Summer Project Report

# Compact Stars and equation of state of nuclear matter

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# Contents

# Introduction

This was a 2 month summer project with the project title "Compact Stars and Equation of State of Nuclear Matter". Over the course of this project, I learned about dense matter, compact stars, numerical solutions of various equations of state, and pressure-mass differential equations. This is a report on the same, covering the material I learned and how I went about learning it.

# 1 Introduction to Dense Matter and Compact Stars

## 1.1 What is Dense Matter?

Matter at incredibly high density typically found inside of neutron stars is known as Dense Matter. This matter is very often described using QCD. The matter I explored during this project is relatively low(on the scale of QCD) in temperature and moderately dense.

## 1.2 Compact Stars

Compact stars are the second densest objects known to us currently, only behind black holes. They have masses of the order of the Sun, but a radius only around $R \sim 10$km. Thus, the mass of the Sun $M_\odot = 1.989 \cdot 10^{33}$ g is concentrated in a sphere with a radius 105 times smaller than that of the Sun, $R_\odot = 6.96 \cdot 10^5$ km. I thus estimate the average mass density in a compact star to be [1]

$$\rho \simeq 7 \cdot 10^{14} g\, cm^{-3} \tag{1}$$

This is denser than the density of heavy nuclei.

Mass and radius of the star is determined by the equation(s) of state(s) of the matter phase inside the star.
We will initially look at the traditional picture of the neutron star, i.e. the matter is neutron-rich nuclear matter. Later, we will also take a look at quark matter compact star.

Some properties of the neutron star are

- Compact stars are born in a supernova
- Compact stars have extreme rotation periods along with density
- Compact stars have huge magnetic fields
- Compact stars are cold on the scale of QCD.

## 2 Mass and Radius of the Star

To theoretically find the mass and radius of the star, we need to connect the microscopic interactions of matter in a compact star to it. This is done by the equation of state which can be used to find the maximum mass of a star. Using a simple estimate, $R > R_S$, where R is the radius of the star and $R_S = 2MG$ is the Schwarzschild radius, with the mass of the star M and the gravitational constant G. Let us take a star made of A nucleons of mass $\simeq$ 939 MeV and a distance $r_0 = 0.5 \cdot 10^{-13}$ cm. We thus cover a volume $\sim r_0^3 A$ and get a radius

$$R \sim r_0 A^{\frac{1}{3}} \tag{2}$$

and a mass

$$M \sim mA \tag{3}$$

From $R_S = 2MG$, we obtain the following.

$$A \sim (\frac{r_0}{2MG})^{\frac{3}{2}} \sim 2.6 \cdot 10^{57} \tag{4}$$

This is the maximum number of nucleons that can be inside our star before it becomes unstable. We can get a maximum radius and mass using this.

$$R \sim 7km \quad M \sim 2.3M_{\odot} \tag{5}$$

Now, let us find the differential pressure and mass at a given radius r

$$dm = \rho(r)dV \text{ where } dV = 4\pi r^2 dr \tag{6}$$

$$dP = \frac{dF}{4\pi r^2} \tag{7}$$

where,

$$dF = -\frac{Gm(r)dm}{r^2} \tag{8}$$

Thus we get, expressing through $\epsilon(r)$

$$\frac{dm}{dr} = 4\pi r^2 \epsilon(r) \tag{9a}$$

$$\frac{dP}{dr} = -\frac{G\epsilon(r)m(r)}{c^2 r^2} \tag{9b}$$

### 2.1 Non Interacting Nuclear Matter

The derivation here is from [1]. I need to write it here because... this is what I learned in the project.
We start with a very simple system, where we neglect all interactions. In this case, all we need are basic statistical physics and thermodynamics. The thermodynamic potential for the grand-canonical ensemble is given by

$$\Omega = E - \mu N - TS \tag{10}$$

with the energy E, chemical potential µ, particle number N, temperature T and entropy S. The pressure is then

$$P = \frac{\Omega}{V} = \epsilon - \mu n - Ts \tag{11}$$

where V is the volume of the system. Number density n = N/V, energy density $\epsilon$ = E/V, and entropy density s = S/V are, for a fermionic system, given by

$$n = 2\int \frac{d^3\mathbf{k}}{(2\pi)^3} f_k \tag{12a}$$

$$\epsilon = 2\int \frac{d^3\mathbf{k}}{(2\pi)^3} E_k f_k, \tag{12b}$$

$$s = -2\int \frac{d^3\mathbf{k}}{(2\pi)^3} \left[(1-f_k)\ln(1-f_k) + f_k \ln f_k\right]. \tag{12c}$$

We will look at a system of neutrons(n), protons(p), and electrons(e), each contributing to pressure. Since they are spin-$\frac{1}{2}$ fermions, there is a factor 2 for the two degenerate spin states. The fermi distribution function is denoted by $f_k$

$$f_k \equiv \frac{1}{e^{(E_k-\mu)/T} + 1} \tag{13}$$

and the single particle energy is

$$E_k = \sqrt{k^2 + m^2} \tag{14}$$

Inserting all this into equation (11c), we get

$$P = 2T \int \frac{d^3\mathbf{k}}{(2\pi)^3} \ln\left[1 + e^{(E_k - \mu)/T}\right] \tag{15}$$

