Nonlinear Model Predictive Control (NMPC) For Surveillance Quadrotor

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Abstract

A nonlinear model predictive control (NMPC) scheme is designed for the surveillance quadrotor for capturing enemy Unmanned Aerial Vehicle (UAV) in prescribed airspace. The controller is capable of capturing enemy UAV in the airspace or returns to the base on successful escape of the enemy UAV. It also rejects the external disturbance applied by the enemy UAV while satisfying control and state constraints. Simulation results are given to demonstrate the efficacy of the proposed control scheme.

1 Introduction

This research is dedicated to formulating and deploying a sophisticated control algorithm tailored for the interception of hostile drones, utilizing a standard quadcopter. The urgency of this pursuit is accentuated by the escalating threat landscape resulting from the widespread use of drones in both civilian and military domains. The profound significance of crafting an effective interception algorithm lies in its pivotal role in fortifying national security, particularly in safeguarding critical infrastructure, borders, and sensitive zones. Addressing concerns related to public safety, this research offers methods to protect against potential harm from malicious drone activities in crowded events and public spaces. Moreover, it addresses privacy apprehensions by ensuring the safeguarding of private spaces from unauthorized drone intrusions. In 2022 alone, the drone intelligence firm DroneSec recorded 2,554 major illicit drone incidents—a 60% increase from 2021 [1]. Examining FAA official records [2] reveals a steep upward trend in illegal drone activity, underscoring the critical need for advanced interception mechanisms. Notably, the incidents encompassed a spectrum of security breaches, including unauthorized surveillance, airspace violations, and potential threats to critical infrastructure.

The research endeavors to develop and implement a sophisticated Model Predictive Control (MPC) algorithm for this urgent purpose. MPC emerges as a superior choice for drone interception compared to commonly used traditional controllers like Proportional-Integral-Derivative (PID), thanks to its distinctive advantages in handling the dynamic and uncertain nature of drone behavior. Unlike PID controllers, which operate reactively based on the current state without considering dynamics, MPC employs a predictive approach, optimizing control actions over a future time horizon. This predictive capability enables MPC to anticipate and adapt to the evolving dynamics of drone movements, facilitating more effective and agile control responses. Furthermore, MPC inherently accommodates constraints and uncertainties, a critical feature for addressing the complex and unpredictable nature of drone interception scenarios. Its ability to optimize trajectories and control inputs over a defined prediction horizon makes MPC particularly adept at achieving precise and rapid interception maneuvers, ensuring a higher degree of success in countering potential threats. In essence, the decision to opt for MPC over other controllers reflects a strategic choice to leverage its advanced predictive and adaptive capabilities, positioning it as a state-of-the-art solution for the intricate challenges posed by drone interception. This is further facilitated by the success of other emerging research, which shows the strengths of using such an approach [3].

2 Nonlinear Model Predictive Control (NMPC)

Nonlinear Model Predictive Control (NMPC) employs a mathematical representation of the system to determine the optimal control actions within a limited prediction horizon (N) through the resolution of a constrained optimization problem (COP). The objective is to minimize a nonlinear cost function while adhering to specified constraints. Despite solving the COP for the entire horizon, only the initial step of the optimized control action is implemented on the system. Subsequently, the starting time is advanced by one step, and the procedure iteratively conducted with the updated state values. This characteristic gives rise to NMPC being referred to as receding horizon control [4], [5].

A block diagram showing the proposed Nonlinear Model Predictive Control (NMPC) scheme is given in 2. The state parameters of the quadrotor is given to the NMPC

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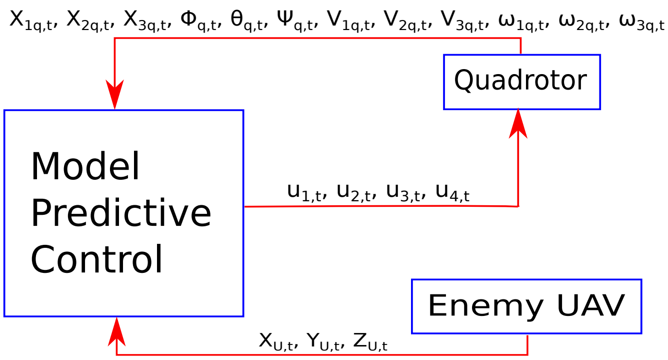


Figure 1: Block diagram of proposed NMPC scheme

block as feedback. Computed control commands from the NMPC block are applied to the quadrotor system. The two core components of the proposed framework, the System Dynamics and the Cost function explained in detail in the following sections.

