RBE550: Motion Planning

Project 2: Point Planning and Discrete Search

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Theoretical Questions:

Solution 1

Compare Visibility Graph and PRM Algorithm.

Visibility Graph	**PRM**
In 2D it always gives optimal/shortest path	No Guarantee on the path found being optimal
Complexity $\implies O(n^2)$ where, n = No of Vertices	Complexity $\implies O(n^2)$ where, n = No of Samples
It is a complete Algorithm	It is Probabilistic Complete (i.e. as $n \to \infty, P(find\_path) \to 1$)

Visibility Graph is better than PRM when:-

1. Visibility Graph is better in environments where there are narrow paths.
2. Visibility Graph is More Efficient in larger maps.
3. Visibility Graph is Less likely to get stuck in local minima (i.e. planner cannot find a path)
4. Visibility Graph is preferred when we want optimal paths.

PRM is better than Visibility Graph when:-

1. When Obstacles have complex structure / high no of edges, PRM can outperform VG in terms of computation.
2. PRM is very versatile and is better in general environments while VG exploits the edges of the obstacle to find a path
3. PRM is better in cases where are want paths that have some distance from obstacles because VG finds paths on obstacle edges and vertices.

It depends of the nature of the problem we are trying to solve on what algorithm is better.

Solution 2

Topology and Dimension of Configuration Space.

1. Two Prismatic Joints $\implies \mathbb{R}^2$
2. Three Revolute Joints $\implies S^1 \times S^1 \times S^1 = \mathbb{T}^3$
3. Two Revolute and a prismatic joint $\implies S^1 \times \mathbb{R} \times S^1 = \mathbb{R} \times \mathbb{T}^2$

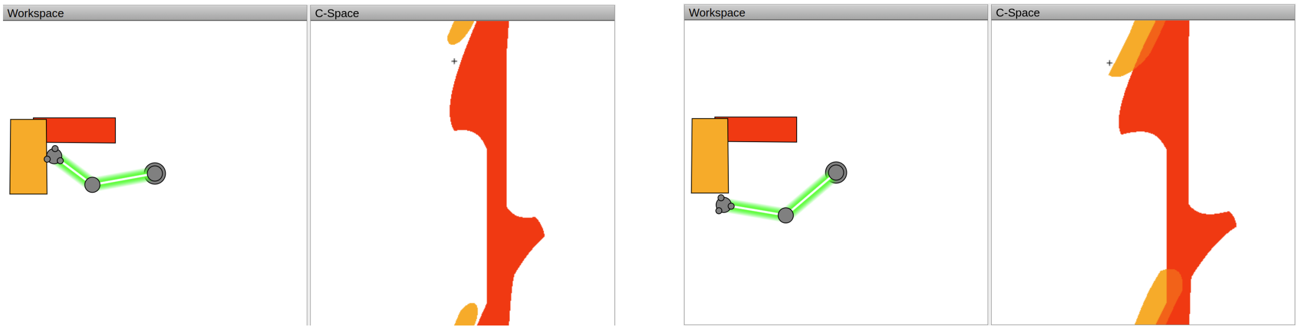
Solution 3

Consider workspace obstacles A and B. If A and B have some intersection, do the configuration space obstacles QA and QB always overlap? If A and B don't intersect, is it possible for the configuration space obstacles QA and QB to overlap? Justify your claims for each question.

Let us consider an example of a 2-joint manipulator to understand the relation between the workspace obstacles A, and B and the configuration space obstacles QA, QB.

Case1: $A \cap B \neq \emptyset$ does configuration space obstacle QA and QB always overlap

As we can see in Figure 1(a) the obstacles are overlapping but in the configuration space, there is no overlap. While in Figure 1(b) when we change the length of the arm without changing obstacles the configuration space obstacles overlap.

