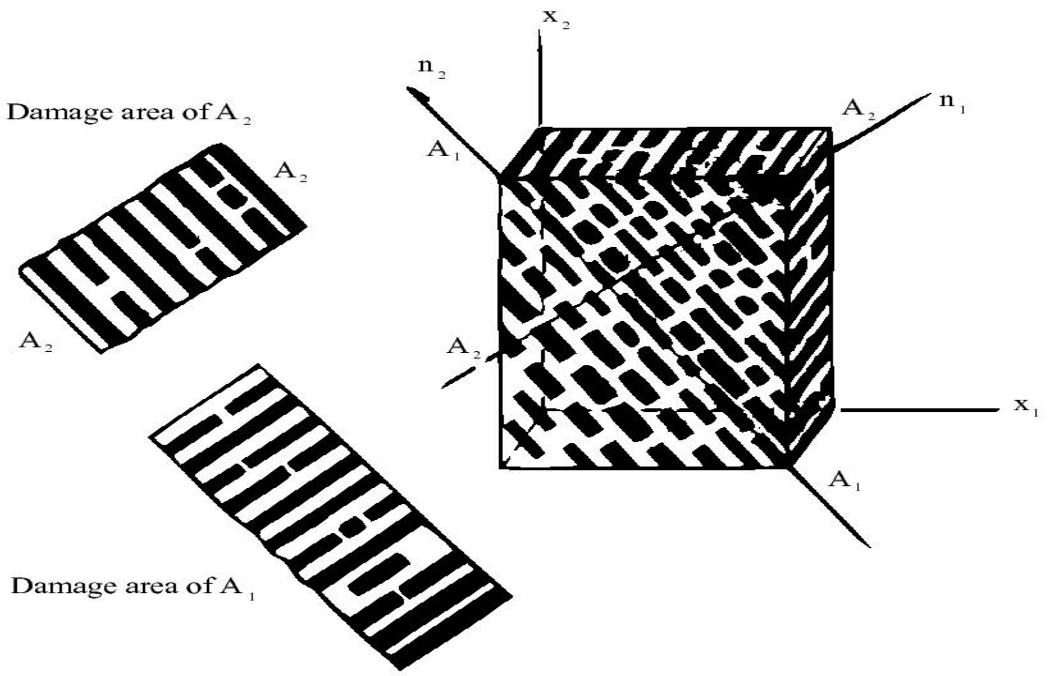

Figure 1. Illustration diagram of damage material

Assuming that the internal forces acting on any damaged section is the one before damage:

$$\sigma_{ij}\delta_{jk}A_k = \sigma_{ij}^*\delta_{jk}A_k^* \tag{25}$$

$$\left\{\sigma^*\right\} = \left\{\psi\right\}\left\{\sigma\right\} \tag{26}$$

where, $\{\psi\}$ is a transformation matrix and can be used to relate a damaged stress matrix with that of undamaged one and is given as below:

$$[\Psi]=\begin{bmatrix} \frac{1}{1-\Gamma_1} & 0 & 0 \\ 0 & \frac{1}{1-\Gamma_2} & 0 \\ 0 & 0 & \frac{1}{1-\Gamma_1} \\ 0 & 0 & \frac{1}{1-\Gamma_2} \end{bmatrix} \qquad \begin{aligned} \{\sigma^*\}^T &= \{\sigma_{11}^* \;\; \sigma_{22}^* \;\; \sigma_{12}^* \;\; \sigma_{21}^*\} \\ \{\sigma\}^T &= \{\sigma_{11} \;\; \sigma_{22} \;\; \sigma_{12}\} \end{aligned}$$

The damage model should not assume the damage tensor to be symmetric in order to define the damage effectively in composites.