2.1 System Dynamics

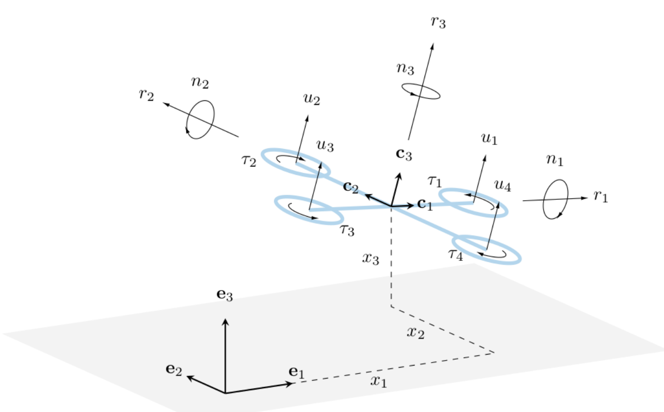


Figure 2: Free-body diagram of the quadrotor

We use a discrete-time model for representing the motion of the quadrotor. The state vector of the quadrotor is represented by $X_Q \in \mathbb{R}^{12}$ and the evolution of the states over the time step t is written as

$$X_{Q,t+1} = X_{Q,t} + f(X_{Q,t}, U_{Q,t})T_s, \tag{1}$$

where T_s is the sampling time, $X_{Q,t} = [X_{q,t}, \alpha_{q,t}, V_{q,t}, \omega_{q,t}]$ and $U_{Q,t} = [u1_{q,t}, u2_{q,t}, u3_{q,t}, u4_{q,t}]$ are the state and control vector of the quadrotor. Omitting the time subscript t for readability, the function $f(X_Q, U_Q) = \dot{X_Q}$ and $\dot{X_Q}$ is defined as:

$$\dot{X_q} = V_q, \tag{2}$$

$$\dot{\alpha_q} = T^{-1}\omega_q, \tag{3}$$

$$\dot{V_q} = -ge_3 + \frac{1}{m}R_{C/E}(u_1+u_2+u_3+u_4)c_3 + \frac{1}{m}R_{C/E}r \tag{4}$$

$$\begin{aligned}\dot{\omega_q} = I^{-1}((u2-u4)lc_1 + (u3-u1)lc_2 \\ + (u_1-u_2+u_3-u_4)\sigma c3 + n - \omega \times I\omega)\end{aligned} \tag{5}$$

where, $c = [c1, c2, c3]$ represents the local coordinate system of the quadrotor, and $e = [e1, e2, e3]$ represents the local coordinate system of the quadrotor. Moreover, each component of the system is defined as follow, linear velocity of the quadrotor:

$$V_q = \sum_{i=1}^{3} \dot{x_i} e_i, \tag{6}$$

Where, x_1, x_2, and x_3 are the distances of the center of mass of quadrotor from global frame. Angular velocity vector $\dot{\alpha_q}$ is defined as:

$$\dot{\alpha_q} = \begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} = T^{-1}\begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{bmatrix} \tag{7}$$

where, T^{-1} is defined as follows:

$$T^{-1} = \begin{bmatrix}1 & \sin\phi\tan\theta & \cos\phi\tan\theta\\ 0 & \cos\phi & -\sin\phi\\ 0 & \sin\phi/\cos\theta & \cos\phi/\cos\theta\end{bmatrix} \tag{8}$$

While rotation matrix which maps from the quadrotor local frame to the global frame of the system is defined as follows:

$$R_{C/E} = \begin{bmatrix}C\theta C\psi & S\phi S\theta C\psi - C\phi S\psi & S\phi S\psi + C\phi S\theta C\psi\\ C\theta S\psi & C\phi C\psi + S\phi S\theta S\psi & C\phi S\theta S\psi - S\phi C\psi\\ -S\theta & S\phi C\theta & C\phi C\theta\end{bmatrix} \tag{9}$$

We used $C = \cos$ and $S = \sin$ in equation (9), Apart from this we used fixed-wing UAV as an enemy UAV, it is because we can adjust the trajector of the enemy UAV by manually feeding the linear and angular velocities of it, moreover, it makes more this problem more realistic.

