

The Mathematical Association of Victoria

Trial Examination 2016

SPECIALIST MATHEMATICS

Written Examination 2

STUDENT NAME _____

Reading time: 15 minutes

Writing time: 2 hours

QUESTION & ANSWER BOOK

Structure of Book

<i>Section</i>	<i>Number of Questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
A	20	20	20
B	6	6	60
		Total	80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 23 pages.
- Formula sheet
- Answer sheet for multiple-choice questions.

Instructions

- Write your **name** in the space provided above on this page.
- Write your **name** on the multiple-choice answer sheet
- Unless otherwise indicated, the diagrams are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION A**Instructions for Section A**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$

Question 1

If $2i$ is a solution to the equation $z^3 - 5z^2 + 4z - mi = 0$ then the value of m is

- A. $-20i$
- B. -20
- C. $-16i$
- D. $16i$
- E. 20

Question 2

If $z = x + iy$, where x and y are non zero real numbers, then which one of the following is a real number

- A. $z - \bar{z}$
- B. $\frac{1}{z} - \frac{1}{\bar{z}}$
- C. z^2
- D. $\frac{1}{z} + \frac{1}{\bar{z}}$
- E. $z - i\bar{z}$

Question 3

For the vectors $\underline{a} = \underline{i} + 2\underline{j} - 2\underline{k}$, $\underline{b} = 3\underline{i} - \underline{j} + 2\underline{k}$ and $\underline{c} = x\underline{i} - 7\underline{j} + 10\underline{k}$ to be linearly dependent, the value of x must be

- A. -2
- B. 7
- C. 4
- D. -7
- E. 2

SECTION A – continued

TURN OVER

Question 4

The vectors \underline{p} and \underline{q} are given by $\underline{p} = 2\underline{i} + x\underline{j} - 3\underline{k}$ and $\underline{q} = -4\underline{i} + \sqrt{3}\underline{j} + y\underline{k}$, where x and y are real numbers. If the magnitude of vector \underline{p} is 4 units and \underline{p} and \underline{q} are perpendicular then possible values of x and y could be

- A. $x = -\sqrt{3}, y = -\frac{11}{3}$
- B. $x = -\sqrt{11}, y = -\frac{5}{3}$
- C. $x = \sqrt{3}, y = \frac{5}{3}$
- D. $x = 3, y = -\frac{5}{3}$
- E. $x = \sqrt{3}, y = \frac{11}{3}$

Question 5

The position vectors of particles A and B are given by

$$\underline{r}_A = 3\underline{i} + (2t - 6)\underline{j} - 10\underline{k}$$

and $\underline{r}_B = (t + 1)\underline{i} + 4\underline{j} + t(t - 7)\underline{k}$

where t is in seconds and $t \geq 0$. The particles

- A. meet when $t = 2$
- B. meet when $t = 3$
- C. meet when $t = 5$
- D. meet when $t = 2$ and $t = 5$
- E. never meet

Question 6

A body moving in a straight line has velocity vms^{-1} given by $v = \frac{1}{3}(1 - 2x^2)^{\frac{3}{2}}$ where x is the displacement in metres from the origin. The acceleration of the body in ms^{-2} is given by

- A. $-2x(1 - 2x^2)^2$
- B. $-4x(1 - 2x^2)^{\frac{1}{2}}$
- C. $\frac{2}{3}(1 - 2x^2)^{\frac{1}{2}}$
- D. $\frac{1}{2}(1 - 2x^2)^{\frac{1}{2}}$
- E. $-\frac{2}{3}x(1 - 2x^2)^2$

SECTION A - continued

Question 7

The position vector of a particle at time t seconds, $t \geq 0$, is given by $\underline{r}(t) = (1+3t)^2 \underline{i} + 3\underline{j} - 18\sqrt{t}\underline{k}$. The speed at time $t = 3$ is

- A. 20.88 ms^{-1}
- B. 99.98 ms^{-1}
- C. 100.1 ms^{-1}
- D. 59.77 ms^{-1}
- E. 60.22 ms^{-1}

Question 8

Using an appropriate substitution, $\int_0^2 (3x-1)\sqrt{(3x+2)} dx$ is equivalent to

- A. $\frac{1}{3} \int_0^2 (u-3)\sqrt{u} du$
- B. $3 \int_2^8 (3u-1)\sqrt{u} du$
- C. $\frac{1}{3} \int_2^8 (u-3)\sqrt{u} du$
- D. $\frac{1}{3} \int_0^2 (3u-1)\sqrt{u} du$
- E. $3 \int_2^8 (u-3)\sqrt{u} du$

Question 9

Let $\frac{dy}{dx} = \sqrt{(x^2 + y + 1)}$ and $(x_0, y_0) = (3, 2)$.

