Burgers' Equation Initial and Boundary Conditions

$$u = -\frac{2\nu}{\phi}\frac{\partial \phi}{\partial x} + 4$$

$$\phi = \exp\left(\frac{-(x-4t)^2}{4\nu(t+1)}\right) + \exp\left(\frac{-(x-4t-2\pi)^2}{4\nu(t+1)}\right)$$

$$\Rightarrow \frac{\partial \phi}{\partial x} = \frac{\partial}{\partial x}\left(\exp\left(\frac{-(x-4t)^2}{4\nu(t+1)}\right) + \exp\left(\frac{-(x-4t-2\pi)^2}{4\nu(t+1)}\right)\right)$$

$$\xRightarrow{\text{Chain rule}} \frac{\partial \phi}{\partial x} = \frac{\partial a}{\partial x}\frac{\partial(\exp(a))}{\partial a} + \frac{\partial b}{\partial x}\frac{\partial(\exp(b))}{\partial b} = \frac{\partial a}{\partial x}\exp(a) + \frac{\partial b}{\partial x}\exp(b)$$

with $a = \dfrac{-(x-4t)^2}{4\nu(t+1)}$ and $b = \dfrac{-(x-4t-2\pi)^2}{4\nu(t+1)}$.

and hence, $\dfrac{\partial \phi}{\partial x} = -\dfrac{x-4t}{2\nu(t+1)}\exp\left(\dfrac{-(x-4t)^2}{4\nu(t+1)}\right) - \dfrac{x-4t-2\pi}{2\nu(t+1)}\exp\left(\dfrac{-(x-4t-2\pi)^2}{4\nu(t+1)}\right)$