Free Vibration Characteristics of Damaged Composite Structures Using FEM

T.R. Jebieshia, V.M. Sreehari and Dipak K. Maiti

Abstract— *The present work deals with the study of free vibration characteristics of damaged cross-ply and angle-ply composite laminates. The formulation is to find out the variation or reduction in natural frequency due to damage in the composite structure using FEM. Finite element method is used to obtain the governing differential equation of thin composite plates. It is observed that the anisotropic damage shows strong orthogonality and in general deteriorates the vibration characteristics. It is also observed that as the damage intensity and damage area increases, there is considerable reduction occurs in the natural frequencies.*

Keywords - *FEM, Anisotropic Damage, Natural Frequency, Laminated Composite Plates*

I. Introduction

Composite materials are widely being used in structures, especially in aircraft and spacecraft, to minimize weight and increase stiffness by tailoring the structures, and also to meet performance requirements. Their behavior of the composites is designed according to their usage; therefore their advantages were fully utilized. Composite materials are highly sensitive to the presence of manufacturing and service related defects that can reach a critical size during service and compromise the safety of the structure.

Unlike isotropic materials that are mostly damaged by cracks or undergo a reduction of stiffness due to fatigue and surface wear and tear from their prolonged usage over time, composite materials usually experience damages in the form of blunt body impact, fibre breakage, and delamination. Composite materials can also exhibit in-homogeneity if there are air pockets or voids in the matrix of the fibre-reinforced composite. Such in-homogeneities are mostly caused by improper or erroneous manufacturing techniques. Analysis of such 'improper' structures can be carried out by incorporating a suitable damage model in the formulation. These damages or in-homogeneities in the structure are reflected in the spectrum as shifts in the natural frequencies and the instability behavior of the structure.

During the service life, defects may appear in the structure. When the structure undergoes some kind of damage, its stiffness is reduced, changing the dynamic response. The occurrence of damage in a structure modifies some of its mass, stiffness or damping properties, changing the vibration response of the structure. Therefore, the knowledge of the vibration behavior of a structure can be used to determine the existence as well as the location and the extent of damage. The damage in composite materials is anisotropic in nature. An anisotropic damage is a damage whose extent is distributed in several orthogonal directions and the extent of damage in any orthogonal direction is independent of the other.

A vast amount of published literature and investigation is available on the vibration and damage modeling of composite structures. Valliappan et al. [1] developed a finite element model of anisotropic damage based on the structural stiffness reduction factor and the elastic constitutive relationships for anisotropic damage mechanics and also found that the reduction of in-plane and bending stiffness and re-distribution of membrane stresses are direct consequences of such damages being present in the system which in turn affect the static and dynamic response characteristics of the system.

Prabhakara and Datta [2] studied the free vibration and the static stability behavior of a rectangular plate with localized zones of damage using a finite element analysis. Joris and Wim [3] found that the gradual deterioration of a fibre reinforced composite – with a loss of stiffness in the damaged zones – leads to a continuous redistribution of stress and a reduction of stress concentration inside a structural component.

Objective

Since as far as any engineering structure under operation is concerned there exists a finite probability of damage occurring at some point of its operating lifetime. So the study of damaged composite structure is important. The free vibration and damage analysis of laminated composites structures is done using finite element method i.e., the governing differential equations have been developed using FEM and then implementing the problem into Matlab coding to obtain the various results. The present study focuses on the effect of damage intensity, damage size on free vibration characteristics of composite structures and to determine the effect of damage orthogonality on free vibration characteristics by choosing suitable values of damage parameters. The presence of damage by itself doesn't mean that the structure has failed. There is a reduction in stiffness value occurs due to the introduction of damage to the structure. The reduction in non-dimensional frequency with respect to the area, size and intensity of damage were analyzed.

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II. Damage Modelling Based On Stiffness Loss

When engineering materials are subjected to unfavourable conditions such as cold and hot working processes, temperature variation, chemical action, radiation, mechanical loading or environmental condition, microscopic defects and cracks may develop inside the materials. This damage causes a reduction in strength that may lead to failure to shorten the operating life of the structures. This deterioration in mechanical properties of a material is known as a damage process. The calculations involved in isotropic damage state are very much simplified due to the scalar nature of the damage variable. For anisotropic damage, the variable is tensorial nature and hence the identification of the models is much more complicated. Ladeveze and LeDantec [4] modelled the fibrous composite laminate damage at the elementary-ply scale. Damage variables are defined and associated with the material stiffness reduction. The behaviour differences between tension and compression in the fibre direction are treated. The understanding vibration characteristics are essential in fail safe design. However, in spite of the complexity involved, the damage mechanics have a promising potential as a means of analyzing and assessing the strength of materials as a complement to fracture mechanics.

