Discretization basics for range of dx

Objective: To compare the first, second and fourth order approximations of the first derivative against the analytical derivative and to plot a range of values of grid spacing (dx) against the error at each approximation.

Given:

- Function, f(x) = sin(x)/x^3 at x = pi/3

- Formula for the fourth order approximation of the first derivative

$$\frac{(f_{j-2} - 8 * f_{j-1} + 8 * f_{j+1} - f_{j+2})}{12dx}$$

The range of dx considered is from pi/4 to pi400 with 20 divisions. I.e., *linspace(pi/4,pi/400,20)*

Solution:

Analytical derivative of the given function is

$$\frac{x^3 \cdot (\cos(x)) - \sin(x) \cdot 3 \cdot x^2}{x^6}$$

First order approximation (Numerical Derivative) is

$$\frac{\left(\frac{\sin(x+dx)}{(x+dx)^3}\right)-\left(\frac{\sin(x)}{x^3}\right)}{dx}$$

Second order approximation (Numerical Derivative) is

$$\frac{\left(\frac{\sin(x+dx)}{(x+dx)^3}\right)-\left(\frac{\sin(x-dx)}{(x-dx)^3}\right)}{2\cdot dx}$$

Fourth order approximation (Numerical Derivative) is

$$((sin(x-2*dx)/(x-2*dx)^3)-8*(sin(x-1*dx)/(x-1*dx)^3)+8*(sin(x+1*dx)/(x+1*dx)^3)-(sin(x+2*dx)/(x+2*dx)^3))/(12*dx)$$

