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Auctions with Endogenous Initiation

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ABSTRACT

We study initiation of takeover auctions by potential buyers and the seller. A bidder's indication of interest reveals that she is optimistic about the target. If bidders' values have a substantial common component, as in takeover battles between financial bidders, this effect disincentivizes bidders from indicating interest, and auctions are seller-initiated. Conversely, in private-value auctions, such as battles between strategic bidders, equilibria can feature both seller- and bidder-initiated auctions, with the likelihood of the latter decreasing in commonality of values and the probability of a forced sale by the seller. We also relate initiation to bids and auction outcomes.

OVER THE LAST SEVERAL DECADES, auction theory has developed into a highly influential field with many important practical results, including those related to applications in finance.[1] To focus on the insights about the auction stage, with rare exceptions, the literature has examined situations in which the asset is already up for sale. In some cases, exogeneity of a sale is an innocuous assumption. For example, the U.S. Treasury auctions off bonds at a known frequency. In many cases, however, the decision to put an asset up for an auction is a strategic one. For example, a firm's board of directors has the right but not an obligation to sell a division. In practice, an auction can be either bidder-initiated, whereby a potential bidder approaches the seller expressing an interest, in which case the seller then decides whether to auction the asset off, or seller-initiated, whereby the seller decides to auction the asset off without first being approached.

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[1] Formal analysis of auctions goes back to Vickrey (1961). An overview of auction theory results can be found in Krishna (2010). Dasgupta and Hansen (2007) review applications of auction theory to corporate finance problems.

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To give a sense of this heterogeneity, consider two examples. The acquisition of Taleo, a provider of cloud-based talent management solutions, by Oracle on February 9, 2012 for $1.9 billion was an outcome of a bidder-initiated auction. In January 2011, the CEO of a publicly traded technology company contacted Taleo expressing an interest in acquiring it. Following this contact, Taleo hired a financial adviser who conducted an auction, engaging four more bidders. Oracle was the winning bidder, and hence acquired Taleo. By contrast, the acquisition of Web security provider Blue Coat Systems by private equity firm Thoma Bravo on December 9, 2011 for $1.1 billion was an outcome of a seller-initiated takeover auction. In early 2011, Elliot Associates, an activist hedge fund, amassed 9% ownership stake in Blue Coat and forced its board to auction off the company. Twelve bidders participated in the auction, and Thoma Bravo was the winner. Overall, in an extensive study of takeover initiations, Eckbo, Norli, and Thorburn (2023) find that 42% of all deals over the 1996 to 2016 period were initiated by the seller's board, 29% by the buyer, and 15% by another bidder.[2]

Which characteristics of auctions and the economic environment determine whether auctions are bidder- or seller-initiated? How do bidding strategies and auction outcomes depend on how the auction was initiated? To address these questions, we develop a theory of the seller's choice to put his asset up for sale and potential buyers' choice to indicate their interest to the seller in the presence of private information about their valuations of the asset.

We begin by considering a two-period model in which a seller owns an asset and faces two potential buyers (bidders); we later extend the model to more periods. Each bidder has a signal about her value of the asset. The signals are independently distributed, but the value of a bidder can depend on both signals. The model therefore nests the cases of pure private values and pure common values, but also allows for mixes of private and common components. Bidders' signals can simultaneously change over time because of an exogenous shock to the asset. In any period, each bidder decides whether to send a "cheap talk" message indicating her interest in buying the asset, which she does if and only if her signal exceeds some endogenous cutoff. After observing the messages, the seller decides whether to put the asset up for an auction. In addition, with some probability, the seller may be hit by a liquidity event, in which case he has no choice but to sell the asset. Thus, the auction can be initiated by a bidder, with the seller auctioning the asset off after receiving an indication of interest, or by the seller, with the seller auctioning the asset off without first receiving any interest. In the former case, ex ante identical bidders can become endogenously asymmetric at the auction stage: the signal of the initiating bidder is drawn from a more optimistic distribution than that of the noninitiating rival. Our base model assumes that the auction format is a first-price auction. In an extension, we consider a rich class of sale formats

[2] The remaining groups (joint effort, seller shareholder, and merger of equals) account for 14%. The working paper of Eckbo, Norli, and Thorburn (2023) is not publicly available at the time of writing this article. See Eckbo, Malenko, and Thorburn (2020) for a summary of this evidence.

that nests first-price and, in the limit, ascending auction formats, and that also allows for intermediate formats.

Our main result is that the degree of commonality of bidders' values is an important determinant of whether the equilibrium features seller- or bidder-initiated auctions. Specifically, we establish the following results:

(i) If the common component of bidders' values is sufficiently high, no bidder indicates her interest to the seller, no matter how high her signal is, and all auctions are seller-initiated in the initial period.
(ii) If the private component of bidders' values is sufficiently high, in equilibrium, both bidder-initiated and seller-initiated auctions can occur. A bidder approaches the seller if her signal is sufficiently high. The seller auctions the asset off upon receiving an indication of interest from at least one bidder or upon being hit by a liquidity shock, or in the terminal period if the sale did not occur before. In addition to this equilibrium, there is also a nonresponsive equilibrium in which no bidder indicates her interest to the seller and all auctions are seller-initiated in the initial period.
(iii) If the seller is highly likely to be hit by a liquidity shock, then regardless of the commonality of values, no bidder indicates her interest and the auction is initiated by the seller in the initial period.
(iv) Under an additional assumption that bidders' values are linear in signals, the likelihood that the auction is bidder-initiated is monotonically decreasing in the extent of commonality of bidders' values and in the likelihood of the seller's liquidity shock.

Applying the model to auctions of companies, we can relate the commonality of values to whether bidders are strategic or financial. Strategic bidders are operating companies, typically in an industry related to the target, such as its competitors, suppliers, or customers. Upon acquiring the target, they usually integrate it into their existing operations with the idea of generating value through synergies. In contrast, financial bidders are typically private equity firms that look for undervalued targets with the potential to improve operations, often with the help of aggressive use of leverage and high-powered managerial incentives. Upon acquiring the target, financial buyers usually keep it as a separate operating company and sell it in a few years to a private buyer or in an IPO. Because synergies are bidder-specific, while the use of leverage and high-powered incentives are less so, it is natural to think about contests among strategic (financial) bidders as auctions that are closer to the private-value (common-value) framework.[3] In auctions of companies, our first two results then imply that battles among financial bidders should be more frequently seller-initiated, while battles among strategic bidders should be more

[3] This interpretation is also used in Bulow, Huang, and Klemperer (1999). Consistent with this view, Gorbenko and Malenko (2014) find that the private component of valuations is quantitatively more important for strategic bidders than for financial bidders.

frequently bidder-initiated. This implication is consistent with empirical evidence: approximately 60% (35%) of strategic (private equity) deals are initiated by bidders (Fidrmuc et al. (2012)).[4] Further, multiplicity of equilibria when the private component of values is high implies that sales of otherwise-similar assets in different markets (e.g., otherwise-similar companies in different countries) may have very different patterns of initiation. Our third result implies that sales of distressed companies, which are expected to go bankrupt and be put up for sale with high probability, are mostly seller-initiated, regardless of whether values are private or common. In contrast, sales of companies that are far away from distress are expected to be seller-initiated if values are common but bidder-initiated if values are private.

The intuition behind these results is as follows. In equilibrium, sending an indication of interest to the seller is a signal that the initiating bidder is sufficiently optimistic about the value of the asset. The rival uses this information to choose her bidding strategy and potentially revalue the asset. The revaluation effect occurs when bidders' values have a common component and is stronger when this common component is larger. As a result, the rival bids aggressively not only because she competes against a strong bidder, but also because of her own higher value. In a pure common-value setting, a bidder earns rents only through informational advantage. The initiating bidder with the lowest signal among those that lead to initiation (the cutoff signal) has no informational advantage, as the rival knows that the bidder's signal is at least equal to the cutoff. Thus, the initiating bidder with the cutoff signal earns zero rents, and consequently prefers not to indicate interest to the seller and instead enter either the rival- or seller-initiated auction, as she would then be able to get information rents. Because this argument holds for any hypothetical equilibrium cutoff signal that leads to initiation, bidder-initiated auctions do not occur in equilibrium. Importantly, this is not a knife-edge result: if values are not purely common but the degree of commonality is sufficiently high, the cutoff type of the initiating bidder still obtains sufficiently low profit from the auction to make bidder initiation suboptimal.

In contrast, if the common component in bidders' values is small, the revaluation effect is small. In this case, a bidder with a sufficiently strong signal may be better off sending an indication of interest and triggering the auction. Intuitively, a bidder's indication of interest is pivotal for the decision of the seller to sell the asset if the rival bidder does not indicate interest. In this event, if the values have a significant private component, the bidder competes with a weak rival and obtains a high surplus. In contrast, not indicating interest can result in the seller delaying the sale, in which case bidders' values may change so that the bidder risks becoming weaker while its rival becomes stronger. In turn, the bidder's surplus may decrease. Thus, if the common component in

[4] These deals include both negotiations and competitive processes (auctions and controlled sales). While our base model assumes that all deals occur via auctions and therefore cannot capture negotiations, we extend the model for costly information acquisition in Section IV.B, which allows for both auctions and single-bidder contests and is consistent with greater prevalence of auctions in private equity deals (Fidrmuc et al. (2012)).

bidders' values is small, both bidder- and seller-initiated auctions may occur in equilibrium.

The results on equilibrium multiplicity and the role of liquidity shocks come from interdependence of initiation decisions by bidders and the seller. To see this, consider the case of pure private values. If a bidder with a high signal perceives a seller-initiated auction to be very unlikely, she will have strong incentives to initiate the auction by indicating interest, as described above. In contrast, if a bidder with a high signal believes that the seller is highly likely to conduct an auction even without receiving any interest, she will have no incentive to indicate interest. Intuitively, by not indicating interest, a strong bidder is able to hide its strength, which leads the rival to bid less aggressively and allows the bidder to win at a lower price. This interdependence among initiation decisions has two implications. First, if the seller is expected to be hit by a liquidity shock that forces him to sell the asset in the near future, no bidder indicates interest, regardless of the commonality of values, and the only equilibrium features immediate initiation of the auction by the seller. Second, if the probability of such a shock and the commonality of values is low, multiple equilibria are possible, implying that different markets with otherwise-similar features can have different patterns of initiation.

In addition to initiation patterns, the model generates implications relating bidders' values to their bidding strategies in the first-price auction and the identity of initiating bidders. For example, auctions are likely to be initiated by stronger bidders, who, on average, will submit higher bids and hence are more likely to win than noninitiating bidders. However, conditional on the same valuation, noninitiating bidders will bid more aggressively than initiating bidders. The model also implies that bidders will bid less aggressively in seller-initiated auctions, even conditional on having the same values.

Our paper belongs to the large literature on auction theory. Virtually all of this literature only considers a stage in which the auction has already taken place. Three exceptions are papers by Board (2007), Cong (2020), and Gorbenko and Malenko (2018), which also feature strategic timing of the auction. Board (2007) and Cong (2020) study the problem of a seller auctioning an option, such as the right to drill oil, where the timing of the sale and option exercise are decision variables. Gorbenko and Malenko (2018) assume that mergers and acquisition (M&A) contests are bidder-initiated and study how their financial constraints affect the timing of initiation. These papers do not study joint initiation by bidders and the seller, and they restrict attention to independent private values, so the issues that we examine in our paper do not arise.

Second, our paper is related to the literature that studies takeover contests as auctions. They have been modeled using both common-value (e.g., Bulow, Huang, and Klemperer (1999), Daley, Geelen, and Green (2023)) and private-value frameworks (e.g., Fishman (1988), Burkart (1995), Povel and Singh (2006), Liu (2012), Liu and Bernhardt (2021)).[5] However, none of these

[5] Bulow and Klemperer (1996, 2009) provide motivations for why running a simple auction is often a good way for the seller to sell the asset.

papers studies endogenous initiation of takeover contests. Initiation has an interesting relation to preemptive bidding, as analyzed by Fishman (1988). Focusing on private values, Fishman (1988) shows that the possibility of preemption gives a first-mover advantage to the initial bidder over an uninformed rival. In an extension, we allow for information acquisition and preemption by bidders, and show that when values are close to common, bidders still prefer not to approach the seller because the aforementioned first-mover advantage shrinks as values become more common.

Finally, our paper is related to models of auctions with asymmetric bidders. Most of the literature on auction theory assumes that bidders are symmetric in that their signals are drawn from the same distribution. Recent literature examines issues that arise when bidders are asymmetric, either in the application to takeover contests (Povel and Singh (2006), Liu (2016)) or more generally.[6] Our solution for equilibrium at the auction stage follows Lebrun (1999) and Kaplan and Zamir (2012, 2015), but extends their solution beyond the case of pure private values. More generally, the novelty of our paper is that asymmetries at the auction stage are not assumed: they arise endogenously and are driven by bidders' incentives to indicate interest to the seller, which differ with the nature of bidders' private information. Endogenous asymmetry is also present in Chen and Wang (2023), who show that the optimal dynamic mechanism can treat ex ante symmetric bidders asymmetrically (favoring the initial bidder) to save on entry costs and compensate the initial bidder for information externality.

The remainder of the paper is organized as follows. Section I describes the setup of the model. Section II solves for the equilibrium bidding in bidder- and seller-initiated auctions and analyzes how the resulting payoffs depend on the commonality of values. Section III characterizes equilibrium initiation strategies. Section IV extends the baseline model to a richer class of selling mechanisms and to costly information acquisition by bidders. Section V discusses the role of additional features of the market for corporate control that are absent from the base model. Section VI concludes.

I. The Model Setup

The economy consists of one risk-neutral seller (male) and two risk-neutral bidders (female), indexed by $i = 1, 2$. There are two periods, $t = 0$ and $t = 1$. All agents value period-1 flows at multiple $\beta \in (0, 1)$. The seller has an asset for sale. For example, the asset can be the whole company or a business unit. The seller's value of the asset is normalized to zero.

At $t = 0$, each bidder randomly draws a private signal. Signals are independent draws from the uniform distribution over $[0, 1]$. Conditional on all signals,

[6] See Maskin and Riley (2000, 2003), Campbell and Levin (2000), Lebrun (1999, 2006), Kaplan and Zamir (2012, 2015), and Kim (2008).

the value of the asset to bidder i is $\alpha v(s_i) + (1-\alpha)v(s_{-i})$, where s_{-i} is the signal of the rival.[7]

ASSUMPTION 1: *Function $v(\cdot)$ is continuous, strictly increasing, and satisfies $v(0) = 0$. Parameter α takes values between $\frac{1}{2}$ and 1.*

Assumption 1 is standard. Continuity means that there are no gaps in possible values of the asset. Strict monotonicity means that a higher private signal is always good news about the bidder's value. Finally, $\alpha \in [\frac{1}{2}, 1]$ means that the model can capture pure private values, pure common values, as well as intermediate cases. At the extremes, it covers two valuation structures commonly used in the literature:

- Pure private values: $\alpha = 1$. A bidder's signal provides information about her own value $v(s_i)$, but not about the value of her competitor $v(s_{-i})$.
- Pure common values: $\alpha = \frac{1}{2}$. Conditional on both signals, bidders have the same value of the asset, $\frac{1}{2}(v(s_1) + v(s_2))$. However, bidders differ in their assessments of the value, because of different private signals.

If $\alpha \in (\frac{1}{2}, 1)$, the valuation structure includes elements of both private and common values. The importance of common values is strictly decreasing in α.

There are three natural interpretations of parameter α in the context of auctions of companies and business units. The first interpretation corresponds to different types of bidders who compete for an asset; we can interpret battles between two financial (strategic) bidders as situations in which α is relatively low (high). Financial bidders use similar strategies after they acquire the target, but may have different estimates of potential gains. In contrast, because synergies that strategic bidders expect to achieve from acquiring the target are often bidder-specific, they are likely to provide little information about the value of the target to the other bidder. Thus, it is natural to expect that the values of financial bidders have a greater common component than the values of strategic bidders. The second interpretation corresponds to different types of targets rather than bidders. Broadly, value in an acquisition can be created because the incumbent target management is inefficient or because the target and the acquirer have synergies that cannot be realized by the stand-alone target. To the extent that many buyers can resolve the inefficiency, acquisitions of the first type are expected to have a higher common component of bidders' values. Finally, commonality of values can also arise endogenously due to downstream interactions among potential strategic buyers (Jehiel and Moldovanu (2000)) or resale opportunities (e.g., Hafalir and Krishna (2008)). In the latter case, parameter α can be related to the degree of frictions in the resale market.

In practice, the environment changes over time, as either external economic shocks arrive or the business nature of the asset for sale changes. In the model, a shock arrives at $t = 1$ with probability $\lambda > 0$, in which case both bidders' values of the asset change. Specifically, each bidder i draws a new independent

[7] Because $v(\cdot)$ is a general function, the assumption of a uniform distribution is, to a large extent, a normalization.

signal from the uniform distribution over $[0, 1]$, denoted as s'_i, $i \in \{1, 2\}$, and her value becomes $\alpha v(s'_i) + (1 - \alpha)v(s'_{-i})$. Independent of this shock, each period with probability $\nu \geq 0$, the seller experiences a liquidity shock and has no choice but to sell the asset immediately. Examples of such liquidity shocks are the arrival of an attractive investment opportunity that is mutually exclusive with the asset under consideration, a change in the firm's strategy, or a bankruptcy in which the judge liquidates the target by auctioning its assets among potential bidders.[8]

In addition to being forced to sell due to a liquidity shock, the seller has the right to auction the asset off to the bidders at any time. Prior to the auction, each bidder communicates with the seller by sending a private message that signals her interest in acquiring the asset. Communication takes the form of cheap talk (Crawford and Sobel (1982)). Messages are costless, and the message space is binary, zero and one. Without loss of generality, we interpret message $m_{i,t} = 1$ from bidder i in period t as an indication of interest in acquiring the asset, and we interpret message $m_{i,t} = 0$ as the lack of such interest. Having observed messages $\mathbf{m}_t = (m_{1,t}, m_{2,t})$, the seller decides whether to place the asset up for sale ($d_t = 1$) or not ($d_t = 0$).

The auction format is sealed-bid first-price with no reserve price. Each bidder simultaneously submits a bid to the seller in a concealed fashion. The two bids are compared, and the bidder with the highest bid acquires the asset and pays her bid. Once the asset is sold, the game ends. The winning bidder obtains the payoff that equals her value less the price she pays.[9] The losing bidder obtains zero payoff. The seller obtains the winning bid.[10] Our focus on the first-price auction is motivated by two factors, besides its popularity. First, it has a unique equilibrium, even if bidders are asymmetric, which makes the analysis unambiguous. Second, it is the simplest auction format that features strategic bidding because the strategy of each bidder depends on her expectations of rivals' bids. As a result, the model delivers interesting implications regarding strategic bidding, which should be also relevant in practice.[11] In Section IV.A, we generalize the analysis to a large class of auctions, which

[8] As an example of an investment opportunity triggering the sale of an existing asset, consider the case of the seller acquiring another firm in a horizontal merger. As a condition of approval, it is common for antitrust authorities to require a spin-off of some of the existing assets to ensure that the product market does not become too concentrated.

[9] Note that if the auction is conducted at $t = 0$, bidders' values depend only on their current signals. In the Internet Appendix, which may be found in the online version of *The Journal of Finance*, we show that our value specification is a reduced form of a more complete cash flow-based specification, in which the asset under management of a bidder generates a stream of cash flows affected by future shocks.

[10] The seller's private value of the asset can also be important for his decision to offer it for sale. Lauermann and Wolinsky (2017) study common-value first-price auctions in which the seller obtains a private signal about his value and solicits a different number of bidders at a cost depending on the signal. Being solicited is thus a signal of the seller's information to bidders.