Anisotropic Damage

Fig 1 shows the illustration of damaged material. In a thin plate, anisotropic damage is parametrically incorporated into the formulation by considering the damage parameter Γ_i , which is a representation of reduction in the effective load bearing area and is given by: $\Gamma_i = \frac{A_i - A_i^*}{A_i}$
Where,

A_i^* is the effective area (with unit normal) after damage

A_i is the area of undamaged material with unit normal n_i

$i \in \{1, 2, 3\}$ are the three orthogonal directions

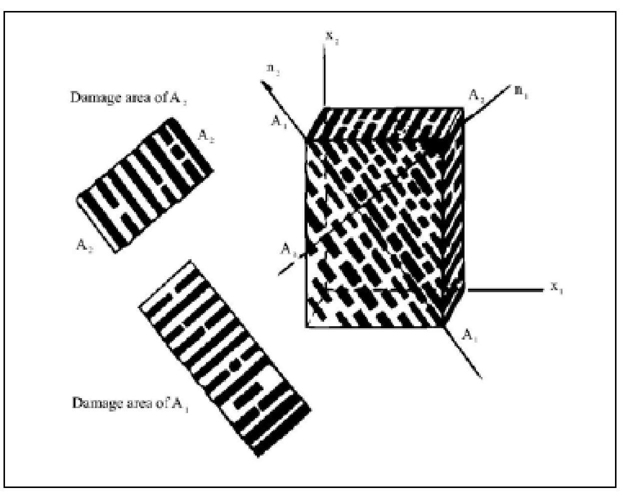

Figure 1. Illustrative diagram of damaged material

III. Mathematical Formulation

Assuming that the internal forces acting on any damaged section is the one before damage:

$$\sigma_{ij} A_k = \sigma_{ij}^* A_k^*$$

$$\{\sigma^*\} = [\Psi]\{\sigma\}$$

$[\Psi]$ is used to relate the damaged stress matrix with the undamaged one

$$[\Psi] = \begin{bmatrix} \frac{1}{1-\Gamma_1} & 0 & 0 \\ 0 & \frac{1}{1-\Gamma_2} & 0 \\ 0 & 0 & \frac{1}{1-\Gamma_1} \\ 0 & 0 & \frac{1}{1-\Gamma_2} \end{bmatrix}$$

The damage model should not assume the damage tensor to be symmetric in order to define the damage effectively in composites.

$$\sigma_{21}^* = \frac{1-\Gamma_2}{1-\Gamma_1}\sigma_{12}^*$$

If it is symmetrized, the effective stress vector is :

$$\{\overline{\sigma}\} = \{\overline{\sigma}_1 \quad \overline{\sigma}_2 \quad \overline{\sigma}_{12}\} = \left\{\sigma_1^* \quad \sigma_2^* \quad \sqrt{\frac{{\sigma_{12}^*}^2 + {\sigma_{21}^*}^2}{2}}\right\}$$

Effective stress can be related to the real stresses by:

$$\begin{bmatrix} \overline{\sigma}_1 \\ \overline{\sigma}_2 \\ \overline{\sigma}_{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{1-\Gamma_1} & 0 & 0 \\ 0 & \frac{1}{1-\Gamma_2} & 0 \\ 0 & 0 & \sqrt{\frac{1}{2}\left(\frac{1}{(1-\Gamma_1)^2} + \frac{1}{(1-\Gamma_2)^2}\right)} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{bmatrix}$$

$$\{\overline{\sigma}\} = [\overline{\Psi}] \{\sigma\}$$

$$[D^*]^{-1} = [\overline{\Psi}]^T [D]^{-1} [\overline{\Psi}]$$

Damaged stress-strain relation can be given as:

$$[\sigma^*] = [Q_{damaged}][\varepsilon^*]$$

$$[Q_{damaged}] = [T]^T [D^*][T]$$

[T] is the transformation matrix and is given by:

$$[T] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & \sin 2\theta \\ \sin^2\theta & \cos^2\theta & -\sin 2\theta \\ -\frac{1}{2}\sin 2\theta & \frac{1}{2}\sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$D_{damaged} = Z^t Q_{damaged} Z$$

where,
$$[Z] = \begin{bmatrix} 1 & 0 & 0 & z & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & z & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & z & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

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Elastic stiffness matrix of the damaged structure is defined as: $K^{(e)}_{damaged} = \int B^{(e)T} D_{damaged} B^{(e)} dA^{(e)} dz$

The governing differential equation for the present problem is:

$$[M]\{\ddot{d}\}+[K_{damage}]\{d\}=0$$

The square roots of the eigenvalues of the above equation are the damaged natural frequencies for different modes. The lowest value of frequency is the fundamental natural frequency.

