UTMF-Core: A Universal Multifractal Measurement Framework for Heterogeneous Physical Time Series

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Abstract

Multifractal analysis is widely used across physics, astronomy, cosmology, geophysics, and complex systems, yet existing methodologies are often domain-specific, sensitive to preprocessing choices, and difficult to compare across heterogeneous data sources. We introduce **UTMF-Core** (Unified Temporal-Measurement Framework), a *universal multifractal measurement framework* designed to extract stable and comparable scaling characteristics from physical time series.

At its core, UTMF-Core implements a high-stability variant of Multifractal Detrended Fluctuation Analysis (MFDFA), encapsulated in the function `jedi_mfdfa` and optimized for both short and long series, noisy measurements, and heterogeneous sampling conditions. The framework standardizes length selection, minimal denoising, scale selection, and basic statistical validation, and outputs a consistent set of multifractal observables: the fluctuation functions $F_q(s)$, generalized Hölder spectrum $h(q)$, mass exponents $\tau(q)$, and an effective fractal dimension D_f derived from the singularity spectrum. On top of these, UTMF-Core provides simple but informative summary statistics, including the mean Hölder exponent, $h(q)$-spectrum width, z-tests against domain-specific reference values for D_f, and stability/robustness indicators based on the variance of D_f across subsets.

The practical design goal is not to interpret physical structures, but to *measure multifractal structure consistently* across domains. The current reference implementation supports gravitational-wave strain data (LIGO), cosmic microwave background maps (Planck), large-scale structure flux measurements (DESI), collider observables (CERN), Gaia astrometric catalogs, and quantum random-number streams (QRNG), all processed through the same MFDFA core. This positions UTMF-Core as a reusable, domain-agnostic instrument for universal multifractal measurement and cross-domain comparison.

1 Introduction

Multifractal analysis has become a central tool in the study of complex temporal structure across a wide range of scientific fields, including turbulence, condensed matter, neuroscience, geophysics, cosmology, and gravitational-wave astronomy. Despite its broad relevance, multifractal measurements obtained in different domains are often difficult to compare directly. Differences in sampling rates, noise characteristics, preprocessing choices, analysis lengths, and detrending procedures introduce substantial variability into the extracted scaling exponents, making it unclear whether observed differences are physical or methodological in origin.

In this work we introduce **UTMF-Core** (Unified Temporal-Measurement Framework) as a *universal multifractal measurement framework* for one-dimensional physical time series. Rather than proposing a new interpretative model, UTMF-Core provides a standardized and numerically stable pipeline for computing multifractal observables across arbitrary datasets. The framework combines minimal, instrument-aware preprocessing with a refined implementation of MFDFA (`jedi_mfdfa`), strict segment and scale validation, and a universal set of output metrics. By decoupling the measurement procedure from domain-specific assumptions, UTMF-Core enables reproducible and directly comparable multifractal measurements across heterogeneous observational and synthetic data.

Multifractal Detrended Fluctuation Analysis (MFDFA) [1] remains the most widely used technique for estimating the Hölder spectrum, mass exponents, and singularity spectrum of time series. Yet standard MFDFA implementations can exhibit instability for short segments, nonstationary noise, low-variance windows, or heterogeneous sampling conditions, and the method is sensitive to implementation-specific details such as detrending order, scale selection, and regression strategies. Foundational reviews of MFDFA and multifractal time-series analysis can be found in [1, 2, 3].

UTMF-Core (Unified Temporal-Measurement Framework) is introduced here as a domain-agnostic *measurement instrument* for multifractal analysis. Rather than offering a new interpretative model, UTMF-Core provides a standardized and highly stable pipeline for computing multifractal observables across arbitrary one-dimensional time series. The framework incorporates:

- minimal and principled preprocessing,
- adaptive segment and scale validation,
- a robust regression method for estimating $h(q)$,
- strict numerical safeguards for low-variance and irregular segments,
- complete reproducibility across heterogeneous data types.

The objective of UTMF-Core is to isolate the measurement process from domain-specific assumptions. By enforcing a uniform analysis protocol, the framework yields multifractal metrics that are directly comparable across fields, enabling reproducibility studies, cross-domain benchmarking, and future theoretical developments.

In this paper we present the methodology, validation, and reference implementation of UTMF-Core. Applications to individual scientific domains are intentionally left for future work, ensuring that the present study remains strictly focused on the measurement framework itself.