[11] A typical selling process in auctions of companies in practice, described in Hansen (2001) and Boone and Mulherin (2007), has several elements of the first-price auction despite having multiple rounds of bidding. See Section IV for a discussion.

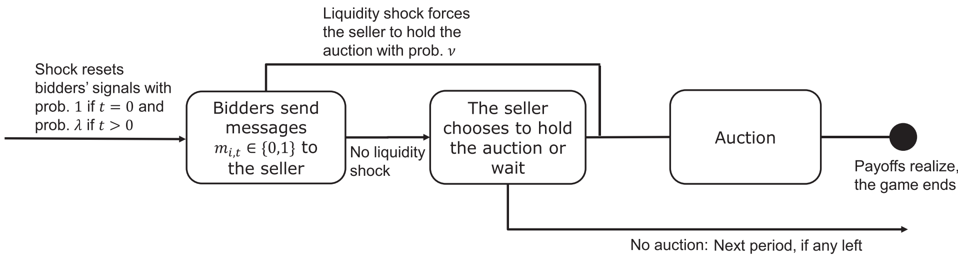

Figure 1. Timing of the model.

includes the first-price auction and (in the limit) the second-price auction. The result that bidders do not indicate their interest for an asset with a sufficiently high common-value component holds for any auction in that class.

We assume that the indications of interest $\mathbf{m}_t$ are publicly observable. The conceptually important assumption is that they become observable to the bidders prior to bidding.[12] This assumption can be justified as follows. The seller may voluntarily disclose whether the auction is bidder- or seller-initiated. In many contexts, whether the auction was bidder- or seller-initiated can be verified ex post—for example, any publicly traded U.S. target is required to report background of the transaction in its SEC filings, and lying has legal consequences. By the standard reasoning (Grossman (1981), Milgrom (1981a)), because it is common knowledge that the seller knows whether the auction is bidder- or seller-initiated and this information is verifiable, he will always disclose it.

Figure 1 summarizes the timing of actions in each period.

A. The Equilibrium Concept

The equilibrium concept is perfect Bayesian equilibrium in pure strategies.[13] In the auction, the bidding strategy of each bidder is a nondecreasing mapping from her own signal s_i and the history of the game into a nonnegative bid. Prior to the auction, the communication strategy of each bidder is a mapping from her own signal s_i and the history of the game into message $m_{i,t} \in \{0, 1\}$, that is, whether to send an indication of interest to the seller or not. Because bidders are ex ante symmetric, we look for equilibria in which they follow symmetric communication strategies.

Furthermore, we look for equilibria in which bidders follow cutoff communication strategies, such that a bidder sends message $m_{i,t} = 1\{s_i \geq \hat{s}_t\}$ for some

[12] In other words, we assume that messages are publicly observed, whether the seller decides to hold an auction or not. Alternatively, we could assume that the messages become observable only if the seller decides to hold the auction. This change affects some off-equilibrium-path payoffs, but the analysis largely remains the same.

[13] In Appendix C, we also consider mixed strategies by the seller. The analysis is more complicated but does not alter the main findings.

time- and history-contingent cutoff $\hat{s}_t \in [0, 1]$.[14] If in equilibrium, all bidder types send the same message at some time t (e.g., if $\hat{s}_t \leq 0$ or $\hat{s}_t \geq 1$), we denote this message by zero without loss of generality.

An equilibrium is *responsive* if there exist message profiles $\mathbf{m}$ and $\mathbf{m}' \neq \mathbf{m}$ that are sent on the equilibrium path such that $d_t(\mathbf{m}) \neq d_t(\mathbf{m}')$ for some t. In other words, an equilibrium is responsive if a bidder's indication of interest affects the decision of the seller to put the asset up for sale. As we shall see below, any responsive equilibrium has the property that a single indication of interest is sufficient to induce an auction. We refer to an auction triggered by an indication of interest as *a bidder-initiated auction*. Otherwise, we refer to an auction as *a seller-initiated auction*.

An equilibrium is *nonresponsive* if $d_t(\mathbf{m}) = d_t(\mathbf{m}')$ for all t and all message profiles $\mathbf{m}$ and $\mathbf{m}'$ that are sent on equilibrium path. In other words, an equilibrium is nonresponsive if the decision of the seller to put the asset up for sale is not affected by bidders' indications of interest. As we will see, in a nonresponsive equilibrium, all types of bidders will send the same message, which will thus convey no information, and thus, all auctions will be seller-initiated.

We solve the model by backward induction. First, we solve for the equilibrium at the auction stage for all possible initiation scenarios. Second, we analyze equilibria at the initiation game, first at $t = 1$, and then at $t = 0$.

II. Auction Stage

We consider the following three cases in turn. First, we consider an auction following an indication of interest from a single bidder. Second, we consider a seller-initiated auction, when the seller puts the asset up for sale without receiving any indications of interest. Finally, we consider an auction following indications of interest from both bidders. We refer to the first and third cases as a bidder-initiated auction and an auction initiated by both bidders, respectively. Throughout this section, we assume that signals are distributed uniformly over $[0, \bar{s}]$ and that indication of interest is sent by bidders with signals $[\hat{s}, \bar{s}]$, where $\hat{s} \leq \bar{s}$ and $\bar{s}$ are arbitrary values in $(0, 1]$ to be determined in the equilibrium of the initiation game.

A. A Bidder-Initiated Auction

In this case, the auction is triggered by one of the two bidders indicating interest to the seller, that is, $m_{1,t} = 1 - m_{2,t}$. Denote the initiating and noninitiating bidders by I and N, respectively. From the perspective of bidder N and the seller, the signal of bidder I is distributed uniformly over $[\underline{s}_I, \bar{s}_I] = [\hat{s}, \bar{s}]$. Similarly, from the perspective of bidder I and the seller, the signal of bidder N

[14] We do not know whether the restriction to equilibria in cutoff strategies is with or without loss of generality, for example, if there can exist equilibria with multiple thresholds. This is because analysis of first-price auctions when distributions of signals have an arbitrary number of gaps is, to our knowledge, an open problem.

is distributed uniformly over $[\underline{s}_N, \bar{s}_N] = [0, \hat{s}]$. Thus, even though bidders are ex ante symmetric, initiation endogenously creates an asymmetry at the auction stage. For brevity, we suppress $\hat{s}$ and $\bar{s}$ from sets of arguments of all functions in this subsection.[15]

Let $a_I(s)$ and $a_N(s)$ be the equilibrium bids of bidder I with signal s and bidder N with signal s, respectively. Conjecture that each bid is strictly increasing and differentiable in s in the relevant range.[16] Denote the corresponding inverses in s by $\phi_I(b)$ and $\phi_N(b)$. Intuitively, $\phi_j(b)$ is the signal of bidder $j \in \{I, N\}$ that submits bid b. Define $\phi_j(b) = \bar{s}_j$ and $\phi_j(b) = \underline{s}_j$ for any b, respectively, above the highest and below the lowest bid submitted by bidder $j \in \{I, N\}$ across signals $[\underline{s}_j, \bar{s}_j]$. Note that the highest bid submitted by both bidders, $a_j(\bar{s}_j) \equiv \bar{a}$ for $j \in \{I, N\}$, must be the same. By contradiction, if, say, $a_I(\bar{s}) > a_N(\hat{s})$, then bidder I with a signal that is sufficiently close to $\bar{s}$ would be better off reducing her bid—she would still win the auction with certainty but her payment would be lower. [17] Then the expected payoff of bidder of type $j \in \{I, N\}$ with signal s from submitting any bid b is

$$\Pi_j(b, s) = \int_{\underline{s}_k}^{\phi_k(b)} \left(\alpha v(s) + (1-\alpha)v(x) - b\right) \frac{1}{\bar{s}_k - \underline{s}_k} dx, \tag{1}$$

where $j \neq k \in \{I, N\}$. The intuition behind (1) is as follows. Consider the initiating bidder who bids b. She wins the auction if and only if the bid of the noninitiating bidder is below b, which happens if such bidder's signal is below $\phi_N(b)$. Conditional on winning when the rival's signal is $x \in [0, \phi_N(b)]$, the value of the asset to the initiating bidder is $\alpha v(s) + (1-\alpha)v(x)$. Integrating over $x \in [0, \phi_N(b)]$ yields (1) for $j = I$.

Taking the first-order conditions of (1), we obtain

$$\frac{\partial \phi_k(b)}{\partial b}\left(\alpha v(s) + (1-\alpha) v\big(\phi_k(b)\big) - b\right) - \left(\phi_k(b) - \underline{s}_k\right) = 0 \tag{2}$$

for $k \in \{I, N\}$ and any signal s such that bidder j wins with positive probability. The first and second terms of (2) represent the trade-off between the marginal benefit and the marginal cost of increasing a bid by a small amount. The marginal benefit is that bidder j wins a marginal event in which the signal of the rival bidder k is exactly $\phi_k(b)$. Her payoff from winning this event is therefore $\alpha v(s) + (1-\alpha)v(\phi_k(b)) - b$. The marginal cost is that bidder j must pay more if she wins. In equilibrium, $b = a_j(s)$ must satisfy (2) for $j \neq k$, implying $s = \phi_j(b)$. Plugging in and rearranging the terms, for $j \in \{I, N\}$, we obtain

$$\frac{\partial \phi_j(b)}{\partial b} = \frac{\phi_j(b) - \underline{s}_j}{\alpha v\big(\phi_k(b)\big) + (1-\alpha) v\big(\phi_j(b)\big) - b}. \tag{3}$$

[15] In particular, equilibrium bids $a_I(\cdot)$ and $a_N(\cdot)$, their inverses $\phi_I(\cdot)$ and $\phi_N(\cdot)$, the range of bids $[\underline{a}, \bar{a}]$, and bidders' payoffs all depend on $\hat{s}$ and $\bar{s}$.

[16] This conjecture is confirmed in the proof of Lemma 1.

[17] The argument for why $a_N(\hat{s})$ cannot exceed $a_I(\bar{s})$ is identical.

The system of two differential equations (3) is solved subject to the appropriate boundary conditions. The first condition is that in equilibrium, the highest bid submitted by both bidders must be the same, $a_j(\bar{s}_j) \equiv \bar{a}$ for $j \in \{I, N\}$. The condition implies $\phi_I(\bar{a}) = \bar{s}$ and $\phi_N(\bar{a}) = \hat{s}$. Next, let $a_I(\hat{s}) \equiv \underline{a}$ be the lowest bid submitted by bidder I, which implies $\phi_I(\underline{a}) = \hat{s}$. The second boundary condition is that in equilibrium, bidder N that submits bid $\underline{a}$ must be indifferent between winning and losing: $\alpha v(\phi_N(\underline{a})) + (1-\alpha)v(\hat{s}) = \underline{a}$. Intuitively, if bidder N with signal $\phi_N(\underline{a})$ strictly preferred to win, then this bidder would be better off deviating to a higher bid that wins with strictly positive probability.

The final boundary condition arises from solving for bid $\underline{a}$, that is, the lowest bid that can potentially win. In principle, this bid can depend on bids of bidder N with very low signals. Because such a bidder wins with probability 0, one can rationalize a variety of her bids. We impose the following natural restriction on equilibrium bids, which will also allow us to pin down the unique equilibrium in the auction.[18]

ASSUMPTION 2: *A bidder's belief about the signal of the rival is weakly monotone in the rival's bid. No bidder bids above her expected (conditional on winning) value in equilibrium.*

Assumption 2 holds trivially for bids in $(\underline{a}, \bar{a}]$, that is, bids that win with positive probability. First, because a bid is monotone in the bidder's signal, a higher bid is indicative of a higher signal. Second, because a bidder can always bid zero and lose the auction with certainty, she would never bid above her expected (conditional on winning) value. Assumption 2 disciplines the bids of bidder N with a very low signal. Consider such a bidder contemplating some bid $b \le \underline{a}$. The first part of Assumption 2 means that the fact that her bid is winning (which never happens in equilibrium) means that the signal of bidder I must be $\hat{s}$. The second part of Assumption 2 means that bidder N with a very low signal s cannot bid above $\alpha v(s) + (1-\alpha)v(\hat{s})$—such a bid is irrational, because bidder N achieves a negative payoff if she wins.[19] As we show in the proof of Lemma 1 in Appendix A, Assumption 2 uniquely pins down the minimum "serious" bid $\underline{a}$ in the following way:[20]

$$\underline{a} = \arg\max_b v^{-1}\left(\frac{b-(1-\alpha)v(\hat{s})}{\alpha}\right)$$

[18] As Kaplan and Zamir (2015) show in the special case of pure private values, without this restriction, multiple equilibria arise in the first-price auction with asymmetric bidders, with some bidders submitting counterintuitive "nonserious" bids (i.e., bids that win with probability 0) above their values.

[19] Note that the second part of Assumption 2 is related to elimination of weakly dominated strategies. If $\alpha = 1$, it is implied by assumption that no weakly dominated strategy is played in equilibrium.

[20] The form of (4) suggests the following interpretation of $\underline{a}$: it is the optimal bid of the initiating bidder with signal $\hat{s}$ if the noninitiating bidder with any low signal x bids her full value $\alpha v(x) + (1-\alpha)v(\hat{s})$.

$$\times\left(\alpha v(\hat{s})+(1-\alpha)\mathbb{E}\left[v(x)|x\leq v^{-1}\left(\frac{b-(1-\alpha)v(\hat{s})}{\alpha}\right)\right]-b\right). \quad (4)$$

The next lemma summarizes the equilibrium in the bidder-initiated auction.

LEMMA 1 (Equilibrium in the bidder-initiated auction): *The bidder-initiated auction has a unique (up to the nonserious bids of the noninitiating bidder with a low signal) equilibrium. The equilibrium bidding strategies of the initiating and noninitiating bidders, $a_j(s)$, $j \in \{I, N\}$, are increasing functions with the lowest serious bid $\underline{a}$ given by (4), such that their inverses satisfy (3) with boundary conditions*

$$\bar{s}=\phi_I(\bar{a}), \quad \hat{s}=\phi_N(\bar{a}), \quad \hat{s}=\phi_I(\underline{a}), \quad \phi_N(\underline{a})=v^{-1}\left(\frac{\underline{a}-(1-\alpha)v(\hat{s})}{\alpha}\right). \quad (5)$$

Examples B1 and B2 in Appendix B solve for bidding strategies in a bidder-initiated auction for $\alpha=1$ (pure private values) and $\alpha=\frac{1}{2}$ (pure common values), when $v(s)=s$, $\bar{s}=1$, and $\hat{s}=\frac{1}{2}$. These bidding strategies are illustrated in Figure 2, Panels A and B.

We denote the equilibrium payoff of bidder $j \in \{I, N\}$ from the bidder-initiated auction by $\Pi_j^*(s,\hat{s},\bar{s})$, where we are now explicit about its dependence on $\hat{s}$ and $\bar{s}$. Equilibrium payoffs of bidders $j \in \{I, N\}$ with cutoff signal $\hat{s}$ (useful in the analysis of the initiation stage) can be computed without knowing equilibrium bids of all types:

$$\Pi_I^*(\hat{s},\hat{s},\bar{s})=\max_y \frac{v^{-1}(y)}{\hat{s}}\left((2\alpha-1)v(\hat{s})+(1-\alpha)\mathbb{E}\left[v(x)|x\leq v^{-1}(y)\right]-\alpha y\right), \quad (6)$$

$$\Pi_N^*(\hat{s},\hat{s},\bar{s})=\alpha v(\hat{s})+(1-\alpha)\mathbb{E}[v(x)|x\in[\hat{s},\bar{s}]]-\bar{a}(\hat{s},\bar{s}). \quad (7)$$

We denote the expected revenue of the seller from the bidder-initiated auction by

$$R_B(\hat{s},\bar{s})=\mathbb{E}[\max(a_I(s_1),a_N(s_2))|s_1\in[\hat{s},\bar{s}],s_2\in[0,\hat{s}]]. \quad (8)$$

B. A Seller-Initiated Auction

Next, consider a seller-initiated auction. In this case, the seller puts the asset up for sale without receiving any indications of interest, that is, $m_{1,t}=m_{2,t}=0$. Thus, from the perspective of all agents, the signal of each bidder is distributed uniformly over $[0,\hat{s}]$ for some $\hat{s}$ to be solved for later. Indeed, if any of the bidders had a signal above $\hat{s}$, she would have indicated her interest to the seller. Because the two bidders are symmetric, the equilibrium is also symmetric. Denote the equilibrium bid by a bidder with signal s by $a_S(s)$ and its inverse in s by $\phi_S(b)$, where we again suppress the possible dependence on $\hat{s}$

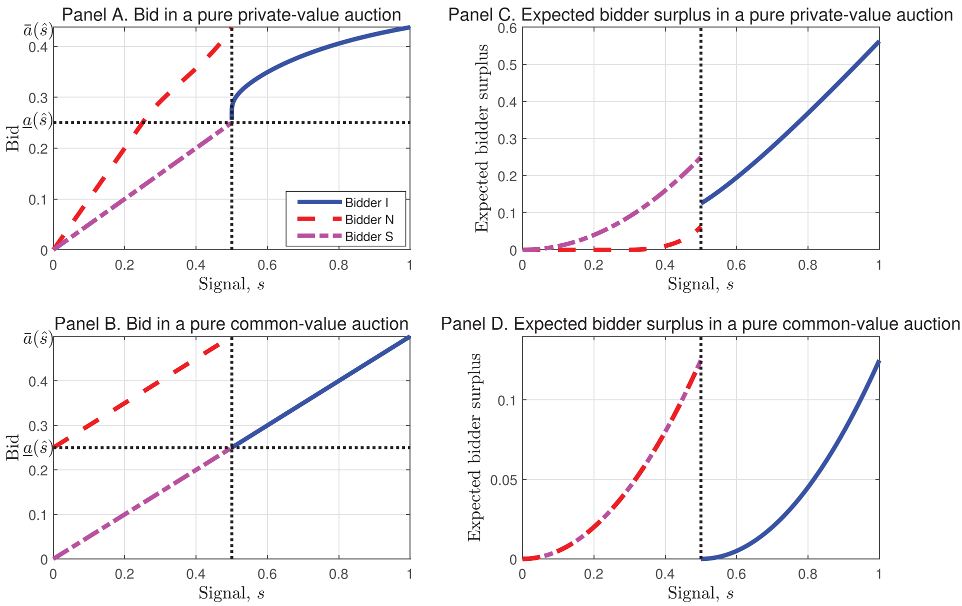

Figure 2. Equilibrium bids and expected payoffs of bidders. For pure private values $\alpha = 1$ and pure common values $\alpha = 1/2$, and for values $v(s) = s$, Panels A and B plot equilibrium bids as functions of signal s for the initiating bidder (solid blue line) and the noninitiating bidder (dashed red line) in a bidder-initiated auction, and either of the two bidders (dash-dotted pink line) in a seller-initiated auction. Panels C and D plot the corresponding expected surpluses. The cutoff signal for initiation, $\hat{s}$, is 0.5. The lowest and highest serious bids are indicated by $\underline{a}(\hat{s})$ and $\bar{a}(\hat{s})$. For signals of the noninitiating bidder below the lowest serious bid $\underline{a}(\hat{s})$, Panel A plots one possible equilibrium bid. (Color figure can be viewed at wileyonlinelibrary.com)

for brevity. The expected payoff of a bidder with signal s from bidding b is

$$\Pi_S(b, s) = \int_0^{\phi_S(b)} \big(\alpha v(s) + (1-\alpha)v(x) - b\big)\frac{1}{\hat{s}}dx. \tag{9}$$

Intuitively, the bidder wins if and only if the signal of the rival bidder x is below $\phi_S(b)$, in which case the asset is worth $\alpha v(s) + (1-\alpha)v(x)$. Taking the first-order condition of (9), we obtain

$$\frac{\partial \phi_S(b)}{\partial b} = \frac{\phi_S(b)}{v\big(\phi_S(b)\big) - b}, \tag{10}$$

which is solved with the initial value condition that the bidder with the lowest signal bids zero: $\phi_S(0) = 0$. This differential equation leads to the following equilibrium bid:

$$a_S(s) = \mathbb{E}[v(x)|x \leq s]. \tag{11}$$

The next lemma summarizes the equilibrium in the seller-initiated auction.

