

Linear Convection with effect of time step

Objective:

To write a function that accepts the time step as an argument and compare the effect of the time step on the numerical solution of linear convection..

1. Set n = 80
2. Time steps to use = 1e-4, 1e-3, 1e-2, and 1e-1

Given data as per previous challenge:

1. Length L=1m
2. Convection velocity, C = 1 and time = 0.4seconds
3. Number of nodes n=80
4. The initial velocity profile is a step function. It is equal to 2m/s between x= 0.1 and 0.3 and 1m/s everywhere else

Solution:

Procedure:

1D Linear convection equation

$$\frac{\partial u}{\partial t} + c * \frac{\partial u}{\partial x} = 0$$

Here, c - convection velocity

Applying first-order forward differencing for the time derivative and first-order backward/rearward differencing for the space term, the equation is as follows

$$u_i^{n+1} = u_i^n - c * \frac{\Delta t}{\Delta x} (u_i^n - u_{i-1}^n)$$

Main Program:

clear all

close all

clc

dt=0.1;

tic;

[x1,u1]=week4_assignment2_function(dt);

t1=toc;

dt=0.01;

tic;

[x2,u2]=week4_assignment2_function(dt);

t2=toc;

dt=0.001;

tic;

[x3,u3]=week4_assignment2_function(dt);

t3=toc;

dt=0.0001;

tic;

[x4,u4]=week4_assignment2_function(dt);

t4=toc;

%plotting

h=figure;

plot(x1,u1,'r','lineWidth', 3);

hold on

plot(x2,u2,'g','lineWidth', 3);

hold on

plot(x3,u3,'b','lineWidth', 0.5);

hold on

```

plot(x4,u4,'y','lineWidth', 2);
axis([0 1 0 2]);
legend ('t1','t2','t3','t4');
title('linear convection for n=80');
xlabel('length/grid spacing');
ylabel('velocity');
grid on

```

Function Program:

```
function [x,u]=week4_assignment2_function(dt)
```

```

L=1;
c=1;
n=80;
x=linspace(0,L,n);
dx=L/(n-1);

%start & end of wave
x_start=0.1;
x_stop=0.3;

%range of velocity u=2
a= (x_start/dx)+1;
b=(x_stop/dx)+1;

%initiating arrays & bc
u=1*ones(1,n);
u(a:b)=2;

```

```
%bc
```

```
u(1)=1;
```

```
uold=u;
```

```
%time fn
```

```
t=0.4;
```

```
nt=t/dt;
```

```
for k=1:nt
```

```
%space fn
```

```
for i=2:n
```

```
     $u(i)=uold(i)-(c*dt/dx)*(uold(i)-uold(i-1));$ 
```

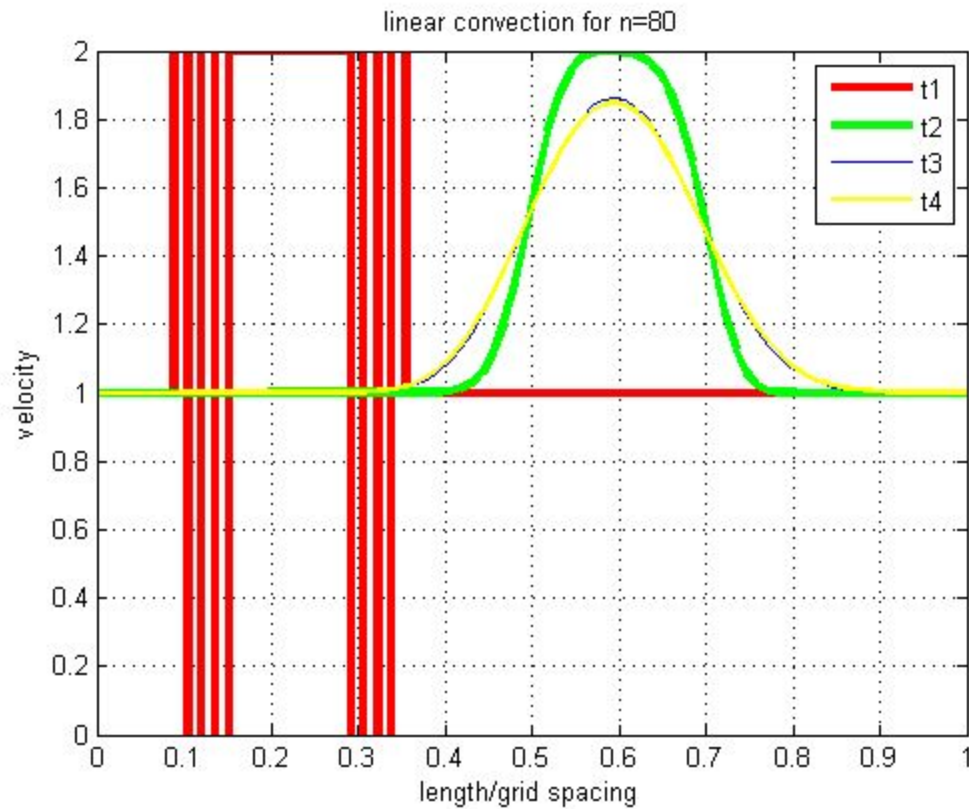
```
end
```

```
%updating new velocity
```

```
uold=u;
```

```
end
```

Results:



1. At $t_1=0.1$ the solution looks like a square wave
2. As the time step is decreased from 0.1 to 0.01, 0.001 and 0.0001, the curve tends to smoothen further and it tries to regain its original flat profile.
3. The below table shows the simulation time for various time steps such as 0.1, 0.01, 0.001, 0.0001.

t1	0.0054
t2	0.0014
t3	0.0028
t4	0.0185

The simulation time is inversely proportional to the time step. (i.e., Higher the time step, lower the simulation time.)