$$\sigma_{21}^* = \frac{1-\Gamma_2}{1-\Gamma_1}\sigma_{12}^* \tag{27}$$

Unified Particle Swarm Optimization

The particle swarm optimization (PSO) algorithms, first proposed by Kennedy and Eberhart [4], are inspired by the collective motion of insects and birds trying to reach an unknown destination, known as "swarm behavior". This process involves both social interaction and intelligence so that birds learn from their own experience and also from the experience of others around them. PSO algorithm has advantages lies with its simplicity in its architecture and convergence speed. However, the main drawback associated with standard PSO is that the algorithm may converge into some local optima. This may lead to prediction of wrong results. Further, to improve its efficiency many alternations and variations are proposed to the original PSO algorithm, among which unified particle swarm optimization (UPSO) one that has the ability to harness both exploration and exploitation capacity simultaneously [Parsopoulos and Vrahatis (5)] by balancing the influence of both global and local search directions simultaneously. Mathematically, for a swarm size of P number of particles, in an S-dimensional search space, let G_{ij}^{t+1} and L_{ij}^{t+1} denotes the velocity update of i^{th} particle in global and local variants of PSO respectively for the $(t+1)^{th}$ iteration as given by,

$$G_{ij}^{t+1} = \chi\left[v_{ij}^t + c_1 r_1\left(pbest_{ij} - x_{ij}^t\right) + c_2 r_2\left(gbest_{ij} - x_{ij}^t\right)\right] \tag{28}$$

$$L_{ij}^{t+1} = \chi\left[v_{ij}^t + c_1 r_3\left(pbest_{ij} - x_{ij}^t\right) + c_2 r_4\left(lbest_{ij} - x_{ij}^t\right)\right] \tag{29}$$

Where, $pbest, gbest$ and $lbest$ respectively denotes the best position explored by individual particle, any particle in the swarm and in the neighborhood of individual swarm. χ denotes the constriction factor which is equals to 0.72984. c_1 and c_2 are two acceleration coefficients and is considered as 2.05 each in present study. Finally, all r terms denote random numbers between [0, 1] and independent of each other. Combining Equations (28) and (29), the aggregate velocity of the particles in the search directions is defined as,

$$V_{ij}^{t+1} = u.G_{ij}^{t+1} + (1-u).L_{ij}^{t+1}, \qquad u \in [0,1] \tag{30}$$

The new position of the particles for $(t+1)^{th}$ iteration is,

$$x_{ij}^{t+1} = x_{ij}^{t} + V_{ij}^{t+1}, \qquad \forall i \in P \ \ and \ \ \forall j \in S \tag{31}$$

The parameter, u in Equation (30) is called unification factor and its value is modified throughout the iterations according to the equation,

$$u(t) = \exp\left(\frac{t.\log(2.0)}{t_{\max}}\right) - 1.0 \tag{32}$$

Objective Function

Objective function is an equation to be optimized under certain given constraints and with variables that need to be minimized or maximized using nonlinear programming techniques. It maps the search space to the function space. Natural frequencies obtained from the damaged structure are used for constructing the objective function for the present problem and is given as:

$$F = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\left(\frac{f_i^m}{f_i^c}\right)-1\right)^2} \tag{33}$$

The terms f_i^m and f_i^c are the measured natural frequencies for damaged structure and the natural frequency obtained from finite element simulation for damaged structure respectively. n is the number of input response parameters (i.e. Natural frequencies or FRF).

Damage orthogonality

The damage parameters determine the extent of damage in a particular orthogonal direction. The direction of damage with respect to the direction of fibre orientation plays an important role in influencing the dynamic instability and dynamic response of the structure. A reduction in the stiffness in the direction of fibre which influences the buckling, vibration and flutter characteristics more profoundly than a reduction of stiffness occurring in a direction perpendicular to the fibre orientation. The ratio of orthogonal damage parameters is varied from 1.0, i.e., mild damage to 9.0, i.e., on the verge of total fibre breakage. In order to understand this; the natural orthotropy exhibited by the formulation, one of these orthogonal parameters must be fixed while the other must be varied to obtain the static and the dynamic characteristics of the structure.