Where, $[M]=\int_V [N]^T [\rho][N] dAdz$

$$[M]=\int_A [N]^T [\bar{\rho}][N] dA \qquad [\bar{\rho}]=\int[\rho]\,dz\ , \qquad [\rho]=\rho\begin{bmatrix}1&0&0&z&0\\0&1&0&0&z\\0&0&1&0&0\\z&0&0&z^2&0\\0&z&0&0&z^2\end{bmatrix}$$

and ρ is the density of the material.

IV. Results and Discussions

A. Undamaged Free Vibration

9x9 mesh is used for the FE modelling of all the undamaged free vibration analysis.

1) For various modes

Non-dimensional natural frequencies for simply supported (SS1) symmetric cross-ply (0/90/0) square plates for various modes m, n are given in Table I and Table II. (E_1/E_2= 25, $G_{12}=G_{13}$= $0.5E_2$, $G_{23}=0.2E_2$, υ_{12} = 0.25) Non dimensional natural frequency: $\bar{\omega}=\omega\frac{b^2}{H}\sqrt{\frac{\rho}{E_{22}}}$

TABLE I. Comparison of Results for Various Modes (b/H) =100

m	n	Present	Ref [23]
1	1	15.1698	15.185
1	2	22.8158	22.822
1	3	40.2468	40.169
2	1	56.0201	56.221
2	2	60.0295	60.230
1	4	66.8789	66.409
2	3	70.6876	70.801

TABLE II. Comparison of Results for Various Modes (b/H=10)

m	n	Present(code)	Ref [23]
1	1	11.569	12.223
1	2	18.5774	18.942
1	3	31.9088	31.421
2	1	31.822	31.131
2	2	33.097	34.794
1	4	44.42977	46.714
2	3	41.332	43.212

2) For various thickness ratios

Non-dimensional free vibration frequencies for anti-symmetric angle ply (-45/45/-45/45) composite plates with various thickness ratios for SS2 boundary conditions are given in Table III. b/h increases as thickness decreases which in turn reduce the rigidity of laminates, thus cause the increase in frequency.

(E_1/E_2=40, $G_{12}=G_{13}=0.6E_2$, $G_{23}=0.5E_2$, $\upsilon_{12} = 0.25$)

SS2 boundary conditions: At $x = 0, a;\ \ u = w = \theta_y = 0$

At $y = 0, b;\ \ v = w = \theta_x = 0$

3) For various boundary conditions

Non-dimensional fundamental natural frequencies of a two-layer cross-ply laminated square plates (0/90) with various boundary conditions are given in Table IV. (E_1/E_2 = 40, b/H = 100, E_1 = 40*10^10 N/m^2, $G_{12} = G_{13}$= $0.5E_2$, $G_{23}=0.6E_2$, $\upsilon_{12} = 0.25$)

4) For different ply orientations

Non-dimensional fundamental natural frequencies for various ply orientations are given in Table V. (E_1/E_2 = 40, b/H = 100, E_1 = 40*10^10 N/m^2, $G_{12} = G_{13}$= $0.6E_2$, $G_{23}=0.5E_2$, $\upsilon_{12} = 0.25$)

5) For various number of layers

It is observed from Table VI that as the number of layers increases, the natural frequency also increases due to bending-stretching coupling. (E_1/E_2 = 40, b/H = 100, E_1 = 40*10^10 N/m^2, $G_{12} = G_{13}$= $0.6E_2$, $G_{23}=0.5E_2$, $\upsilon_{12} = 0.25$)

TABLE III. Comparison of Results for Various Thickness Ratios

b/h	Present	Ref [23]
5	12.6273	10.799
10	18.45	19.289
20	21.8596	23.259
100	23.446	25.176