LEMMA 2 (Equilibrium in the seller-initiated auction): *The seller-initiated auction has a unique equilibrium. A bidder with signal* $s \in [0, \hat{s}]$ *bids* $a_S(s) = \mathbb{E}[v(x)|x \leq s]$.

Panels A and B of Figure 2 illustrate bidding strategies in the seller-initiated auction for $\alpha = 1$ (pure private values) and $\alpha = \frac{1}{2}$ (pure common values) when $v(s) = s$, $\bar{s} = 1$, and $\hat{s} = \frac{1}{2}$. Note that the equilibrium in the seller-initiated auction depends neither on whether the valuation structure is closer to common or private values nor on the cutoff $\hat{s}$. Intuitively, $\hat{s}$ is irrelevant, because when choosing a bid, a bidder conditions on the rival's signal below the bidder's signal. The degree of commonality of values α is irrelevant because the marginal event that a bidder with signal s wins has the opponent with exactly the same signal, and therefore, the bidder infers that the value of the asset in this marginal event is $v(s)$. In contrast, both the valuation structure and the initiation cutoff matter for the equilibrium in the bidder-initiated auction.

We denote the equilibrium payoff of a bidder from the seller-initiated auction by $\Pi_S^*(s, \hat{s})$, where we are now explicit about the payoff's dependence on $\hat{s}$. The equilibrium payoff of a bidder with cutoff signal $\hat{s}$ is

$$\Pi_S^*(\hat{s}, \hat{s}) = \alpha v(\hat{s}) - \alpha \mathbb{E}\big[v(x)|x \leq \hat{s}\big]. \tag{12}$$

We denote the expected revenues of the seller from the seller-initiated auction by

$$R_S(\hat{s}) = \mathbb{E}[v(\min{(s_1, s_2)})|s_i \leq \hat{s}, i \in \{1, 2\}]. \tag{13}$$

Note that $R_S(\hat{s})$ does not depend on α because bidders' equilibrium bidding strategies in the seller-initiated auction do not depend on α, as shown above.

C. An Auction Initiated by Both Bidders

The remaining possibility is that the seller puts the asset up for sale after both bidders indicate their interest, that is, $m_{1,t} = m_{2,t} = 1$. In this case, from the perspective of all agents, the signal of each bidder is distributed uniformly over $[\hat{s}, \bar{s}]$ for some $\hat{s}$ and $\bar{s}$ to be determined later. Denoting the equilibrium bid by a bidder with signal s by $a_D(s)$ and following the derivation analogous to the case of a seller-initiated auction, we obtain the following equilibrium bid (as before, we suppress dependence on $\hat{s}$):

$$a_D(s) = \mathbb{E}\big[v(x)|x \in [\hat{s}, s]\big]. \tag{14}$$

As in the seller-initiated auction, the equilibrium bid is independent of the degree of commonality of values α and the highest signal $\bar{s}$. However, it depends on the cutoff signal $\hat{s}$: it becomes common knowledge that the value of the asset is at least $v(\hat{s})$. In turn, a bidder with signal $\hat{s}$ bids $a_D(\hat{s}) = v(\hat{s})$, wins only if the rival bidder's signal is also $\hat{s}$, and obtains zero surplus from the auction. Formally, letting $\Pi_D^*(s,\hat{s},\bar{s})$ denote the equilibrium payoff of a bidder with signal s from such an auction, $\Pi_D^*(\hat{s},\hat{s},\bar{s})=0$.

D. Ranking of Bidders' Payoffs

Given the equilibria in the bidder- and seller-initiated auctions, we next compare the payoffs of a bidder with the cutoff signal $\hat{s}$ in three cases: (i) in a bidder-initiated auction that she initiates, (ii) in a bidder-initiated auction initiated by the rival bidder, and (iii) in a seller-initiated auction. The case of a dual bidder-initiated auction is trivial because in that case, the payoff of a bidder with signal $\hat{s}$ is zero. The next proposition establishes this comparison.

PROPOSITION 1 (Ranking of payoffs for the cutoff type): *Consider any cutoff type $\hat{s}\in(0,1)$. The expected payoffs of a bidder with signal $\hat{s}$ from the seller-initiated auction, $\Pi_S^*(\hat{s},\hat{s})$, the auction initiated by her, $\Pi_I^*(\hat{s},\hat{s},\bar{s})$, and the auction initiated by the rival bidder, $\Pi_N^*(\hat{s},\hat{s},\bar{s})$, compare as follows:*

- *(a) $\Pi_S^*(\hat{s},\hat{s})>\Pi_I^*(\hat{s},\hat{s},\bar{s})$ for any α.*
- *(b) There exists $\alpha'\in(\frac{1}{2},1)$ such that for any $\alpha>\alpha'$, $\Pi_I^*(\hat{s},\hat{s},\bar{s})>\Pi_N^*(\hat{s},\hat{s},\bar{s})$.*
- *(c) There exists $\alpha''\in(\frac{1}{2},1)$ such that for any $\alpha<\alpha''$, $\Pi_I^*(\hat{s},\hat{s},\bar{s})<\Pi_N^*(\hat{s},\hat{s},\bar{s})$. Furthermore, $\Pi_I^*(\hat{s},\hat{s},\bar{s})$ approaches zero as $\alpha\to\frac{1}{2}$.*

Panels C and D of Figure 2 illustrate these payoffs for $\alpha=1$ (pure private values) and $\alpha=\frac{1}{2}$ (pure common values) when $v(s)=s$, $\bar{s}=1$, and $\hat{s}=\frac{1}{2}$. Proposition 1 is a key step to the results about initiation derived in the next section, so it is worth explaining the intuition in detail. Result (a) shows that a bidder with signal $\hat{s}$ is always better off in the auction initiated by the seller than in the auction that she initiates. This result may seem surprising because the distribution of the signal of the other, noninitiating bidder is the same in both cases. However, the important difference is in the noninitiating bidder's perception of the strength of her rival. In a bidder-initiated auction, the noninitiating bidder believes that she competes against a strong rival whose signal is distributed over $[\hat{s},\bar{s}]$. In contrast, in a seller-initiated auction, each bidder believes that she competes against a weak rival whose signal is distributed over $[0,\hat{s}]$. In response, the noninitiating bidder bids more aggressively in the bidder-initiated auction. As a result, a bidder with signal $\hat{s}$ obtains a higher payoff in the seller-initiated auction. Note that this argument and the result do not depend on the degree of commonality of bidders' values.

Result (b) shows that a bidder with signal $\hat{s}$ is better off in the auction that she initiates than in the auction initiated by the rival bidder if the private-value component is sufficiently high. In contrast, if the common-value component is sufficiently high, then the opposite is true, as shown in result (c). Intuitively, there are two opposite effects. It is easiest to see them in the two extreme cases, pure common values ($\alpha=\frac{1}{2}$) and pure private values ($\alpha=1$).

The first effect concerns asset revaluation by the noninitiating bidder. If bidders' values have a common component, the noninitiating bidder updates the belief about her value from the fact that the auction is initiated by the rival bidder—she learns that the signal of the rival bidder is at least $\hat{s}$. In the case of pure common values, the noninitiating bidder learns that the value of the asset is at least $\frac{1}{2}(v(s) + v(\hat{s}))$, where s is her signal. In particular, it becomes common knowledge that the value of the asset is at least $\frac{1}{2}v(\hat{s})$ because the signal of the noninitiating bidder cannot be below zero and the signal of the initiating bidder cannot be below $\hat{s}$. Thus, no bidder bids below $\frac{1}{2}v(\hat{s})$, and bid $\frac{1}{2}v(\hat{s})$ is submitted by the bidder of each type with the lowest possible signal ($\hat{s}$ for the initiating bidder and zero for the noninitiating bidder). The initiating bidder with signal $\hat{s}$ therefore obtains zero surplus. In contrast, the noninitiating bidder with signal $\hat{s}$ preserves her rents, as the initiating bidder does not know that her signal is that high. This effect is absent in the case of pure private values but is particularly strong if values are close to common, leading to part (c) of Proposition 1.

The second effect concerns the strength of the initiating bidder's rival. The fact that the rival has not indicated interest implies that her signal is distributed over $[0, \hat{s}]$, so that the rival has a lower value than the initiating bidder as long as values contain some private component. In contrast, if a bidder with signal $\hat{s}$ does not initiate but participates in the auction initiated by the rival, she competes against a strong rival whose signal is distributed over $[\hat{s}, \bar{s}]$, which lowers the bidder's surplus. If values are close to private, only this effect is present as the asset revaluation effect is small, leading to part (b) of Proposition 1.

Related to the argument of Proposition 1, the next proposition generates a set of implications about bidding behavior in bidder-initiated and seller-initiated auctions.[21]

PROPOSITION 2 (Implications about bidding and values): *Consider any cutoff type $\hat{s} \in (0, 1)$, and suppose that $\alpha > \frac{1}{2}$, that is, values are not pure common. Then:*

- *(a) In a bidder-initiated auction, conditional on the same signal, the noninitiating bidder bids more aggressively than the initiating bidder: $a_N(\hat{s}) > a_I(\hat{s})$.*
- *(b) In a bidder-initiated auction, unconditional on the signal, the initiating bidder bids more aggressively and wins more often: $\mathbb{E}[a_I(s)|\hat{s} \leq s \leq \bar{s}] > \mathbb{E}[a_N(s)|s \leq \hat{s}]$.*
- *(c) Any noninitiating bidder who makes a serious bid in the bidder-initiated auction (i.e., $a_N(s) \geq \underline{a}$) bids less aggressively in a seller-initiated auction: $a_S(s) < a_N(s)$.*

[21] See Landsberger et al. (2001) and section 4.3 of Krishna (2010) for related results on the comparison of bidding strategies of strong and weak bidders in the first-price auction under independent private values when asymmetry is assumed. Proposition 2 relates asymmetry to initiation and goes beyond private values.

(d) *The value of the winning bidder is, on average, higher in a bidder-initiated auction than in a seller-initiated auction, whether she is an initiating bidder or not.*

The last statement of Proposition 2 is supported by evidence in Masulis and Simsir (2018), who find that takeover premia are about 10 to 12 percentage points lower in target-initiated deals than in bidder-initiated deals. We are not aware of existing tests of the other implications of Proposition 2.

III. Initiation Game

Having solved for the equilibria in the auction for all combinations of bidders' messages, we analyze the initiation game. We solve the model by backward induction. First, we consider the bidders' and the seller's initiation decisions in the terminal period, $t = 1$. It turns out that this subgame has a unique and very simple equilibrium. Using this equilibrium, we analyze initiation decisions in period $t = 0$, which is our main object of interest. Finally, we derive the key properties of initiation equilibria.

A. Initiation in the Terminal Period

Consider the initiation game in period $t = 1$, conditional on the seller choosing to not hold the auction at $t = 0$. At $t = 1$, each bidder's signal is distributed uniformly over $[0, \bar{s}]$, where either $\bar{s} = 1$ if a shock arrives earlier in the period, leading to a reset of bidders' signals, or $\bar{s} = \hat{s}_0$ if no such shock arrives. Here, $\hat{s}_0$ denotes the cutoff signal at $t = 0$, such that bidders with all signals above $\hat{s}_0$ indicate their interest to the seller at $t = 0$. This cutoff will be determined below.

The next proposition shows that for all α and $\bar{s}$, the terminal period has a very simple equilibrium: no bidder sends an indication of interest to the seller regardless of how high her signal is, and the seller always initiates the auction.

PROPOSITION 3 (Equilibrium in the initiation game at $t = 1$): *There is a unique nonresponsive equilibrium in which neither bidder sends an indication of interest ($m_{i,1} = 0\ \forall s_i, i \in \{1, 2\}$), and the seller holds the auction at $t = 1$ regardless of the messages: $d_1(\mathbf{m}_1) = 1\ \forall \mathbf{m}_1$.*

The intuition is as follows. Because the seller values the asset less than bidders and the game ends at $t = 1$, there is no benefit for him to not hold an auction at $t = 1$ regardless of the indications of interest. Expecting this reaction from the seller, no bidder finds it optimal to indicate her interest, as by Proposition 1, the payoff of the bidder with any hypothetical cutoff signal is higher when the rival perceives her to be weak. Thus, the only equilibrium at $t = 1$ is that no bidder indicates her interest and the auction is seller-initiated.

Given Proposition 3, we can compute the continuation payoffs in period $t = 1$ for the seller and each bidder. Suppose that at $t = 0$, each bidder sends message $m_{i,0} = 1$ if and only if $s_i \geq \hat{s}_0$ for some $\hat{s}_0 \in [0, 1]$, and the seller holds the auction unless $m_{1,0} = m_{2,0} = 0$, which will be shown to be the case in the next subsection. From Lemma 2, whether bidders' signals reset at $t = 1$ or not, they follow the same bidding strategy $a_S(s) = \mathbb{E}[v(x)|x \leq s]$ at $t = 1$. Thus, before the signal-resetting shock, the expected revenues of the seller are

$$R_1(\hat{s}_0) = \lambda \mathbb{E}\big[v\big(\min(s_1, s_2)\big)\big] + (1-\lambda)\mathbb{E}\big[v\big(\min(s_1, s_2)\big)|s_i \leq \hat{s}_0, i \in \{1, 2\}\big]. \quad (15)$$

Likewise, before the shock, the expected payoff of a bidder with signal $\hat{s}_0$ is

$$\Pi_1(\hat{s}_0) = (1-\lambda)\Pi_S^*(\hat{s}_0, \hat{s}_0) + \lambda \Pi_R = (1-\lambda)\alpha(v(\hat{s}_0) - \mathbb{E}[v(x)|x \leq \hat{s}_0]) + \lambda \Pi_R, \quad (16)$$

where Π_R is the expected payoff of a bidder conditional on a reset of signals at $t = 1$:

$$\Pi_R = \frac{\alpha}{2}\mathbb{E}[v(\max(s_1, s_2)) - v(\min(s_1, s_2))]. \quad (17)$$

Intuitively, after the shock but before learning a new signal, a bidder expects to win with a 50% chance; her expected value conditional on winning is $\mathbb{E}[\alpha v(\max(s_1, s_2)) + (1-\alpha)v(\min(s_1, s_2))]$; and her expected payment is $\mathbb{E}[v(\min(s_1, s_2))]$.

B. Initiation in the Initial Period

Given the equilibrium in the terminal period and the continuation payoffs (15) and (16), consider the initiation game in period $t = 0$. Denoting the seller's decision of whether to hold the auction in response to bidders' messages by $d_0(\mathbf{m}_0) \in \{0, 1\}$, the next proposition characterizes all possible equilibria of the model.

PROPOSITION 4 (Equilibria in the initiation game at $t = 0$): *The set of possible equilibria is as follows:*

- *(a) A nonresponsive equilibrium, in which neither bidder sends an indication of interest ($m_{i,0} = 0\ \forall s_i, i \in \{1, 2\}$), and the seller holds the auction at $t = 0$ regardless of the messages: $d_0(\mathbf{m}_0) = 1\ \forall \mathbf{m}_0$.*
- *(b) A responsive equilibrium, in which each bidder sends an indication of interest if and only if $s_i \geq \hat{s}_0$ for $\hat{s}_0 \in (0, 1)$, and the seller who is not hit by the liquidity shock holds the auction unless $\mathbf{m}_0 = (0, 0)$:*

$$d_0(\mathbf{m}_0) = \begin{cases} 1, & \text{if } \mathbf{m}_0 \neq (0, 0), \\ 0, & \text{if } \mathbf{m}_0 = (0, 0). \end{cases} \quad (18)$$

The cutoff signal $\hat{s}_0$ satisfies

$$\Pi_I^*(\hat{s}_0, \hat{s}_0, 1) = \left(\frac{1}{\hat{s}_0} - 1\right)\Pi_N^*(\hat{s}_0, \hat{s}_0, 1) + \nu \Pi_S^*(\hat{s}_0, \hat{s}_0) + (1-\nu)\beta \Pi_1(\hat{s}_0). \quad (19)$$

The nonresponsive equilibrium always exists. The responsive equilibrium exists if and only if the solution to (19) satisfies

$$R_S(\hat{s}_0) \leq \beta((1-\lambda)R_S(\hat{s}_0) + \lambda R_S(1)) \leq R_B(\hat{s}_0, 1), \tag{20}$$

and inequalities (A.11) and (A.12) in Appendix A.

Proposition 4 shows that the equilibrium is either nonresponsive, so that all auctions are seller-initiated, or responsive, so that a single indication of interest triggers a bidder-initiated auction. Interestingly, there cannot be a responsive equilibrium in which the seller not hit by the liquidity shock holds an auction only if both bidders indicate their interest, that is, does not hold an auction when only one of the bidders indicates her interest. Intuitively, if such an equilibrium did exist, then an initiating bidder with the cutoff signal $\hat{s}_0$ would obtain zero payoff in such an auction, as she would have the lowest value and thus would win with zero probability. Furthermore, if the rival's signal is below $\hat{s}_0$, the bidder with signal $\hat{s}_0$ would be worse off indicating her interest: when the seller finally holds the auction at $t = 1$, the interest of the bidder with signal $\hat{s}_0$ reveals her high signal and leads to more aggressive bidding by the rival. Across these two possibilities, the bidder with signal $\hat{s}_0$ is strictly better off not indicating her interest. Hence, any responsive equilibrium is such that a single indication of interest suffices to trigger an auction.

The nonresponsive equilibrium always exists from the following argument. If bidders expect the seller to put the asset up for sale in the absence of interest, they do not benefit from indicating their interest. Moreover, if the lack of interest does not reveal any negative information about bidders' signals, then the seller does not benefit from delaying the auction until $t = 1$ because he does not expect an improvement in the bidders' willingness to pay.

To see the intuition for responsive equilibria, consider equation (19), which determines the cutoff signal $\hat{s}_0$ and is key for analyzing the existence of a responsive equilibrium. In the responsive equilibrium, a bidder with signal $\hat{s}_0$ is indifferent between indicating her interest ($m_{i,0} = 1$), which leads to the auction with certainty, and not indicating it ($m_{i,0} = 0$), in which case the auction occurs only if the rival indicates her interest or if the seller is hit by the liquidity shock. Table I shows the expected payoffs of the bidder with signal $\hat{s}_0$ from each strategy. Suppose such a bidder indicates interest. With probability $1 - \hat{s}_0$, the rival also indicates interest. Because in this case, the rival's signal is distributed over $[\hat{s}_0, 1]$, bidder i loses the auction with certainty and gets a payoff of zero. With probability $\hat{s}_0$, the rival does not indicate interest. In this case, we have an asymmetric auction in which bidder i's expected payoff is $\Pi_I^*(\hat{s}_0, \hat{s}_0, 1)$. Thus, sending $m_{i,0} = 1$ yields the expected payoff of $\hat{s}_0 \Pi_I^*(\hat{s}_0, \hat{s}_0, 1)$.

Next, suppose that bidder i with signal $\hat{s}_0$ does not indicate her interest. With probability $1 - \hat{s}_0$, the rival indicates her interest and the auction occurs with certainty. In this case, bidder i obtains the expected payoff of $\Pi_N^*(\hat{s}_0, \hat{s}_0, 1)$. With probability $\hat{s}_0$, the rival does not indicate her interest, and the auction occurs

Table I
Expected Payoffs of the Bidder with Cutoff Signal $\hat{s}_0$

This table shows the expected payoffs of a bidder with signal $\hat{s}_0$ from indicating and not indicating interest, when the signal of the rival bidder is low (below $\hat{s}_0$, so that the rival does not indicate interest) versus high (above $\hat{s}_0$, so that the rival indicates interest.