III. RESULTS AND DISCUSSIONS

1. NUMERICAL STUDY

For the validation of present numerical method, non-dimensional fundamental natural frequencies of undamaged composite structures for various boundary conditions are compared with available results (Table I). Material properties of a two-layer cross-ply laminated square plates (0/90) are given as: (E_1/E_2 = 40, b/H = 100, E_1 = 40*10^10 N/m_2, G_{12} = G_{13} = $0.5E_2$, G_{23} = $0.6E_2$, υ_{12} = 0.25)

TABLE I: COMPARISON OF NON-DIMENSIONAL NATURAL FREQUENCIES FOR VARIOUS BOUNDARY CONDITIONS

BOUNDARY CONDITION		**NON – DIMENSIONAL FREQUENCIES**			
		a/h = 5	**a/h = 10**	**a/h = 20**	**a/h = 50**
SSSS	**Present**	7.3890	8.8999	9.4744	9.6597
	Ref [6]	7.3901	8.9003	9.4750	9.6600
SCSS	**Present**	8.1292	10.6118	11.8052	12.2357
	Ref [6]	8.1295	10.6120	11.8054	12.2358
SCSC	**Present**	8.9333	12.6225	14.8750	15.8101
	Ref [6]	8.9334	12.6225	14.8748	15.8097

TABLE II: VARIATION OF FUNDAMENTAL NATURAL FREQUENCY WITH VARIOUS INTENSITIES OF DAMAGE OF A CENTRALLY DAMAGED, SIMPLY SUPPORTED, CROSS-PLY (0/90/0) COMPOSITE PLATE WITH INDIVIDUAL DAMAGE PARAMETERS Γ_1 AND Γ_2 WHILE OTHER KEPT FIXED AT 0.1. THICKNESS RATIO IS B/H = 100. DAMAGE IS IN 36% AREA OF ALL THE LAYERS.

($E_1/E_2 = 40$, $E_1 = 40*10$^10 N/M^2, $G_{12} = G_{13} = 0.5\ E_2$, $G_{23} = 0.2E_2$, $\upsilon_{12} = 0.25$)

$\Gamma_1 = 0.1$		$\Gamma_2 = 0.1$	
Γ_2	Non-dimensional natural frequency (ω)	Γ_1	Non-dimensional natural frequency (ω)
0	10.0938	0	10.4328
0.2	10.0737	0.2	9.7594
0.4	10.0572	0.4	9.1993
0.6	10.0442	0.6	8.7752
0.8	10.0348	0.8	8.5085
0.9	10.0315	0.9	8.4390

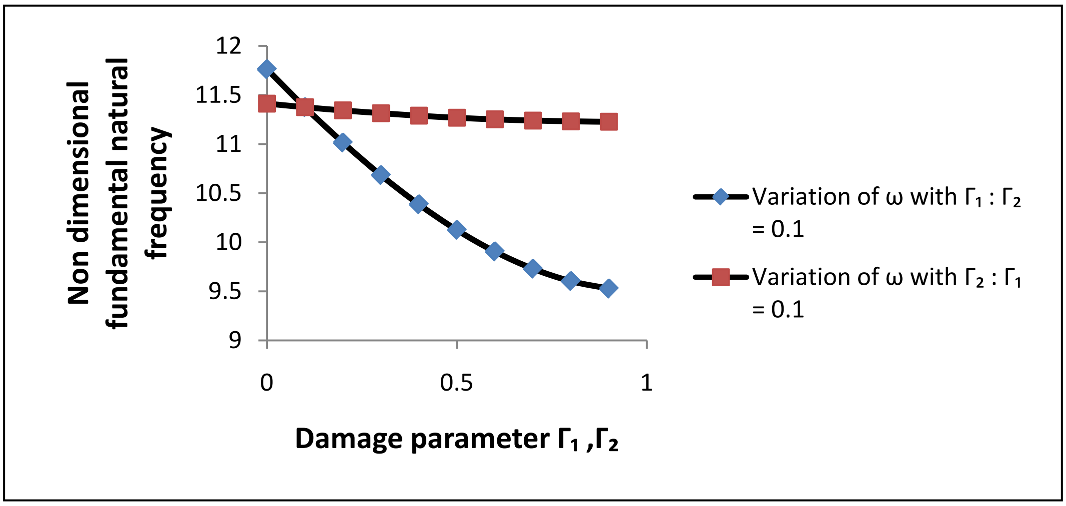

Figure 2. Reduction of fundamental natural frequency with increase in intensity of damage of a damaged clamped-simply supported composite plate (45/-45/45) with individual damage parameters Γ_1 and Γ_2 whileanotherr kept fixed at 0.1. Thickness ratio is b/h = 100. The damage is in the 25 % area of the plate, throughout the thickness. ($E_1/E_2 = 25$, $E_1 = 25*10$^10 N/m^2, $G_{12} = G_{13}= 0.5\ E_2$, $G_{23} = 0.2E_2$, $\upsilon_{12} = 0.25$)