TABLE IV. Comparison of Results for Various Boundary Conditions

Boundary condition		Non – dimensional frequencies			
		a/h = 5	a/h = 10	a/h = 20	a/h = 50
SSSS	*Present*	7.3890	8.8999	9.4744	9.6597
	Ref [22]	7.3901	8.9003	9.4750	9.6600
SCSS	*Present*	8.1292	10.6118	11.8052	12.2357
	Ref [22]	8.1295	10.6120	11.8054	12.2358
SCSC	*Present*	8.9333	12.6225	14.8750	15.8101
	Ref [22]	8.9334	12.6225	14.8748	15.8097

TABLE V. COMPARISON WITH DIFFERENT PLY ORIENTATIONS

	Ply orientations		Mesh division		
			4x4	*8x8*	*10x10*
Non-dimensional frequencies for different ply orientations	5/-5/5/-5	*Present*	14.304	14.3	14.3
		Ref [22]	14.7545	14.7428	14.742
	15/-15/15/-15	*Present*	15.493	15.497	15.499
		Ref [22]	15.6192	15.6066	15.606
	30/-30/30/-30	*Present*	17.85	17.873	17.88
		Ref [22]	17.6471	17.6331	17.633
	45/-45/45/-45	*Present*	18.89	18.91	18.913
		Ref [22]	18.4747	18.4601	18.459

TABLE VI. COMPARISON WITH NUMBER OF LAYERS

a/h	45/-45/45/-45		$(45/-45/45/-45)_2$	
	Present	*Ref [22]*	*Present*	*Ref [22]*
5	12.775	12.627	12.9226	12.889
10	18.913	18.4596	19.380	19.285
20	22.620	21.870	23.430	23.257
25	23.230	22.425	24.110	23.924
50	24.130	23.237	25.122	24.909
100	24.387	23.455	25.399	25.176

B. *Effect Of Damage On Free Vibration*

1) *Comparison of results*

The equivalent cross-ply model is used to simulate the woven laminate. A small surface crack (about 16mm length and 1mm depth) on the model is analyzed. The dimension of the laminate panel is 310 mm length, 222 mm width and 2.2 mm thickness. The density of the material is 1447 kg/m³. A cross-ply model is established by assigning laminae [0/90] for each element with different tensile moduli E_1 and E_2 in fiber and matrix directions. The equivalent cross-ply properties are E_1 = 87.6 GPa, E_2 = 5.13 GPa, G_{12} = 4.08 GPa, υ_{12} = 0.315. 14x10 mesh is used for the FE analysis. Comparison of frequencies is shown in Table VII.

2) *Influence of orthogonality of damage parameters*

Since the model of damage being investigated is anisotropic in nature, it is essential to understand the nature of orthotropy exhibited by this formulation. 9x9 mesh is used for FE analysis. It may be inferred from Fig. 2 that, the contribution of damage in the direction of fibre may not be equal to that in the orthogonal direction. It can be observed that damage in the direction of fibre, Γ_1 shows a steeper deterioration of natural frequencies.

a= b= 300mm, h= 3mm

(E_1/E_2=40, E_1=40*10^10 N/m², G_{12} = G_{13}= 0.6E_2, G_{23}=0.5E_2, υ_{12} = 0.25)

TABLE VII. COMPARISON OF FREQUENCY OF DAMAGED CROSS-PLY PLATE

Mode	Without damage		With damage	
	Present	*Ref [8]*	*Present*	*Ref [8]*
(1,1)	34.474	-	34.3772	-
(2,2)	56.593	55.54	56.40	55.56
(3,1)	144.750	142.7	144.189	142.6
(3.2)	215.90	229.5	215.292	229.5
(1,3)	247.005	253.1	246.265	253.1
(3,3)	333.215	347.3	331.835	347.3

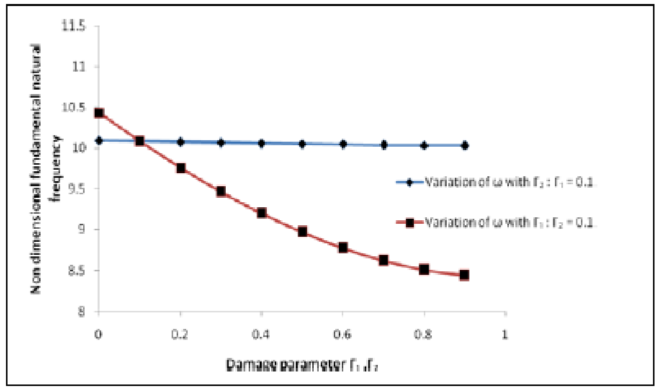

Figure 2. Variation of fundamental natural frequency of a damaged simply supported, cross-ply (0/90/0) composite plate with individual damage parameters Γ_1 and Γ_2 while the other kept fixed at 0.1. Thickness ratio is b/h = 100. The damage is in the 36 % area of all the layers throughout the thickness.