Using Euler's methods with a step size of 0.1, the value of y_2 , correct to four decimal places is

- A. 2.7063
- B. 3.2839
- C. 2.7064
- D. 3.2828
- E. 2.7238

SECTION A – continued

TURN OVER

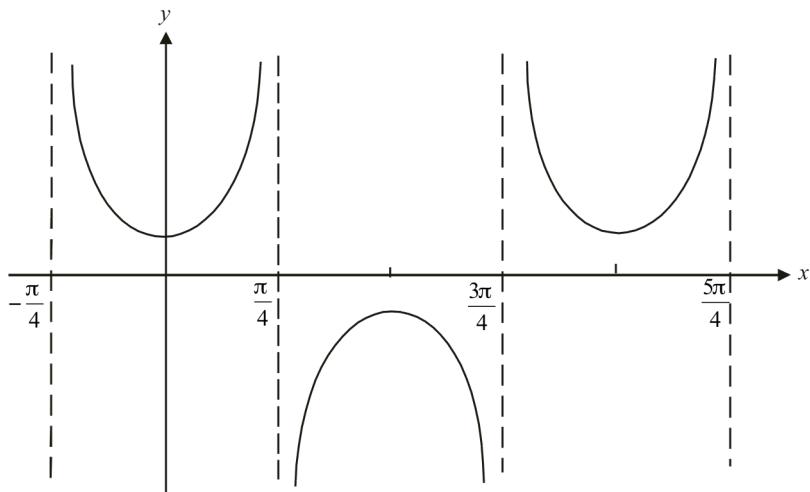
Question 10

For the graph of $y = \frac{1-2x^2}{3x}$

- A. $x = 0$ and $y = -\frac{2}{3}x$ are the only straight line asymptotes
- B. there are no straight line asymptotes
- C. $x = 0$ is the only straight line asymptotes
- D. $y = 0$ and $y = -\frac{2}{3}x$ are the only straight line asymptotes
- E. $y = 3x$ and $x = 0$ are the only straight line asymptotes

Question 11

The graph of $y = \operatorname{cosec}(a(x-b))$ for $x \in (-\frac{\pi}{4}, \frac{5\pi}{4})$ is shown below.



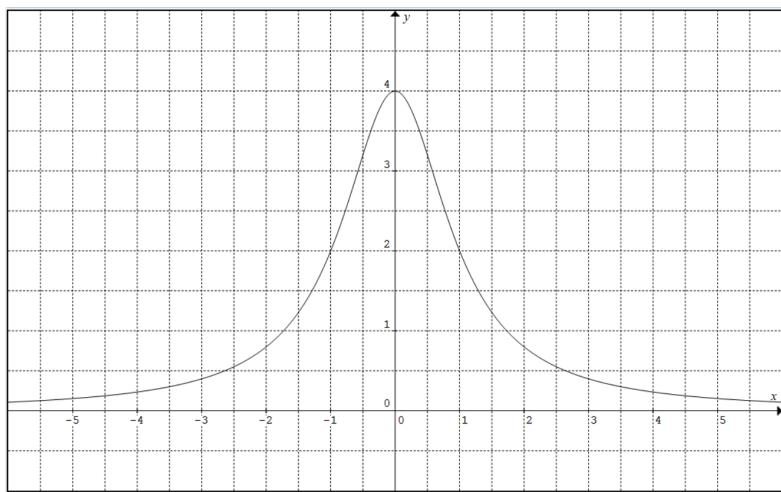
Possible values of a and b are

- A. $a = \frac{1}{2}$ $b = \frac{\pi}{2}$
- B. $a = 2$ $b = -\frac{\pi}{4}$
- C. $a = \frac{1}{2}$ $b = \frac{\pi}{4}$
- D. $a = 2$ $b = 0$
- E. $a = \frac{1}{2}$ $b = -\frac{\pi}{4}$

Question 12

The equation $x^2 + 2px + y^2 + 1 = 0$, where $p \in R$, will represent a circle if