		Rival Bidder's Signal	
		Low ($s < \hat{s}_0$)	High ($s > \hat{s}_0$)
Strategy of bidder with type $\hat{s}_0$	Indicate	$\Pi_I^*(\hat{s}_0, \hat{s}_0, 1)$	0
	Not indicate	$\nu\Pi_S^*(\hat{s}_0, \hat{s}_0) + (1-\nu)\beta\Pi_1(\hat{s}_0)$	$\Pi_N^*(\hat{s}_0, \hat{s}_0, 1)$

only if the seller is hit by the liquidity shock. In this case, bidder i obtains the expected payoff of $\Pi_S^*(\hat{s}_0, \hat{s}_0) = \alpha(v(\hat{s}_0) - \mathbb{E}[v(x)|x \leq \hat{s}_0])$. With probability $\hat{s}_0(1-\nu)$, the auction does not occur at $t = 0$, and the game proceeds to $t = 1$. In this case, bidder i obtains the expected payoff of $\beta\Pi_1(\hat{s}_0)$, where $\Pi_1(\hat{s}_0)$ is given by (16). To be indifferent between indicating and not indicating her interest, cutoff $\hat{s}_0$ must satisfy the indifference condition (19).

Condition (20) ensures that the seller prefers to hold the auction upon receiving a single indication of interest (the right inequality) and prefers to wait until $t = 1$ if she receives no interest at all (the left inequality). If the condition does not hold, the responsive equilibrium does not exist because the seller's action is not responsive to bidders' messages. For example, if the seller discounts the future very strongly ($\beta \to 0$), he prefers to hold the auction immediately even in the absence of any interest, so only the nonresponsive equilibrium exists.[22]

C. Equilibrium Properties

Equation (19) highlights several equilibrium properties. The first property relates to the role of common versus private values. Recall from Proposition 1 that $\Pi_I^*(\hat{s}_0, \hat{s}_0, 1) < \Pi_N^*(\hat{s}_0, \hat{s}_0, 1)$ when a common component of values is sufficiently high, and that $\Pi_I^*(\hat{s}_0, \hat{s}_0, 1)$ becomes close to zero as values get closer to common. As discussed in Section II.D, this happens because of the asset revaluation effect. Intuitively, sending an indication of interest to the seller signals that the initiating bidder is sufficiently optimistic about the value of the asset. When values have a significant common component, the rival bidder updates her value based on this information and bids more aggressively, leaving the initiating bidder with the cutoff signal with little surplus.

The following result links the commonality of values to the existence of the responsive equilibrium.

[22] Additional conditions (A.11) and (A.12) ensure that if the bidder with signal $\hat{s}_0$ is indifferent between indicating her interest and not, then bidders with signals below $\hat{s}_0$ prefer to not indicate their interest, while those with signals above $\hat{s}_0$ prefer to indicate it.

PROPOSITION 5 (Equilibrium when values are close to common): *There exists* $\hat{\alpha} \in (\frac{1}{2}, 1)$*, such that for any* $\alpha < \hat{\alpha}$*, only the nonresponsive equilibrium exists at* $t = 0$.

For a stark example, consider (19) and the case of almost pure common values, $\alpha \to \frac{1}{2}$. In this case, revaluation by the rival erodes almost all payoff of the initiating bidder with signal $\hat{s}_0$. Intuitively, in the case of pure common values, a bidder earns her payoff only through informational advantage over the rival. However, the initiating bidder with the cutoff signal loses this informational advantage because the rival knows that the initiating bidder's signal is at least at the cutoff level. Because the initiating bidder with the cutoff signal earns almost zero payoff and because not indicating interest results in a strictly positive payoff (as shown in Table I), the initiating bidder with the cutoff signal is better off not indicating her interest to the seller. Therefore, only the nonresponsive equilibrium exists.[23]

The second property highlighted by equation (19) is the role of liquidity shocks for the seller. Proposition 1 shows that $\Pi_I^*(\hat{s}_0, \hat{s}_0, 1) < \Pi_S^*(\hat{s}_0, \hat{s}_0)$. In turn, if the seller is likely to initiate the auction in the absence of any interest, each bidder's incentives to indicate interest and face more aggressive bidding by the rival are low. The following result shows that irrespective of the commonality of values, only seller-initiated auctions occur in equilibrium if the seller is sufficiently likely to receive liquidity shocks:

PROPOSITION 6 (Equilibrium when liquidity shocks are likely): *For any* $\alpha \in [\frac{1}{2}, 1]$*, there exists* $\hat{\nu} \in (0, 1)$*, such that for any* $\nu > \hat{\nu}$*, only the nonresponsive equilibrium exists at* $t = 0$.

For a stark example, consider (19) and $\nu \to 1$. For any $\hat{s}_0$, the benefits of waiting are above the benefits of initiation, so the bidders do not indicate their interest and the seller sells the asset immediately. As a practical application, Proposition 6 suggests that in the market for distressed assets, sales would tend to be seller-initiated. Bidders are reluctant to approach the seller who is close to liquidation, because they expect him to put the asset up for sale soon regardless of the expressed demand for it. This result holds regardless of whether the asset is commonly or privately valued by market participants.

It is natural to extend the model by assuming that the seller can have some disutility $C > 0$ from selling the asset. In the application to auctions of companies, C can be interpreted as the level of entrenchment of the target's board. Suppose that $C > R_S(1)$, so that the seller would not sell the asset unless he is hit by the liquidity shock or receives some indications of interest. The following result, a corollary to Proposition 5, shows that if values are close to common and bidders are reluctant to show their interest, then in equilibrium, the asset will be sold only if the seller is hit by the liquidity shock.

[23] The fact that the lowest type gets zero surplus in a common-value auction is a known result in the auction theory literature, but we are not aware of earlier papers that connect this result to bidders' initiation incentives.

COROLLARY 1 (No strategic initiation): *If $C > R_S(1)$, there exists $\hat{\alpha} \in (\frac{1}{2}, 1)$ such that for any $\alpha < \hat{\alpha}$, only the nonresponsive equilibrium exists in each period, whereby the seller does not put the asset up for sale unless he is hit by the liquidity shock.*

In contrast, when values are close to private, responsive equilibria can exist even if $C > R_S(1)$. Intuitively, in this case, the bidder with signal $\hat{s}_0$ is eager to indicate her interest if doing so leads to the auction. If C does not exceed $R_S(1)$ by too much and $\hat{s}_0$ is relatively high, the seller finds it optimal to put the asset up for sale upon receiving interest and inferring that one of the bidder's signals is above $\hat{s}_0$.

D. Special Case: Linear Values

For the special case of $v(s) = s$, all expressions of bidders' payoffs $\Pi_I^*(\hat{s}, \hat{s}, 1)$, $\Pi_N^*(\hat{s}, \hat{s}, 1)$, $\Pi_S^*(\hat{s}, \hat{s})$, and Π_R admit closed-form solutions. Thus, we can solve for equilibria in closed form, illustrate Propositions 5 and 6, and obtain general comparative statics in α and ν.

For details of the solution, we refer the reader to Appendix B, where we show that

$$\Pi_I^*(\hat{s}, \hat{s}, 1) = \frac{(2\alpha - 1)^2}{2(3\alpha - 1)}\hat{s},\ \ \Pi_N^*(\hat{s}, \hat{s}, 1) = \frac{1-\alpha}{2}\left(\hat{s} - \hat{s}^2\right) + \frac{\alpha^2}{2(3\alpha - 1)}\hat{s}^2,\ \ \Pi_S^*(\hat{s}, \hat{s}) = \frac{\alpha}{2}\hat{s},\ \ \Pi_R = \frac{\alpha}{6}. \tag{21}$$

Plugging (21) into (19) yields a quadratic equation whose positive solution is

$$\hat{s}_0 = \frac{(3\alpha - 1)\left((1 + \nu + (1-\nu)\beta(1-\lambda))\alpha - 1 + \sqrt{(1 - (1 + \nu + (1-\nu)\beta(1-\lambda))\alpha)^2 + 4\frac{(2\alpha-1)^2}{3\alpha-1}\left(1 - \alpha + (1-\nu)\beta\lambda\frac{\alpha}{3}\right)}\right)}{2(2\alpha - 1)^2}. \tag{22}$$

If the parameters are such that $\hat{s}_0 \geq 1$, then only the nonresponsive equilibrium exists.

The next proposition studies the effects of the commonality of values α, and the probability of the seller's liquidity shock ν.

PROPOSITION 7 (Comparative statics): *Consider parameters for which the responsive equilibrium exists. Then:*

(a) A marginal increase in α strictly decreases $\hat{s}_0$.
(b) A marginal increase in ν strictly increases $\hat{s}_0$.

The probability of a bidder-initiated auction is increasing in α and decreasing in ν.

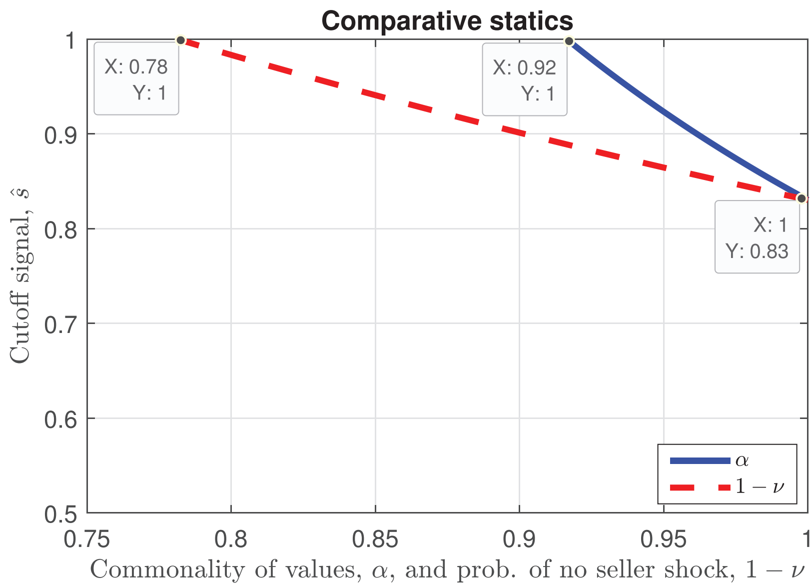

Figure 3. Comparative statics of the initiation cutoff signal. For values $v(s) = s$ and parameters $\beta = 0.9$ and $\lambda = 0.9$, the figure plots comparative statics of the period-0 responsive-equilibrium initiation cutoff signal $\hat{s}_0$ with respect to commonality of values α and the probability of no exogenous liquidity shocks, $1 - \nu$. The responsive equilibrium when $\alpha = 1$ and $\nu = 0$ is $\hat{s}_0 = 0.83$. The responsive equilibrium ceases to exist when $\alpha < 0.92$ and $\nu > 0.22$. (Color figure can be viewed at wileyonlinelibrary.com)

Proposition 7 generalizes Propositions 5 and 6 beyond existence results: the likelihood of a bidder-initiated sale monotonically decreases in the degree of commonality of potential buyers' values and in the likelihood of the seller's liquidity shock. Figure 3 illustrates how the initiation cutoff $\hat{s}$ changes with respect to α and ν when $v(s) = s$, $\beta = 0.9$, and $\lambda = 0.9$.

IV. Extensions

A. Generalization to Other Auction Formats

We assume that the sale proceeds as a first-price auction for two reasons. First, under regularity conditions, the first-price auction has a unique equilibrium even when signals of bidders are distributed asymmetrically. In contrast, the difficulty with other auction formats, such as the ascending (English) "clock" auction in which the price increases continuously until one bidder remains,[24] is that in the common-value setting, there exist multiple equilibria (see, e.g., Milgrom (1981b)). When bidders are symmetric, it is natural to focus on the equilibrium in which both bidders play identical strategies, which is

[24] When there are two bidders, the ascending "clock" auction and the second-price auction are equivalent.

typically unique. However, when bidders are asymmetric, as is the case in this paper, there is no clear way to select one equilibrium from many. Second, a first-price auction is the simplest format that highlights how bidders respond to the perceived aggressiveness of the rival. Such strategic considerations are common to a variety of auction formats but are often absent from the ascending "clock" auction. For example, if the ascending auction has jump bids (bids that lead to a discontinuous increase in the running price), which are common in takeover bidding, then its outcome will have properties of both the first-price and the second-price auctions in both the common-value setting (Avery (1998)) and the private-value setting (Liu (2012), Daniel and Hirshleifer (2018)). The practice of typical takeover auctions, described in Hansen (2001) and Boone and Mulherin (2007), is as follows. A few rounds of informal preliminary bids are followed by a final round of formal sealed bids, after which the merger agreement is signed. Sometimes, rival bidders submit bids even after the merger agreement is signed. This process has features of both first-price and ascending "clock" auctions. Like in an ascending auction, there are multiple rounds of bidding. But unlike an ascending "clock" auction, bid revisions occur in jumps, and the number of bidding rounds is usually very small. Furthermore, like in a first-price auction, there is often a final round in which bidders submit formal sealed bids.

In light of these practical concerns, it is important to explore whether the results of our analysis generalize beyond the first-price auction. In this section, we show that the main implication of the baseline model generalizes to a large class of auction formats, which covers first-price and (in the limit) second-price auctions, as well as intermediate formats. Specifically, consider the base model but assume that the auction format is a combination of first- and second-price auctions. Bidders simultaneously submit bids in a concealed fashion. The bidder with the highest bid wins the auction and pays the weighted average of her bid and the rival's bid with weights f and $1-f$, respectively, where $f \in (0,1]$ captures the proximity of the auction format to first price. If $f=1$, the format is identical to the first-price auction, and the model reduces to the base model. If $f \to 0$, the format is arbitrarily close to the second-price and ascending (English) auctions. We maintain Assumption 2 of the base model.

In Appendix A, we analyze this model in detail. When the auction format is not first-price ($f<1$), there is a possibility of multiple equilibria.[25] Because multiple equilibria can exist, a detailed analysis of equilibrium properties would require additional equilibrium selection. In the example below, we offer one selection criterion. However, the main implication of our base model holds regardless of the equilibrium selection. The next proposition shows that if the commonality of values is sufficiently high, then the equilibrium features

[25] A loose intuition for equilibrium uniqueness in the first-price auction and multiplicity in the auction with $f<1$ is as follows. In the first-price auction, "nonserious" bids of very low types of the noninitiating bidder play a limited role, as they never win and the initiating bidder pays her own bid. In contrast, here such bids play an important role, as they can affect the payment of the initiating bidder.

no bidder-initiated auctions, regardless of the equilibrium that is expected to be played in a bidder-initiated auction.

PROPOSITION 8 (Equilibrium with seller-initiated auctions when values are close to common: robustness to auction format): *Fix any* $f \in (0, 1)$. *There exists* $\tilde{\alpha} \in (\frac{1}{2}, 1)$ *such that for any* $\alpha < \tilde{\alpha}$, *only the nonresponsive equilibrium exists at* $t = 0$, *whereby the seller holds the auction immediately.*

The specific equilibrium in a bidder-initiated auction affects the payoff of the initiating bidder with signal $\hat{s}$, $\Pi_I^*(\hat{s}, \hat{s}, \bar{s})$. However, Proposition 8 shows that this payoff converges to zero as the common component of bidders' values becomes large ($\tilde{\alpha} \rightarrow \frac{1}{2}$) in any equilibrium. As a result, the initiating bidder with signal $\hat{s}$ prefers not to approach the seller for any cutoff $\hat{s}$, leading to the seller's rational response to initiate the auction himself.

Once multiple auction formats are allowed, it is natural to ask which format should optimally be chosen by a seller to maximize his revenues. The analysis of revenues depends on the specific equilibrium in a bidder-initiated auction and has to be performed numerically, as bidding strategies in the bidder-initiated auction do not have closed-form solutions for intermediate auction formats. The equilibrium selection criterion that we propose is that a nonserious bid by a noninitiating bidder with sufficiently low signal x is simply equal to her value, taking into account the fact that the initiating bidder's signal is at least $\hat{s}$: $a_N(x) = \alpha v(x) + (1 - \alpha)v(\hat{s}) \leq \underline{a}$.[26] Our numerical analysis shows that for reasonable model parameters, the seller prefers the first-price auction to all other formats. In fact, we were unable to find parameters for which the second-price format is preferred, and only identified a small subset of parameters for which an intermediate format is preferred. As an example, when $\alpha = 1$, $v(s) = s^{0.35}$, $\nu = 0$, $\beta = 0.936$, and $\lambda = 0.965$, the first-price format is preferred to the second-price format in both the responsive and nonresponsive equilibrium, and the optimal format in the responsive equilibrium has $f = 0.64$ (the seller is indifferent among formats in the nonresponsive equilibrium). The implication is that the optimal auction weighs heavily on features that make its format closer to the first-price format.

B. Costly Information Acquisition and Preemption

We assume that potential bidders always know their signals. As Fishman (1988) shows, if potential bidders need to acquire this information at a cost, in a sequential sale process, the first bidder may preempt the second potential bidder from acquiring information and participating in the auction. Because such a sale process only features a single participating bidder, it would typically be classified as a negotiation by empirical researchers. Although Fishman's model

[26] This criterion is reasonable when bidders, on the margin, prefer winning the auction to losing and when bidders may make mistakes in their bids with small probability. These two conditions allow noninitiating bidders with very low signals to win at their value with a vanishingly small probability.

has independent private values, it is natural to ask if this first-mover advantage is also relevant when values have a significant common component and if it can overturn the result of the base model that sales are seller-initiated when values are close to common.

To explore this question, we alter the base model to capture the key ingredients of Fishman (1988). We assume that initially at $t = 0$, one bidder knows her signal, while the other does not. At $t = 0$, the informed bidder sends message $m_{i,0} \in \{0, 1\}$, where $m_{i,0} = 1$ is interpreted as an indication of interest and $m_{i,0} = 0$ is interpreted as lack of such indication. After observing the message, the seller decides whether to put the asset up for sale; with probability ν, she is hit by the liquidity shock and has no choice but to sell the asset. In a responsive equilibrium, the informed bidder sends $m_{i,0} = 1$ if and only if her signal exceeds some cutoff $\hat{s}_0 \in (0, 1)$, and the seller sells voluntarily if and only if the informed bidder sends $m_{i,0} = 1$.

If the sale is bidder-initiated (i.e., if it is triggered by the informed bidder sending $m_{i,0} = 1$), it proceeds via a sequential format.[27] At the first stage, the informed bidder chooses bid b to submit.[28] Having observed this bid, the uninformed bidder decides whether to pay cost Ψ to observe her signal (if she does not already know it). If she chooses not to pay the cost, the informed bidder acquires the asset at price b in a single-bidder sale. In this case, the bid of the initiating bidder preempts the rival from participating. If the uninformed bidder chooses to pay the cost to learn her signal, the two bidders compete in the first-price auction with the initiating bidder's first-round bid b serving as the reservation price. This timing is similar to Fishman (1988), with the difference being that we assume that the second-round format is first-price, while Fishman assumes an English auction.[29] If the sale is seller-initiated (i.e., it is not triggered by the informed bidder sending $m_{i,0} = 1$), the uninformed bidder decides whether to pay cost Ψ to learn her signal, after which the two bidders also compete in the first-price auction. We assume that Ψ is not too high, so that the uninformed bidder prefers to learn her signal if she knows nothing about the signal of her rival beyond her prior. The other assumptions are as in the base model.