It can be observed from Table II, that the fundamental natural frequency decreases with the increase in damage ratio. The damage ratio Γ_1/Γ_2 takes values between 0.0 and 9.0. A mild damage may be represented with a damage ratio $0.0 \leq \Gamma_1/\Gamma_2 \leq 3.0$ while a heavy damage may be denoted by the range of values, $7.0 \leq \Gamma_1/\Gamma_2 \leq 9.0$. It is observed from the following table as the damage intensity increases the natural frequency decreases.

Since the model of damage being investigated is anisotropic in nature, it is essential to understand the nature of orthotropy exhibited by this formulation. It may be inferred from Fig. 2 that, the contribution of damage in the direction of fibre may not be equal to that in the orthogonal direction. It can be observed that damage in the direction of fibre, Γ_1 shows a steeper deterioration of natural frequencies.

To study the effect of damaged area, various damaged, clamped-simply supported composite plates with different ply layout have been analyzed. Table III tabulates the variation of non-dimensional fundamental natural frequencies with damage area with thickness ratio b/h = 100, having central damage having damage intensities Γ_1/Γ_2 = 1.0 (mild damage) and Γ_1/Γ_2 = 9.0 (heavy damage). The drop in fundamental natural frequency in the heavily damaged case is more as compared to the mild one. The fundamental natural frequency drops for all plies as the damage area increases. The material properties are given as: $E_1/E_2 = 25$, $E_1 = 25*10^{10}$ N/m^2, $G_{12} = G_{13} = 0.5 E_2$, $G_{23} = 0.2E_2$, $\upsilon_{12} = 0.25$.

TABLE III. VARIATION OF NON-DIMENSIONAL FUNDAMENTAL NATURAL FREQUENCIES WITH % OF DAMAGED AREA FOR CSCS SQUARE COMPOSITE PLATES WITH THICKNESS RATIO B/H = 100 AND B/H = 10 AND $\Gamma_1/\Gamma_2 = 9$.

Ply orientation	Damaged area in %				
	0	20	40	60	80
b/h = 100					
0/90/0/90	17.15	15.457	13.552	11.327	8.523
45/-45/45/-45	16.505	14.981	13.273	11.288	8.794
b/h = 10					
0/90/0/90	10.512	9.4309	8.2069	6.7613	4.889
45/-45/45/-45	10.109	9.0971	7.9520	6.5979	5.240

1.1. Comparative study of damaged structure

To study the effect of damage on the dynamics of a structure and to validate the correctness of present finite element code for damaged structure developed in MATLAB environment; a comparative study is made with a published literature [13]. A surface crack is created to represent nominal damage which is comparatively small and does not significantly affect the global stiffness of the woven laminate. The dimension of the laminate panel is 310 mm length, 222 mm width and 2.2 mm thickness. Density of the material is 1447 kg/m^3. A cross-ply model is established by assigning laminae [0/90] for each element with different tensile modulus E_1 and E_2 in fiber and matrix directions. The equivalent cross-ply properties are E_1 = 87.6 GPa, E_2 = 5.13 GPa, G_{12} = 4.08 GPa, υ_{12} = 0.315. First five frequencies in Hz are compared with present result. From the comparison results shown in Table VI., it is clear that, the results obtained from present MATLAB code shows similar values as that of the referenced literature. Thus from all these validation studies, the correctness of FEM formulation and corresponding MATLAB code for undamaged and damaged structure may be ascertained.