- A. $-1 < p < 1$
- B. $p^2 > 2$
- C. $|p| > 1$
- D. $p < -2$ and $p > 2$
- E. $-1 < p$ and $p > 1$

Question 13

If the graph above represents $y = f'(x)$, the equation for $f(x)$ could be

- A. $\frac{1}{2x} \log_e(x^2 + 1)$
- B. $2 \tan^{-1}\left(\frac{x}{2}\right)$
- C. $\tan^{-1}(4x)$
- D. $\log_e(x^2 + 1)$,
- E. $4 \tan^{-1}(x) + 2$

Question 14

The maximal range of the function with the rule $f(x) = 4 \cos^{-1}(3x + 1) + \frac{\pi}{2}$ is

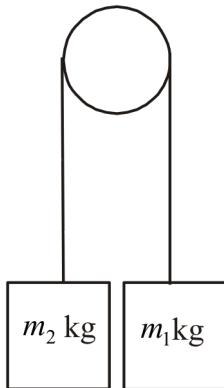
- A. $\left[0, \frac{9\pi}{2}\right]$
- B. $\left[0, \frac{\pi}{2}\right]$
- C. $\left[-\frac{2}{3}, 0\right]$
- D. $\left[0, \frac{5\pi}{2}\right]$
- E. $\left[\frac{\pi}{2}, \frac{9\pi}{2}\right]$

SECTION A – continued
TURN OVER

Question 15

A body of mass 5kg is acted upon by three concurrent coplanar forces \underline{F}_1 , \underline{F}_2 and \underline{F}_3 where $\underline{F}_1 = \underline{i} - 3e^{t-3}\underline{j}$, $\underline{F}_2 = (t^2 - 7)\underline{i} + \underline{j}$ and $\underline{F}_3 = -7\underline{i} + (t+2)\underline{j}$. The forces are measured in Newtons and t is the time in seconds. The magnitude of the acceleration of the body in ms^{-2} when $t = 3$ is

- A. 5
- B. $\frac{13}{5}$
- C. 2
- D. 1
- E. $\frac{7}{5}$

Question 16

The diagram above shows a particle of mass m_1 kg which is connected by a light inelastic string which passes over a smooth pulley to a particle of mass m_2 kg, where $m_1 > m_2$. If the magnitude of the acceleration of the particle with mass m_1 is $\frac{g}{2}$ ms^{-2} then

- A. $T = \frac{3m_1g}{2}$
- B. $\frac{m_1}{m_2} = 3$
- C. $m_1 - m_2 = \frac{g}{2}(m_1 + m_2)$
- D. $\frac{m_1}{m_2} = \frac{3}{2}$
- E. $T = \frac{m_2g}{2}$

Question 17

A golf ball is hit from the ground at an angle of 60° to the horizontal. If its initial velocity is 30ms^{-1} , the time, in seconds, that the golf ball will take to reach the ground again is

- A. $\frac{15}{g}$
- B. $\frac{30}{g}$
- C. $\frac{15\sqrt{3}}{g}$
- D. $\frac{30\sqrt{3}}{g}$
- E. $\frac{60}{g}$

Question 18

X is a random variable with mean 3.6 and standard deviation 0.68, and Y is a random variable with mean 12.3 and standard deviation 5.1. If X and Y are independent and $T = 3X - Y$, then $E(T)$ and $\text{sd}(T)$ are

- A. $E(T) = -1.5$, $\text{sd}(T) = 5.23$
- B. $E(T) = 23.1$, $\text{sd}(T) = 5.23$
- C. $E(T) = -1.5$, $\text{sd}(T) = 5.49$
- D. $E(T) = 1.5$, $\text{sd}(T) = 5.49$
- E. $E(T) = 23.1$, $\text{sd}(T) = 7.14$

Question 19

The waiting time at a dental surgery is known to follow a normal distribution with mean μ and standard deviation $\frac{\mu}{2}$. The surgery wants to collect a sample so that they can construct a 95% confidence interval for μ which has width of at most 0.2μ . What is the minimum sample size necessary to achieve this level of accuracy?