For T = 0, our fermi distribution is a step function $f_k = \Theta(k_F - k)$, and thus will be cut off at the Fermi momentum $k_F$

$$n = \frac{1}{\pi^2}\int_0^{k_F} dk\, k^2 = \frac{k_F^3}{3\pi^2}, \tag{16a}$$

$$\begin{aligned}\epsilon &= \frac{1}{\pi^2}\int_0^{k_F} dk\, k^2\sqrt{k^2+m^2} \\ &= \frac{1}{8\pi^2}\left[2k_F^3 + m^2 k_F\sqrt{k_F^2+m^2} - m^4 \ln\left(\frac{k_F + \sqrt{k_F^2+m^2}}{m}\right)\right].\end{aligned} \tag{16b}$$

$$\begin{aligned}P &= \frac{1}{\pi^2}\int_0^{k_F} dk\, k^2(\mu - \sqrt{k^2+m^2}) \\ &= \frac{1}{24\pi^2}\left[(2k_F^3 - 3m^2k_F)\sqrt{k_F^2+m^2} + 3m^4 \ln\frac{k_F + \sqrt{k_F^2+m^2}}{m}\right].\end{aligned} \tag{16c}$$

For n, p, e matter, the total pressure is

$$P = \frac{1}{\pi^2}\sum_{i=n,p,e}\int_0^{k_{F,i}} dk k^2(\mu_i - \sqrt{k^2 + m_i^2}). \tag{17}$$

The Fermi momenta can be thought of as variational parameters which have to be determined from maximising the pressure, i.e. the conditions

$$\frac{\partial P}{\partial k_{F,i}} = 0, \qquad i = n, p, e \tag{18}$$

which implies

$$\mu_i = \sqrt{k_{F,i}^2 + m_i^2} \tag{19}$$

There are some additional constraints. First, the star must be electrically neutral, i.e.

$$n_e = n_p \tag{20}$$

Hence,

$$k_{F,e} = k_{F,p} \tag{21}$$

Second, there must be chemical equilibrium with respect to the weak process

$$n \to p + e + \bar{v}_e \tag{22a}$$
$$p + e \to n + \bar{v}_e \tag{22b}$$

The first of these equations is called the $\beta-$decay, the second is called the inverse $\beta$ - decay or electron capture. We shall assume that the neutrino chemical potential vanishes, $\mu_{v_e} = 0$. This is equivalent to assuming that neutrinos and antineutrinos, once created by the above processes, simply leave the system without further interaction. This assumption is justified for compact stars since the neutrino mean free path is of the order of the size of the star or larger (except for the very early stages in the life of the star). Thus we get,

$$\mu_n = \mu_p + \mu_e \tag{23}$$

which then gives

$$\sqrt{k_{F,n}^2 + m_n^2} = \sqrt{k_{F,p}^2 + m_p^2} + \sqrt{k_{F,e}^2 + m_e^2} \tag{24}$$

We can use charge neutrality to get

$$k_{F,p}^2 = \frac{(k_{F,n}^2 + m_n^2 - m_e^2)^2 - 2(k_{F,n}^2 + m_n^2 + m_e^2)m_p^2 + m_p^4}{4(k_{F,n}^2 + m_n^2)}. \tag{25}$$

To illustrate the physical meaning of this relation, let us consider some limit cases. First, assume a vanishing proton contribution, $k_{F,p} = 0$. Then the equation gives

$$k_{F,n}^2 = (m_p + m_e)^2 - m_n^2 < 0. \tag{26}$$

This expression is negative because the neutron is slightly heavier than the electron and the proton together, $m_p \simeq 938.3$MeV, $m_n \simeq 939.6$MeV, $m_e \simeq 0.511$MeV. Therefore, $k_{F,p} = 0$ is impossible, and there always has to be at least a small fraction of protons or electrons. Now let's assume $k_{F,n} = 0$, which leads to

$$k_{F,p}^2 = \left(\frac{m_n^2 + m_e^2 - m_p^2}{2m_n}\right)^2 - m_e^2 \approx 1.4\,\mathrm{MeV}^2. \tag{27}$$

This is the threshold below which there are no neutrons, and the charge-neutral system in $\beta$-equilibrium contains only protons and electrons of equal number density.