The next proposition shows that if values are close to common, then the equilibrium can only be nonresponsive, and thus, all sales are seller-initiated.

PROPOSITION 9 (Equilibrium with costly information acquisition and sequential bidding): *Suppose that $\Psi \leq \bar{\Psi}$, defined in the proof in Appendix A. There exists $\tilde{\alpha} \in (\frac{1}{2}, 1)$ such that for any $\alpha < \tilde{\alpha}$, only the nonresponsive equilibrium exists at $t = 0$, in which the seller holds the auction immediately.*

[27] Alternatively, one could assume that the informed bidder makes an initial bid and submits an indication of interest simultaneously.

[28] Since b is the choice of the informed bidder, it can depend on the signal of the informed bidder.

[29] As in the base model, the English auction format would result in equilibrium multiplicity with equilibrium selection being nonobvious due to asymmetric bidders. In Fishman (1988), multiplicity does not arise because of the restriction to independent private values.

The intuition is as follows. Consider a hypothetical responsive equilibrium in which the informed bidder sends $m_{i,0} = 1$ if and only if her signal exceeds some cutoff $\hat{s}_0$, and the seller puts the asset up for sale at $t = 0$ if either the informed bidder indicates her interest or the liquidity shock occurs. When values are close to common, the initiating bidder with cutoff signal $\hat{s}_0$ cannot successfully preempt the rival: the rival already knows that the signal of the initiating bidder is at least $\hat{s}_0$ and updates her value accordingly. Therefore, the argument of the base model applies: the initiating bidder with signal $\hat{s}_0$ obtains very small rents from indicating her interest and is better off not approaching the seller. Because the argument holds for any $\hat{s}_0$, the responsive equilibrium cannot exist, and the sale is initiated by the seller at $t = 0$. Thus, while the ability to preempt gives first-mover advantage to the initiating bidder when values are close to private, there is no such advantage to the initiating bidder with the cutoff signal when values are close to common. If one interprets an outcome in which the initiating bidder preempts the rival from participating as a "negotiation" (as opposed to an "auction"), Proposition 9 is consistent with a greater prevalence of negotiations among sales to strategic bidders than to financial bidders (Fidrmuc et al. (2012)).

C. A Model with a General Number of Periods

The base model is two-period, but its results extend to the multiperiod setting. To show this, we consider a $T + 1$-period model. The model setup is the same as in the base model. The only difference is that a signal-resetting shock arrives with probability λ in each period $t \geq 1$, in which case both bidders' values of the asset for sale change as described in the base model. Similarly to the two-period model, this extended model can be solved by backward induction. Because this analysis of the model is quite technical and the intuition behind it is similar to the two-period model, we refer the reader to the Internet Appendix for the details.

V. Additional Discussion

The base model provides a general theory of endogenous initiation of auctions, with auctions of companies being our lead application. In this section, we discuss additional features that are relevant in this application but missing from the base model.

A. Shareholder Activists as Facilitators of Auctions of Companies

Corollary 1 shows that if a company is underperforming (inducing common value of restructuring in potential bidders), its management is entrenched, and the likelihood of a liquidity shock is low, such a company can remain without a change in ownership, even though the value-added from the change is large. Thus, the role of takeovers as a corporate governance mechanism can be limited. The lack of an incentive for bidders to initiate auctions when the common component of their values is large gives rise to alternative ways of promoting takeovers, such as shareholder activism. An activist can find it beneficial to buy a fraction of the company's shares and undertake a campaign, which results in selling the firm, so seller-initiated takeovers can occur in equilibrium even in the presence of an entrenched management.[30] The model can be extended to capture this feature (see the Internet Appendix for a simple extension). The implication is that shareholder activism and the market for corporate control are not simply two alternative mechanisms for disciplining the management but rather complement each other: activists use the market for corporate control to facilitate transactions of targets, inefficiencies in which would not be corrected otherwise.

Thus, the paper points to a novel mechanism through which activism and the market for corporate control interact, and relates to several other recent papers that highlight other aspects of the interaction—Burkart and Lee (2022) focus on the free-rider problem in tender offers, and Corum and Levit (2019) focus on the commitment problem of the bidder in a proxy fight. Greenwood and Schor (2009), Jiang, Li, and Mei (2018), and Boyson, Gantchev, and Shivdasani (2017) provide empirical evidence on interactions between activism and the market for corporate control, which is broadly consistent with shareholder activists creating value in the M&A market.

B. Toeholds

When the common component of bidders' values is large, the initiating bidder with the cutoff signal can obtain a positive expected profit if she secretly acquires a toehold (a block of shares) in the target prior to the auction (see the Internet Appendix for a simple extension). Such a bidder can therefore find it optimal to indicate her interest to the company in the first place. Thus, while toeholds are often considered a source of inefficiency, they help bidders initiate positive-value deals that would not occur otherwise.

C. Investment Banks

The lack of indications of interest from bidders when values are close to common relies on the assumption that whether the auction is bidder- or seller-initiated is publicly known. While it is in the ex post interest of the seller to disclose the interest of the initiating bidder to rivals, such disclosure can be ex ante suboptimal for all parties combined as it impedes initiation. This inconsistency between ex ante and ex post objectives can create a role for an intermediary, such as an investment bank, to alleviate the lack-of-commitment problem. The investment bank can centralize communication among all participating parties, and, because it is a long-run player that

[30] This role of shareholder activism also applies to settings with significant private values if managerial entrenchment is so high that the management of the target prefers not to sell it even conditional on learning about a high value of the initiating bidder.

interacts with buyers and sellers over time across a range of services, it can incentivize the seller not to disclose the ex post beneficial but ex ante harmful (for all parties combined) information in the context of a single transaction.[31]

VI. Conclusion

In this paper, we theoretically examine endogenous initiation of auctions by potential buyers and the seller. Our model aims to capture many real-world environments in which initiation of an auction is a strategic choice. Our lead application is to auctions of companies and intercorporate asset sales. We show that approaching the seller signals that the initiating bidder is sufficiently optimistic about her value of the asset. This leads to two opposite effects, one of which reduces bidders' incentives to approach the seller, while the other improves them. The asset revaluation effect implies that the noninitiating bidder updates her value of the asset upwards, which reduces the surplus of the initiating bidder and discourages her from approaching the seller. The rival selection effect implies that not initiating the auction in hopes that the rival will initiate instead is costly for a bidder because it results in competition against a stronger rival. When bidders' values are common, only the first effect is present. When values are private, only the second effect is present.

The model establishes three main results about the equilibrium patterns of initiation. First, if the common component of bidders' values is sufficiently important, the asset revaluation effect dominates, so that no potential buyer indicates her interest to the seller and all auctions are seller-initiated. Second, if the private component of values is important, both bidder- and seller-initiated auctions can occur in equilibrium. At the same time, there is also an equilibrium in which all auctions are seller-initiated. Finally, if potential buyers expect the seller to be forced to sell his asset in the near future, no potential buyer indicates her interest and all auctions are seller-initiated, regardless of the commonality of values. We derive a set of testable implications relating the identity of the initiating party to bids and auction outcomes.

Several extensions of the paper could be interesting. First, it would be interesting to introduce private types of the seller, capturing the seller's value from not selling the company, which can be interpreted as the degree of private benefits of control. Such an extension would be able to capture the empirical regularity that sellers often reject some of the bidder-initiated offers and would potentially lead to richer implications. Second, it would be interesting to allow the seller to choose reserve prices in the auction. Our conjecture is that the outcome will depend on the ability of the seller to commit to reserve prices prior to receiving indications of interest from bidders. If the seller cannot commit to a reserve price prior to receiving bidders' messages but can commit to any reserve price thereafter, our conjecture is that bidders will not indicate their

[31] Hiding whether the auction is bidder- or seller-initiated gives positive rents to the initiating bidder with the cutoff signal because the rival believes that, with some probability, the auction is seller-initiated and the initiating bidder's signal is below the cutoff.

interest, bidder-initiated auctions will never happen, and the seller will put the asset up for sale in the initial period. In contrast, if the seller can commit to a reserve price prior to receiving indications of interest, we expect the equilibria to look conceptually similar to the equilibria of the base model. Intuitively, it is in the interest of the seller to commit to a lower reserve price than ex post optimal to preserve the rents of the marginal initiating bidder. However, preservation of these rents is only feasible if the private component of bidders' values is substantial, as otherwise, competition from the rival bidder erodes the payoff of the marginal initiating bidder. Third, in the context of the model with preemption in Section IV.B, it would be interesting to analyze the role of lockup clauses as means of transferring rents to the initiating bidder and thereby encouraging bidders to approach the target. Che and Lewis (2007) analyze lockups in a static model based on Fishman (1988). In future work, it would be interesting to incorporate them into a model of endogenous initiation. Fourth, it would be useful to consider bids in securities, such as a bidder's stock, and the interaction of securities used in a bid and initiation decisions. For example, it would be interesting to adopt the assumption of Liu and Bernhardt (2021) that the seller has stronger commitment power in seller-initiated relative to bidder-initiated auctions and develop a theory that makes joint predictions about who initiates takeovers and the means of payment. Fifth, the asset for sale could be made divisible: for example, bankrupt companies are often sold piecemeal in a liquidation auction. Finally, it would be interesting to consider multiple sellers and allow bidders to choose which asset to pursue and sellers to choose which bidders to invite to an auction in a dynamic model of matching.

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Appendix A: Proofs

PROOF OF LEMMA 1: First, we prove that an equilibrium must satisfy a system of two equations (3) for $\phi_I(b)$ and $\phi_N(b)$ with four boundary conditions (5) and $\underline{a}$ given by (4). To do so, we first prove that each bidder's strategy $a_i(s)$, $i \in \{I, N\}$ cannot have a jump in the range of bids that win with nonzero probability. By contradiction, suppose that there is a jump in $a_i(s)$ at some s' from a' to a''. Then, types s of the other bidder that bid between a' and a'' benefit from a deviation to bidding infinitesimally close to a': this deviation has no effect on the probability of winning and the value conditional on winning but discontinuously reduces the payment. However, in this case type, $s = a'' + \varepsilon$ of bidder i for an infinitesimal positive ε benefits from a deviation to bidding just above a' : this deviation has no effect on the probability of winning and the value conditional on winning but discontinuously reduces the payment. Second, we prove that each bidder's strategy $a_i(s)$, $i \in \{I, N\}$ must be strictly increasing in the range of bids that win with nonzero probability. By contradiction, suppose not, that is, $\Pr(a_i(s) = b) > 0$ for some b. Then, no type of bidder j wants to bid $b - \varepsilon$ for an infinitesimal ε: bidding b rather than

$b-\varepsilon$ leads to a discontinuous increase in the probability of winning, in the value conditional on winning, and only an infinitesimal increase in the price paid. However, this contradicts the result above that there is no jump in $a_i(s)$, $i \in \{I, N\}$ in the range of bids that win with nonzero probability. Therefore, $\phi_I(b)$ and $\phi_N(b)$ are continuous and strictly increasing. Optimality then implies that equations (3) are satisfied. The proof of the boundary conditions is in the main text with the exception of (4).

Let us prove (4). Because bid $\underline{a}$ must be optimal for the initiating bidder with signal $\hat{s}$,

$$\begin{aligned}&\phi_N(\underline{a})\big(\alpha v(\hat{s}) + (1-\alpha)\mathbb{E}\big[v(x)|x \le \phi_N(\underline{a})\big] - \underline{a}\big)\\ &\ge \phi_N(b)\,(\alpha v(\hat{s}) + (1-\alpha)\mathbb{E}\big[v(x)|x \le \phi_N(b)\big] - b) \quad \forall b.\end{aligned}$$

Because no noninitiating bidder with signal s bids above $\alpha v(s) + (1-\alpha)v(\hat{s})$, it must be the case that $\phi_N(b) \ge v^{-1}(\frac{b-(1-\alpha)v(\hat{s})}{\alpha})$. Using the fact that the right-hand side of the above inequality is a strictly increasing function of $\phi_N(b)$ and the fact that $\phi_N(\underline{a})$ is described by (5), the inequality implies

$$\begin{aligned}&v^{-1}\left(\frac{\underline{a}-(1-\alpha)v(\hat{s})}{\alpha}\right)\left(\alpha v(\hat{s}) + (1-\alpha)\mathbb{E}\left[v(x)|x \le v^{-1}\left(\frac{\underline{a}-(1-\alpha)v(\hat{s})}{\alpha}\right)\right] - \underline{a}\right)\\ &\ge v^{-1}\left(\frac{b-(1-\alpha)v(\hat{s})}{\alpha}\right)\left(\alpha v(\hat{s}) + (1-\alpha)\mathbb{E}\left[v(x)|x \le v^{-1}\left(\frac{b-(1-\alpha)v(\hat{s})}{\alpha}\right)\right] - b\right) \forall b,\end{aligned}$$

which implies (4). □

Second, we prove existence of equilibrium. For this, we use the results of Athey (2001). To satisfy her assumptions, limit the action (bid) space to $[0, v(\bar{s})]$. Because no bidder bids above her expected value by Assumption 2, this restriction is without loss of generality. The problem also satisfies Assumption A1 in Athey (2001) (bounded and atomless-type distribution). Next, we verify the single-crossing condition (Definition 3 in Athey (2001)): whenever an opponent uses an increasing strategy ($a_j(s_j)$ increasing in s_j), player i's objective function $\Pi_i(b_i, s_i)$ satisfies SCP-IR in (b_i, s_i). The expected payoffs of bidders are

$$\Pi_I(b, s) = \int_0^{\phi_N(b)} \big(\alpha v(s) + (1-\alpha)v(x) - b\big)\frac{1}{\hat{s}}dx, \tag{A.1}$$

$$\Pi_N(b, s) = \int_{\hat{s}}^{\phi_I(b)} \big(\alpha v(s) + (1-\alpha)v(x) - b\big)\frac{1}{\bar{s}-\hat{s}}dx. \tag{A.2}$$

Their cross-partial derivatives in b and s are

$$\frac{\partial \Pi_I}{\partial b \partial s}(b, s) = \frac{\alpha v'(s)}{\hat{s}}\frac{\partial \phi_N(b)}{\partial b} \ge 0, \quad \frac{\partial \Pi_N}{\partial b \partial s}(b, s) = \frac{\alpha v'(s)}{\bar{s}-\hat{s}}\frac{\partial \phi_I(b)}{\partial b} \ge 0.$$

Therefore, the problem satisfies the single-crossing condition. The payoffs also satisfy Assumptions A2 and A3 (A3 follows from Theorem 7, Part 1). By Theorem 6 in Athey (2001), we conclude that there exists an equilibrium in nondecreasing strategies. Because equilibrium bidding strategies cannot have flat regions, as shown above, there exists an equilibrium in increasing strategies.

Finally, we prove uniqueness of the solution to (3). Let $v(\phi_j(b)) \equiv v_j(b)$ for $j \in \{I, N\}$. Then system (3) can be rewritten as

$$\frac{d}{db}\ln\left(v_j^{-1}(v_k(b)) - \underline{s}_j\right) = \frac{1}{\alpha v_k(b) + (1-\alpha)v_j(b) - b},$$

and boundary conditions (5) can be rewritten as $v_I(\bar{a}) = v(\bar{s})$, $v_N(\bar{a}) = v(\hat{s})$, $v_I(\underline{a}) = v(\hat{s})$, and $v_N(\underline{a}) = \frac{\underline{a}-(1-\alpha)v(\hat{s})}{\alpha}$. This system of equations satisfies the standard assumptions of the theory of differential equations, and thus, admits a unique solution. The specific argument is the same as in Lebrun (2006).

PROOF OF LEMMA 2: The derivation of the equilibrium is in the main text. Uniqueness is established in, for example, Lebrun (2006). □

PROOF OF PROPOSITION 1: First, compare $\Pi_S^*(\hat{s}, \hat{s})$ and $\Pi_I^*(\hat{s}, \hat{s}, \bar{s})$. Consider two cases. Case 1 is $a_S(\hat{s}) \le \underline{a}$, where we omit the dependence of $\underline{a}$ on $\hat{s}$ and $\bar{s}$ for brevity. Then,

$$\begin{aligned}
\Pi_S^*(\hat{s}, \hat{s}) &\ge \alpha v(\hat{s}) + (1-\alpha)\mathbb{E}[v(x)|x \le \hat{s}] - \underline{a} \\
&> \alpha v(\hat{s}) + (1-\alpha)\mathbb{E}\left[v(x)|x \le v^{-1}\left(\frac{\underline{a}-(1-\alpha)v(\hat{s})}{\alpha}\right)\right] - \underline{a} \\
&\ge \frac{\phi_N(\underline{a})}{\hat{s}}\left(\alpha v(\hat{s}) + (1-\alpha)\mathbb{E}\left[v(x)|x \le v^{-1}\left(\frac{\underline{a}-(1-\alpha)v(\hat{s})}{\alpha}\right)\right] - \underline{a}\right) \\
&= \Pi_I^*(\hat{s}, \hat{s}, \bar{s}),
\end{aligned}$$

where the second inequality follows from $v(\hat{s}) \ge \underline{a}$, which, in turn, follows from the optimality of type $\hat{s}$ of the initiating bidder bidding $\underline{a}$. The inequality is strict if $\hat{s} > 0$. The last inequality follows from $\phi_N(\underline{a}) \le \hat{s}$. Case 2 is $a_S(\hat{s}) > \underline{a}$. Then,

$$\begin{aligned}
\Pi_S^*(\hat{s}, \hat{s}) &> \frac{\phi_S(\underline{a})}{\hat{s}}\left(\alpha v(\hat{s}) + (1-\alpha)\mathbb{E}\left[v(x)|x \le \phi_S(\underline{a})\right] - \underline{a}\right) \\
&> \frac{1}{\hat{s}}v^{-1}\left(\frac{\underline{a}-(1-\alpha)v(\hat{s})}{\alpha}\right)\left(\alpha v(\hat{s}) + (1-\alpha)\mathbb{E}\left[v(x)|x \le v^{-1}\left(\frac{\underline{a}-(1-\alpha)v(\hat{s})}{\alpha}\right)\right] - \underline{a}\right) \\
&= \Pi_I^*(\hat{s}, \hat{s}, \bar{s}).
\end{aligned}$$

The first inequality follows from the fact that a bidder with signal $\hat{s}$ must prefer bidding $a_S(\hat{s})$ over bidding $\underline{a}$. The second inequality is implied by $\phi_S(\underline{a}) > v^{-1}(\frac{\underline{a}-(1-\alpha)v(\hat{s})}{\alpha})$. Therefore, $\Pi_S^*(\hat{s}, \hat{s}) > \Pi_I^*(\hat{s}, \hat{s}, \bar{s})$.