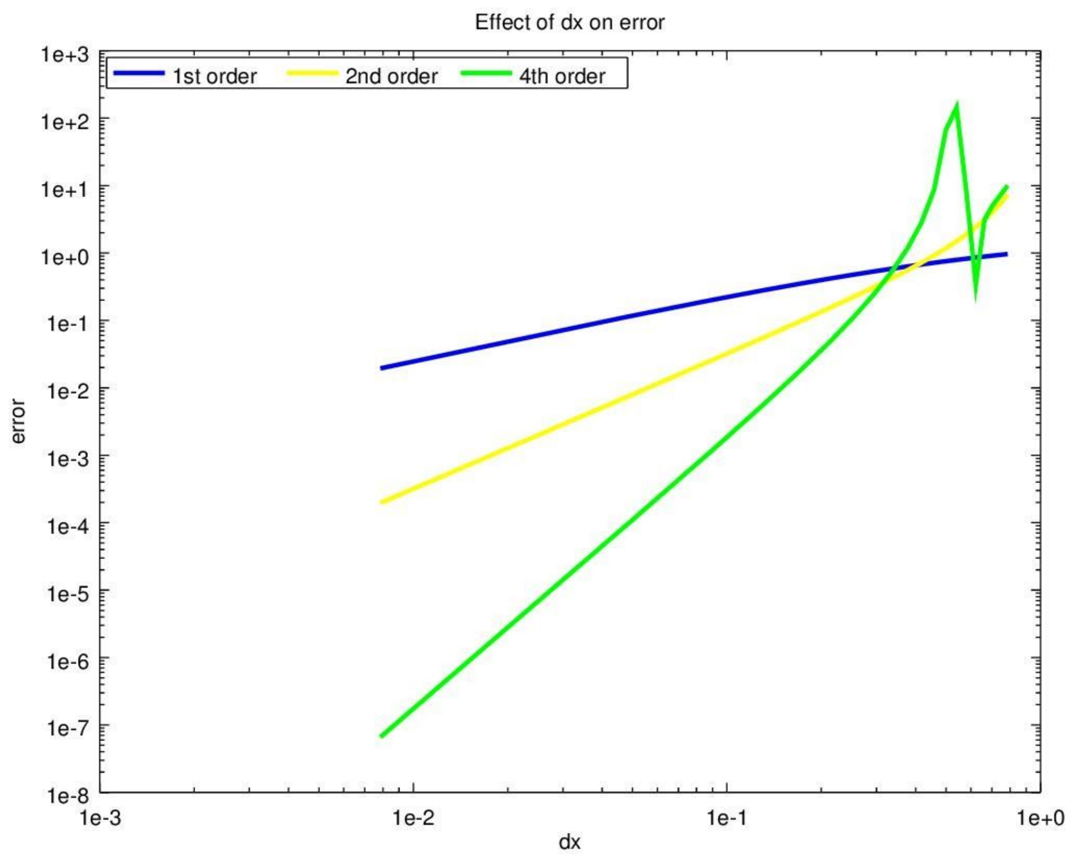
Program:

```
%function = sin(x)/x^3;
x=pi/3;
dx=linspace(pi/4,pi/400,20); %mesh grid spacing

for i=1:length(dx)
%%analytical derivative:
analytical_derivative(i)=(x^3*(cos(x))-sin(x)*3*x^2)/x^6;
slopedx(i)=log(dx(i)); %for slope calculation from log plot
%numerical derivative-forward differencing
%first order approx
forward_diff(i)=((sin(x+dx(i))/(x+dx(i))^3)-(sin(x)/x^3))/dx(i);
error_firstorder(i)=abs(forward_diff(i)-analytical_derivative(i));
slope1 (i)=log(error_firstorder(i));  %for slope calculation from log plot

% %second order approx
central_diff(i)=((sin(x+dx(i))/(x+dx(i))^3)-(sin(x-dx(i))/(x-dx(i))^3))/(2*dx
(i));
error_secondorder(i)=abs(central_diff(i)-analytical_derivative(i));
slope2 (i)=log(error_secondorder(i));  %for slope calculation from log
plot
%fourth order approx
central_diff2(i)=((sin(x-2*dx(i))/(x-2*dx(i))^3)-8*(sin(x-1*dx(i))/(x-1*dx(i)
)^3)+8*(sin(x+1*dx(i))/(x+1*dx(i))^3)-(sin(x+2*dx(i))/(x+2*dx(i))^3))/(12
*dx(i));
error_fourthorder(i)=abs(central_diff2(i)-analytical_derivative(i));
slope3 (i)=log(error_fourthorder(i));    %for slope calculation from log
plot
end
```

```
%plotting
loglog(dx,error_firstorder,'b','linewidth' ,2);
hold on;
loglog(dx,error_secondorder,'y','linewidth' ,2);
hold on;
loglog(dx,error_fourthorder,'g','linewidth' ,2);
title('Effect of dx on error');
legend({'1st order', '2nd order','4th order'},'Location','northwest','Orientation','horizontal')
xlabel('dx');
ylabel('error');
```

Output Plot:

Below is the log plot of the relation between error & dx(grid spacing).



Inference based on log plot:

1. The first order approximation has a higher error value right from the initiation of dx=pi/4 till the least value of grid spacing is reached dx=pi/400. Even though the curve goes on a decreasing trend, the decrease in error is not significant. Hence first order approximation is least accurate.
2. The fourth order approximation has a higher error value than the first order approximation at dx=pi/4 and the error value increases steeply for the next 5-6 values of dx and goes down decreasing

in a highly significant manner and almost makes the error insignificant(i.e., at pi/400, error = 6.6150e-08). Hence fourth order approximation is highly accurate.

3. The second order approximation lies in between first and fourth order approximation in terms of accuracy.

<u>Based on Slope:</u>

Note: Slope of a straight line is constant but for a curve, slope varies. So, in order to correlate the effect of slope on error, slope is taken at the log values of 19th and 20th terms of slopedx(i), slope1 (i), slope2 (i), slope3 (i) and the slope values are compared to establish a relation.

General formula of slope = dy/dx=(y2-y1)/(x2-x1)

- **<u>Slope of first order approx. curve</u>**
  (-3.9397+2.1638)/(-4.8467+3.0205)= 0.97
- **<u>Slope of second order approx. curve</u>**
  (-8.5374+4.8818)/(-4.8467+3.0205)= 2.00
- **<u>Slope of third order approx. curve</u>**
  (-16.5313+9.2122)/(-4.8467+3.0205)= 4.00

Hence, higher the slope, lower the error.