- A. 10
- B. 97
- C. 9
- D. 96
- E. 192

SECTION A – continued

TURN OVER

Question 20

The amount of saturated fat in a serving of Star Wars breakfast cereal is normally distributed with mean 35g and a standard deviation 4g. If Yoda eats it every day for a week, the probability that his average daily saturated fat intake for the week is more than 38g is

- A. 0.0236
- B. 0.9763
- C. 0.0472
- D. 0.2266
- E. 0.0036

END OF SECTION A

SECTION B**Instructions for Section B**

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$

Question 1 (12 marks)

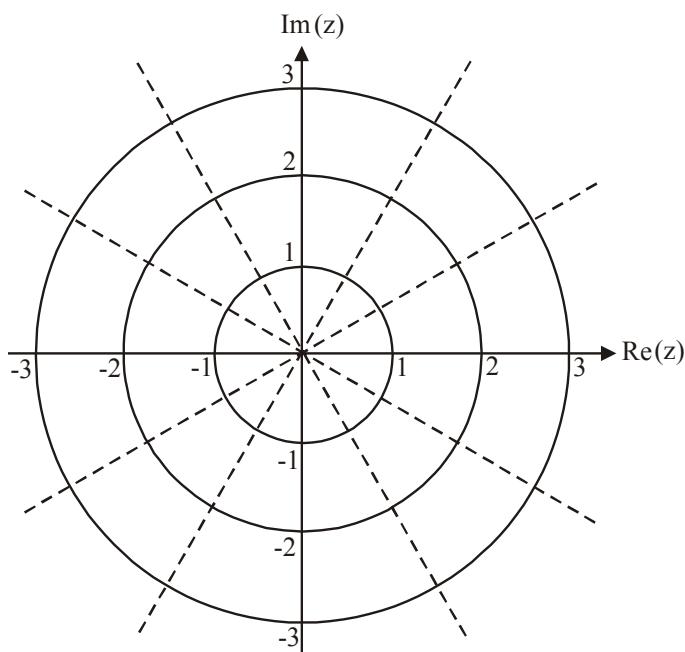
- a. Let $w^4 = -2 - 2\sqrt{3}i$

- i. Express w^4 in polar form

1 mark

- ii. Find the roots of the equation $w^4 = -2 - 2\sqrt{3}i$ in polar form and plot them on the argand diagram below.

3 marks



SECTION B – Question 1 – continued
TURN OVER

b. Find a and b where $a, b \in R$ such that $w^8 + aw^4 + b = 0$

2 marks

c. If $v = -1+i$ and $\text{Arg}(\bar{u}) = \frac{\pi}{6}$ calculate

i. $\frac{v^2}{(\bar{w})^4}$ in Cartesian form

1 mark

ii. $\text{Arg}\left(\overline{\left(\frac{v^2}{(\bar{w})^4}\right)} \cdot u^3\right)$

2 marks

SECTION B – Question 1 – continued

- d. The complex relationship $|z - w^4| = 2|z - \bar{w}^4|$ represents a circle. Find the Cartesian equation of the circle and state the exact coordinates of its centre and the value of the radius.

3 marks

SECTION B – continued
TURN OVER

Question 2 (11 marks)

A curve is defined by the following parametric equations

$$x = 2 \cos(t) \text{ and } y = 3 \sin(2t) \text{ for } 0 \leq t \leq 2\pi$$

- a. Find p and q , where p and q are integers and $\frac{dy}{dx} = p \operatorname{cosec}(t) + q \sin(t)$.

2 marks

- b. Find the equation of the normal to the curve at the point where $t = \frac{\pi}{6}$

2 marks

- c. Show that the curve can be expressed in Cartesian form as $y^2 = kx^2(4 - x^2)$ and state the value of k .