In general, we may consider a given baryon density $n_B = n_n + n_p$ to express the neutron Fermi momentum as

$$k_{F,n} = (3\pi^2 n_B - k_{F,p}^3)^{1/3}. \tag{28}$$

Inserting this into the equation 25 yields an equation for $k_{F,p}$ as a function of the baryon density. In the ultrarelativistic limit, i.e., neglecting all masses, equation 25 obviously yields $k_{F,p} = k_{F,n}/2$ and thus $n_p = n_n/8$ or

$$n_p = \frac{n_B}{9}. \tag{29}$$

As an approximation, we may thus consider the simple case of pure neutron matter. We also consider the non-relativistic limit, $m_n \gg k_{F,n}$. In this case, the energy density 16b and the pressure 16c become

$$\epsilon \approx \frac{m_n^4}{3\pi^2}\left[\frac{k_{F,n}^3}{m_n^3} + \mathcal{O}\left(\frac{k_{F,n}^5}{m_n^5}\right)\right], \tag{30a}$$

$$P \approx \frac{m_n^4}{15\pi^2}\left[\frac{k_{F,n}^5}{m_n^5} + \mathcal{O}\left(\frac{k_{F,n}^7}{m_n^7}\right)\right]. \tag{30b}$$

Consequently, keeping the terms in the lowest order in $k_{F,n}/m_n$,

$$P(\epsilon) \approx \left(\frac{3\pi^2}{m_n}\right)^{5/3} \frac{\epsilon^{5/3}}{15 m_n \pi^2}. \tag{31}$$

Thus, we have found the non-relativistic EoS for non-interacting nuclear matter.

Now, to find the equation of state in the relativistic case, we can take the opposite limit, i.e. $k_{F,n} \gg m_n$. In this case, the energy density 16b and pressure 16c become

$$\epsilon \approx \frac{k_F^4}{4\pi^2} \tag{32a}$$

$$P \approx \frac{k_F^4}{12\pi^2} \tag{32b}$$

Thus we get our relativistic EoS

$$P = \frac{\epsilon}{3} \tag{33}$$

#### 2.1.1 Numerical solution of non relativistic EoS

Our equation of state is

$$P = K\epsilon^{\gamma} \tag{34}$$

where $K = \frac{h^2}{15\pi^2 m_n}\left(\frac{3\pi^2}{m_n c^2}\right)^{5/3}$ and $\gamma = 5/3$. We also know the differential equations for pressure and mass in radius r

$$\frac{dm}{dr} = \frac{4\pi}{K^{1/\gamma}} r^2 P(r)^{1/\gamma} \tag{35a}$$

$$\frac{dP}{dr} = -\frac{G}{K^{1/\gamma}} \frac{P(r)^{1/\gamma} m(r)}{c^2 r^2} \tag{35b}$$

Using the above equations, we can find the mass and radius of compact stars for a given set of initial conditions. I coded everything in Python 3.10.

Transcribing the above equations to Python and trying to run an integrator will result in failure, as the value of the differential pressure is far too small compared to the value of initial pressure that you could realistically provide, and hence it will result in a star with infinite mass and radius. To avoid running into this issue, we will be adjusting the values of all the constants using the appropriate units and dimensions. This method was taken from [2], I'm explaining it here because... this is what I learned in the project.

We shall first define $\bar{\mathcal{M}} = \mathcal{M}/M_\odot$. Equation 9a becomes

$$\frac{dP}{dr} = -R_0 \frac{\epsilon(r)\bar{\mathcal{M}}(r)}{r^2} \tag{36}$$

where $R_0 = GM_\odot/c^2 = 1.47km$. In this equation, P and $\epsilon$ still carry units of ergs/cm$^3$. Let us define dimensionless energy density and pressure as follows,

$$p = \epsilon_0 \bar{p} \quad \epsilon = \epsilon_0 \bar{\epsilon} \tag{37}$$

where $\epsilon_0$ has the dimensions of the energy density. This will allow us to ensure that the values of pressure are convenient. For the polytrope, we shall write

$$\bar{p} = \bar{K}\bar{\epsilon} \text{ where } \bar{K} = K\epsilon_0^{1/\gamma} \tag{38}$$

Recasting it in the form of $\bar{p}$

$$\bar{\epsilon} = (\bar{p}/\bar{K})^{1/\gamma} \tag{39}$$

Equation 36 then becomes

$$\frac{d\bar{p}(r)}{dr} = -\frac{\alpha\, \bar{p}^{1/\gamma}\bar{\mathcal{M}}(r)}{r^2} \tag{40}$$

where the constant $\alpha$ is

$$\alpha = R_0/\bar{K}^{1/\gamma} \tag{41}$$

Equation 40 has dimensions of 1/km, with $\alpha$ in km (since $R_0$ is). That is, it is to be integrated with respect to r, with r also in km.

We can choose any convenient value for $\alpha$ since $\epsilon_0$ is also free. For a given value of $\alpha$, $\epsilon_0$ is fixed as

$$\epsilon_0 = \left[ \frac{1}{K} \left( \frac{R_0}{\alpha} \right) \right]^{1/\gamma} \tag{42}$$

Equation 35a becomes

$$\frac{d\bar{\mathcal{M}}}{dr} = \beta r^2 \bar{p}(r)^{1/\gamma} \tag{43}$$

where $\beta$ is

$$\beta = \frac{4\pi\epsilon_0}{M_\odot c^2 \bar{K}^{1/\gamma}} \tag{44}$$

Now that the appropriate units and dimensions are being used, we can write a program in any Python using a define function to write the equations. We can then give initial values for pressure and mass(=0) and use the scipy.solve_ivp function to numerically solve differential equations. Then we can graph the radius on X-axis with mass and radius on Y-axis to observe our results.