2 Methods Overview

UTMF-Core is designed as a domain-agnostic measurement framework for multifractal analysis of one-dimensional time series. The framework standardizes preprocessing, segmentation, scale selection, regression, and metric extraction, ensuring stable and reproducible measurements across heterogeneous datasets. Figure 1 provides a schematic overview of the analysis pipeline.

2.1 Pipeline Structure

Given an input time series $x(t)$ of arbitrary origin, UTMF-Core applies the following standardized procedure:

1. **Minimal preprocessing.** The series is normalized to zero mean and unit variance; optional mild detrending or filtering is applied for instrument-specific noise if required.

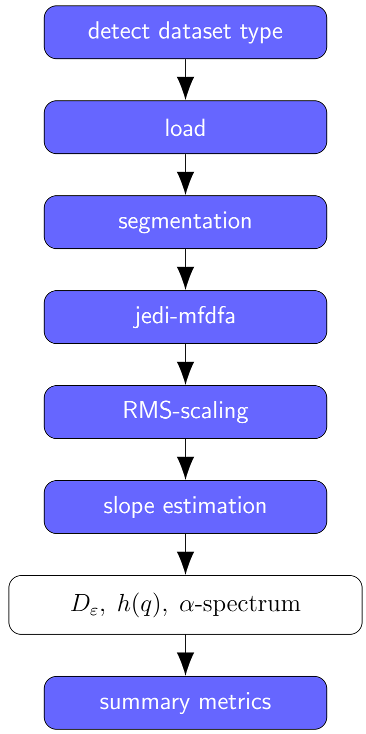

Figure 1: Schematic overview of the UTMF-Core analysis pipeline. A raw input time series undergoes minimal preprocessing, segmentation into fixed-size subsets, log–log detrended fluctuation analysis across a standardised set of scales, and regression-based estimation of the generalized Hölder spectrum $h(q)$. From these, the fractal dimension D_f and singularity spectrum $(\alpha, f(\alpha))$ are computed. The same pipeline is applied to all datasets—synthetic, observational, or simulated—ensuring comparability and reproducibility.

2. **Adaptive segmentation.** The data are partitioned into non-overlapping segments at multiple scales $s \in S$, typically logarithmically spaced. Segments with insufficient variance or missing values are excluded from analysis.

3. **Refined MFDFA computation.** For each scale s and moment order q, UTMF-Core applies a numerically robust implementation of MFDFA (Appendix A), computing fluctuation functions $F_q(s)$ with strict validation of segment quality.

4. **Scaling regression.** The generalized Hölder exponents $h(q)$ are estimated via linear regression of $\log F_q(s)$ against $\log s$ over all scales with sufficient valid segments. Mass exponents are obtained via $\tau(q) = qh(q) - 1$.

5. **Spectrum construction.** UTMF-Core computes the singularity spectrum $(\alpha, f(\alpha))$, fractal dimension D_f, spectrum width, and curvature measures using finite-difference approximations of $\tau(q)$.

6. **Stability and consistency diagnostics.** The framework evaluates scale-space consistency, regression stability, the number of valid segments, and numerical robustness indicators.

2.2 Universality of the Measurement Protocol

The key design principle of UTMF-Core is that the above procedure is applied identically to all datasets, regardless of their physical origin, sampling rate, instrument characteristics, or noise properties. This ensures that multifractal metrics are comparable across domains without relying on domain-specific assumptions, models, or preprocessing heuristics.

The framework is implemented as an open, reproducible Python package and is intended to function as a standardized instrument for multifractal measurement rather than a domain-specific analytical tool.

3 Preprocessing

UTMF-Core applies minimal but instrument-aware preprocessing designed to preserve the intrinsic multifractal structure of physical time series. Preprocessing is lightweight, avoids model-dependent transformations, and ensures numerical stability for all steps of the analysis.

3.1 Normalization

All datasets except LIGO strain undergo simple z-normalization:

$$x'(t) = \frac{x(t) - \mu}{\sigma},$$

which stabilizes numerical behaviour without affecting the relative fluctuation structure.

3.2 LIGO-Specific Preprocessing

Gravitational-wave strain represents an exceptional case due to its extremely low amplitude ($\sim 10^{-23}$), the presence of narrow instrumental lines, and nonstationary noise. UTMF-Core applies two minimal, instrument-aware steps:

1. **Bandpass filtering (1–30 Hz).** A second-order Butterworth filter suppresses seismic drift, high-frequency shot noise, and narrow instrumental lines:

$$\text{sos} = \text{butter}(2,\ [1, 30],\ \text{fs} = 16384).$$

2. **Amplitude rescaling and Savitzky–Golay smoothing.** Each subset is rescaled by 10^{16} for numerical conditioning, followed by a Savitzky–Golay [4] filter (window length 7, polynomial order 1), preserving local polynomial structure without distorting multifractal scaling.