3) *Effect of damage intensity on dynamic characteristics*

It can be observed from Table VIII, Fig.3 and Fig.4 that the fundamental natural frequency decreases with the increase in damage ratio. The damage ratio Γ_1/Γ_2 takes values between 0.0 and 9.0. A mild damage may be represented with a damage ratio $0.0 \le \Gamma_1/\Gamma_2 \le 3.0$ while a heavy damage may be denoted by the range of values, $7.0 \le \Gamma_1/\Gamma_2 \le 9.0$. It is observed from the following table as the damage intensity increases the natural frequency decreases. 9x9 mesh is used for FE analysis.

a= b= 300mm, h= 3mm

(E_1/E_2 =40, E_1 = 40*10^10 N/m², G_{12} =G_{13}=0.6E_2, G_{23}=0.5E_2, υ_{12} = 0.25)

4) *Effect of damaged area on dynamic characteristics*

To study the effect of damaged area, various damaged, clamped-simply supported composite plates with different ply layout have been analyzed. 5x5 mesh is used for FE analysis. Table IX and Table X tabulates the variation of non-dimensional fundamental natural frequencies with damage area with thickness ratio b/h = 100, having central damage having damage intensities Γ_1/Γ_2 = 1.0 (mild damage) and Γ_1/Γ_2 = 9.0 (heavy damage). The drop in fundamental natural frequency in the heavily damaged case is more as compared to the mild one. The fundamental natural frequency drops for all plies as the damage area increases.

TABLE VIII. VARIATION OF FUNDAMENTAL NATURAL FREQUENCY WITH VARIOUS INTENSITIES OF DAMAGE OF A DAMAGED SIMPLY SUPPORTED, CROSS-PLY (0/90/0) COMPOSITE PLATE WITH INDIVIDUAL DAMAGE PARAMETERS Γ_1 AND Γ_2 WHILE THE OTHER KEPT FIXED AT 0.1. THICKNESS RATIO IS B/H = 100. THE DAMAGE IS IN THE 36 % AREA OF ALL THE LAYERS.

$\Gamma_1 = 0.1$		$\Gamma_2 = 0.1$	
Γ_2	*Natural frequency*	*Γ_1*	*Natural frequency*
0	10.0938	0	10.4328
0.2	10.0737	0.2	9.7594
0.4	10.0572	0.4	9.1993
0.6	10.0442	0.6	8.7752
0.8	10.0348	0.8	8.5085
0.9	10.0315	0.9	8.4390

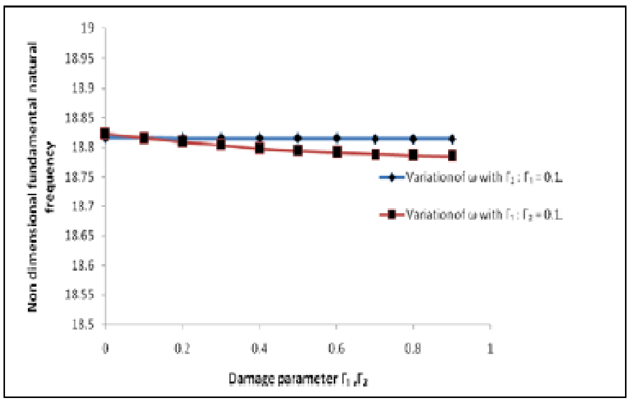

Figure 3. Variation of fundamental natural frequency of a damaged simply supported, cross-ply (0/90/0) composite plate with individual damage parameters Γ_1 and Γ_2 while the other kept fixed at 0.1. Thickness ratio is b/h = 100. The damage is in the 11 % area of the middle layer. ($E_1/E_2 = 40$, $E_1 = 40*10^{10}$ N/m^2, $G_{12} = G_{13} = 0.6 E_2$, $G_{23}=0.5E_2$, $\upsilon_{12} = 0.25$)