2 marks

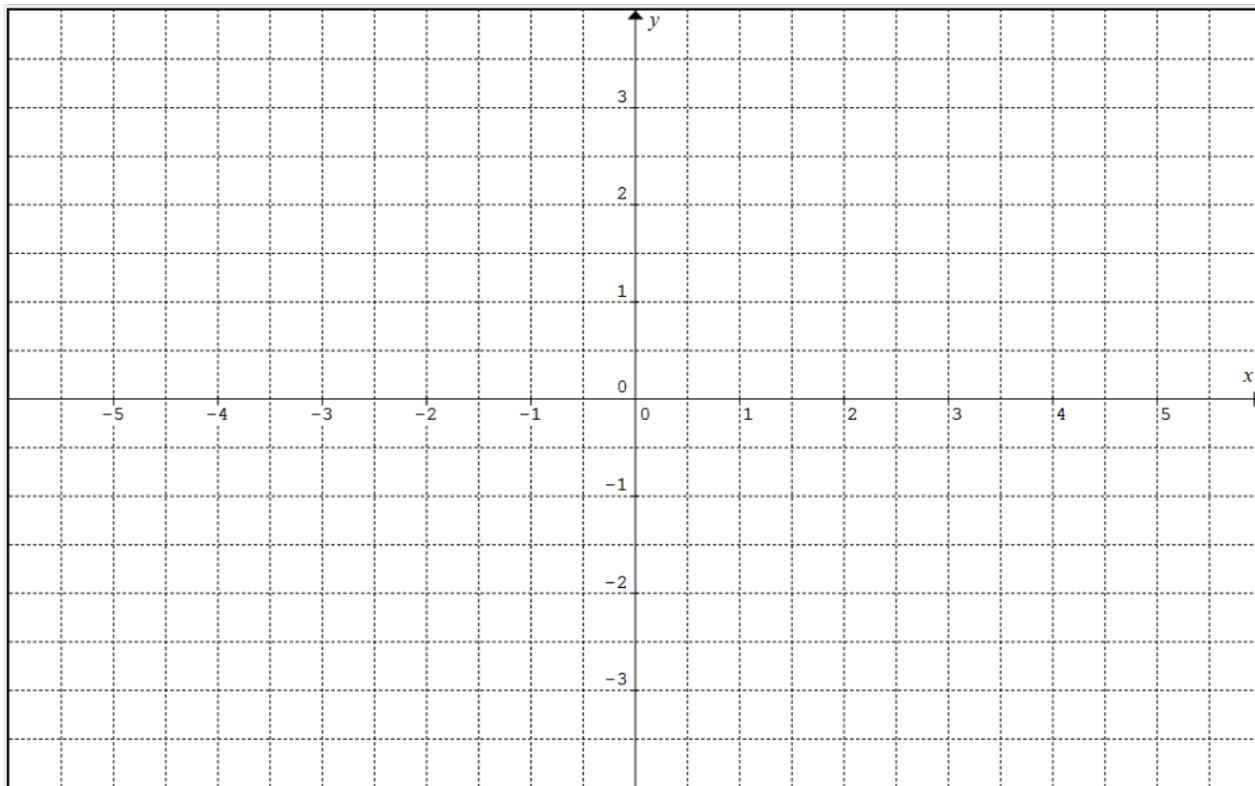
SECTION B – Question 2 – continued

- d. Use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y .

2 marks

- e. Sketch the curve on the axes below.

1 mark



SECTION B – Question 2 – continued
TURN OVER

- f. i. Write down a definite integral which gives the total length of the curve.

1 mark

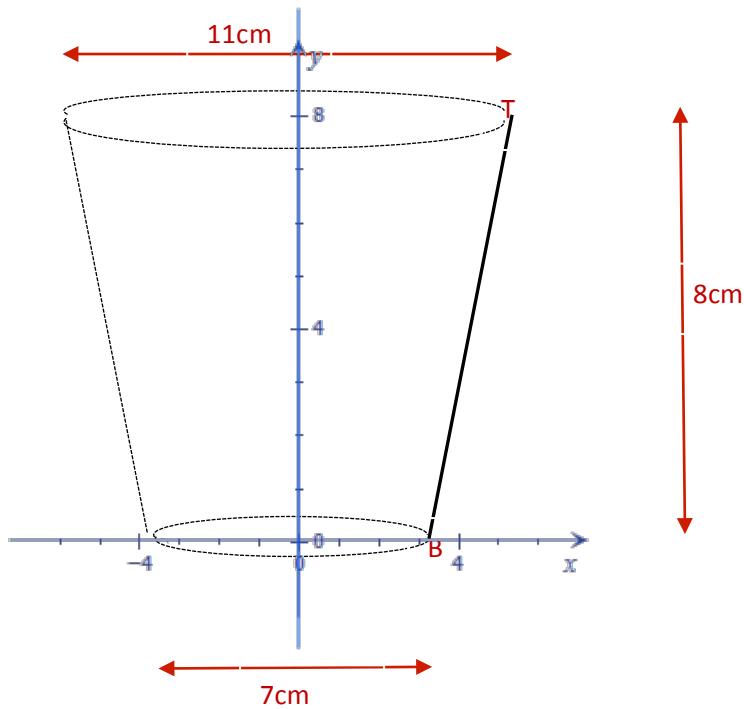
- ii. Find the total length of the curve, giving your answer to two decimal places.

