Proceedings of ICTACEM 2014
International Conference on Theoretical, Applied, Computational and Experimental Mechanics
December 29-31, 2014, IIT Kharagpur, India
ICTACEM-2014/ 279

Free Vibration Characterictics and Damage Assesment of Composite Structures

Jebieshia T R[a1], Bharadwaj Nanda[b], D K Maiti[c] and D Maity[d]

[a, c] Department of Aerospace Engineering, Indian Institute of Technology Kharagpur, West Bengal, India
[b, d] Department of Civil Engineering, Indian Institute of Technology Kharagpur, West Bengal, India

ABSTRACT

During the service life, defects may appear in composite structure. When the structure undergoes some kind of damage, its stiffness is reduced, changing the dynamic response. In order to avoid safety issues early detection of damage is necessary. The knowledge of the vibration behavior of a structure is necessary and can be used to determine the existence as well as the location and the extent of damage.

1. INTRODUCTION & OBJECTIVE

Composite materials are widely being used in structures, especially in aircraft and spacecraft, since composite materials are having high strength to weight ratio and high stiffness to weight ratio. Their behavior of the composites is designed according to their usage; therefore their advantages such as enhanced fatigue life and low thermal expansion were fully utilized. Beyond these advantages, composite structures may develop damage in the form of matrix cracks, delamination, fiber-matrix debonds and fiber breakage due to the manufacturing process or fatigue loading during service, affects the safety of the structure.

The occurrence of damage in a structure modifies some of its mass, stiffness or damping properties, and thus changing the vibration response of the structure. This deterioration in mechanical properties of a material is known as a damage process. This damage causes a reduction in strength that may lead to failure to shorten the operating life of the structures. The damage in composite materials is anisotropic in nature. An anisotropic damage is a damage whose extent is distributed in several orthogonal directions and the extent of damage in any orthogonal direction is independent of the other. The variable is tensorial in nature for

[1] Email: rstrjebieshia@aero.iitkgp.ernet.in (corresponding author)

the case of anisotropic damage, and hence the damage modelling is much more complicated. Anisotropic damage is parametrically incorporated into the formulation by considering the damage parameter Γ_i, which is a representation of reduction in the effective load bearing area.

Since as far as any engineering structure under operation is concerned there exists a finite probability of damage occurring at some point of its operating lifetime. So the study of damaged composite structure is important. The free vibration and damage analysis of laminated composites structures is done using finite element method i.e., the governing differential equations have been developed using FEM and then implementing the problem into Matlab coding to obtain the various results. The present study focuses on the effect of damage intensity, damage size on free vibration characteristics of composite structures and to determine the effect of damage orthogonality on free vibration characteristics by choosing suitable values of damage parameters. The presence of damage by itself doesn't mean that the structure has failed. There is a reduction in stiffness value occurs due to the introduction of damage to the structure. The reduction in non-dimensional frequency with respect to the area, size and intensity of damage were analyzed.

Different approaches are used to detect and assess damages in structures. Damage detection method has four stages; firstly to determine occurrence of damage and secondly to determine the location of damage. In the next stage, the severity of the damage is determined; and finally, the remaining service life of the structure is predicted. When damages occur in an element of a structure, the stiffness as well as the global frequency of the whole structure decreases. Generally, when some damage appears in a structure, stiffness matrix can offer more information than the mass matrix since the changes of mass matrix may be considered negligible. So it is useful to apply the changes in stiffness matrix to detect damage. A mathematical relationship is established in the form of an objective function between damages in the structure and the corresponding changes in the vibration characteristics. Then this objective function is solved with some swarm based optimization techniques which are popular now and are used to identify the damage using the vibration response.

1.1.Damage Modelling Based On Stiffness Loss

When engineering materials are subjected to unfavorable conditions such as cold and hot working processes, temperature variation, chemical action, radiation, mechanical loading or environmental condition, microscopic defects and cracks may develop inside the materials.

This damage causes reduction in strength that may lead to failure to shorten the operating life of the structures. This deterioration in mechanical properties of a material is known as a damage process.