TABLE IV. FREQUENCIES OF DAMAGED CROSS-PLY PLATE

Mode	Without damage		With damage	
	Present	Ref [13]	Present	Ref [13]
(1,1)	34.474	-	34.3772	-
(2,2)	56.593	55.54	56.400	55.56
(3,1)	144.750	142.7	144.189	142.6
(3.2)	215.900	229.5	215.292	229.5
(1,3)	247.005	253.1	246.265	253.1
(3,3)	333.215	347.3	331.835	347.3

1.2.Damage Detection of composite structures

The damage detection procedures outlined are used for assessment of damage in the composite beam and plate like structures by developing computer program in MATLAB environment. The developed procedure is demonstrated by considering few numerical and experimental examples. Two numerical examples considered for the study includes damage assessment in a numerical beam and a numerical plate structure.

Damage assessment in beam like structures:

To demonstrate the efficacy of the present damage assessment algorithm, for the evaluation of damage in composite beam like structures, two numerical examples are considered: a cantilever beam and a fixed beam. The material properties for the beam are given in Table V which is obtained experimentally. The physical dimensions of the beam are given as: 500 mm in length, 30 mm in width and 2.8 mm in thickness. Damage is introduced to the structure in terms of reduction of effective load bearing area Γ_1 and Γ_2. The algorithm is verified for damage assessment of both single case and multiple element damage cases. For finite element simulation the beams are modelled with 10 orthotropic elements as shown in Figure 3. Numerically evaluated natural frequency is considered for constructing the objective function. First six natural frequencies are used for damage assessment. Four damage conditions are considered for each beam type i.e., two single damage condition and two multiple damage conditions. These damage cases and the values of first six natural frequencies for cantilever beam are presented in Tables VI. and that of fixed beam is shown in Table VII.

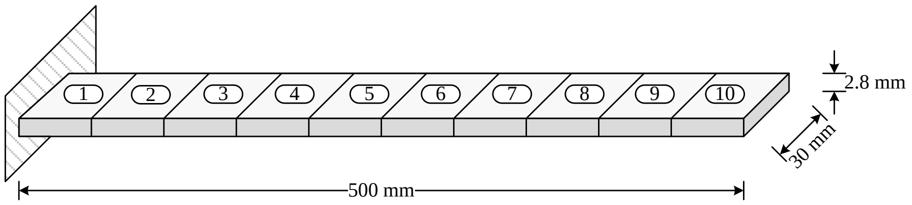

Figure 3. Cantilever beam made of composite material.

TABLE V. MATERIAL PROPERTIES OF EXPERIMENTAL COMPOSITE PLATE

E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	υ_{12}	ρ (kg/m^3)
9.51	6.82	1.4	0.14	1761

TABLE VI. DAMAGE CASES SELECTED FOR SIMULATION STUDIES IN CANTILEVER BEAM STRUCTURE AND THE RESPECTIVE NATURAL FREQUENCIES

Damage Id.	Damage Conditions [Element No. (Γ_1, Γ_2)]	Natural Frequencies (Hz)					
		1st	2nd	3rd	4th	5th	6th
Undamaged		4.17	26.10	44.41	73.09	82.92	143.42
J1	5 (0.30, 0.00)	4.03	24.09	42.98	72.05	80.59	135.73
J2	5 (0.00, 0.20)	4.16	25.9	44.5	73.3	81.6	144
J3	2 (0.25, 0.00) 6 (0.30, 0.00)	3.78	23.25	39.69	70.67	78.86	134.28
J4	2 (0.00, 0.30) 6 (0.00, 0.40)	4.1	25.9	44.6	73.6	77.8	143.1

TABLE VII. DAMAGE CASES SELECTED FOR SIMULATION STUDIES IN FIXED BEAM STRUCTURE AND THE RESPECTIVE NATURAL FREQUENCIES

Damage Id.	Damage Conditions [ElementNo. (Γ_1, Γ_2)]	Natural Frequencies (Hz)					
		1st	2nd	3rd	4th	5th	6th
Undamaged		26.5	73.2	143.6	170	238.3	271.1
J5	5 (0.30, 0.00)	25.1	72.1	135.6	170.7	231.2	255.3
J6	5 (0.00, 0.20)	26.5	73.5	142.8	170.3	239.3	273.1
J7	2 (0.25, 0.00) 6 (0.30, 0.00)	24.6	71.2	132.9	166.3	219.5	252.6
J8	2 (0.00, 0.30) 6 (0.00, 0.40)	26.7	73.7	143.9	164.3	237.8	269.8