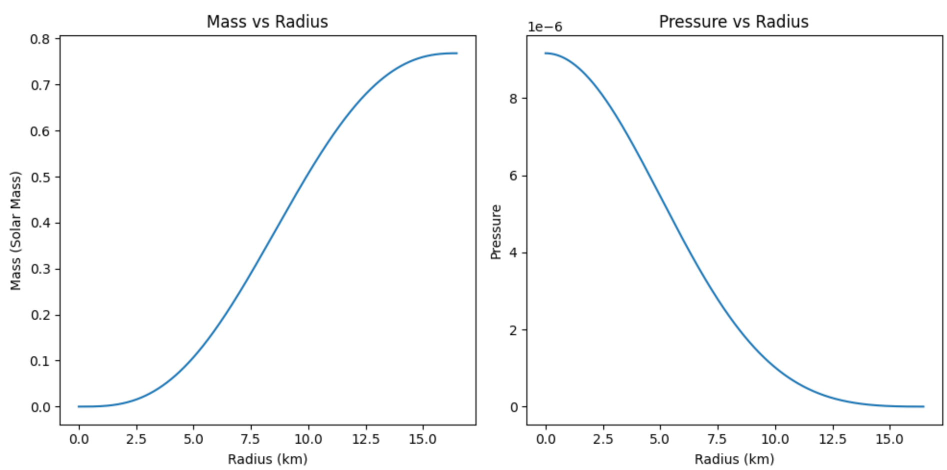


Figure 1: Mass vs Radius, Pressure vs Radius for a given initial pressure.

This is the output of the code that I get for the initial value of $\bar{p}(0) = 10^{-4}$(depending on your implementation, you may have to give the initial value of p instead of $\bar{p}$, so keep that in mind).

| $\bar{p}(0)$ | M($M_\odot$) | R(km) |
|---|---|---|
| $10^{-4}$ | 0.7682 | 16.47 |
| $10^{-5}$ | 0.3850 | 20.73 |
| $10^{-6}$ | 0.1930 | 26.10 |

Table 1: Values of Mass($M_\odot$) and Radius(km) for different values of initial pressure

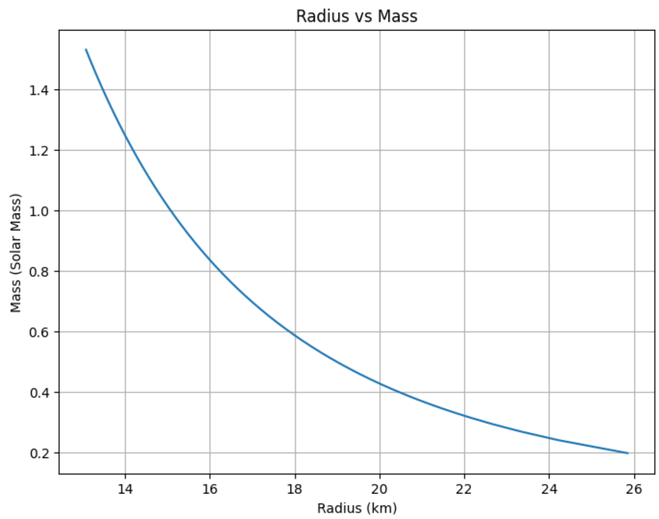


Figure 2: Mass vs Radius for various values of initial pressures.

As we can see from the above figure, this EoS does not have a limit towards the top, as it is purely made from the non relativistic part. To account for this, we have to consider the full EoS.

#### 2.1.2 Numerical solution of Full EoS

To derive the full EoS, we simply need to combine the elements of both the non-relativistic EoS and relativistic EoS. Since we know both,

$$P = K_1 \epsilon^{3/5} \tag{45a}$$

$$P = K_2 \epsilon \tag{45b}$$

We can take a hint from the two polytropes, and try to fit the energy density as a function of pressure, each given as a transcendental function of $k_F$ , with the form

$$\bar{\epsilon}(p) = A_{NR}\bar{p}^{3/5} + A_R\bar{p} \tag{46}$$

For low pressures, the non-relativistic term dominates over the second. We shall redefine our $\epsilon_0$ value to fit our equations appropriately here.

$$\epsilon_0 = \frac{m_n^4 c^5}{3\pi^2 \hbar^3} = 5.346 \times 10^{36} \frac{\text{ergs}}{\text{cm}^3} = 0.003006 \frac{M_\odot c^2}{\text{km}^3} \tag{47}$$

We can find the values of $A_{NR}$ and $A_R$ by plotting the full EoS and fitting equation 46 to it. Doing so gives us,

$$A_{NR} = 2.4216, \quad A_R = 2.8663 \tag{48}$$

Now we can implement the full EoS to find the mass and radius of a neutron star.

| $\bar{p}(0)$ | M($M_\odot$) | R(km) |
|---|---|---|
| $10^{-2}$ | 0.7551 | 13.16 |
| $10^{-3}$ | 0.5142 | 18.39 |
| $10^{-4}$ | 0.2957 | 24.27 |

Table 2: Values of Mass($M_\odot$) and Radius(km) for different values of initial pressure

As we can see, the solution works for both relativistic and non-relativistic values of pressure.