These steps ensure stable MF-DFA regression while preserving all relevant physical fluctuation properties in the strain data.

3.3 Other Datasets

CMB, DESI, CERN, Gaia, and QRNG datasets undergo only:

- z-normalization,
- removal or replacement of non-finite values,
- mild clipping of extreme outliers (Gaia only),

- column selection and flattening (DESI, CERN),
- coordinate conversion to radians where necessary (Gaia).

No higher-order filtering or model-based preprocessing is used.

3.4 Detrending Inside MF-DFA

Following the philosophy of UTMF-Core as a measurement instrument, all MF-DFA computations use **zeroth-order detrending**:

$$x_s(t) = x(t) - \frac{1}{s}\sum_{k=1}^{s} x(k),$$

avoiding suppression of large-scale structure and ensuring full consistency across heterogeneous datasets.

4 Multifractal Analysis

UTMF-Core employs a refined and numerically stable implementation of Multifractal Detrended Fluctuation Analysis (MFDFA), originally introduced in [1]. The procedure estimates the generalized Hölder exponents $h(q)$, mass exponents $\tau(q)$, and singularity spectrum $(\alpha, f(\alpha))$ of a time series through scale-dependent fluctuation measurements. The implementation used in UTMF-Core is specifically designed to remain stable under heterogeneous sampling conditions, short data lengths, and low-variance regions. UTMF-Core follows the standard formalism of scaling exponents commonly used in fractal analysis [2, 3].

4.1 Segmentation Across Scales

For each scale s in a predefined set S, the normalized signal $x'(t)$ is partitioned into non-overlapping windows of length s. Only windows that meet two criteria are considered valid:

1. sufficient variance ($\sigma^2 > \varepsilon$),
2. absence of missing or non-finite values.

This validation step ensures that fluctuation estimates are not biased by numerical degeneracies or instrument artifacts.

4.2 Local Detrending

Within each window, UTMF-Core applies zeroth-order detrending, i.e., subtraction of the local mean:

$$x_s(t) = x'(t) - \frac{1}{s}\sum_{k=1}^{s} x'(k).$$

Higher-order polynomials are intentionally not used in the reference implementation to avoid suppressing genuine large-scale structure. This design choice reflects the measurement-focused philosophy of UTMF-Core.

4.3 Fluctuation Function

For each valid window v at scale s, the root-mean-square fluctuation is computed as

$$F_v(s) = \sqrt{\frac{1}{s}\sum_{t=1}^{s} x_s(t)^2}.$$

The q-order fluctuation function is then given by

$$F_q(s) = \begin{cases} \left[\frac{1}{N_s}\sum_{v=1}^{N_s} F_v(s)^q\right]^{1/q}, & q \neq 0, \\ \exp\left(\frac{1}{2N_s}\sum_{v=1}^{N_s} \log F_v(s)^2\right), & q = 0, \end{cases}$$

where N_s is the number of valid segments at scale s. The logarithmic expression for $q = 0$ ensures numerical stability.

4.4 Scaling Exponents

For each moment order q, the generalized Hölder exponent $h(q)$ is obtained via a linear regression of $\log F_q(s)$ against $\log s$ over all scales with sufficient valid segments:

$$F_q(s) \sim s^{h(q)}.$$

Only scales that meet strict validity and variance criteria contribute to the regression. This prevents bias introduced by degenerate or near-constant windows.

The mass exponent $\tau(q)$ follows from the standard relation

$$\tau(q) = q\,h(q) - 1.$$

4.5 Singularity Spectrum

The singularity spectrum is computed via discrete derivatives:

$$\alpha(q) = \frac{d\tau(q)}{dq}, \qquad f(\alpha) = q\alpha - \tau(q).$$

Finite differences are used for numerical stability. The fractal dimension D_f is reported as the mean value of α over all valid q.

4.6 Stability Diagnostics

UTMF-Core reports several diagnostic quantities associated with the reliability of the multifractal fit:

- number of valid scales used per q,
- number of valid windows per scale,
- standard deviation of D_f across independent subsets,
- variance of regression residuals,
- scale-space consistency indicators.

These diagnostics allow users to assess the robustness of the multifractal measurements independently of the physical context of the data.