The assumption of isotropic damage is often sufficient in practice to give a good prediction of load carrying capacity, the number of cycles to (local) failure and others. The calculations involved in isotropic damage state are very much simplified due to the scalar nature of the damage variable. For anisotropic damage, the variable is tensorial nature as mentioned and hence the identification of the models is much more complicated. Reduction of in-plane and bending stiffness and re-distribution of membrane stresses occurs due to the presence of damages in the system which inturn affect the static and dynamic response characteristics of the system. So understanding vibration characteristics is essential in fail safe design. Due to the complexities involved in an anisotropic damage, the use of numerical methods such as the finite element method based on damage mechanics theory has been proved to be very effective.

For obtaining the frequency response, the forward method is used and to determine the location and severity of damage inverse technique is used. The reverse technique is a numerical procedure to detect and quantify damage in a composite structure from changes natural frequencies using unified particle swarm optimization (UPSO) technique. Natural frequency based objective function is used for this optimization technique. PSO has found its application in many complex optimization problems in engineering including structural design optimization and structural damage detection because of its simplicity and increased convergence speed.

2. THEORETICAL FORMULATION

2.1. Anisotropic damage

In a thin plate, anisotropic damage is parametrically incorporated into the formulation by (Valliappan, 1990) considering the damage parameter which is a representation of reduction in effective area and is given by:

$$\Gamma_i = \frac{A_i - A_i^*}{A_i} \tag{1}$$

Where A_i^* is the effective area (with unit normal) after damage

Ai is the area of damaged material with unit normal ni

i ϵ {1, 2, 3} are the three orthogonal directions

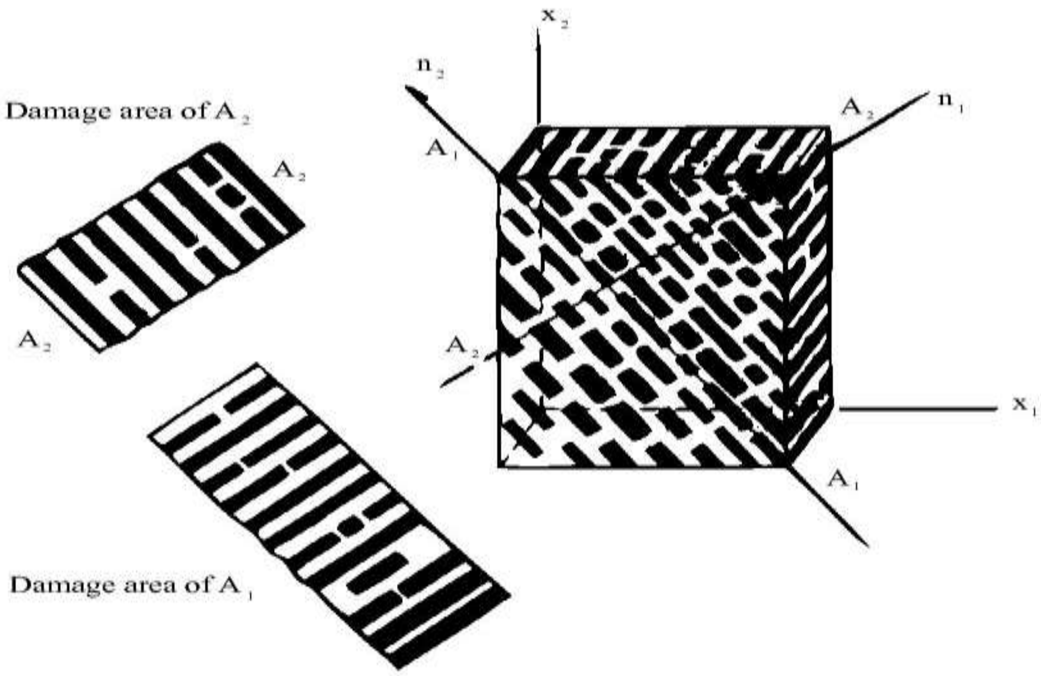

Figure 1. Illustration diagram of damage material

Assuming that the internal forces acting on any damaged section is the one before damage:

$$\sigma_{ij}\delta_{jk}A_k = \sigma^*_{ij}\delta_{jk}A^*_k \tag{2}$$

$$\{\sigma^*\} = \{\psi\}\{\sigma\} \tag{3}$$

where, $\{\psi\}$ is a transformation matrix and can be used to relate a damaged stress-strain matrix with undamaged one and is given as below:

For 2D:

$$[\Psi] = \begin{bmatrix} \frac{1}{1-\Gamma_1} & 0 & 0 \\ 0 & \frac{1}{1-\Gamma_2} & 0 \\ 0 & 0 & \frac{1}{1-\Gamma_1} \\ 0 & 0 & \frac{1}{1-\Gamma_2} \end{bmatrix} \qquad \begin{aligned} \{\sigma^*\}^T &= \{\sigma^*_{11}\ \sigma^*_{22}\ \sigma^*_{12}\ \sigma^*_{21}\} \\ \{\sigma\}^T &= \{\sigma_{11}\ \ \sigma_{22}\ \ \sigma_{12}\} \end{aligned}$$

The damage model should not assume the damage tensor to be symmetric in order to define the damage effectively in composites.

$$\sigma^*_{21} = \frac{1-\Gamma_2}{1-\Gamma_1}\sigma^*_{12} \tag{4}$$

2.2. Unified Particle Swarm Optimization

The particle swarm optimization (PSO) algorithms, first proposed by Kennedy and Eberhart (1995), are inspired by the collective motion of insects and birds trying to reach an unknown destination, known as "swarm behavior". This process involves both social interaction and intelligence so that birds learn from their own experience and also from the experience of others around them. PSO algorithm has advantages lies with its simplicity in its architecture and convergence speed. However, the main drawback associated with standard PSO is that the algorithm may converge into some local optima. This may lead to prediction of wrong results. Further, to improve its efficiency many alternations and variations are proposed to the original PSO algorithm, among which unified particle swarm optimization (UPSO) one that has the ability to harness both exploration and exploitation capacity simultaneously [Parsopoulos and Vrahatis (2005)] by balancing the influence of both global and local search directions simultaneously. Mathematically, for a swarm size of P number of particles, in an S-dimensional search space, let G_{ij}^{t+1} and L_{ij}^{t+1} denotes the velocity update of i^{th} particle in global and local variants of PSO respectively for the $(t+1)^{th}$ iteration as given by,

$$G_{ij}^{t+1} = \chi\left[v_{ij}^{t} + c_1 r_1\left(pbest_{ij} - x_{ij}^{t}\right) + c_2 r_2\left(gbest_{ij} - x_{ij}^{t}\right)\right] \tag{5}$$

and, $$L_{ij}^{t+1} = \chi\left[v_{ij}^{t} + c_1 r_3\left(pbest_{ij} - x_{ij}^{t}\right) + c_2 r_4\left(lbest_{ij} - x_{ij}^{t}\right)\right] \tag{6}$$

Where, $pbest, gbest$ and $lbest$ respectively denotes the best position explored by individual particle, any particle in the swarm and in the neighborhood of individual swarm. χ denotes the constriction factor which is equals to 0.72984. c_1 and c_2 are two acceleration coefficients and is considered as 2.05 each in present study. Finally, all r terms denote random numbers between [0, 1] and independent of each other. Combining Equations (5) and (6), the aggregate velocity of the particles in the search directions is defined as,

$$V_{ij}^{t+1} = u.G_{ij}^{t+1} + (1-u).L_{ij}^{t+1}, \qquad u \in [0,1] \tag{7}$$

The new position of the particles for $(t+1)^{th}$ iteration is,

$$x_{ij}^{t+1} = x_{ij}^{t} + V_{ij}^{t+1}, \qquad \forall i \in P \ and \ \forall j \in S \tag{8}$$

