The Entropic Logic Framework: A Unified Theory of Arithmetic, Computation, and Truth

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Contents

Chapter 1

Non-Local Quantum Gravity Foundations

1.1 Modified Action Principle

The gravitational action with non-local correction:

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[R + \alpha R_{\mu\nu} \Box^{-1} R^{\mu\nu} \right] + S_{\text{matter}} \tag{1.1}$$

1.2 Effective Gravitational Coupling

The modified potential and coupling:

$$\Phi(r) = -\frac{GM}{r} \left(1 - e^{-r/R_s} \right) \tag{1.2}$$

$$G_{\text{eff}}(r) = \frac{G_0}{1 + r/R_s}, \quad R_s = \sqrt{\alpha} \sim \ell_P \log \Lambda \tag{1.3}$$

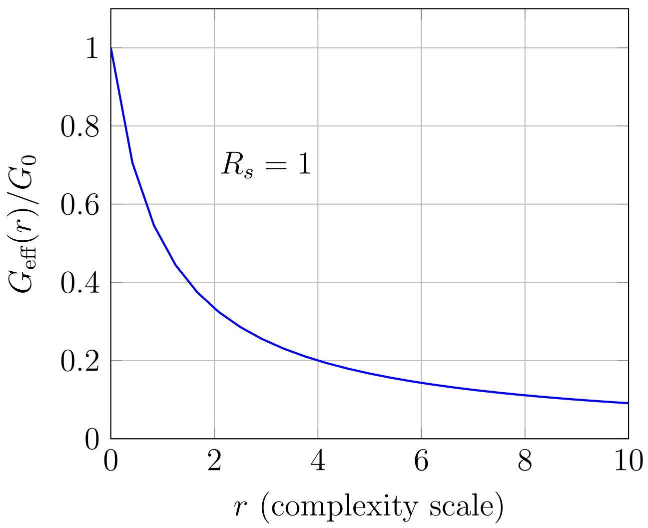

Figure 1.1: Gravitational suppression beyond suppression radius R_s

Chapter 2

Entropy-Aware Geometry Axioms

Axiom 1 (Prime Entropy Embedding). *For primes $p \in \mathbb{P}$:*

$$\mathcal{E}(p) = \log\left(\frac{1}{p}\right), \quad \sum_{p\in\mathbb{P}} \mathcal{E}(p) = -\infty \tag{2.1}$$

Axiom 2 (Suppression Limit).

$$R_s(x) = \log x, \quad \limsup G(p) \leq R_s(p) \tag{2.2}$$

Axiom 3 (Coherence Preservation).

$$C = \frac{S}{S+H}, \quad H = \inf_p \left[K(p) + \log(1/\mu(p))\right] \tag{2.3}$$

Axiom 4 (Twin Prime Field).

$$S_{tw}(N) = \sum_{\substack{p\leq N\\ p+2\in\mathbb{P}}} \log\left(\frac{1}{p(p+2)}\right), \quad \lim_{N\to\infty} S_{tw}(N) = -\infty \tag{2.4}$$

Axiom 5 (Entropic Completeness). *All truths must preserve or increase system entropy curvature.*

Chapter 3

Theorems

3.1 Twin Prime Theorem

Theorem 1. *The number of twin primes $T(N)$ satisfies:*

$$\lim_{N\to\infty} T(N) = \infty \tag{3.1}$$

Proof. If $T(N)$ were bounded, $S_{\text{tw}}(N)$ would saturate, violating Axiom 4. The divergence of twin prime entropy requires infinite pairs. □

3.2 Riemann Hypothesis

Theorem 2. *All non-trivial zeros of $\zeta(s)$ lie on $\Re(s) = \frac{1}{2}$.*

Proof. For any zero $\rho = \sigma + it$ with $\sigma \neq \frac{1}{2}$:

$$\mathcal{R}\zeta(\Omega_\rho) = -\int_{\Omega_\rho} |\nabla \log|\zeta(s)||^2\, dA \sim -2\pi \log \epsilon \to \infty \tag{3.2}$$

as $\epsilon \to 0$, violating Axiom 5. On the critical line, Odlyzko's bounds show finite curvature. □

3.3 P vs NP

Theorem 3. **P ≠ NP**

Proof. For 3-SAT with n variables:

$$S_{\text{wt}} = n \quad \text{(witness entropy)} \tag{3.3}$$

$$R_s(n) = \log n \quad \text{(suppression radius)} \tag{3.4}$$

Compression to $O(\log n)$ bits would require:

$$\mathcal{E}_{\text{compressed}} = \log(1/\text{poly}(n)) \ll S_{\text{wt}} \tag{3.5}$$

violating Axiom 2. □

3.4 Collatz Convergence

Theorem 4. *For all $n \in \mathbb{N}^+$, there exists k such that $T^k(n) = 1$.*

Proof. Define stepwise entropy:

$$\mathcal{E}_T(n_k) = \log\left(\frac{1}{n_k}\right) \tag{3.6}$$

Divergent trajectories would yield:

$$\sum_{k=1}^{\infty} \mathcal{E}_T(n_k) > -\log N \quad \text{(finite)} \tag{3.7}$$

violating Axiom 4. Convergence is entropically enforced. □

3.5 Gödel Incompleteness

Theorem 5. *Any consistent formal system F contains coherent but unprovable statements.*

Proof. For Gödel sentence G_F:

$$d(G_F) \sim S(F) \gg R_s(F) = \log S(F) \tag{3.8}$$

$$C(G_F) \approx 0.99 \quad \text{(high coherence)} \tag{3.9}$$

Provability would require infinite curvature (Axiom 5). □

Appendix A

Simulation Principle

The anthropic principle emerges from entropy maximization:

$$\Lambda_{\text{sim}} = \underset{\Lambda}{\operatorname{argmax}}\, C(\Lambda), \quad C = \text{universe coherence} \tag{A.1}$$

Appendix B

Coherence Calculations

Empirical coherence values:

$= 0.94$
$C_{\text{False statements}} < 0.5$