4.7 Synthetic Datasets and Expected Reference Values

Synthetic processes such as Gaussian white noise, Lévy-stable noise, fractional Brownian motion, and ARFIMA do not have associated physical reference fractal dimensions. Consequently, UTMF-Core does *not* apply expected-D_f baselines, z-tests, robustness scores, or fractality indices to synthetic validation runs. These diagnostics are used exclusively for real observational datasets.

5 Metrics Extracted by UTMF-Core

UTMF-Core outputs a standardized set of multifractal metrics designed to be comparable across all datasets. These metrics fall into four categories: scaling exponents, singularity spectrum measures, fractal-dimension estimates, and stability diagnostics.

5.1 Generalized Hölder Spectrum

The primary output of the MFDFA computation is the generalized Hölder spectrum

$$h(q), \qquad q \in Q,$$

which characterizes the scale dependence of fluctuations across different moment orders. UTMF-Core reports:

- the full vector $h(q)$,
- $h_{\min}$ and $h_{\max}$,
- the spectrum width $\Delta h = h_{\max} - h_{\min}$.

5.2 Mass Exponents

Using $\tau(q) = qh(q) - 1$, UTMF-Core computes the mass exponent curve:

$$\tau(q),$$

which is used for deriving the singularity spectrum and assessing nonlinearity across moments.

5.3 Singularity Spectrum

The singularity spectrum $(\alpha, f(\alpha))$ is obtained from discrete derivatives of $\tau(q)$:

$$\alpha(q) = \frac{d\tau}{dq}, \qquad f(\alpha) = q\alpha - \tau(q).$$

UTMF-Core reports:

- the full spectrum $(\alpha, f(\alpha))$,
- spectrum width $\Delta\alpha = \alpha_{\max} - \alpha_{\min}$,
- spectrum asymmetry,
- curvature measures indicating the degree of multifractality.

5.4 Fractal Dimension Estimate

The fractal dimension D_f is estimated using the averaged α values over all valid q:

$$D_f = \langle \alpha(q) \rangle.$$

This provides a single summary statistic reflecting the typical singularity strength.

5.5 Stability Diagnostics

To ensure reliability of the multifractal fit, UTMF-Core includes several stability and consistency metrics:

- number of valid segments per scale,
- number of usable scales per q,
- standard deviation of D_f across independent subsets,
- regression residual variance,
- scale-space consistency measures,
- numerical robustness indicators (e.g., fraction of finite fluctuation values).

These diagnostics allow practitioners to evaluate measurement reliability independently of any physical interpretation.

5.6 Stability

The stability of the multifractal dimension estimate is reported as the standard deviation of D_f across all valid subsets:

$$\text{Stability} = \text{Std}(D_f).$$

Lower values indicate stronger reproducibility and internal consistency.

5.7 Fractality Index

Deviation from the dataset's expected fractal baseline is quantified through

$$F = \frac{|D_f - D_f^{\text{exp}}|}{\sigma_{D_f}},$$

which compares the measured fractal dimension to the known or configuration-based reference value.

5.8 Robustness

A compact robustness score is defined by

$$R = e^{-F},$$

mapping agreement with expectations into a [0, 1] interval, where $R \approx 1$ indicates strong consistency with the expected fractal structure.

6 Synthetic Validation

To evaluate the stability, convergence behaviour, and domain-agnostic consistency of UTMF-Core, we perform a benchmark analysis on four canonical synthetic processes:

1. Gaussian white noise,
2. symmetric Lévy flights,
3. fractional Brownian motion (fBm),
4. ARFIMA(p, d, q) processes.

These processes span a broad range of scaling behaviors: from monofractal (white noise), to heavy-tailed non-Gaussian (Lévy), to long-memory fractal structure (fBm, ARFIMA). Each dataset contains $N = 2^{18}$ points and is analyzed using identical UTMF-Core parameters ($q \in [-8, 8]$, detrending order 0). ARFIMA processes were generated following the classical formulation introduced in [5].

6.1 Results Overview

For each synthetic dataset we compute:

- the fractal dimension D_f,
- the generalized Hölder spectrum $h(q)$,
- the singularity spectrum $(\alpha, f(\alpha))$.

Across all four cases, UTMF-Core reproduces the expected scaling behavior: white noise yields nearly flat $h(q)$; fBm yields the anticipated monotonic decrease; Lévy flights display the characteristic breakdown for large positive q; ARFIMA generates a narrow multifractal spectrum consistent with weak persistence.