Second, compare $\Pi_I^*(\hat{s},\hat{s},\bar{s})$ and $\Pi_N^*(\hat{s},\hat{s},\bar{s})$. The payoff of type $\hat{s}$ of the initiating bidder solves

$$\begin{aligned}\Pi_I^*(\hat{s},\hat{s},\bar{s}) &= \max_b \frac{1}{\hat{s}} v^{-1}\left(\frac{b-(1-\alpha)v(\hat{s})}{\alpha}\right)\left(\alpha v(\hat{s}) + (1-\alpha)\mathbb{E}\left[v(x)|x \le v^{-1}\left(\frac{b-(1-\alpha)v(\hat{s})}{\alpha}\right)\right] - b\right) \\ &= \max_y \frac{v^{-1}(y)}{\hat{s}}\left((2\alpha-1)v(\hat{s}) + (1-\alpha)\mathbb{E}\left[v(x)|x \le v^{-1}(y)\right] - \alpha y\right),\end{aligned}$$

where $y \equiv \frac{b-(1-\alpha)v(\hat{s})}{\alpha}$. Therefore,

$$\Pi_I^*(\hat{s},\hat{s},\bar{s}) \le (2\alpha-1)\max_y \frac{v^{-1}(y)}{\hat{s}}(v(\hat{s})-y) = (2\alpha-1)\max_x \frac{x}{\hat{s}}(v(\hat{s})-v(x)). \quad \text{(A.3)}$$

It immediately follows that as $\alpha \to \frac{1}{2}$, $\Pi_I^*(\hat{s},\hat{s},\bar{s})$ converges to zero. Next, the payoff of type $\hat{s}$ of the noninitiating bidder solves

$$\begin{aligned}\Pi_N^*(\hat{s},\hat{s},\bar{s}) &= \alpha v(\hat{s}) + (1-\alpha)\mathbb{E}[v(x)|x \le \hat{s}] - \bar{a} \\ &= \max_b \frac{\phi_N(b,\hat{s}) - \hat{s}}{\bar{s}-\hat{s}}\left(\alpha v(\hat{s}) + (1-\alpha)\mathbb{E}\left[v(x)|x \in \left[\hat{s}, \phi_N(b,\hat{s})\right]\right] - b\right).\end{aligned}$$

Because $\Pi_I^*(\hat{s},\hat{s},\bar{s})$ converges to zero as $\alpha \to \frac{1}{2}$, but $\Pi_N^*(\hat{s},\hat{s},\bar{s})$ does not, $\Pi_N^*(\hat{s},\hat{s},\bar{s}) > \Pi_I^*(\hat{s},\hat{s},\bar{s})$ for any α sufficiently close to $\frac{1}{2}$.

Finally, compare $\Pi_I^*(\hat{s},\hat{s},\bar{s})$ and $\Pi_N^*(\hat{s},\hat{s},\bar{s})$ for $\alpha = 1$:

$$\begin{aligned}\Pi_I^*(\hat{s},\hat{s},\bar{s}) &= \frac{\phi_N(\underline{a})}{\hat{s}}\left(v(\hat{s}) - \underline{a}\right) = \max_b \frac{\phi_N(b)}{\hat{s}}\left(v(\hat{s}) - b\right) \\ &> \frac{\phi_N(\bar{a})}{\hat{s}}(v(\hat{s}) - \bar{a}) = v(\hat{s}) - \bar{a} = \Pi_N^*(\hat{s},\hat{s},\bar{s}).\end{aligned}$$

Therefore, $\Pi_I^*(\hat{s},\hat{s},\bar{s}) > \Pi_N^*(\hat{s},\hat{s},\bar{s})$ for $\alpha = 1$. By continuity of $\Pi_I^*(\hat{s},\hat{s},\bar{s})$ and $\Pi_N^*(\hat{s},\hat{s},\bar{s})$ in α, $\Pi_I^*(\hat{s},\hat{s},\bar{s}) > \Pi_N^*(\hat{s},\hat{s},\bar{s})$ for any α sufficiently close to one. □

PROOF OF PROPOSITION 2: As before, we omit the dependence of all functions on $\hat{s}$ for brevity. Type $\hat{s}$ of the noninitiating bidder bids $\bar{a}$. Type $\hat{s}$ of the initiating bidder bids $\underline{a}$. Because $\bar{a} > \underline{a}$, $a_N(\hat{s}) > a_I(\hat{s})$, proving part (a) of the proposition.

To prove part (b), let $H_k(b)$ and $h_k(b)$ denote the cumulative distribution function (c.d.f.) and probability density function (p.d.f.), respectively, of bids of type $k \in \{I, N\}$. Note that $H_k(b) = \frac{\phi_k(b)-\underline{s}_k}{\bar{s}_k-\underline{s}_k}$, and therefore, $h_k(b) = \frac{1}{\bar{s}_k-\underline{s}_k}\frac{\partial \phi_k(b)}{\partial b}$. Using the equilibrium condition for optimality of bids, (3),

$$\frac{H_I(b)}{h_I(b)} = \alpha v\left(\phi_N(b)\right) + (1-\alpha)v\left(\phi_I(b)\right) - b < \alpha v\left(\phi_I(b)\right) + (1-\alpha)v\left(\phi_N(b)\right) - b = \frac{H_N(b)}{h_N(b)},$$

where the inequality follows from $\phi_I(b) > \phi_N(b)$ and $\alpha > 1-\alpha$. Inequality $\phi_I(b) > \phi_N(b)$ holds because $\phi_I(b) \ge \hat{s} > \phi_N(b)$ for any $b < \bar{a}$ and $\bar{s} = \phi_I(b) >$

$\hat{s} = \phi_N(b)$ for $b = \bar{a}$. Therefore, the distribution of bids of the initiating bidder dominates the distribution of bids of the noninitiating bidder in terms of the reverse hazard rate. In turn, dominance in terms of the reverse hazard rate implies first-order stochastic dominance (e.g., Krishna (2010)). Therefore, $\mathbb{E}[a_I(s)|s \in [\hat{s}, \bar{s}]] > \mathbb{E}[a_N(s)|s \leq \hat{s}]$.

To prove part (c), let $\tilde{s}$ denote the type of the noninitiating bidder who submits the lowest serious bid: $a_N(\tilde{s}) = \underline{a}$. In the seller-initiated auction, this type bids

$$a_S(\tilde{s}) = \mathbb{E}[v(x)|x \leq \tilde{s}] < v(\tilde{s}) < \alpha v(\tilde{s}) + (1-\alpha)v(\hat{s}) = \underline{a}.$$

Therefore, type $\tilde{s}$ bids less aggressively in the seller-initiated auction: $a_S(\tilde{s}) < a_N(\tilde{s})$. We next prove that $a_S(s) < a_N(s)$ $\forall s \in (\tilde{s}, \hat{s}]$ by contradiction. Suppose that there exists type z for which $a_S(z) \geq a_N(z)$. Because $a_S(\tilde{s}) < a_N(\tilde{s})$ and both $a_S(\tilde{s})$ and $a_N(\tilde{s})$ are continuous, there exists type $z' \in (\tilde{s}, z)$ for which $a_S(z') = a_N(z')$. Pick the lowest such z'. Then, because $a_S(\tilde{s}) < a_N(\tilde{s})$, $a_N(s)$ crosses $a_S(s)$ from above at $s = z'$. Using (3) and (9) and $\phi_I(b) > \hat{s} > z'$ for any $b \in [\underline{a}, \bar{a}]$,

$$\frac{\partial a_N(s)}{\partial s}\Big|_{s=z'} = \frac{\alpha v(\phi_I(a_N(s))) + (1-\alpha)v\big(z'\big) - a_N\big(z'\big)}{z'} > \frac{v\big(z'\big) - a_S\big(z'\big)}{z'} = \frac{\partial a_S(s)}{\partial s}\Big|_{s=z'}.$$

Therefore, at any intersection point, $a_N(s)$ must cross $a_S(s)$ from below, which is a contradiction. Therefore, $a_S(s) < a_N(s)$ $\forall s \in [\tilde{s}, \hat{s}]$.

Finally, part (d) trivially follows from (i) the fact that the distribution of the initiating bidder's signal is uniform over $[\hat{s}, \bar{s}]$, while the distribution of the noninitiating bidder's signal and each bidder's signal in the seller-initiated auction is uniform over $[0, \hat{s}]$, and (ii) the fact that, conditional on winning, the expected noninitiating bidder's signal exceeds the expected winning bidder's signal in the seller-initiated auction. □

PROOF OF PROPOSITION 3: The proposition follows from the following two observations. The first observation is that the strategy profile described in the proposition constitutes an equilibrium. Indeed, because the seller is expected to auction the asset off for any set of messages, each bidder finds it optimal to send $m_{i,1} = 0$. In addition, because the game ends at $t = 1$, it is optimal for the seller to auction the asset off for any set of messages he receives. The second observation is that there is no other equilibrium. By contradiction, suppose that there exists $\hat{s}_1 \in (0, \bar{s})$, such that bidder i sends message $m_{i,1} = 1\{s_i \geq \hat{s}_1\}$. Because the game ends at $t = 1$, it is optimal for the seller to auction the asset off for any pair of messages he receives, including $(0, 0)$. By Proposition 1, $\Pi^*_S(\hat{s}_1, \hat{s}_1) > \Pi^*_I(\hat{s}_1, \hat{s}_1, \bar{s})$ and $\Pi^*_N(\hat{s}_1, \hat{s}_1, \bar{s}) > \Pi^*_D(\hat{s}, \hat{s}, \bar{s}) = 0$. Consider a bidder with signal $\hat{s}_1$: because the seller's decision to auction the asset off is the same but the bidder's auction payoff is strictly higher if she sends message $m_{i,1} = 0$, the bidder (and any bidder with the signal just above $\hat{s}_1$) is better off deviating to sending message $m_{i,1} = 0$. Hence, the equilibrium described in the proposition is unique. □

PROOF OF PROPOSITION 4: First, we prove that the nonresponsive equilibrium always exists by showing that neither bidders nor the seller benefit from any deviation from the equilibrium strategies. A bidder does not benefit from sending message $m_{i,0} = 0$because the seller's reaction does not change: the seller will hold the auction. The seller does not benefit from delaying the auction until $t = 1$because the distribution of values of bidders will not change (bidders will have the same signals if there is no shock at $t = 1$, or will draw new signals from the same distribution if there is a shock at $t = 1$) and the payoff will be discounted.

Second, we solve for the responsive equilibria. We proceed in the following three steps. □

Step 1: Any responsive equilibrium must have the seller's reaction described by equation (18).

By symmetry, the seller's reaction must be the same for $\mathbf{m}_0 = (1, 0)$ and $\mathbf{m}_0 = (0, 1)$. Thus, step 1 means that there cannot be a responsive equilibrium in which the seller's reaction is

$$d_0(\mathbf{m}_0) = \begin{cases} 1, & \text{if } \mathbf{m}_0 = (1, 1), \\ 0, & \text{if } \mathbf{m}_0 \neq (1, 1). \end{cases}$$

By contradiction, suppose that such an equilibrium exists with some cutoff $\hat{s}_0 \in (0, 1)$. Consider bidder i with signal $\hat{s}_0$. Suppose she sends message $m_{i,0} = 1$. With probability $1 - \hat{s}_0$, the rival also sends $m_{-i,0} = 1$. Because conditional on this event, the rival's signal s_{-i} is distributed uniformly over $[\hat{s}_0, 1]$, and bidder i loses the auction with certainty and obtains the payoff of zero. With probability $\hat{s}_0$, the rival sends $m_{-i,0} = 0$. In this case, the seller waits until the next period, so the bidder's payoff becomes $\beta((1 - \lambda)\Pi_I^*(\hat{s}_0, \hat{s}_0, 1) + \lambda\Pi_R)$. Hence, the bidder's expected payoff from message $m_{i,0} = 1$ is

$$\hat{s}_0\beta\big((1 - \lambda)\Pi_I^*(\hat{s}_0, \hat{s}_0, 1) + \lambda\Pi_R\big). \tag{A.4}$$

Suppose instead that, bidder i with signal $\hat{s}_0$sends message $m_{i,0} = 0$. In this case, the auction will take place at $t = 1$ with certainty. The bidder's payoff will be

$$\beta\big((1 - \hat{s}_0)(1 - \lambda)\Pi_N^*(\hat{s}_0, \hat{s}_0, 1) + \hat{s}_0(1 - \lambda)\Pi_S^*(\hat{s}_0, \hat{s}_0) + \lambda\Pi_R\big). \tag{A.5}$$

By Proposition 1, $\Pi_S^*(\hat{s}_0, \hat{s}_0) > \Pi_I^*(\hat{s}_0, \hat{s}_0, 1)$. Therefore, (A.5) is strictly higher than (A.4). Thus, bidders with signals at or just above $\hat{s}_0$ are better off deviating to sending message $m_{i,0} = 0$. We have reached a contradiction.

Step 2: Optimality conditions for the bidder.

The indifference condition (19) for the cutoff signal $\hat{s}_0$ was derived in the main text. It remains to show that if a bidder with signal $\hat{s}_0$ is indifferent between sending messages $m_{i,0} = 0$ and $m_{i,0} = 1$, then any bidder with signal $s < \hat{s}_0$ prefers to send message $m_{i,0} = 0$ and any bidder with signal $s > \hat{s}_0$

prefers to send message $m_{i,0} = 1$:

$$\Pi_I^*(s, \hat{s}_0, 1) \geq \left(\frac{1}{\hat{s}_0} - 1\right)\big(\Pi_N^*(s, \hat{s}_0, 1) - \Pi_D^*(s, \hat{s}_0, 1)\big) + \nu \Pi_S^*(s, \hat{s}_0) + (1-\nu)\beta\big((1-\lambda)\Pi_S^*(s, \hat{s}_0) + \lambda \Pi_R\big) \text{ for any } s > \hat{s}_0, \tag{A.6}$$

$$\Pi_I^*(s, \hat{s}_0, 1) \geq \left(\frac{1}{\hat{s}_0} - 1\right)\Pi_N^*(s, \hat{s}_0, 1) + \nu \Pi_S^*(s, \hat{s}_0) + (1-\nu)\beta\big((1-\lambda)\Pi_S^*(s, \hat{s}_0) + \lambda \Pi_R\big) \text{ for any } s < \hat{s}_0. \tag{A.7}$$

Note that $\Pi_D^*(s, \hat{s}_0, 1) = 0$ for all $s \leq \hat{s}_0$. Because equality is achieved at $s = \hat{s}_0$, it is sufficient to prove

$$\frac{\partial}{\partial s}\Pi_I^*(s, \hat{s}_0, 1) \geq \left(\frac{1}{\hat{s}_0} - 1\right)\left(\frac{\partial}{\partial s}\Pi_N^*(s, \hat{s}_0, 1) - \frac{\partial}{\partial s}\Pi_D^*(s, \hat{s}_0, 1)\right) + \big(\nu + (1-\nu)\beta(1-\lambda)\big)\frac{\partial}{\partial s}\Pi_S^*(s, \hat{s}) \text{ for all } s. \tag{A.8}$$

First, consider $s > \hat{s}_0$. In this range,

$$\Pi_N^*(s, \hat{s}_0, 1) = \alpha v(s) + (1-\alpha)\mathbb{E}[v(x)|x \in [\hat{s}_0, 1]] - \bar{a}(\hat{s}_0) \ \Rightarrow \frac{\partial}{\partial s}\Pi_N^*(s, \hat{s}_0, 1) = \alpha v'(s),$$

$$\Pi_S^*(s, \hat{s}) = \alpha v(s) + (1-\alpha)\mathbb{E}[v(x)|x \leq \hat{s}_0] - a_S(\hat{s}_0, \hat{s}_0) \Rightarrow \frac{\partial}{\partial s}\Pi_S^*(s, \hat{s}_0) = \alpha v'(s),$$

$$\Pi_D^*(s, \hat{s}_0, 1) = \frac{s - \hat{s}_0}{1 - \hat{s}_0}\alpha\big(v(s) - \mathbb{E}\big[v(x)|x \in [\hat{s}, s]\big]\big) \Rightarrow \frac{\partial}{\partial s}\Pi_D^*(s, \hat{s}_0, 1) = \alpha v'(s)\frac{s - \hat{s}_0}{1 - \hat{s}_0}.$$

Thus, the derivative of the right-hand side of (A.6) in s is

$$\left(\frac{1-s}{\hat{s}_0} + \nu + (1-\nu)\beta(1-\lambda)\right)\alpha v'(s).$$

To calculate the derivative of the left-hand side of (A.6), apply the envelope theorem to

$$\Pi_I^*(s, \hat{s}_0, 1) \ = \ \max_b \left\{\frac{\phi_N\big(b, \hat{s}_0, 1\big)}{\hat{s}_0}\big(\alpha v(s) + (1-\alpha)\mathbb{E}\big[v(x)|x \leq \phi_N\big(b, \hat{s}_0, 1\big)\big] - b\big)\right\}$$
$$\Rightarrow \frac{\partial}{\partial s}\Pi_I^*(s, \hat{s}) = \alpha v'(s)\frac{\phi_N(a_I(s, \hat{s}_0, 1), \hat{s}_0, 1)}{\hat{s}_0}. \tag{A.9}$$

Thus, (A.8) is equivalent to

$$\frac{\phi_N(a_I(s, \hat{s}_0, 1), \hat{s}_0, 1)}{\hat{s}_0} \geq \frac{1-s}{\hat{s}_0} + \nu + (1-\nu)\beta(1-\lambda). \tag{A.10}$$

Because the left-hand side is strictly increasing in $s \in [\hat{s}_0, 1]$ and the right-hand side is strictly decreasing in s, it is sufficient to verify the inequality for $s \downarrow \hat{s}_0$, in which case it reduces to

$$\nu < 1 - \frac{1 - v^{-1}\left(\frac{\underline{a}(\hat{s}_0, 1) - (1-\alpha)v(\hat{s}_0)}{\alpha}\right)}{\hat{s}_0\big(1 - \beta(1-\lambda)\big)}, \tag{A.11}$$

where we used the final initial value condition in (5) and rearranged (A.10) for $s \downarrow \hat{s}_0$. Next, consider $s < \hat{s}_0$. In this case, condition (A.7) must be satisfied in equilibrium. Because nonserious bids of low types of the noninitiating bidder are not uniquely pinned down in equilibrium, there can be multiple equilibrium values of $\Pi_I^*(s, \hat{s}_0, 1)$ for $s < \hat{s}_0$. However, it is necessary and sufficient to verify (A.7) for a specific value of $\Pi_I^*(s, \hat{s}_0, 1)$:

$$\begin{aligned}\tilde{\Pi}_I^*(s, \hat{s}_0, 1) \leq \left(\frac{1}{\hat{s}_0} - 1\right)\Pi_N^*(s, \hat{s}_0, 1) + \nu\Pi_S^*(s, \hat{s}_0) \\ + (1-\nu)\beta\big((1-\lambda)\Pi_S^*(s, \hat{s}) + \lambda\Pi_R\big) \text{ for all } s < \hat{s}_0,\end{aligned} \tag{A.12}$$

where

$$\begin{aligned}\tilde{\Pi}_I^*(s, \hat{s}_0, 1) \equiv \max\Bigg\{0, \max_{b \geq (1-\alpha)v(\hat{s}_0)} \Bigg\{ v^{-1}\left(\frac{b - (1-\alpha)v(\hat{s}_0)}{\alpha}\right) \\ \times \left(\alpha v(s) + (1-\alpha)\mathbb{E}\left[v(x) | x \leq v^{-1}\left(\frac{b-(1-\alpha)v(\hat{s}_0)}{\alpha}\right)\right] - b\right)\Bigg\}\Bigg\}.\end{aligned} \tag{A.13}$$

Intuitively, this expression is the payoff of the bidder with signal $s < \hat{s}_0$ from deviating and indicating the interest under the assumption that the noninitiating bidder with a low signal s bids $\alpha v(s) + (1-\alpha)v(\hat{s}_0)$. Thus, this expression is the lowest possible payoff of the deviating bidder. If (A.12) holds, then there exist equilibrium bids of the noninitiating bidder with a low signal that satisfy (A.7): for example, the equilibrium bids of the noninitiating bidder with a low signal that win with probability zero could be $\alpha v(s) + (1-\alpha)v(\hat{s}_0)$. Of course, there can be other nonserious bids of the noninitiating bidder (which imply higher values of $\Pi_I^*(s, \hat{s}_0)$ for $s < \hat{s}_0$) that also satisfy (A.7). In contrast, if (A.12) does not hold for some $s < \hat{s}_0$, then it cannot hold for any equilibrium $\Pi_I^*(s, \hat{s}_0, 1)$ for the same s. Note that (A.12) trivially holds for sufficiently small values of s (as $\tilde{\Pi}_I^*(s, \hat{s}_0, 1) = 0$ in this case) and holds for $s \uparrow \hat{s}_0$ (it is implied by (A.11) in this case). However, we were unable to prove that these results imply that (A.12) necessarily also holds for intermediate values of s, so it needs to be verified numerically.