The parameter, u in Equation (7) is called unification factor and its value is modified throughout the iterations according to the equation,

$$u(t) = \exp\left(\frac{t.\log(2.0)}{t_{\max}}\right) - 1.0 \tag{9}$$

2.3. Objective Function

Objective function is an equation to be optimized under certain given constraints and with variables that need to be minimized or maximized using nonlinear programming techniques. It maps the search space to the function space. Natural frequencies obtained from the damaged structure are used for constructing the objective function for the present problem and is given as:

$$F = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\left(\frac{f_i^m}{f_i^c}\right)-1\right)^2} \tag{10}$$

The terms f_i^m and f_i^c are the measured natural frequencies for damaged structure and the natural frequency obtained from finite element simulation for damaged structure respectively. n is the number of input response parameters (i.e. Natural frequencies).

2.4. Damage orthogonality

The damage parameters determine the extent of damage in a particular orthogonal direction. The direction of damage with respect to the direction of fibre orientation plays an important role in influencing the dynamic instability and response of the plate. A reduction in the stiffness in the direction of fibre which influences the buckling, vibration and flutter characteristics more profoundly than a reduction of stiffness occurring in a direction perpendicular to the fibre orientation. The ratio of orthogonal damage parameters is varied from 1.0, i.e., mild damage to 9.0, i.e., on the verge of total fibre breakage. In order to understand this; the natural orthotropy exhibited by the formulation, one of these orthogonal parameters must be fixed while the other must be varied to obtain the static and the dynamic characteristics of the structure.

3. RESULTS AND DISCUSSION

3.1. Damage assessment results

The current study deals with damage at the macro level that can be described using anisotropic parameters and its influence on free vibration has been observed. Moreover identification of damage and its severity has been done using the present formulation with the help of PSO.

A composite beam of length, breadth and thickness respectively of 500 mm, 30 mm and 2.8 mm is considered for the demonstration of the developed algorithm. The material properties for the beam are given in the Table 1.

Table 1. Material properties considered for the numerical composite beam

E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	υ_{12}	ρ (kg/m^3)
9.5	6.8	1.4	0.14	1761

The algorithm is verified for both single and multiple element damage cases. For finite element simulation the beams are modeled with 10 orthotropic elements. First six numerically evaluated natural frequencies (Table 2) are used for constructing the objective function. It can be observed from Table 2 that the fundamental natural frequency decreases with the increase in damage ratio. Up to 1.0% noise is added to the numerical natural frequency to simulate the experimental condition. UPSO algorithm is employed to search the actual damaged element and damage amount. For optimization purpose, 50 swarms are considered for the study. Maximum number of iteration allowed is 300. The damage conditions considered for the study and the results of damage assessment for various noise levels are evaluated and produced in Table 3.

Table 2. Damage cases selected for simulation studies in beam structure and the corresponding natural frequencies

Damage Id.	**Damage Conditions** $[\textbf{Element No.} (\Gamma_1, \Gamma_2)]$	**Natural Frequencies**					
		1st	**2nd**	**3rd**	**4th**	**5th**	**6th**
Undamaged		26.57	73.24	143.76	170.11	238.52	271.29
J5	4 (0.60, 0.00)	23.92	62.08	137.31	162.80	209.29	245.05
J6	4 (0.00, 0.50)	26.58	73.29	143.77	165.99	238.66	270.41
J7	3 (0.50, 0.00) 7 (0.35, 0.00)	25.36	60.14	128.21	157.19	214.74	258.05
J8	3 (0.00, 0.40) 7 (0.00, 0.65)	26.58	73.35	143.81	156.25	238.78	267.94

