Balanced Multiway Tree

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Balanced Multiway Tree

- The binary search techniques could be used directly for organizing table on disks: the LEFT and RIGHT pointers become addresses on the disk instead of addresses in internal memory
- We would require disk access whenever a LEFT/RIGHT pointer was followed, essentially making one probe per disk access

Continue…

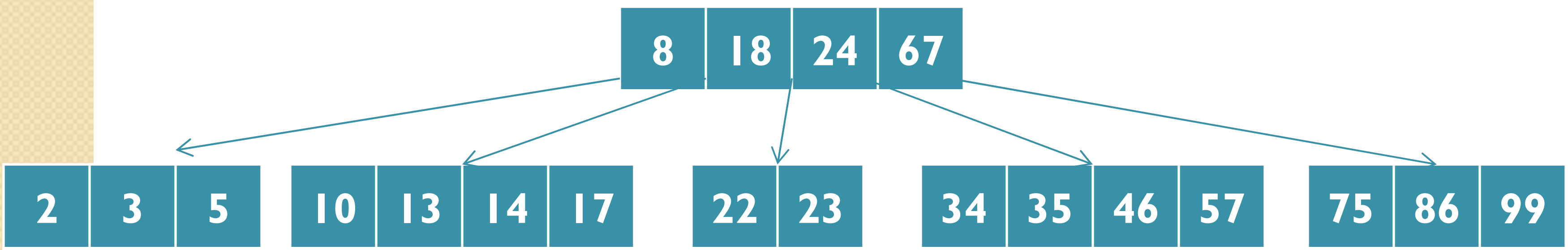
- Since disk access are costly compared to probes, it is preferable to make a number of probes for each disk access
- We can do so if nodes in tree contain m-way branches instead of 2-way branches (binary tree).
- In analogy with binary trees, we define an m-way tree, such a tree T either is empty or consists of a distinguished node called the root and k subtrees T1,T2,…,Tk, 2<=k<=m

Continue…

T1	X1	T2	X2	T3	X3	T4	. . .	T(k-1)	X(k-1)	Tk

- In an m-way tree: $X1<X2<\ldots<X(k-1)$ and that elements in subtree Ti be greater than X(i-1) and less than Xi

Five Way Search Tree Example



Balanced Multiway Tree Compromise

- It is time consuming to keep the tree perfectly balanced under insertion and deletion, so we need a compromise like that a height or a weight balanced trees
- A good compromise: all paths from the root to an external node are of equal length and that each node except the root has at least $\lceil m/2 \rceil$ subtrees

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Continue…

- Balanced multiway tree of order m or B-tree as an m-way tree:

1. All external nodes are at the same level
2. The root has anywhere from 2 to m subtrees
3. Other internal nodes have anywhere from $\lceil m/2 \rceil$ to m subtrees

- For purpose of the organization of table on disks, we will be interested in m being around several hundreds

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Insertion

- Consider inserting 15
- We begin with an unsuccessful search for 15, as the search proceeds, a record is kept on a stack of the nodes visited. This lets us retrace the path up the tree. The search for 15 fails at the bottom level of internal nodes in the tree

Continue…

- If the node at which it fails contains less than m-1 elements, the new element we inserted just simply inserted into its proper place in that node (For example, we insert 20).