2.2 Cost Function

NMPC is used to determine the optimal control inputs for the quadrotor such that the duadrotor can track and catch the enemy UAV or return to the base if enemy escape the airspace while being within the airspace. We designed cost function in a way that while finding the optimal control actions NMPC decrease the distance between the enemy UAV and eventually catches it. The cost function is defined as:

$$\min_{U_t} J(X_t, U_t) = \sum_{i=t}^{t+N} W_1 G(X_{Q,i}, X_{U,i}), \tag{10}$$

subject to:

$$X_{t+1} = G(X_{Q,i}, U_{Q,i}), \tag{11}$$

$$X_{Q,i} \in [X^-, X^+], \tag{12}$$

$$U_{Q,i} \in [U^-, U^+] \tag{13}$$

Quadrotor State Constraints		
variable	min	max
$X1_q(m)$	-5	5
$X2_q(m)$	-5	5
$X3_q(m)$	0	10
Quadrotor Control Constraints		
variable	min	max
$u1_q(N)$	0	3
$u2_q(N)$	0	3
$u3_q(N)$	0	3
$u4_q(N)$	0	3

Table 1: Constraints on the system: (1) State Constraints - Respects the Airspace and Keeps Quadrotor Within it (2) Control Constraints - Respects the Control Signal Bounds

where $U_{Q,t} = [u1_{q,t}, u2_{q,t}, u3_{q,t}, u4_{q,t}]$ is the control vector of the quadrotor, W_1 is the weighting coefficients, X^-, X^+ are the lower and upper bounds on the states, and U^-, U^+ are the lower and upper bounds of the control inputs, respectively. Where, function $G(X_{Q,i}, X_{U,i})$ is the euclidean distance between enemy UAV and quadrotor which is defined as follows:

$$G(X_{Q,i}, X_{U,i}) = \sqrt{(X1_{q,i} - X_{U,i})^2 + (X2_{q,i} - Y_{U,i})^2 + (X3_{q,i} - Z_{U,i})^2} \tag{14}$$

Where, $(X1_q, X2_q, X3_q)$ are the coordinates of the quadrotor and (X_U, Y_U, Z_U) are the coordinates of the enemy UAV.

The reason behind designing this cost function is we can decrease the distance between the enemy drone and quadrotor, eventually quadrotor ends-up at catching it or enemy UAV is able to escape the airspace.

3 Results

The different simulation parameters are tested and optimal parametrs are mentioned here, which was given the best results while having less computation time. The sampling time for the NMPC is selected as $T_s = 0.2\,\text{s}$ and the length of the prediction horizon $N = 20$ which is $0.2 * 20 = 4s$ for all simulations. The enemy UAV can be enter from any plane in the airspace at any randomly generated linear and angular velocity and that the initial position for the enemy UAV. The initial positions of the quadrotor is selected as $(X1_q, X2_q, X3_q) = (0, 0, 0)$. The bounds on the states and controls are given in Table 1.

3.1 Simulation Setup

The simulation is conducted for a quadrotor system with the following set of parameters. The distance from the center of mass to each rotor (L) is $0.2\,\text{m}$, and the total mass of the quadrotor (m) is $0.5\,\text{kg}$. The mass moment of inertia about the global X and Y axes $(I_{11},\ I_{22})$ is $1.24\,\text{kg}\cdot\text{m}$, while about the global Z axis (I_{33}) is $2.48\,\text{kg}\cdot\text{m}$. The gravitational acceleration (g) is $9.8\,\text{m/s}^2$.

In addition to these parameters, external forces are introduced on the quadrotor after it catches the enemy UAV. The magnitude of the external force vector $\mathbf{r}$ is limited to be less than or equal to $2\,\text{N}$, and the magnitude of the vector $\mathbf{n}$ is limited to be less than or equal to $1\,\text{N}$, we selected this forces on the random bases within bound to test the robustness of the controller.