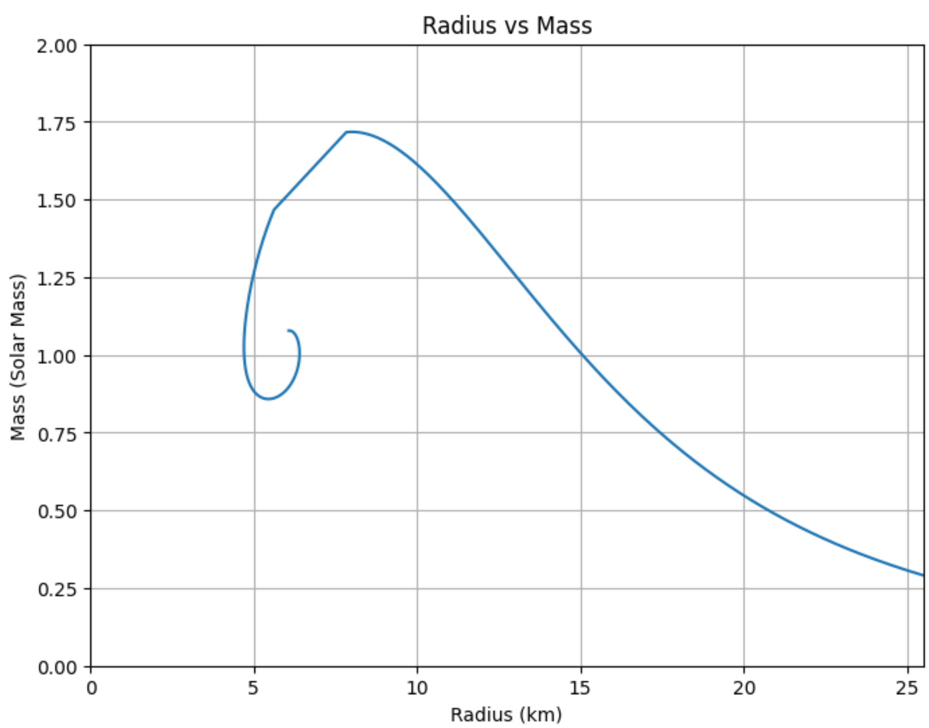


Figure 3: Mass vs Radius for various values of initial pressures.

As we see here, the mass vs radius after the top of the graph keep on spiralling in towards a certain point. They are actually unstable and collapse in on itself due to gravity.

#### 2.1.3 Why is there a limiting mass?

The limiting mass, also known as the maximum mass, is a property of the star. But why should there be a limiting mass?

An increase in density results in a proportional increase in energy density. This results in a corresponding increase in the gravitational attraction. To balance this, we require the increment in pressure to be large enough. However, the rate of change of pressure with respect to the energy density is related to the speed of sound. In a purely Newtonian world, this is in principle unbounded. However, the speed of all propagating signals cannot exceed the speed of light. This then puts a bound on the pressure increment associated with changes in density. Once we accept this bound, we can safely conclude that all cold compact objects will eventually run into the situation in which any increase in density will result in an additional gravitational attraction that cannot be compensated for by the corresponding increase in pressure. This leads naturally to the existence of a limiting mass for the star.

### 2.2 Non Interacting Quark Matter

The derivation here is from [1]. I need to write it here because... this is what I learned in the project.

We shall only look at up(u), down(d), and strange(s) quarks in this report, as the chemical potential inside of a star is too low for the other three flavours of quarks. We shall ignore the masses of the up and down quark which are $m_u \simeq m_d \simeq 5MeV \ll \mu \simeq (300-500)MeV$. The mass of the strange quark, however, is not negligible which is $m_s \simeq 90MeV$.

We only have to remember that there are three colors for each quark flavor, $N_c = 3$, i.e., the degeneracy factor for a single quark flavor is $2N_c = 6$, where the factor 2 counts the spin degrees of freedom. Consequently, for each quark flavor f = u,d,s we have at zero temperature

$$n_f = \frac{k_{F,f}^3}{3\pi^2}, \tag{49a}$$

$$\epsilon_f = \frac{3}{\pi^2}\int_0^{k_{F,f}} dk\, k^2\sqrt{k^2+m_f^2} \tag{49b}$$

$$P_f = \frac{3}{\pi^2}\int_0^{k_{F,f}} dk\, k^2(\mu_f - \sqrt{k^2+m_f^2}) \tag{49c}$$

We need to impose equilibrium conditions with respect to the weak interactions. Here, the relevant processes are the leptonic processes.

$$d \to u + e + \bar{v}_e, \qquad s \to u + e + \bar{v}_e, \tag{50a}$$

$$u + e \to d + v_e, \qquad u + e \to s + v_e. \tag{50b}$$

and the non leptonic processes

$$s + u \to d + u. \tag{51}$$

This gives us the following chemical potential relations

$$\mu_d = \mu_e + \mu_u, \qquad \mu_d = \mu_e + \mu_u. \tag{52}$$

The charge neutrality is written as

$$\sum_{f=u,d,s} q_f n_f - n_e = 0, \tag{53}$$

with the electric quark charges

$$q_u = \frac{2}{3} \qquad q_d = q_s = -\frac{1}{3}. \tag{54}$$

and the electron density $n_e$.