Continue…

- 15 cannot be inserted because node

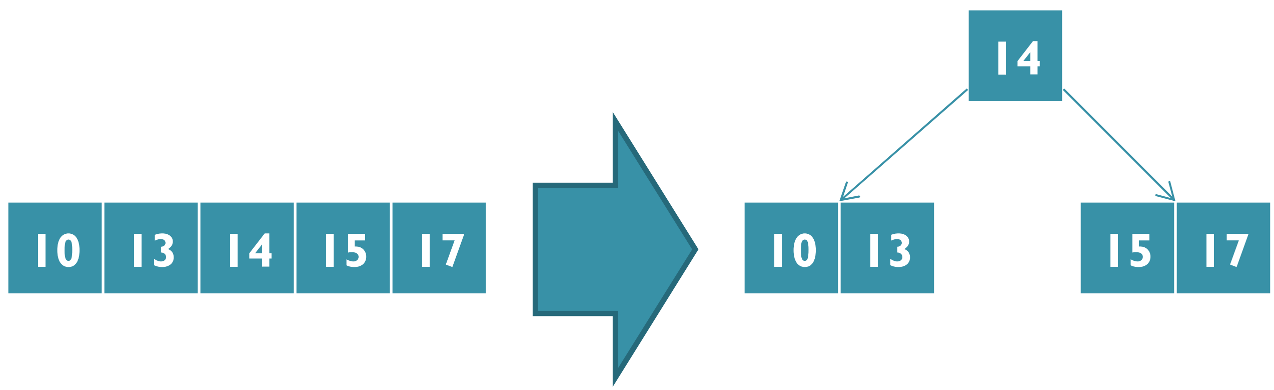
10	13	14	17

contains the maximum allowable number of elements

Continue…

- In such a case the m elements, consisting of m-1 in the node and the new element, are split into 2 node containing the smallest [m/2]-1 elements and the largest [m/2] elements, the median element is pushed up into the parent node to be the separator element between the two halves.

Continue…



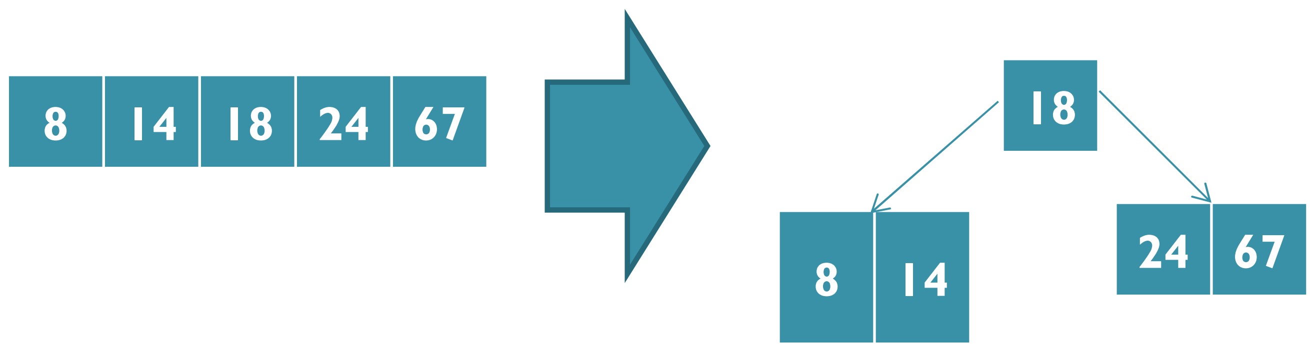
Continue…

- With 14 inserted into node

- That node is split in turn:

  this process continues up the tree, as needed

- When the root split, a new node is created that becomes the root of the tree and the tree becomes one level taller.



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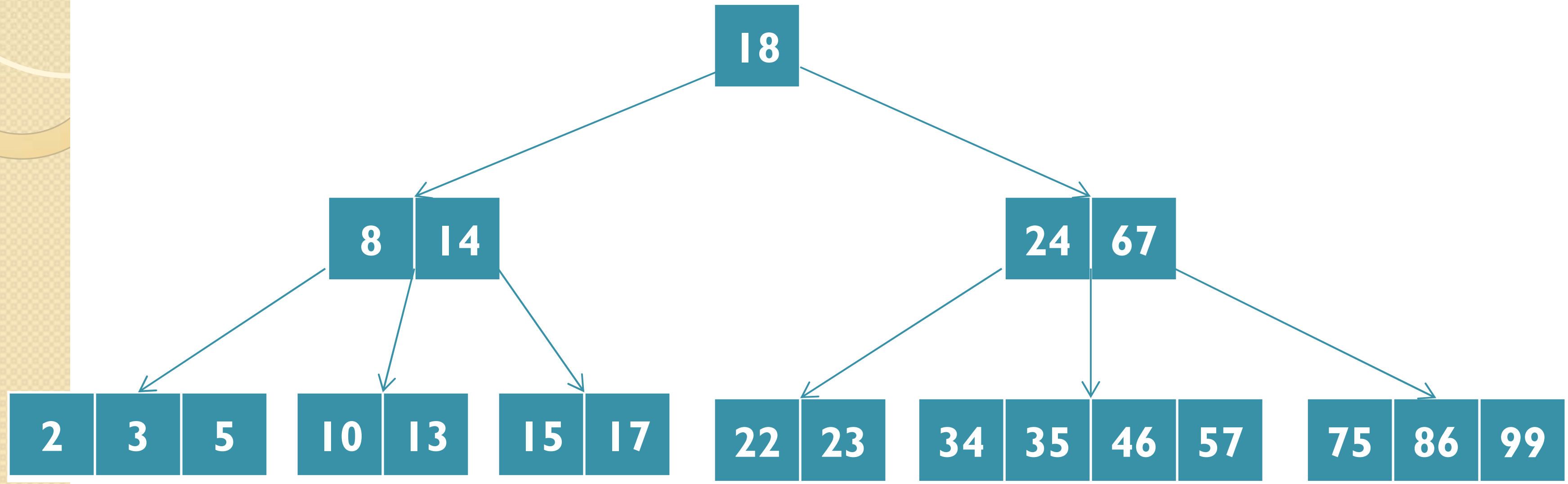
Tree Result