TABLE VIII. DAMAGE CASES SELECTED FOR SIMULATION STUDIES AND ASSESSMENT RESULTS OF CANTILEVER BEAM

Damage Case	**Damage Cases**	**Damage Assessment Results** $\left[\text{Element No. } (\Gamma_1, \Gamma_2)\right]$	
		Actual	**Present code**
One element damage	J1	5 (0.30, 0.00)	5 (0.31, 0.00)
	J2	5 (0.00, 0.20)	5 (0.00, 0.199)
Two element damage	J3	2 (0.25, 0.00)	2 (0.249, 0.00)
		6 (0.30, 0.00)	6 (0.299, 0.00)
	J4	2 (0.00, 0.30)	2 (0.00, 0.295)
		6 (0.00, 0.40)	6 (0.00, 0.399)

TABLE IX. DAMAGE CASES SELECTED FOR SIMULATION STUDIES AND ASSESSMENT RESULTS OF FIXED BEAM

Damage Case	**Damage Cases**	**Damage Assessment Results** $\left[\text{Element No. } (\Gamma_1, \Gamma_2)\right]$	
		Actual	**Present code**
One element damage	J5	5 (0.30, 0.00)	5 (0.30, 0.00)
	J6	5 (0.00, 0.35)	5 (0.00, 0.345)
Two element damage	J7	2 (0.35, 0.00)	2 (0.351, 0.00)
		6 (0.25, 0.00)	6 (0.249, 0.00)
	J8	2 (0.00, 0.30)	2 (0.00, 0.31)
		6 (0.00, 0.40)	6 (0.00, 0.39)

UPSO algorithm is employed to search the actual damaged element and damage amount. For optimization purpose, 50 swarms are considered for the study. Maximum number of iteration allowed is 300. Each experiment is run for 3 times and the run that provided minimum objective function value is considered as the actual damage condition. The results of damage assessment of cantilever and fixed beam are presented in Tables VIII. and IX. respectively.

It may be observed from the above tables that, for all considered cases the algorithm could detect and quantify the damages to a significant accuracy. This signifies the efficacy and robustness of the present algorithm for damage assessment in composite beams. Thus from these numerical simulation studies it may be concluded that, the present algorithm is able to identify the damage present in a composite beam or plate like structures from first few natural frequency values.

2. EXPERIMENTAL STUDY OF DAMAGED COMPOSITE BEAM

The glass/polyester composite specimens are fabricated using hand lay-up technique. The constituent materials used for fabricating the glass fiber plates are: E-glass woven roving as reinforcement, unsaturated polyester as resin amounting two times the weight of Fiber, calcium carbonate (10% of the weight of resin), aerosol powder (7% of the weight of resin), catalyst (1% of the weight of resin) and accelerator (1% of the weight of resin).

For measuring the Young's modulus and other material properties, the specimen is loaded in INSTRON 1342 universal testing machine, with a recommended rate of extension (rate of loading) of 0.1kN/minute. Specimens were fixed in the upper jaw first and then gripped in the movable jaw (lower jaw). Gripping of the specimen was as much as possible to prevent the slippage. For that 50mm in each end is taken for gripping. Initially strain was kept at zero. The load, as well as the strain, was recorded digitally with the help of a strain gauge indicator. From these data, engineering stress vs. strain curve was plotted; the initial slope of which gives the Young's modulus. The ratio of transverse to longitudinal strain directly gives the Poisson's ratio by using two strain gauges (Quarter Bridge) in longitudinal and transverse direction. Table X. gives the material constants measured for the test specimens.