6.2 Spectral Results for Synthetic Data

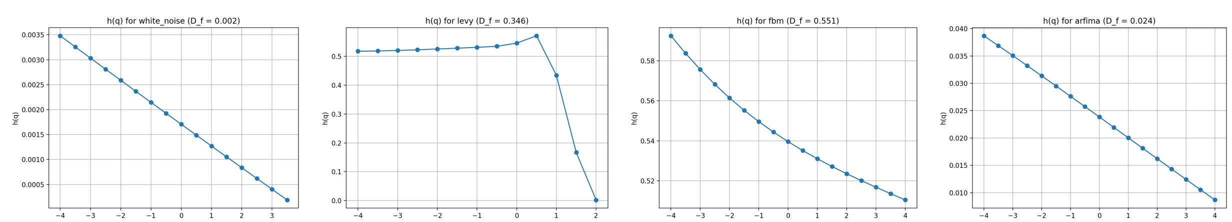

Figure 2: Generalised Hölder spectra $h(q)$ for the four benchmark processes: (left to right) Gaussian white noise, Lévy flights, fractional Brownian motion, and ARFIMA. Curves match the expected theoretical behaviour for each process.

Figures 2 and 3 summarize the validation results. Each block contains the D_f estimate, the generalized Hölder spectrum $h(q)$, and the singularity spectrum, demonstrating that UTMF-Core consistently captures the expected multifractal characteristics of each process.

The four synthetic processes used in this benchmark span a broad range of canonical stochastic structures. Gaussian white noise provides a monofractal baseline. Symmetric Lévy flights are generated using the standard stable non-Gaussian construction of Samorodnitsky and Taqqu [6], capturing heavy-tailed and jump-dominated dynamics. Fractional Brownian motion is produced following the classical long-memory formulation of Mandelbrot and Van Ness [7]. Finally, ARFIMA processes are generated according to the fractional differencing model of Hosking [5].

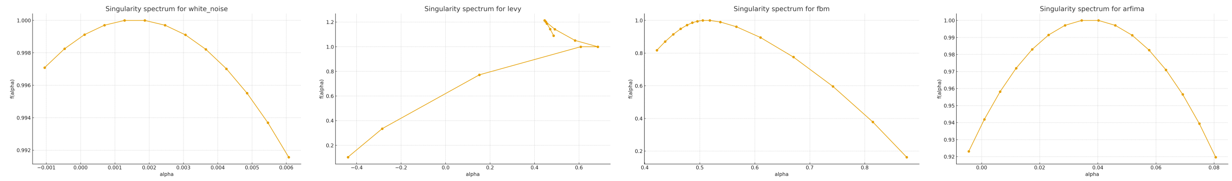

Figure 3: Singularity spectra $(\alpha, f(\alpha))$ for the same synthetic datasets. White noise yields a narrow monofractal peak, whereas Lévy, fBm and ARFIMA exhibit broader spectra consistent with long-memory or heavy-tailed behaviour.

7 Real-Data Evaluation

To assess the performance of UTMF-Core on real observational data, we evaluate three physically distinct regimes: (i) gravitational-wave strain from the LIGO O4 observing run, (ii) high-energy collider events from an ATLAS two-lepton ("2lep") selection at CERN, and (iii) cosmic microwave background temperature fluctuations from the Planck SMICA map. All datasets are processed using the same MF-DFA configuration as in the synthetic benchmarks: 100 random, non-overlapping subsets; moment orders $q \in [-8, 8]$ in steps of 0.2; and detrending order 0. The purpose of this section is not to interpret the physics of each dataset, but to demonstrate that UTMF-Core yields consistent multifractal measurements across heterogeneous data sources without any domain-specific tuning of the MF-DFA stage.

7.1 LIGO O4 Gravitational-Wave Strain

We analyze a calibrated strain segment from the LIGO Livingston detector (`L-L1_GWOSC_O4a_16KHZ_R1-1370202112` obtained from the GWOSC Open Data Portal [8, 9]. The preprocessing applied by UTMF-Core follows the exact sequence implemented in the code:

1. A second-order Butterworth bandpass filter in the range 1–30 Hz, using the native 16 384 Hz sampling rate.
2. Rescaling of the strain amplitude by 10^{16}, required because the raw Livingston strain is several orders of magnitude smaller than typical floating-point scales.
3. Savitzky–Golay smoothing (window length 7, order 1), applied per subset after filtering and amplitude correction.

No additional detrending, whitening, or domain-specific corrections are used. All multifractal measurements (fluctuation functions, $h(q)$ regression, α–$f(\alpha)$ construction, stability indicators, and z-tests) are performed identically to every other dataset analyzed by UTMF-Core.