For the quark matter EoS, we first need to be introducted to the bag model. In quarks, it is observed that when they are very close together, they're free to move around, but as they get farther and farther apart, the force binding them together becomes stronger. Thus, the bag model is a simplistic model to explain this action. It can be included in our model as

$$P + B = \sum_f P_f, \tag{55a}$$

$$\epsilon = \sum_f \epsilon_f + B \tag{55b}$$

Now, let us first consider a model with massless quarks, $m_f = 0$. This gives us

$$n_f = \frac{\mu_f^3}{\pi^2}, \quad \epsilon_f = \frac{3\mu_f^4}{4\pi^2}, \quad P_f = \frac{\mu_f^4}{4\pi^2}. \tag{56}$$

Thus we get,

$$P_f = \frac{\epsilon_f}{3} \tag{57}$$

Hence, we get the equation of state for massless quarks as

$$P = \frac{\epsilon - 4B}{3} \tag{58}$$

Now, we can consider a more realistic approximation, where the mass of the strange quark is taken into account, while masses of the up and down quark is taken as 0.

We include the effect of the strange quark mass to the lowest order and also include electrons. It is convenient to express the quark chemical potentials in terms of an average quark chemical potential $\mu = \mu_u + \mu_d + \mu_s/3$ and the electron chemical potential $\mu_e$,

$$\mu_u = \mu - \frac{2}{3}\mu_e, \tag{59}$$

$$\mu_d = \mu + \frac{1}{3}\mu_e, \tag{60}$$

$$\mu_s = \mu + \frac{1}{3}\mu_e. \tag{61}$$

Taking into account the strange quark mass, the Fermi momenta for the approximately massless up and down quark and the massive strange quark are given by

$$k_{F,u} = \mu_u, \tag{62}$$

$$k_{F,d} = \mu_d, \tag{63}$$

$$k_{F,s} = \sqrt{\mu_s^2 - m_s^2}. \tag{64}$$

The energy density and the pressure are

$$\sum_{i=u,d,s,e} \epsilon_i = \frac{3\mu_u^4}{4\pi^2} + \frac{3\mu_d^4}{4\pi^2} + \frac{3}{\pi^2}\int_0^{k_{F,s}} dk\, k^2 \sqrt{k^2 + m_s^2} + \frac{\mu_e^4}{4\pi^2}, \tag{65}$$

$$\sum_{i=u,d,s,e} P_i = \frac{\mu_u^4}{4\pi^2} + \frac{\mu_d^4}{4\pi^2} + \frac{3}{\pi^2}\int_0^{k_{F,s}} dk\, k^2 \left(\mu_s - \sqrt{k^2 + m_s^2}\right) + \frac{\mu_e^4}{12\pi^2}, \tag{66}$$

where we have neglected the electron mass. The neutrality condition can now be written as

$$0 = \frac{\partial}{\partial \mu_e} \sum_{i=u,d,s,e} P_i = -\frac{2}{3}n_u + \frac{1}{3}n_d + \frac{1}{3}n_s + n_e. \tag{67}$$

(Note that $\mu_e$ is defined as the chemical potential for negative electric charge.) Solving this equation to lowest order in the strange quark mass yields

$$\mu_e \simeq \frac{m_s^2}{4\mu}. \tag{68}$$

Consequently, the quark Fermi momenta become

$$k_{F,u} \simeq \mu - \frac{m_s^2}{6\mu}, \tag{69}$$

$$k_{F,d} \simeq \mu + \frac{m_s^2}{12\mu}, \tag{70}$$

$$k_{F,s} \simeq \mu - \frac{5m_s^2}{12\mu}. \tag{71}$$

Now, we can insert this into the pressure and energy density formulas.

$$\sum_i \epsilon_i \simeq \frac{9\mu^4}{4\pi^2} - \frac{3\mu^2 m_s^2}{4\pi^2}, \tag{72}$$

$$\sum_i P_i \simeq \frac{3\mu^4}{4\pi^2} - \frac{3\mu^2 m_s^2}{4\pi^2}. \tag{73}$$

Consequently,

$$\sum_i \epsilon_i \simeq 3\sum_i P_i + \frac{3\mu^2 m_s^2}{2\pi^2}. \tag{74}$$

Now taking into account the bag constant, we get,

$$P \simeq \frac{3\mu^4}{4\pi^2} - \frac{3\mu^2 m_s^2}{4\pi^2} - B, \tag{75}$$

and the EoS becomes,

$$P \simeq \frac{\epsilon - 4B}{3} - \frac{\mu^2 m_s^2}{2\pi^2}. \tag{76}$$

This is the equation of state of noninteracting, unpaired strange quark matter within the bag model with strange quark mass corrections to the lowest order.