Table 3. Damage assessment results

Damage Case	Damage Assessment Results [Element No., (Γ_1, Γ_2)]			
	Actual	**0.00 % Noise**	**0.50% Noise**	**1% Noise**
Single element damage	4 (0.60, 0.00)	7 (0.60, 0.00)	4 (0.60, 0.06)	7 (0.60, 0.00)
	4 (0.00, 0.50)	4 (0.00, 0.50)	4 (0.00, 0.50)	4 (0.00, 0.49)
Two element damage	3 (0.50, 0.00)	3 (0.50, 0.00)	3 (0.50, 0.00)	3 (0.50, 0.00)
	7 (0.35, 0.00)	7 (0.35, 0.00)	7 (0.35, 0.00)	7 (0.35, 0.09)
	3 (0.00, 0.40)	3 (0.00, 0.40)	3 (0.00, 0.40)	4 (0.00, 0.33)
	7 (0.00, 0.65)	7 (0.00, 0.65)	7 (0.00, 0.63)	8 (0.00, 0.68)

CONCLUSION

A numerical procedure is presented to detect and quantify the damages in a composite beam like structure based on changes in natural frequency data using unified particle swarm optimization technique. The proposed methodology is demonstrated using a numerically simulated composite beam structure containing single and multiple damages. As indicated by the simulation results, the proposed method is able to detect and quantify the damage accurately using first six natural frequencies for considered damage cases.

Damage in composite plates shows strong orthogonality. It has been observed that a damage in the direction of fibre results in steeper deterioration of natural frequencies. Damage in orthogonal direction to fibre has little influence on fundamental natural frequency.

REFERENCES

1. B. Nanda., D. Maity, and D. K. Maiti., “Vibration-based Structural Damage Detection Technique using Particle Swarm Optimization with Incremental Swarm Size”, *International Journal of Aeronautical & Space Sciences*, 13.3, pp. 323–331, 2012.
2. Valliappan.S, Murti.V, and Zhang Wohua. , “Finite element analysis of anisotropic damage mechanics problems”., *Engineering Fracture Mechanics*, Vol 35, No 6, pp-1061- 1071, 1990.

3. D.L.Prabhakara and P.K.Datta. “Vibration and static stability characteristics of rectangular plates with a localized flaw”., *Computers and Structures*, vol.49, No.5 pp- 825-836, 1993.
4. Huiwen Hu and Jieming Wang, “Damage detection of a woven fabric composite laminate using a modal strain energy method”, *Engineering Structures*., Vol 31, pp-1042-1055, 2009.
5. J.N. Reddy, “Mechanics of laminated composite plates and shells theory and analysis”- 2nd edition , CRC Press, 2004.
6. Parsopoulos, K.E. and Vrahatis, M.N., “Unified Particle Swarm Optimization for Solving Constrained Engineering Optimization Problems,” *Proceedings of the First international conference on Advances in Natural Computation* - Volume Part III, Berlin, Heidelberg, 582–591, 2005.
7. Salawu, O.S., “Detection of Structural Damage through Changes in Frequency: a Review” *Engineering Structures*. 19.9, 718–723, 1997.
8. George Z. Voyiadjis and Peter I. Kattan, Damage Of Finer-Reinformed Composrte Materials With Micromechanical Characterization, *Solid structures* Vol. 30, No. 20, w. 2757-2778, 1993.
9. R. M. Jones., “Mechanics of composite materials”. McGraw Hill, New York, 1975.
10. J.N. Reddy, “An Introduction to the Finite Element Method”, 3rd ed., McGraw-Hill Education (2005).
11. J.V.Araujo dos Santos, C.M. Mota Soares, C.A. Mota Soares, H.L.G. Pina. Development of a numerical model for the damage identification on composite plate structures. Composite Structures 48 (2000) 59-65.
12. Yam L.H., Yan Y.J., and Jiang J.S., “Vibration-based damage detection for composite structures using wavelet transform and neural network identification,” Composite Structures, Vol. 60, pp. 403–412, 2003.
13. Ma Ge and Eric M. Lui, “Structural damage identification using system dynamic properties,” Computers and Structures, Vol. 83 pp. 2185–2196, 2005.
14. Li J., Law S.S. and Ding Y., “Substructure damage identification based on response reconstruction in frequency domain and model updating,” Engineering Structures, Vol. 41, pp. 270–284, 2012.