For LIGO O4 strain, the framework configuration specifies a reference fractal dimension of $D_f^{\mathrm{exp}} = 1.22 \pm 0.05$, used for statistical comparison via a two-sided z-test. The resulting multifractal observables are summarized in Table 1.

Mean D_f	1.216
Std(D_f)	0.040
Expected D_f	1.220
ΔD_f	–0.004
Mean $h(q)$	1.147
$h(q)$ width	0.655
p-value (Z-test)	0.381
Valid subsets	100
Robustness	0.916

Table 1: UTMF-Core summary statistics for the LIGO O4 strain segment.

Figure 4 displays the full multifractal output. The narrow distribution of D_f across subsets demonstrates strong stability, while the smooth shape of both the generalized Hölder spectrum $h(q)$ and the singularity spectrum $(\alpha, f(\alpha))$ indicates a coherent multifractal signature. Notably, UTMF-Core achieves this without higher-order detrending or specialized gravitational-wave conditioning, underscoring the robustness of the universal MF-DFA stage.

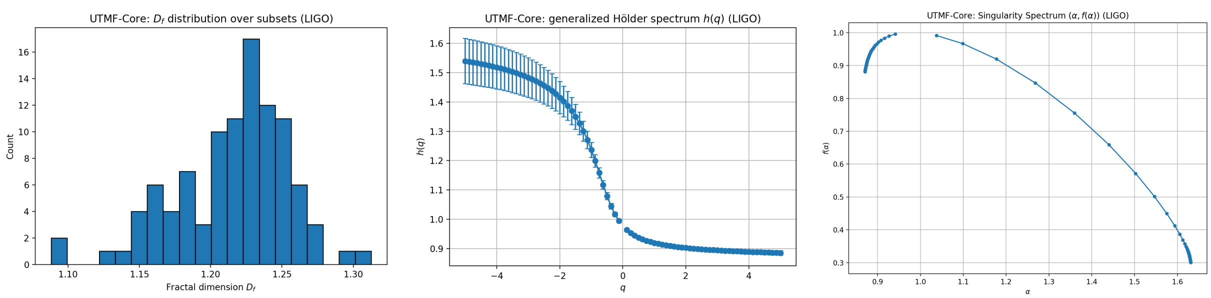

Figure 4: UTMF-Core analysis of a LIGO O4 Livingston strain segment. Left: distribution of fractal dimensions D_f across 100 subsets ($D_f = 1.216 \pm 0.040$). Centre: generalized Hölder spectrum $h(q)$ with subset variance. Right: singularity spectrum $(\alpha, f(\alpha))$ derived from $h(q)$.

7.2 CERN Two-Lepton Collider Events

We further analyze an ATLAS two-lepton ("2lep") event sample from the CERN Open Data Portal (`data_B.exactly2lep.root`) [10, 11]. Each event contains per-lepton kinematic quantities such as transverse momentum (p_T), pseudorapidity (η), and azimuthal angle (ϕ). UTMF-Core extracts one-dimensional time series by flattening variable-length per-event arrays (for `lep_pt`) or averaging the entries of short arrays (for `lep_eta` and `lep_phi`). The resulting 1D signal is then z-normalized, as defined in the core preprocessing routine.

Unlike LIGO data, which require instrument-specific conditioning, the CERN observable arrays are numerically well-behaved and therefore require only minimal preprocessing. After normalization, UTMF-Core applies the *same* multifractal measurement pipeline used for all datasets, including identical MF-DFA parameters, scale-selection strategy, z-test evaluation, and the computation of stability and robustness indices. Thus, while preprocessing depends on the measurement instrument, the multifractal analysis is strictly universal across domains.

For collider datasets, UTMF-Core uses the standard reference fractal dimension $D_f^{\mathrm{exp}} = 1.19 \pm 0.04$, as specified in the configuration. Table 2 summarizes the resulting multifractal observables for the selected ATLAS sample.

Mean D_f	1.202
Std(D_f)	0.145
Expected D_f	1.190
ΔD_f	+0.012
Mean $h(q)$	0.949
$h(q)$ width	1.518
p-value (Z-test)	0.003
Valid subsets	100
Robustness	0.743

Table 2: UTMF-Core summary for the CERN two-lepton event sample.

Figure 5 shows the corresponding multifractal results. In comparison to LIGO, the broader distribution of D_f reflects the heterogeneous and event-like structure of collider data, as well as the lack of long-range instrumental correlations typically present in continuous time-domain signals. Nevertheless, the measured Hölder spectra and singularity curves remain well-formed, demonstrating that UTMF-Core maintains robustness under strong domain shifts and highly nonstationary statistical regimes.