- The insertion process is remarkable in that trees grow taller by adding levels to the root, rather than to the leaves

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Deletion

- The deletion of elements in m-way tree is no harder than an insertion
- Suppose we want to delete 34
- We just delete 34 from its node in the tree, since that node has only empty subtrees and since it has enough elements, nothing else needed to be done

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Continue…

- If we wanted to delete 22, the deletion would leave the node

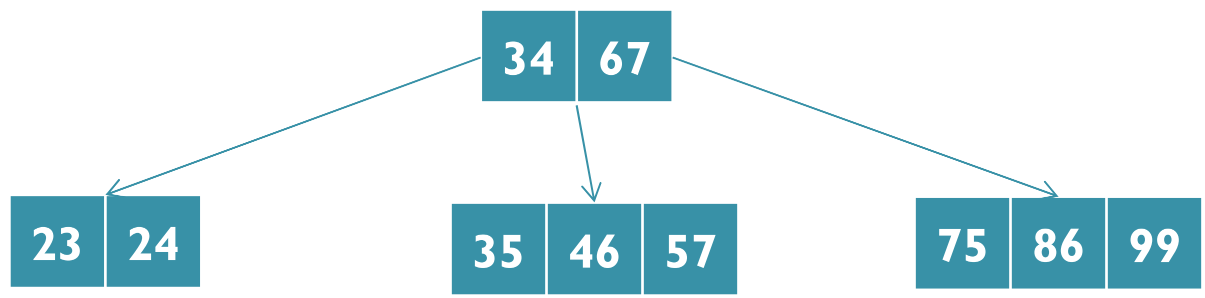
22	23

Insufficiently full

Continue…

- In this case we could take an element from neighboring

and use it to remake the tree into

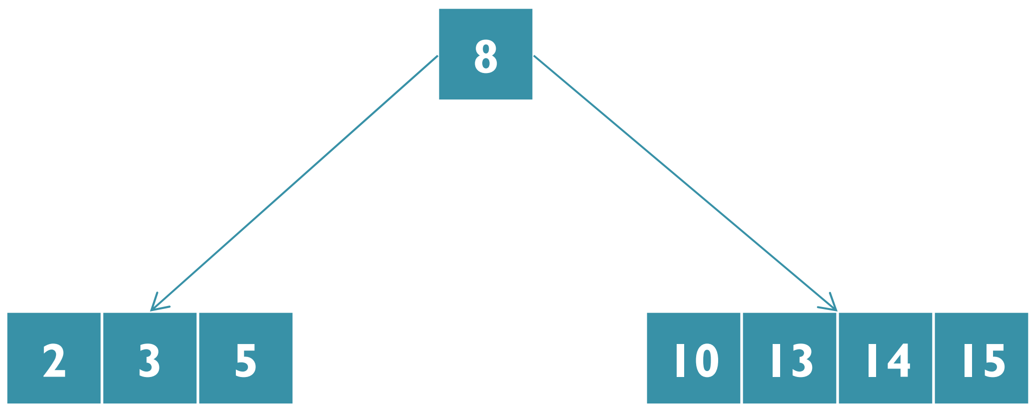


Continue…

- If we are deleting FOR, the neighboring node does not have an element to spare
- In such a case a minimally full node with a minimally full neighbor, the node, its neighbor, and the element that separates them in their parent node can be combined into an acceptably full node

Continue…

- Thus to delete 17 from the B-tree we would replace the left subtree with:



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Continue…

- Pushing the problem of the deletion up to the next higher level, where it is handled in precisely the same way: the neighbor

34	67

- Has insufficiently many elements to give one up,

8 and 34 | 67

- And their separator are combined into a single node.
- Notice that as in an insertion, the height of the tree changes at the root, not at the leaves

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