### 2.2.1 Numerical solution of quark matter EoS for massless quarks

The equation of state in this case is

$$P = \frac{\epsilon - 4B}{3} \quad \text{where} \quad B = \frac{3\mu^4}{4\pi^2} \tag{77}$$

We can take the value of B as given and take it as $B^{1/4} = 160MeV$. Now, we can implement this similarly to how we have implemented every other equation of state.

But, there's a catch! If you directly try to implement it, you will notice that there is always some error(if you implemented it in the same way I did, there are different ways to implement it which avoid this altogether). In my case, it just refused to integrate it all. At first, I thought the issue was that

$\epsilon_0$ was defined in terms of alpha, and alpha here was a very small value, so $\epsilon_0$ will be very close to 1, throwing away the whole point of making it dimensionless out of the window. But I wasn't exactly correct. So what was the real problem?

If you look back at the derivation of these dimensionless quantities, you'll notice that since it's not a polytrope, things will be slightly different now. Very obviously, alpha and $\epsilon_0$ do not have a direct relationship in this case, in fact, they are completely independent, which means that alpha has a definite value of $R_0/3$. So, we're completely free to take whatever value of $\epsilon_0$ we want. You can experiment around with this value, I found that $1.6 \times 10^{38}$ $\frac{\text{ergs}}{\text{cm}^3}$ worked the best for me.

If we implement it like this, using the proper conversions for B as well, we will get the following mass vs radius graph for various values of initial pressure.

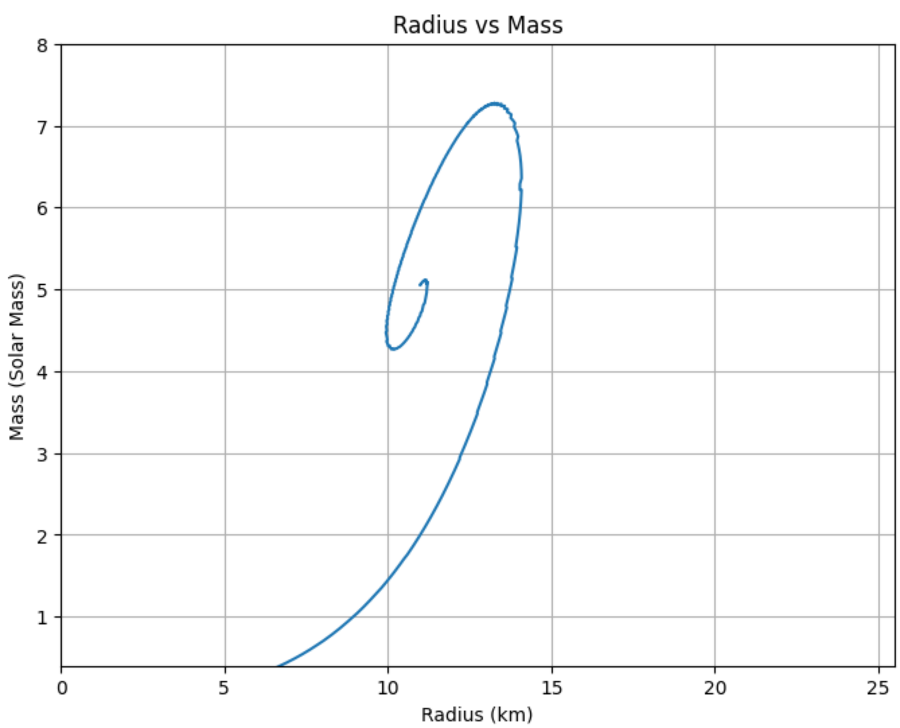


Figure 4: Mass vs Radius for various values of initial pressures.

It is very clear that this is not very correct theoretically, but this is because we are using Newtonian equations.

#### 2.2.2 Numerical solution of quark matter EoS with mass of strange quark

The equation of state in this case will be

$$P = \frac{\epsilon - 4B}{3} - \frac{\mu^2 m_s^2}{2\pi^2} \quad \text{where} \quad B = \frac{3\mu^4}{4\pi^2} \tag{78}$$

This is pretty much the same as the above case, so I won't be elaborating too much on the implementation of this. The mass of strange quark is taken here as 90 MeV and $\mu$ is taken as 400 MeV.

The graph below is what we end up getting with these new equations.