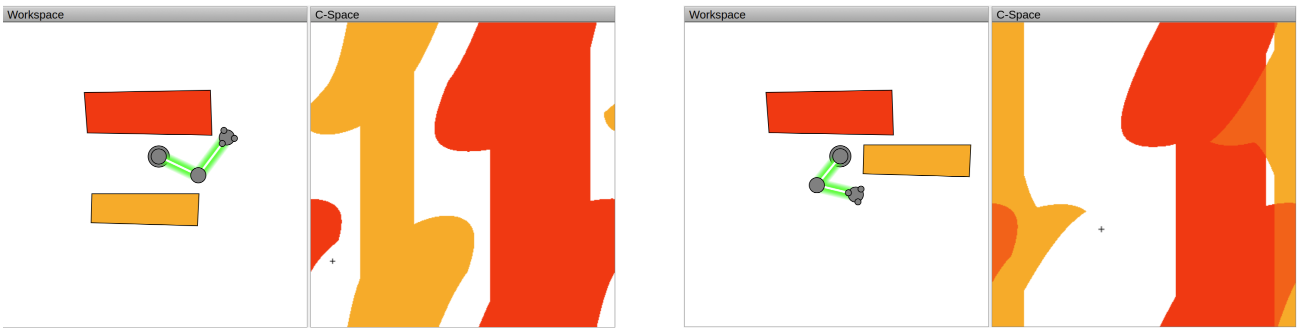


(a) No Overlap (b) Overlap

Figure 1: Obstacle overlap

This helps us conclude that even if there is overlap in obstacles A and B, the robot's geometry can be such that there is no overlap in configuration space obstacles QA and QB. But in general, if A and B overlap it is more likely that QA and QB also overlap in configuration space.

Case2: $A \cap B = \emptyset$ is it possible for configuration space obstacle QA and QB to overlap



(a) No Overlap (b) Overlap

Figure 2: No Obstacle Overlap

In conclusion, even if there is no overlap in obstacles A and B, the robot's geometry can be such that there is some overlap in configuration space obstacles QA and QB.

Possible Reasoning: The overlap/intersection in the configuration space obstacles QA and QB has a direct relation location of obstacles A and B in the workspace and the specific robot geometry. To conclusively state something we need more information on robot geometry, obstacle size and location, etc.
Reference link for diagrams :- https://www.cs.unc.edu/ jeffi/c-space/robot.xhtml

Solution 4

Suppose five polyhedral bodies float freely in a 3D world. Each is capable of rotating and translating.

Configuration space of one polyhedral body $\implies \mathbb{R}^3 \times SO(3) = SE(3)$.

Now, we have five such polyhedral bodies as a one composite robot.

Topology of the Configuration space of five polyhedral bodies is given by (assume that the bodies are not attached to each other)
$\implies (\mathbb{R}^3 \times SO(3)) \times (\mathbb{R}^3 \times SO(3)) \times (\mathbb{R}^3 \times SO(3)) \times (\mathbb{R}^3 \times SO(3)) \times (\mathbb{R}^3 \times SO(3))$.
$\implies SE(3) \times SE(3) \times SE(3) \times SE(3) \times SE(3)$
$\implies SE(3)$

The Dimension of the composite configuration space is 6 DOF per body so = 6*5 = 30 Dimensions

Programming Component:

Solution 1 - PRM Implementation

Check the Code and Output folder for code and results.

Explanation of how different sampling methods lead to the different sample points in the graph:

1. **Uniform Sampling Method -** This method samples the point in the entire map and then finds the best possible path from the sampled points. This method is computationally expensive as it checks the collusion between two points, and finds the nearest neighbors even if those points are not going to be useful for finding the final path.
2. **Gaussian Sampling Method -** This method first finds one sample point and with the Gaussian sampled radius it samples another point on the circle of that radius, it checks which one of the sampled points is in free space while another should be on an obstacle, then it takes free spaced point as a sample. By using this strategy this method is able to sample points on both sides of the obstacle, so, it can find the path in either way of the obstacle is available. This method is computationally less expansive as compared to the uniform sampling method.
3. **Bridge Sampling Method -** This method samples the two sampling points and checks if both of these points are on the obstacle then it finds the midpoint of both these two points, if the midpoint is in free space, it takes it as a sample. By using this strategy this method samples the points that are in the middle of the obstacle, so, less number of samples are required to find the path using this method, that is why it is computationally less expansive compared to both previous ones.

Solution 2 - RRT Implementation

Check the Code and Output folder for code and results