1 mark

SECTION B – continued

Question 3 (8 marks)

A machine pours coke into large paper cups. Each paper cup has a base diameter of 7cm, a top diameter of 11cm and height of 8cm. The shape of the cup can be modelled by rotating the line BT around the y -axis.



- a. If the line segment BT is given by the equation $y = mx + c$, find m and c

1 mark

- b. If the vertical height of the coke in the cup is denoted by h cm, find V the volume of coke in the cup in terms of h .

2 marks

- c. If the cup is completely filled with coke which is levelled at the top, how much drink would it contain? Give your answer to two decimal places.

1 mark

SECTION B – Question 3 – continued
TURN OVER

d. Shamira buys one of these cups of coke and drinks it at the rate of $3\sqrt{h}$ cm³/sec.

- i. Find $\frac{dh}{dt}$ in terms of h .

2 marks

- ii. Find the time in seconds that it will take her to drink all of her coke, correct to two decimal places.

2 marks

SECTION B – continued

Question 4 (12 marks)

OAB is an equilateral triangle such that the points A and B have the coordinates $(2, -2)$ and (u, v) respectively, where u and v are both positive

- a. Express \overrightarrow{OA} and \overrightarrow{OB} in vector form

2 marks

- b. Show that $u^2 + v^2 = 8$ and $u - v = 2$

2 marks

- c. Find u and v

1 mark

- d. Find the coordinates of point C such that OABC is a rhombus.

1 mark

- e. Prove that the diagonals of the rhombus are perpendicular to each other.

2 marks

SECTION B – Question 4 – continued
TURN OVER

Let P be the point which divides \overline{OB} in the ratio 1:2.

- f. Find the area of triangle OAP.

1 mark

- g. X is the point on the line segment AB which closest to P. Find the coordinates of X.

3 marks

SECTION B – continued

Question 5 (9 marks)

In 2011 Facebook reported that the number of Facebook friends a person had was normally distributed with a mean of 175.5 and a standard deviation of 90.57.

- a. Assuming that the distribution of the number of Facebook friends, X , has not changed since 2011, calculate the probability that the mean number of Facebook friends for a random sample of 20 people would be less than 150? Give your answer correct to 4 decimal places.

1 mark

- b. In a more recent study involving 160 participants the sample mean was calculated to be 186.9

- i. Write down a suitable null and alternative hypothesis to test whether the average number of Facebook friends has increased since 2011.

1 mark

- ii. Find the p -value for this test, correct to four decimal places.

1 mark

- iii. State whether or not the null hypothesis should be rejected at the 5% level of significance.

1 mark

SECTION B – Question 5 – continued
TURN OVER

- c. If the amount of time people spend on Facebook each day, T , is normally distributed with a mean of 71 minutes and a standard deviation of 4.7 minutes, would you be more likely to select a random person that spends more than 76 minutes per day on Facebook, or a group of 4 people that spend an average of more than 74 minutes on Facebook? Show working to justify your answer.

2 marks

- d. For females the number of ‘likes’ that a person will give in a day is normally distributed with a mean of 7.1 and standard deviation of 4.7, whereas for males the mean is 5.8 and the standard deviation is 3.5. If W represents the number of likes given by females in a day and Y represents the number of likes given by males, find the probability that on random day, the number of likes given by females will be more than twice the number of likes given by the males. Give your answer to 4 decimal places.

3 marks

SECTION B – continued

Question 6 (8 marks)

A parachutist with mass 80kg jumps out of a plane. The parachute opens at time $t = 0$ s when he has a speed of 58.8 ms^{-1} , and he experiences an air resistance force of magnitude $320v$, where $v \text{ ms}^{-1}$ is his speed at time t .

- a. Find the resultant force acting on the parachutist in terms of v .

1 mark

- b. Hence, solve to find v in terms of t .

2 marks

- c. If it takes 5 mins for the parachutist to reach the ground, how far up was he when he opened his parachute? Calculate this to two decimal places.

2 marks

On the next day the parachutist jumps from the same place with the same initial speed on opening his parachute, but this time the air resistance force is $40v^2$ newtons.

- d. Find the speed at which he hits the ground, to two decimal places.

3 marks

END OF QUESTION AND ANSWER BOOK