TABLE X. MATERIAL PROPERTIES OF EXPERIMENTAL COMPOSITE PLATE

E_1 (GPa)	**E_2 (GPa)**	**G_{12} (GPa)**	**υ_{12}**	**ρ (kg/m^3)**
9.51	6.82	1.4	0.14	1761

The vibration data such as frequency, FRF, modal damping etc has been extracted with the help of NVGate (Noise and Vibration) software. Impact hammer force is used to excite the structure and uniaxial accelerometers are used to record the acceleration response of the test structure. The first eight natural frequencies of undamaged and damaged composite beams which obtained from the experiment are given in Table XI.

TABLE XI. EXPERIMENTAL NATURAL FREQUENCIES FOR THE COMPOSITE BEAM

Mode	**Natural Frequencies (in Hz)**	
	Undamaged beam	**Damaged Beam**
1st	-	-
2nd	32.5	32.5
3rd	93.75	92.5
4th	183.75	181.25
5th	302.5	300
6th	451.25	447.5
7th	630	615
8th	838.75	828.75

The numerical model has been considered based on the material and geometric properties of the test structure. But the prediction of the finite element model is quite different from the measurements which are obtained experimentally. This may be due to the factors such as variation in material property, dimensional tolerance, defects during fabrication and noise in the measurements. In such cases, the numerical model update is carried out to ensure that the numerical results match with the measured response of the experimental results within acceptable limits. For this purpose, the young's modulus and density of the beam are considered as the variables for updating process. The updating of these parameters has been carried out using UPSO in which objective function is to reduce the difference between the natural frequencies of experimental model and updated numerical model. The initial and updated model parameters are given in Table XII.

TABLE XII. INITIAL AND UPDATED MODEL PARAMETERS OF UNDAMAGED BEAM

Parameter	Initial model	Updated model
E_{11} (GPa)	9.51	12.3
E_{22} (GPa)	6.82	8
Density (kg/m^3)	1761	1761

From Table XIV, it is inferred that experimental values are almost matching with the updated natural frequencies. It also shows the comparison to validate the numerical results with experimental values. The frequencies obtained using ANSYS are also presented to validate the developed numerical code in MATLAB environment.

TABLE XIV. NATURAL FREQUENCIES OF THE UNDAMAGED BEAM BEFORE AND AFTER MODEL UPDATE

Mode	**Natural frequencies of undamaged beam (Hz)**								
	Experimental (Hz)	**Initial FE Results**		**Initial ANSYS Results**		**Updated FE Results**		**Updated ANSYS Results**	
		Freq (Hz)	Error (%)	Freq (Hz)	Error (%)	Freq (Hz)	Error (%)	Freq (Hz)	Error (%)
1st	-	4.5	-	4.66	-	5.2	-	5.3	-
2nd	32.5	28.3	12.92	29.198	10.16	32.8	0.92	33.2	2.15
3rd	93.75	86.5	7.73	86.711	7.51	91.9	1.97	92.9	0.90
4th	183.75	155.6	15.32	160.12	12.86	180.2	1.93	181.99	0.95
5th	302.5	292.0	3.47	298.86	1.20	299	1.15	301.04	0.48
6th	451.25	438.8	2.76	442.94	1.84	449.9	0.29	451.17	0.01
7th	630	623.9	0.97	632.58	0.41	636.6	1.04	630.28	0.04
8th	838.75	818.9	2.37	833.08	0.68	842.9	0.49	835.86	0.34

IV. CONCLUSION AND SUMMARY

The present study deals with damage at the macro level that can be described using anisotropic parameters and its effects on free vibration has been observed. The present formulation can successfully compute the free vibration characteristics and the results have been compared with the available literature.

Damage in composite plates shows strong orthogonality. It has been observed that damage in the direction of fibre results in steeper deterioration of natural frequencies and damage in orthogonal direction to fibre has little influence on fundamental natural frequency.

It can be observed that as the intensity of damage or damage area or size increases, deteriorates the natural frequency irrespective of ply orientation.

A numerical procedure is presented to detect and quantify damages in a composite structures based on changes natural frequency data using unified particle swarm optimization technique. The proposed methodology is demonstrated using a numerically simulated composite beam like structures containing single and multiple damages. As indicated by the simulation results, the proposed method is able to detect and quantify the damage accurately using first six natural frequencies for considered damage cases.

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