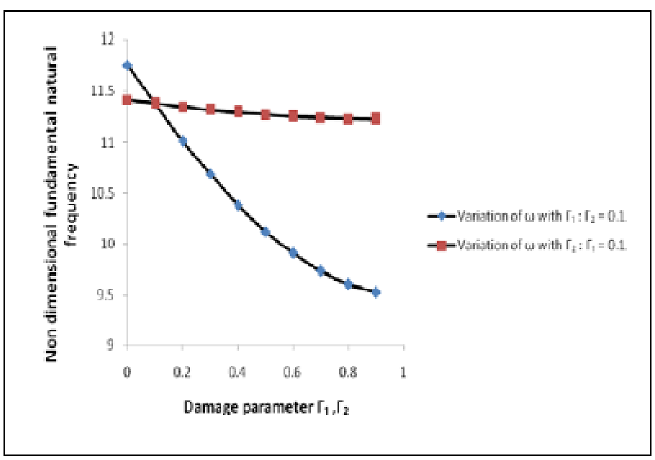

Figure 4. Reduction of fundamental natural frequency with increase in intensity of damage of a damaged clamped-simply supported composite plate (45/-45/45) with individual damage parameters Γ_1 and Γ_2 whileanotherr kept fixed at 0.1. Thickness ratio is b/h = 100. The damage is in the 25 % area of the plate, throughout the thickness. ($E_1/E_2 = 25$, $E_1 = 25*10^{10}$ N/m^2, $G_{12} = G_{13} = 0.5 E_2$, $G_{23}=0.2E_2$, υ_{12} 0.25)

TABLE IX. VARIATION OF NON-DIMENSIONAL FUNDAMENTAL NATURAL FREQUENCIES WITH PERCENTAGE OF DAMAGED AREA FOR CSCS COMPOSITE PLATE WITH THICKNESS RATIO B/H = 100 AND DAMAGE PARAMETER $\Gamma_2 = 0.1$

Damaged area in %	Ply orientations			
	0/90/0	*15/-15/15*	*30/-30/30*	*45/-45/45*
0% undamaged	22.92	21.5831	19.8837	16.652
	$\Gamma_1/\Gamma_2 = 1$			
4 %	22.1998	21.5310	19.118	16.0772
36%	21.3483	20.8940	18.518	15.0810
40%	17.4120	17.1235	15.929	14.097
	$\Gamma_1/\Gamma_2 = 9$			
4%	21.8545	21.1732	18.698	15.5990
36%	17.9110	17.4066	15.547	13.1645
40%	15.5750	15.2030	13.722	11.7185

($E_1/E_2 = 40$, $E_1 = 40*10^{10}$ N/m^2, $G_{12} = G_{13} = 0.6E_2$, $G_{23}=0.5E_2$, $\upsilon_{12} = 0.25$)

TABLE X. VARIATION OF NON-DIMENSIONAL FUNDAMENTAL NATURAL FREQUENCIES WITH % OF DAMAGED AREA FOR CSCS SQUARE COMPOSITE PLATES WITH THICKNESS RATIO B/H = 100 AND B/H = 10 AND $\Gamma_1/\Gamma_2 = 9$.

Ply orientation	Damaged area in %				
	0	20	40	60	80
b/h = 100					
0/90/0/90	17.15	15.457	13.552	11.327	8.523
45/-45/45/-45	16.505	14.981	13.273	11.288	8.794
b/h = 10					
0/90/0/90	10.512	9.4309	8.2069	6.7613	4.889
45/-45/45/-45	10.109	9.0971	7.9520	6.5979	5.240

($E_1/E_2=25$, $E_1=25*10^{10}$ N/m^2, $G_{12}=G_{13}= 0.5 E_2$, $G_{23}=0.2E_2$, $\upsilon_{12} = 0.25$)

V. CONCLUSION

The current study deals with damage at the macro level that can be described using anisotropic parameters and its influence on free vibration has been observed. The present formulation can successfully compute the free vibration characteristics and the results of the limiting cases have been verified with the available literature.

SUMMARY OF THE RESULTS

- It is observed that as the thickness ratio (a/h) increases, the fundamental natural frequency increases irrespective of boundary conditions.
- The effect of different parameter like number of layers, thickness ratio, fiber orientation and boundary conditions is presented in details.

- The strong dependence of the natural frequencies on fibre orientations is presented.
- Damage in composite plates shows strong orthogonality. It has been observed that a damage in the direction of fibre results in steeper deterioration of natural frequencies. Damage in orthogonal direction to fibre has little influence on fundamental natural frequency.
- It can be observed that as the extension of damage or damage area or size increases, deteriorates the natural frequency irrespective of ply orientation.

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