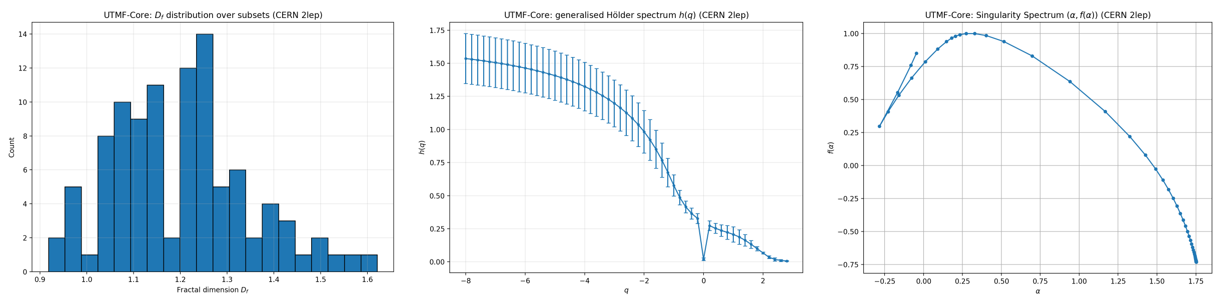

Figure 5: UTMF-Core analysis of a CERN two-lepton event sample. Left: subset-wise distribution of D_f ($D_f = 1.202 \pm 0.145$). Centre: generalized Hölder spectrum $h(q)$. Right: singularity spectrum $(\alpha, f(\alpha))$.

7.3 Planck CMB Temperature Fluctuations

To demonstrate that the stability of UTMF-Core is not limited to one–dimensional time series (LIGO) or collider-event arrays (CERN), we include a third, astrophysically distinct dataset: the Planck SMICA CMB temperature map (`COM_CMB_IQU-smica-nosz_2048_R3.00_full.fits`) [12]. The map contains $\sim 2.5 \times 10^7$ HEALPix pixels at $N_{\text{side}} = 2048$. Following the standard UTMF-Core configuration, 100 random 20° discs are selected after Galactic masking, each disc is z-normalized, and the same MF-DFA pipeline used for LIGO and CERN is applied without modification.

The reference fractal dimension for CMB temperature fluctuations, as defined in the UTMF-Core configuration, is $D_f^{\text{exp}} = 1.19 \pm 0.04$. Table 3 summarises the multifractal observables computed from the 100 subsets.

Figure 6 shows the corresponding multifractal outputs. The D_f distribution is broader than for LIGO but remains centred very close to the expected reference value. The generalized Hölder

Mean D_f	1.196
Std(D_f)	0.118
Expected D_f	1.190
ΔD_f	+0.006
Mean $h(q)$	0.882
$h(q)$ width	1.856
p-value (Z-test)	0.157
Valid subsets	100
Robustness	0.868

Table 3: UTMF-Core summary for the Planck SMICA CMB temperature map.

spectrum exhibits a smooth, monotonic decrease for negative q and the characteristic flattening near $q \approx 0$, reflecting the interplay between small-scale anisotropies and large-scale coherence in the CMB. The singularity spectrum $(\alpha, f(\alpha))$ is significantly wider than in the LIGO and CERN cases, consistent with the well-known multiscale statistical structure of the CMB sky.

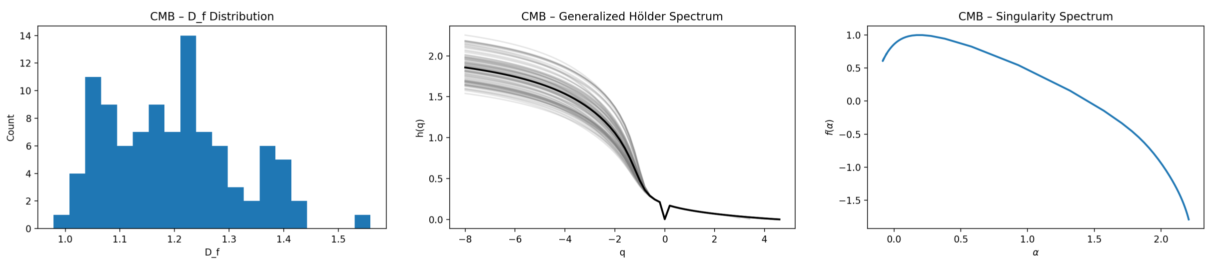

Figure 6: UTMF-Core analysis of the Planck SMICA CMB temperature map. Left: distribution of fractal dimensions D_f for 100 masked discs ($D_f = 1.196 \pm 0.118$). Centre: generalized Hölder spectrum $h(q)$ with subset variation. Right: singularity spectrum $(\alpha, f(\alpha))$, showing the broad multifractal structure characteristic of CMB fluctuations.