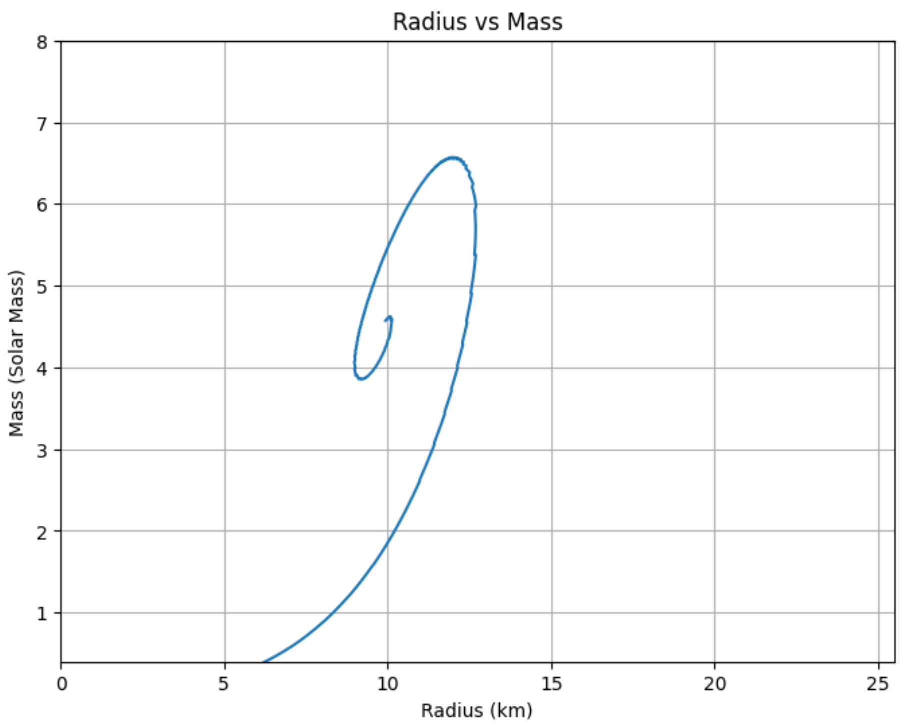


Figure 5: Mass vs Radius for various values of initial pressures.

As we can see, this is a bit more sensible compared to the previous implementation, but still way over the mass of any observed neutron star. If instead of Newtonian equations, the TOV equation is directly implemented, we get a much more realistic answer.

# A Fitting the equation of state equation to find the appropriate constants

In Section 2.1.2, I simply state the fitted equation of state, without any further description on the matter. Here, I will give a rough outline of how I came to these specific numbers. First, we take equations 16b and 16c, and transcribe them into the program. Then calculate the pressure and energy density values for various values of the Fermi momentum. Then, a plot of $\epsilon$ vs P is made. Now, I have a numerical solution of the full pressure and energy density. Now you can take equation 46 and fit it to the graph you get by solving the equations. This can be done with the scipy module from Python.

Do keep in mind that the equation 46 uses the barred values of pressure and energy density, so you need to adjust the pressure vs. energy density solutions accordingly. That is how I got the values of $A_{NR}$ and $A_R$. Initially, I only got it from [2], but then decided to calculate it independently and got values close to theirs.

# B Use of ChatGPT in the project.

I used ChatGPT for several purposes in this project. I did not use it to produce results for me, I rather used it to make my life easier. The following are the ways I used ChatGPT,

1. During the initial reading, a lot of mathematical nuances were lost to me. I used ChatGPT to help me understand a few derivations if I could not manage it myself. E.g. I could not understand why equation 16a cut off at Fermi momentum $k_F$ initially. Looking back, it seems simple now but I was thoroughly confused at the time. Another one is equation 30a and 30b, I did not know what $\mathcal{O}$ meant. ChatGPT helped me to understand the concept in these situations.

2. I also used ChatGPT for help with the programming part of the project. E.g. I did not know how to export the mass vs radius values for various values of initial pressure to a file. I did not use it to directly write any programs. Additionally, GPT 3.5 cannot write code for the project work I've done here generally.

3. Debugging. Sometimes, I might make some silly mistake, e.g. forgetting to close the bracket in an equation. It can fix bugs, but it isn't consistent.

4. I broke my left hand in June. I could not type properly for a few weeks. ChatGPT was helpful in this case as I could ask it to give me a rough outline of the things I wanted to do. It was usually incorrect, but fixing the incorrect parts was faster than writing it from scratch when my hand was broken. Under normal circumstances, I would recommend doing it by hand.

A few limitations I've noticed in my experience are

1. It is very inconsistent. Sometimes it will give a correct answer, and sometimes it will give an incorrect answer. The inconsistency is proportional to the difficulty/rarity of the questions.

2. It gives incorrect answers very confidently. So, you can't take what it says for granted, and need to verify it always.

3. It ignores instructions. This is relatively rare, but it can happen frustratingly often, depending on the prompts.

# References

[1] Andreas Schmitt. *Dense matter in compact stars: A pedagogical introduction*. Vol. 811. Springer, 2010.

[2] Richard R Silbar and Sanjay Reddy. "Neutron stars for undergraduates". In: *American journal of physics* 72.7 (2004), pp. 892–905.