Step 3: Optimality condition for the seller.

Finally, in the responsive equilibrium, the seller must have incentives to hold an auction if he gets at least one indication of interest, and wait until $t = 1$ if he gets no indications of interest. We need to consider three cases: (i) the seller deviates to not holding the auction upon receiving $\mathbf{m}_0 = (1, 0)$, (ii) the seller deviates to not holding the auction upon receiving $\mathbf{m}_0 = (1, 1)$, and (iii) the seller deviates to holding the auction upon receiving $\mathbf{m}_0 = (0, 0)$. By symmetry, the case of $\mathbf{m}_0 = (0, 1)$ is identical to the case of $\mathbf{m}_0 = (1, 0)$.

Consider the case of $\mathbf{m}_0 = (1, 0)$. If the seller holds an auction, he obtains an expected payoff of $R_B(\hat{s}_0, 1)$. If the seller does not hold an auction, the game proceeds to $t = 1$, when the seller-initiated auction occurs with certainty by the argument identical to Proposition 3. The seller's discounted payoff from not holding the auction at $t = 0$ is therefore $\beta((1 - \lambda)R_S(\hat{s}_0) + \lambda R_S(1))$, where $R_S(\hat{s})$ are the expected revenues of the seller from the seller-initiated auction given by (13). Thus, the seller finds it optimal to hold an auction upon receiving $\mathbf{m}_0 = (1, 0)$ if and only if

$$R_B(\hat{s}_0, 1) \geq \beta((1 - \lambda)R_S(\hat{s}_0) + \lambda R_S(1)). \tag{A.14}$$

Because $R_B(\hat{s}_0, 1) > R_S(\hat{s}_0)$ and $\beta < 1$, (A.14) holds for $\hat{s}_0$ sufficiently close to one, but may be violated for low values of $\hat{s}_0$. As can be seen, it always holds for λ sufficiently close to zero.

Next, consider the case of $\mathbf{m}_0 = (1, 1)$. The expected revenues of the seller from immediate initiation of the auction are $R_D(\hat{s}_0) = \mathbb{E}[v(\min(s_1, s_2))|s_i \in [\hat{s}_0, 1], i \in \{1, 2\}]$. If the seller does not hold an auction, the game proceeds to $t = 1$, when the seller-initiated auction occurs with certainty by the argument identical to Proposition 3. The seller's discounted payoff from this option is $\beta((1 - \lambda)R_D(\hat{s}_0) + \lambda R_S(1))$. Because $R_D(\hat{s}_0) > R_S(1)$ and $\beta < 1$, $R_D(\hat{s})$ exceeds $\beta((1 - \lambda)R_D(\hat{s}_0) + \lambda R_S(1))$. Thus, the seller always finds it optimal to hold the auction upon receiving two indications of interest.

Finally, consider the case of $\mathbf{m}_0 = (0, 0)$. If the seller waits until $t = 1$, his expected payoff would be $\beta((1 - \lambda)R_S(\hat{s}_0) + \lambda R_S(1))$. If the seller holds the auction immediately, his expected revenues would be $R_S(\hat{s}_0)$. Thus, the seller finds it optimal to wait if and only if

$$R_S(\hat{s}_0) \leq \beta((1 - \lambda)R_S(\hat{s}_0) + \lambda R_S(1)).$$

Combining with (A.14), we obtain (20).

PROOF OF PROPOSITION 5: Consider a single bidder-initiated auction. The payoff of the initiating bidder with signal $\hat{s}_0$ is $\Pi_I^*(\hat{s}_0, \hat{s}_0, 1) \leq (2\alpha - 1)\max_x \frac{x}{\hat{s}_0}(v(\hat{s}_0) - v(x))$, as shown by (A.3) in the proof of Proposition 1. Note that $\max_x \frac{x}{\hat{s}_0}(v(\hat{s}_0) - v(x))$ is the profit of the bidder with signal $\hat{s}_0$ in the first-price auction with pure private values, in which the rival bids according to schedule $v(x)$ for x distributed uniformly over interval $[0, \hat{s}_0]$. Suppose instead that, the rival with signal x bids $\mathbb{E}[v(s)|s \leq x]$. Because the bid of the rival under this schedule is lower than $v(x)$ for any realization of x, the payoff of the bidder under consideration goes up. Therefore,

$$\max_x \frac{x}{\hat{s}_0}(v(\hat{s}_0)-v(x)) \leq \max_x \frac{x}{\hat{s}_0}\big(v(\hat{s}_0)-\mathbb{E}\big[v(s)|s\leq x\big]\big) = v(\hat{s}_0)-\mathbb{E}[v(s)|s\leq \hat{s}_0],$$

where the equality comes from the fact that $\mathbb{E}[v(s)|s \leq \hat{s}_0]$ is the optimal bid for the bidder with signal $\hat{s}_0$ in the symmetric first-price auction with pure private values (see Section II.B).

Consider equation (19) for the cutoff signal $\hat{s}_0$. For the left-hand side and the right-hand side of this equation, we have

$$\begin{aligned} LHS &\leq (2\alpha-1)(v(\hat{s}_0)-\mathbb{E}[v(s)|s\leq \hat{s}_0]),\\ RHS &> \alpha(\nu+(1-\nu)\beta(1-\lambda))(v(\hat{s}_0)-\mathbb{E}[v(s)|s\leq \hat{s}_0]). \end{aligned}$$

Therefore, a sufficient condition for there to be no responsive equilibrium is

$$2\alpha-1 \leq \big(\nu+(1-\nu)\beta(1-\lambda)\big)\alpha \quad \Rightarrow \quad \alpha \leq \hat{\alpha} \equiv \frac{1}{2-\nu-(1-\nu)\beta(1-\lambda)}.$$

Note that $\hat{\alpha} > \frac{1}{2}$. □

PROOF OF PROPOSITION 6: From the proof of Proposition 5, a sufficient condition for there to be no responsive equilibrium is $\alpha \leq \frac{1}{2-\nu-(1-\nu)\beta(1-\lambda)}$. We can alternatively rewrite a sufficient condition as

$$\nu \geq \hat{\nu} \equiv 1-\frac{1-\alpha}{\alpha\big(1-\beta(1-\lambda)\big)},$$

which proves the proposition. □

PROOF OF COROLLARY 1: If $C > R_S(1)$, the seller's revenues from the seller-initiated auction are below C for any $\hat{s}_0$. Hence, the seller never initiates the auction voluntarily. Because $\Pi_I^*(\hat{s}_0, \hat{s}_0, 1)$ approaches zero as $\alpha \to \frac{1}{2}$, the payoff of the bidder with cutoff signal $\hat{s}_0$ from sending an indication of interest to the seller is close to zero, while the payoff from not sending it is strictly positive as long as $\nu > 0$. Thus, no responsive equilibrium exists. □

PROOF OF PROPOSITION 7: If the responsive equilibrium exists, then the bidder with the cutoff signal $\hat{s}_0$ is given by (22). First, consider the comparative statics with respect to α. Differentiating the quadratic equation (B.3) for $\hat{s}_0$ in α and rearranging terms yields

$$\frac{d\hat{s}_0}{d\alpha} = \frac{(1+\nu+(1-\nu)\beta(1-\lambda))\hat{s}_0-\big(1-(1-\nu)\beta\lambda\frac{1}{3}\big)-\left[\frac{(2\alpha-1)^2}{3\alpha-1}\right]'\hat{s}_0^2}{\frac{(2\alpha-1)^2}{3\alpha-1}2\hat{s}_0+(1-(1+\nu+(1-\nu)\beta(1-\lambda))\alpha)}.$$

The denominator is positive, and as can be seen from (B.3), it is equal to $\frac{(1-\alpha)+(1-\nu)\beta\lambda\frac{\alpha}{3}}{\frac{(2\alpha-1)^2}{3\alpha-1}\hat{s}_0}+\frac{(2\alpha-1)^2}{3\alpha-1}\hat{s}_0 > 0$. Hence, it is sufficient to analyze the numerator.

From (B.3),

$$\alpha\left[(1+\nu+(1-\nu)\beta(1-\lambda))\hat{s}_0-\left(1-(1-\nu)\beta\lambda\frac{1}{3}\right)\right]=\frac{(2\alpha-1)^2}{3\alpha-1}\hat{s}_0^2-1.$$

Because $[\frac{(2\alpha-1)^2}{3\alpha-1}]'=\frac{2\alpha-1}{(3\alpha-1)^2}[24\alpha-7-12\alpha^2]>0$ for $\alpha\in[\frac{1}{2},1]$, $\frac{(2\alpha-1)^2}{3\alpha-1}$ is strictly increasing in α. Using the fact that $\hat{s}_0\in(0,1)$, as otherwise the responsive equilibrium would not exist,

$$\alpha\left[(1+\nu+(1-\nu)\beta(1-\lambda))\hat{s}_0-\left(1-(1-\nu)\beta\lambda\frac{1}{3}\right)\right]<\frac{1}{2}\hat{s}_0^2-1<0.$$

We can now sign the numerator:

$$\left[(1+\nu+(1-\nu)\beta(1-\lambda))\hat{s}_0-\left(1-(1-\nu)\beta\lambda\frac{1}{3}\right)\right]-\left[\frac{(2\alpha-1)^2}{3\alpha-1}\right]'\hat{s}_0^2<0,$$

because both terms are negative. Therefore, $\hat{s}_0$ is strictly decreasing in α. □

Second, consider the comparative statics with respect to ν. Differentiating the quadratic equation (B.3) for $\hat{s}_0$ in ν and rearranging terms yields

$$\frac{d\hat{s}_0}{d\nu}=\frac{(1-\beta(1-\lambda))\alpha\hat{s}_0-\beta\lambda\frac{\alpha}{3}}{\frac{(2\alpha-1)^2}{3\alpha-1}2\hat{s}_0+(1-(1+\nu+(1-\nu)\beta(1-\lambda))\alpha)}.$$

The denominator is the same as above, so it is positive. Hence, it is sufficient to analyze the numerator. Rearranging the terms in (B.3) yields

$$\begin{aligned}(1-\nu)\left[(1-\beta(1-\lambda))\alpha\hat{s}_0-\beta\lambda\frac{\alpha}{3}\right]&=(1-\alpha)+(2\alpha-1)\hat{s}_0-\frac{(2\alpha-1)^2}{3\alpha-1}\hat{s}_0^2\\ &>\min\left\{1-\alpha,\alpha-\frac{(2\alpha-1)^2}{3\alpha-1}\right\},\end{aligned}$$

where the inequality comes from the fact that the right-hand side of the equation is inverted U-shaped in $\hat{s}_0\in[0,1]$. Finally,

$$\begin{aligned}\min\left\{1-\alpha,\alpha-\frac{(2\alpha-1)^2}{3\alpha-1}\right\}&=\min\left\{1-\alpha,\frac{3\alpha-\alpha^2-1}{3\alpha-1}\right\}\\ &>\min\left\{1-\alpha,\frac{\frac{3}{2}-\frac{1}{4}-1}{3\alpha-1}\right\}>0,\end{aligned}$$

where the first inequality is from the fact that $[3\alpha-\alpha^2-1]'=3-2\alpha>0$ for all $\alpha\in[\frac{1}{2},1]$. Hence, the numerator is positive. Therefore, $\hat{s}_0$ is strictly increasing in ν.

PROOF OF PROPOSITION 8: We start by deriving the first-order conditions for equilibrium bids in the auctions. Consider a seller-initiated auction.[32] Denote the equilibrium bidding strategy by $a_S(s)$ and its inverse by $\phi_S(b)$. The expected payoff of a bidder with signal s and bid b is

$$\Pi_S(b,s) = \int_0^{\phi_S(b)} \left(\alpha v(s) + (1-\alpha)v(x) - fb - (1-f)a_S(x)\right)\frac{1}{\hat{s}}dx.$$

Differentiating in b and using the equilibrium condition that the maximum is reached at $b = a_S(s)$, or, equivalently, that $s = \phi_S(b)$, yields

$$\frac{\partial \phi_S(b)}{\partial b} = \frac{f\phi_S(b)}{v(\phi_S(b)) - b}, \tag{A.15}$$

subject to the initial value condition $\phi_S(0) = 0$. Note that (A.15) is equivalent to (10) in the base model but has multiple f in the numerator on the right-hand side. Intuitively, bidders bid more aggressively if f is lower, as they expect the payment upon winning to weigh less on their own bid and more on the (lower) bid of the rival. Equation (A.15) is equivalent to

$$\frac{ds}{da_S(s)} = \frac{fs}{v(s) - a_S(s)}.$$

It can be further rewritten as

$$\frac{d}{ds}\left[s^{\frac{1}{f}}a_S(s)\right] = s^{\frac{1}{f}-1}\frac{v(s)}{f},$$

which is solved by

$$a_S(s) = \int_0^s v(x)d\left[\left(\frac{x}{s}\right)^{\frac{1}{f}}\right] = \tilde{\mathbb{E}}[v(x)|x \le s],$$

where $\tilde{\mathbb{E}}[v(x)]$ denotes the expectation of $v(x)$ when random variable x is distributed with c.d.f. $x^{\frac{1}{f}}$. We can now calculate the expected payment of a bidder with signal s, conditional on winning:

$$\begin{aligned} &f\tilde{\mathbb{E}}[v(x)|x \le s] + (1-f)\int_0^s \tilde{\mathbb{E}}[v(y)|y \le x]\frac{1}{s}dx \\ &= \frac{1}{s^{\frac{1}{f}}}\int_0^s x^{\frac{1}{f}-1}v(x)dx - \frac{1}{s^{\frac{1}{f}}}\int_0^s v(x)\left(x^{\frac{1}{f}-1} - s^{\frac{1}{f}-1}\right)dx \\ &= \frac{1}{\hat{s}}\int_0^{\hat{s}} v(x)dx = \mathbb{E}[v(x)|s \le \hat{s}], \end{aligned}$$

[32] As in Sections II.A and II.B, we omit the dependence of bidding functions on $\hat{s}$ and $\bar{s}$ to keep notation simpler.

where we use integration by parts to calculate the second term. Thus, there is revenue equivalence of seller-initiated auction formats. Consequently, the expected payoff of a bidder with signal $\hat{s}$ in the seller-initiated auction is the same as in the base model: $\Pi_S^*(\hat{s},\hat{s}) = \alpha(v(\hat{s}) - \mathbb{E}[v(s)|s \le \hat{s}])$. The analysis of the auction initiated by both bidders is very similar.

Next, consider a bidder-initiated auction, similar to Section II.A. Now the expected payoff of bidder of type $j \in \{I, N\}$ with signal s from bidding b is

$$\Pi_j(b,s) = \int_{\underline{s}_k}^{\phi_k(b)} \left(\alpha v(s) + (1-\alpha)v(x) - fb - (1-f)a_k(x)\right)\frac{1}{\bar{s}_k - \underline{s}_k}dx. \quad \text{(A.16)}$$

Taking the first-order condition of (A.16), we obtain

$$\frac{\partial \phi_j(b)}{\partial b} = \frac{f\left(\phi_j(b) - \underline{s}_j\right)}{\alpha v(\phi_k(b)) + (1-\alpha)v(\phi_j(b)) - b} \quad \text{(A.17)}$$

for $j \neq k \in \{I, N\}$. This system of differential equations reduces to (3) if $f = 1$. It must be solved subject to boundary conditions $\phi_I(\bar{a}) = \bar{s}$, $\phi_N(\bar{a}) = \hat{s}$, $\phi_I(\underline{a}) = \hat{s}$, and $\phi_N(\underline{a}) = v^{-1}(\frac{\underline{a}-(1-\alpha)v(\hat{s})}{\alpha})$. These boundary conditions are the same as boundary conditions in the base model and follow from the same arguments, so we omit them here. In the base model, we had the additional condition (4), which pinned down $\underline{a}$, leading to the unique equilibrium. When the auction format is not first-price ($f < 1$), the argument from the base model does not apply. Consequently, there is a possibility of multiple equilibria.

Consider the payoff of the initiating bidder with signal $\hat{s}$:

$$\begin{aligned}
\Pi_I^*(\hat{s},\hat{s},\bar{s}) &\le \frac{v^{-1}(y)}{\hat{s}}\left(\alpha v(\hat{s}) + (1-\alpha)\mathbb{E}\left[v(x)|x \le v^{-1}(y)\right] - f(\alpha y + (1-\alpha)v(\hat{s}))\right.\\
&\quad \left. - (1-f)(1-\alpha)v(\hat{s})\right)\\
&= \frac{v^{-1}(y)}{\hat{s}}\left((2\alpha-1)v(\hat{s}) + (1-\alpha)\mathbb{E}\left[v(x)|x \le v^{-1}(y)\right] - f\alpha y\right)\\
&\le (2\alpha-1)\max_y \frac{v^{-1}(y)}{\hat{s}}\left(v(\hat{s}) - \frac{(f+1)\alpha - 1}{2\alpha-1}y\right)\\
&= (2\alpha-1)\max_x \frac{x}{\hat{s}}\left(v(\hat{s}) - \frac{(f+1)\alpha-1}{2\alpha-1}v(x)\right).
\end{aligned}$$

As in Proposition 1, $\Pi_I^*(\hat{s},\hat{s},\bar{s})$ converges to zero as $\alpha \to \frac{1}{2}$ for any f. Because the payoff of the bidder with signal $\hat{s}$ in the seller-initiated auction is the same as in the first-price auction ($\Pi_S^*(\hat{s},\hat{s}) = \alpha(v(\hat{s}) - \mathbb{E}[v(s)|s \le \hat{s}])$), the final part of the proof of Proposition 5 applies, leading to the statement of the proposition. □

PROOF OF PROPOSITION 9: As in the base model, the seller initiates the auction in the terminal period. Consider a single bidder-initiated auction at $t = 0$.

With a slight abuse of notation, we use the same notation to denote the payoffs of bidders as in the base model. We show that the payoff of the initiating bidder with the cutoff signal $\hat{s}_0$, $\Pi_I^*(\hat{s}_0, \hat{s}_0, 1)$, approaches zero in the sequential format as $\alpha \to \frac{1}{2}$. Once this result is established, the final part of the proof of Proposition 5 applies, leading to the statement of the proposition.

Let $\hat{b}$ denote the lowest bid that preempts the uninformed noninitiating bidder from learning her signal. Let $\tilde{s} \geq \hat{s}_0$ be the lowest signal of the initiating bidder who leads her to preempt the uninformed bidder by submitting bid $b \geq \hat{b}$. There are two possible cases, $\tilde{s} > \hat{s}_0$ and $\tilde{s} = \hat{s}_0$. If $\tilde{s} > \hat{s}_0$, then the initiating bidder with signal $\hat{s}_0$ does not preempt the uninformed bidder from learning her own signal. In this case, we have the first-price auction in which the signal of one bidder is drawn from the uniform distribution over $[\hat{s}_0, \tilde{s}]$, while the signal of the other bidder is drawn from the uniform distribution over $[0, 1]$. Aside from a different distribution of signals of the rival bidder ($[0, 1]$ instead of $[0, \hat{s}_0]$), the argument of the base model (Proposition 5) applies in the same way to show that the payoff of the initiating bidder with the cutoff signal $\hat{s}_0$ converges to zero as $\alpha \to \frac{1}{2}$.