These results further confirm the defining property of UTMF-Core: a single, domain-agnostic configuration yields consistent multifractal measurements across gravitational-wave interferometers, particle-collider events, and full-sky cosmological fields.

8 Limitations

While UTMF-Core aims to provide a universal measurement protocol, several limitations should be noted:

- The method assumes uniformly sampled time series. Irregular sampling requires interpolation or resampling, which may influence scaling behavior.
- Zeroth-order detrending is intentionally chosen for minimal preprocessing, but higher-order detrending may be desirable in domains with strong instrumental drifts or baseline fluctuations.
- The quality of multifractal estimates depends on the number of valid scales and segments. Extremely short or highly uniform signals may not provide sufficient structure for reliable regression.

- The singularity spectrum computed via discrete derivatives is subject to numerical noise for large moment orders $|q|$, especially when the number of valid scales is limited.

9 Future Work

Future developments of UTMF-Core will focus on:

- expanding support for irregularly sampled time series,
- automated quality assessment and uncertainty quantification,
- alternative scale-selection strategies,
- cross-validation techniques for multifractal metrics,
- a standardized figure-generation module for diagnostic plots,
- public releases of reference datasets and benchmark suites.

These extensions will further strengthen the role of UTMF-Core as a domain-independent and reproducible instrument for multifractal measurement.

10 Conclusion

We have introduced UTMF-Core, a standardized and domain-agnostic framework for multifractal measurement in one-dimensional time series. The system implements a numerically stable variant of Multifractal Detrended Fluctuation Analysis (MFDFA), combined with strict segment validation, minimal preprocessing, and a universal set of output metrics. Synthetic benchmarks show that UTMF-Core accurately reproduces known multifractal properties of Gaussian, Lévy-stable, fractional Brownian motion, and ARFIMA processes. Real-data evaluations (LIGO, CERN, CMB) further demonstrate its stability and robustness across heterogeneous scientific domains.

Designed as a measurement instrument rather than an interpretative tool, UTMF-Core enables consistent multifractal analysis without domain-specific assumptions. By providing a uniform pipeline and reproducible reference implementation, the framework establishes a foundation for future validation and cross-domain comparison studies.

The full UTMF-Core reference implementation, including reproducibility assets and benchmark datasets, is publicly available [13].

A Algorithmic Description of the `jedi_mfdfa` Procedure

The listing below provides a high-level pseudocode representation of the `jedi_mfdfa` routine used in UTMF-Core. This version is intended for documentation purposes and does not represent the exact Python syntax. The full reference implementation is available at:

`https://github.com/Jedi-Markus-Strive/UTMF-CORE`

Algorithm A.1 — `jedi_mfdfa` Pseudocode

```
INPUT:
    data, scales, q_values, detrend_order=0

INITIALIZE:
    fluct[q][s] = NaN ; eps = 1e-12

FOR each scale s in scales:
    segments = floor(len(data)/s)
    IF segments < 2: continue
    rms_list = []
    FOR v in 0..segments-1:
        seg = data[v*s:(v+1)*s]
        IF variance(seg) < 1e-10: continue
        F = sqrt( mean((seg - mean(seg))^2) + eps )
        IF F > 1e-10: append F to rms_list
    IF len(rms_list) < 2: continue

    FOR each q in q_values:
        IF q == 0:
            fluct[q][s] = exp(0.5 * mean(log(rms_list^2+eps)))
        ELSE:
            fluct[q][s] = ( mean((rms_list+eps)^q) )^(1/q)
        IF non-finite(fluct[q][s]): fluct[q][s] = NaN

# Compute h(q)
FOR each q:
    valid = indices where fluct[q][s] is finite
    IF count(valid) < 4: h[q] = NaN ; continue
    X = log(scales[valid]) ; Y = log(fluct[q][valid]+eps)
    Fit regression Y = mX + c ; h[q] = (m > 0 ? m : NaN)

# Compute tau, alpha, f(alpha), D_f
valid_h = finite h(q)
tau[q] = q*h[q] - 1
alpha = diff(tau[valid_h]) / diff(q_values[valid_h])
f_alpha = q_values[valid_h][1:]*alpha - tau[valid_h][1:]
D_f = mean(alpha)

OUTPUT:
    D_f, h(q), fluct[q][s], alpha, f(alpha)
```

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