If $\tilde{s} = \hat{s}_0$, then the initiating bidder with signal $\hat{s}_0$ preempts the uninformed noninitiating bidder from learning her own signal. The highest bid that the initiating bidder with signal $\hat{s}_0$ is willing to make cannot exceed the expected payoff of the asset:

$$\hat{b} \leq \alpha v(\hat{s}_0) + (1 - \alpha)\mathbb{E}[v(s)] \equiv \bar{b}(\hat{s}_0).$$

If the uninformed bidder competes against bid $\hat{b}$, her expected payoff from participating in the auction, denoted by $E\Pi_N(\hat{s}_0, \hat{b})$, satisfies $E\Pi_N(\hat{s}_0, \hat{b}) \geq E\Pi_N(\hat{s}_0, \bar{b}(\hat{s}_0)) > 0$ (this expected payoff is positive because there exist realizations of the signal close to one such that the noninitiating bidder with this signal obtains positive information rents). Therefore, if $\Psi \leq \bar{\Psi} \equiv \min_{\hat{s}_0 \in [0,1]} E\Pi_N(\hat{s}_0, \bar{b}(\hat{s}_0))$, the initiating bidder with signal $\hat{s}_0$ cannot preempt the noninitiating bidder in equilibrium. In this case, $\Pi_I^*(\hat{s}, \hat{s}, \bar{s})$ approaches zero as $\alpha \to \frac{1}{2}$ according to the same argument as in the base model. □

Appendix B: Special Case of Linear Valuations

We first derive the closed-form solutions of (21) for the special case $v(s) = s$. Using $\mathbb{E}[v(x)|x \leq s] = \frac{s}{2}$, expression (12) specializes to $\Pi_S^*(\hat{s}, \hat{s})$ in (21). Consider optimization (4). Rewrite it as

$$\Pi_I^*(\hat{s}, \hat{s}, 1) = \max_x \frac{x}{\hat{s}}\left((2\alpha - 1)\hat{s} - \left(\frac{3\alpha}{2} - \frac{1}{2}\right)x\right).$$

The first-order condition yields $x = \frac{2\alpha-1}{3\alpha-1}\hat{s}$. This gives $\Pi_I^*(\hat{s}, \hat{s}, 1)$ in (21). It follows that the minimum bid is

$$\underline{a}(\hat{s}) = \alpha\frac{2\alpha - 1}{3\alpha - 1}\hat{s} + (1 - \alpha)\hat{s} = \frac{3\alpha - 1 - \alpha^2}{3\alpha - 1}\hat{s}.$$

To obtain $\Pi_N^*(\hat{s}, \hat{s}, 1)$, rewrite differential equations (3) that determine functions $\phi_j(b, \hat{s})$, $j \in \{I, N\}$ for $v(s) = s$ as

$$\frac{\partial \phi_j(b, \hat{s})}{\partial b}\left(\alpha \phi_k(b, \hat{s}) + (1-\alpha)\phi_j(b, \hat{s}) - b\right) = \phi_j(b, \hat{s}) - \underline{s}_j.$$

Here, $\underline{s}_I = \hat{s}$ and $\underline{s}_N = 0$. Adding up both equations,

$$\begin{aligned} &\left[\alpha \phi_I(b, \hat{s})\phi_N(b, \hat{s})\right]' + \left[\frac{1-\alpha}{2}\phi_I(b, \hat{s})^2\right]' + \left[\frac{1-\alpha}{2}\phi_N(b, \hat{s})^2\right]' \\ &\quad = \left[\left(\phi_I(b, \hat{s}) + \phi_N(b, \hat{s}) - \hat{s}\right)b\right]'. \end{aligned}$$

Integrating,

$$\alpha \phi_I(b, \hat{s})\phi_N(b, \hat{s}) + \frac{1-\alpha}{2}\phi_I(b, \hat{s})^2 + \frac{1-\alpha}{2}\phi_N(b, \hat{s})^2 = \left(\phi_I(b, \hat{s}) + \phi_N(b, \hat{s}) - \hat{s}\right)b + c, \tag{B.1}$$

where c is the constant of integration. Evaluating it at $b = \underline{a}(\hat{s})$ and using $\underline{a}(\hat{s}) = \frac{3\alpha-1-\alpha^2}{3\alpha-1}\hat{s}$, $\phi_N(\underline{a}(\hat{s}), \hat{s}) = \frac{2\alpha-1}{3\alpha-1}\hat{s}$, and $\phi_I(\underline{a}(\hat{s}), \hat{s}) = \hat{s}$ yields the constant of integration in closed form:

$$c = \frac{\alpha^2}{2(3\alpha-1)}\hat{s}^2. \tag{B.2}$$

Evaluating (B.1) at $b = \bar{a}(\hat{s})$ and using $\phi_N(\bar{a}(\hat{s}), \hat{s}) = \hat{s}$, $\phi_I(\bar{a}(\hat{s}), \hat{s}) = 1$, and c from (B.2) yields $\bar{a}(\hat{s})$:

$$\bar{a}(\hat{s}) = \alpha \hat{s} + \frac{1-\alpha}{2} + \frac{1-\alpha}{2}\hat{s}^2 - \frac{\alpha^2}{2(3\alpha-1)}\hat{s}^2.$$

Now we can obtain the payoff of the noninitiating bidder with signal $\hat{s}$. This bidder submits bid $\overline{a}(\hat{s})$, wins with certainty, and values the asset at $\alpha\hat{s} + (1-\alpha)\frac{1+\hat{s}}{2}$. This gives $\Pi_N^*(\hat{s}, \hat{s}, 1)$ in (21):

$$\Pi_N^*(\hat{s}, \hat{s}) = \alpha \hat{s} + (1-\alpha)\frac{1+\hat{s}}{2} - \overline{a}(\hat{s}) = \frac{1-\alpha}{2}\left(\hat{s} - \hat{s}^2\right) + \frac{\alpha^2}{2(3\alpha-1)}\hat{s}^2.$$

Finally,

$$\Pi_R = \int_0^1 \Pi_S^*(s, 1)ds = \alpha \int_0^1 \frac{s^2}{2}ds = \frac{\alpha}{6}.$$

Plugging expressions (21) into equation (19), which determines threshold $\hat{s}_0$ in the responsive equilibrium, we obtain the following quadratic equation:

$$\frac{(2\alpha-1)^2}{3\alpha-1}\hat{s}_0^2 + (1 - (1 + \nu + (1-\nu)\beta(1-\lambda))\alpha)\hat{s}_0 - (1-\alpha) - (1-\nu)\beta\lambda\frac{\alpha}{3} = 0. \tag{B.3}$$

It has two roots, but one of them is negative. The positive root is given by (22).

Next, we consider three examples of the model with linear values.

EXAMPLE B1 (Auction stage, pure private values): Suppose that $\alpha = 1$ and $v(s) = s$. This type of asymmetric auction with private values was analyzed by Kaplan and Zamir (2012). The solution in inverse bidding strategies is

$$\phi_I(b) = \hat{s} + \frac{\hat{s}^2}{(\hat{s} - 2b)c_I e^{-\frac{\hat{s}}{\hat{s}-2b}} - 4b}, \phi_N(b) = \frac{\hat{s}^2}{(\hat{s} - 2b)c_N e^{\frac{\hat{s}}{\hat{s}-2b}} + 4(\hat{s} - b)},$$

where constants c_N and c_I are determined from the first two boundary conditions in (5), namely, $\phi_I(\hat{s} - \frac{\hat{s}^2}{4}) = 1$ and $\phi_N(\hat{s} - \frac{\hat{s}^2}{4}) = \hat{s}$. The range of bids is $b \in [\frac{\hat{s}}{2}, \hat{s} - \frac{\hat{s}^2}{4}]$ for both bidders. For $\hat{s} = \frac{1}{2}$, Figure 2, Panel A, illustrates these bidding strategies as well as bidding strategies $a_S(s) = \frac{s}{2}$ in the seller-initiated auction.

EXAMPLE B2 (Auction stage, pure common values): Suppose that $\alpha = \frac{1}{2}$ and $v(s) = s$. To find the equilibrium, let $\phi_j(b) = \gamma_j + \beta_j b$, $j \in \{I, N\}$. Then the first two boundary conditions in (5) become

$$1 = \gamma_I + \beta_I \bar{a}, \quad \hat{s} = \gamma_N + \beta_N \bar{a} \quad \Rightarrow \quad 1 + \hat{s} = \gamma_I + \gamma_N + (\beta_I + \beta_N)\bar{a}. \tag{B.4}$$

Using (4), $\underline{a} = \frac{\hat{s}}{2}$. The final two conditions in (5) become

$$\hat{s} = \gamma_I + \beta_I \frac{\hat{s}}{2}, \quad 0 = \gamma_N + \beta_N \frac{\hat{s}}{2} \quad \Rightarrow \quad \hat{s} = \gamma_I + \gamma_N + (\beta_I + \beta_N)\frac{\hat{s}}{2}. \tag{B.5}$$

Note that, particular to pure common values, the lowest serious noninitiating bidder has signal 0, so there are no nonserious bids. The difference between (B.4) and (B.5) yields $\beta_I + \beta_N = \frac{1}{\bar{a} - \hat{s}/2}$. Next, plugging $\phi_j(b)$ into differential equations (3) yields

$$\beta_j = \frac{\alpha_j + \beta_j b - \underline{s}_j}{\frac{1}{2}(\gamma_I + \gamma_N + (\beta_I + \beta_N)b) - b}. \tag{B.6}$$

Adding the two equations up at $b = \bar{a}$ results in $\beta_I + \beta_N = \frac{1}{(1+\hat{s})/2 - \bar{a}}$. Combining the two equations for $\beta_I + \beta_N$ yields $\bar{a} = \frac{1+2\hat{s}}{4}$ and the range of bids $b \in [\frac{\hat{s}}{2}, \frac{1+2\hat{s}}{4}]$ for both bidders. With $\bar{a}$ known, the coefficients in $\phi_j(b, \hat{s})$ can be found from boundary conditions. The resulting inverses of bidding strategies are

$$\phi_I(b) = \hat{s}(2\hat{s} - 1) + 4(1 - \hat{s})b, \quad \phi_N(b) = -2\hat{s}^2 + 4\hat{s}b.$$

The bidding strategies given signals are, in turn, inverses of $\phi_j(b)$. They are linear in own signals:

$$a_I(s) = \frac{s + \hat{s}(1 - 2\hat{s})}{4(1 - \hat{s})}, \quad a_N(s) = \frac{s + 2\hat{s}^2}{4\hat{s}}.$$

For $\hat{s} = \frac{1}{2}$, Figure 2, Panel B, illustrates these bidding strategies as well as bidding strategies $a_S(s) = \frac{s}{2}$ in the seller-initiated auction.

EXAMPLE B3 (Initiation stage, pure private values): Suppose that $\alpha = 1$ and $v(s) = s$. Then (22) simplifies to

$$\hat{s}_0 = \nu + (1-\nu)\beta(1-\lambda) - 1 + \sqrt{(\nu + (1-\nu)\beta(1-\lambda))^2 + (1-\nu)\beta\lambda\frac{2}{3}}.$$

Appendix C: Equilibria in Mixed Strategies

In this section, we look at responsive equilibria in mixed seller strategies. An equilibrium is responsive if there exist bidder message profiles $\mathbf{m}$ and $\mathbf{m}' \neq \mathbf{m}$ on equilibrium path such that $h(\tilde{d}_t(\mathbf{m})) \neq h(\tilde{d}_t(\mathbf{m}'))$ for some t, where $\tilde{d}_t \in \{0, 1\}$ is the possibly randomized strategy of the seller to put the auction up for sale or not, and $h(\cdot)$ is its p.d.f.

While the presence of mixed seller strategies does not affect pure-strategy equilibria at $t = 0$ and $t = 1$ and does not add new equilibria at $t = 1$ (the only equilibrium remains nonresponsive), it can add new responsive equilibria at $t = 0$. The following modification of Proposition 4 characterizes all possible mixed-strategy equilibria of the extended model.

PROPOSITION C1 (Mixed-strategy equilibria in the initiation game at $t = 0$): *In addition to the pure-strategy equilibria described in Proposition 4, the set of possible mixed-strategy equilibria is as follows:*

(1) A responsive equilibrium, in which each bidder sends an indication of interest if and only if $s_i \geq \hat{s}_0$ for some $\hat{s}_0 \in (0, 1)$, and the seller holds the auction with certainty if $\max_i \mathbf{m}_{i,0} = 1$ *and with probability $\mu_0 \geq \nu$ if $\mathbf{m}_0 = (0, 0)$:*

$$h\left(\tilde{d}_0(\mathbf{m}_0) = 1\right) = \begin{cases} 1, & \textit{if } \mathbf{m}_0 \neq (0, 0), \\ \mu_0, & \textit{if } \mathbf{m}_0 = (0, 0). \end{cases} \tag{C.1}$$

The cutoff signal $\hat{s}_0$ satisfies

$$\Pi_I^*(\hat{s}_0, \hat{s}_0, 1) = \left(\frac{1}{\hat{s}_0} - 1\right)\Pi_N^*(\hat{s}_0, \hat{s}_0, 1) + \mu_0 \Pi_S^*(\hat{s}_0, \hat{s}_0) + (1-\mu_0)\beta\Pi_1(\hat{s}_0). \tag{C.2}$$

This equilibrium exists if and only if the solution to (C.2) satisfies

$$R_S(\hat{s}_0) = \beta((1-\lambda)R_S(\hat{s}_0) + \lambda R_S(1)) \leq R_B(\hat{s}_0, 1) \tag{C.3}$$

and inequalities similar to (A.11) and (A.12) in Appendix A, in which ν is substituted with μ_0.

(2) A responsive equilibrium, in which each bidder sends an indication of interest if and only if $s_i \geq \hat{s}_0$ for some $\hat{s}_0 \in (0, 1)$, and the seller holds the

auction with certainty if $\mathbf{m}_0 = (1,1)$*, with probability* $\mu_0 \geq \nu$ *if* $\mathbf{m}_0 = (0,1)$ *or (1,0), and only if he is hit by the liquidity shock if* $\mathbf{m}_0 = (0,0)$*:*

$$h\Big(\tilde{d}_0(\mathbf{m}_0) = 1\Big) = \begin{cases} 1, & \text{if } \mathbf{m}_0 = (1,1), \\ \mu_0, & \text{if } \mathbf{m}_0 = (1,0) \text{ or } (0,1), \\ \nu, & \text{if } \mathbf{m}_0 = (0,0). \end{cases} \tag{C.4}$$

The cutoff signal $\hat{s}_0$ *satisfies*

$$\mu_0 \Pi_I^*(\hat{s}_0, \hat{s}_0, 1) + (1-\mu_0)\beta \Pi_{I,1}(\hat{s}_0) = \mu_0 \left(\frac{1}{\hat{s}_0} - 1\right) \Pi_N^*(\hat{s}_0, \hat{s}_0, 1)$$
$$+ (1-\mu_0)\beta \left(\frac{1}{\hat{s}_0} - 1\right) \Pi_{N,1}(\hat{s}) + \nu \Pi_S^*(\hat{s}_0, \hat{s}_0) + (1-\nu)\beta \Pi_{S,1}(\hat{s}_0), \tag{C.5}$$

where $\Pi_{j,1}(\hat{s}_0)$*,* $j \in \{I, N, S\}$ *are continuation values for bidder i if* $(m_{i,0}, m_{-i,0}) = (1,0)$*, (0,1), and (0,0), respectively, when the initiation cutoff at* $t = 0$ *is* $\hat{s}_0$*. Specifically,* $\Pi_{j,1}(\hat{s}_0) = (1-\lambda)\Pi_j^*(\hat{s}_0, \hat{s}_0, 1) + \lambda \Pi_R$ *for* $j \in \{I, N\}$*, where* Π_R *is defined in (17), and* $\Pi_{S,1}(\hat{s}_0) = \Pi_1(\hat{s}_0)$*. This equilibrium exists if and only if the solution to (C.2) satisfies*

$$R_S(\hat{s}_0) < \beta((1-\lambda)R_B(\hat{s}_0, 1) + \lambda R_S(1)) = R_B(\hat{s}_0, 1), \tag{C.6}$$

and modified inequalities (A.11) and (A.12) in Appendix A.

The proof and intuition for the existence of the first of the two mixed-strategy responsive equilibria is the same as the proof of Proposition 4. This mixed equilibrium exists only if the pure-strategy responsive equilibrium of Proposition 4 exists. The proof for the existence of the second of the two mixed-strategy responsive equilibria is different in details but fundamentally still follows the proof of Proposition 4. In most numerical examples we tried, this mixed-strategy equilibrium does not exist: in order to delay the auction with some likelihood despite interest from one of the bidders, the benefit from a signal reset at $t = 1$ to the seller must be large. This only occurs when the initiation cutoff $\hat{s}_0$ is low. However, at a low hypothetical equilibrium cutoff, the bidder with the cutoff signal finds it not worthwhile to indicate her interest for most parameters.

For the case of $\alpha = 1$, $v(s) = s$, $\nu = 0$, $\beta = 0.9$, and $\lambda = 0.9$, Figure C.1 illustrates the bidders' and seller's best responses, and two responsive equilibria, in pure and mixed seller strategies at $t = 0$: $(\hat{s}_0, \mu_0) = (0.83, 0)$ and (0.89,0.09), respectively. Additionally, there is a nonresponsive equilibrium $(\hat{s}_0, \mu_0) = (1, 1)$ in which the seller puts the asset up for sale immediately. The responsive equilibrium, in which the seller delays the auction with some likelihood despite interest from one of the bidders, does not exist.

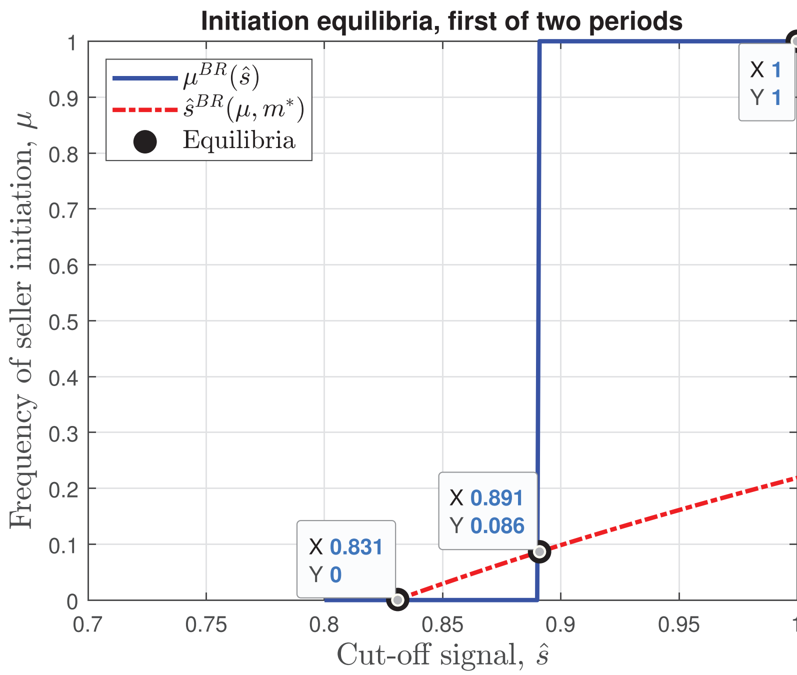

Figure C.1. Equilibria with seller- and bidder-initiated auctions, pure private values. For pure private values $\alpha = 1$, values $v(s) = s$, no exogenous liquidity shocks $\nu = 0$, and parameters $\beta = 0.9$ and $\lambda = 0.9$, the figure plots period-0 best responses (BR) of the seller, who chooses the probability of putting the asset up for sale without receiving any indications of interest μ (solid black line), and each bidder, who chooses the cutoff signal for initiation $\hat{s}$ (dashed red line), as well as multiple pure and mixed equilibria (circle markers). (Color figure can be viewed at wileyonlinelibrary.com)

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Appendix S1: Internet Appendix.
Replication Code.