3.2 Simulation Results

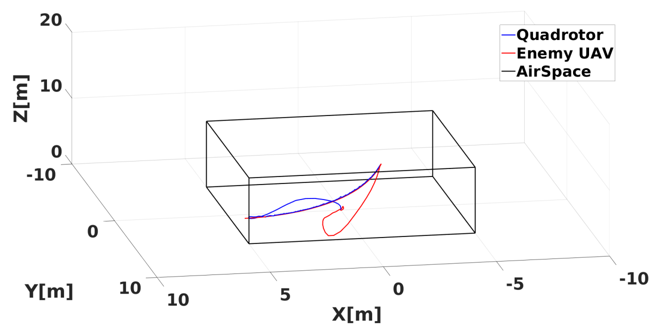


Figure 3: Quadrotor Demonstrating Catch Maneuver

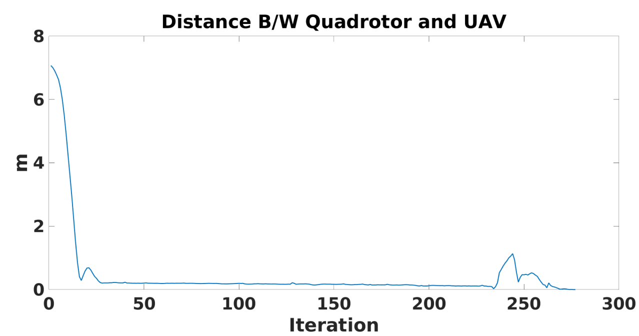
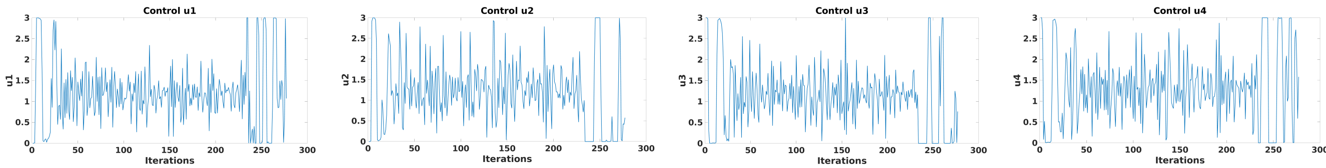


Figure 4: Distance B/W Quadrotor and Enemy UAV

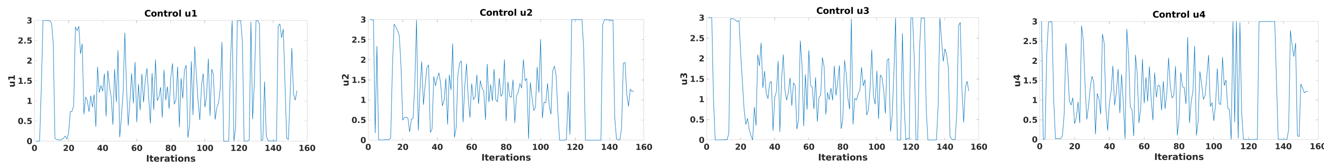
Above figure (3) demonstrates the catch maneuver of the quadrotor where quadrotor is able to capture the target UAV and bringing it back to the base smoothly, figure (4) show the distance between the enemy UAV and quadrotor, figure (5) shows the control inputs of the quadrotor in catch scenario. It's clearly visible from figure (4) that after 270 iteration distance becomes the zero means quadrotor successfully capturing UAV.

Above figure (3) demonstrates the escape maneuver of the quadrotor, where quadrotor successfully returns to the base after enemy UAV escape from the airspace, figure (4) show the distance between the enemy UAV and quadrotor, figure (5) shows the control inputs of the quadrotor in the escape scenario. It's clearly visible from figure (8) that after 110 iteration distance starts increasing exponentially means UAV escaped and quadrotor is returning to the base.



(a) (b) (c) (d)

Figure 5: NMPC Generated Control Signals of Quadrotor in Catch Scenario: (A) Control Signal u1, (B) Control Signal u2, (C) Control Signal u3, and (D) Control Signal u4



(a) (b) (c) (d)

Figure 6: NMPC Generated Control Signals of Quadrotor in Escape Scenario: (A) Control Signal u1, (B) Control Signal u2, (C) Control Signal u3, and (D) Control Signal u4

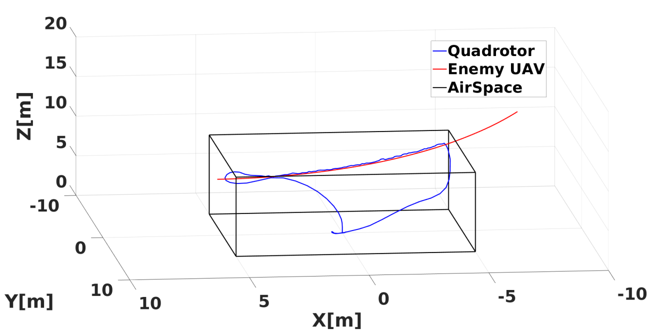


Figure 7: Quadrotor Returns to Base After Successful Evade of Enemy UAV

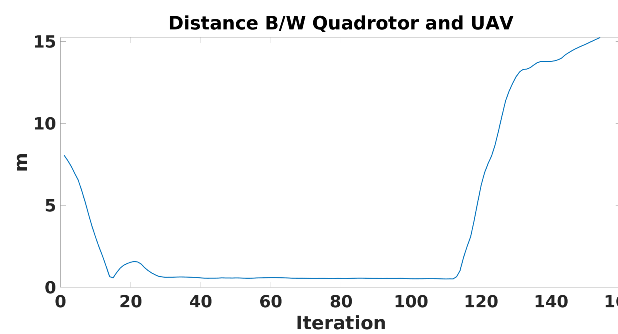


Figure 8: Distance B/W Quadrotor and Enemy UAV

4 Conclusions

The quadrotor system is capable to meticulously track, follow, and capture enemy Unmanned Aerial Vehicle (UAV). During the simulation, the quadrotor executes these maneuvers flawlessly. In the event that an enemy UAV manages to evade airspace successfully, the quadrotor autonomously initiates a return to the base, which is situated at the origin of the global frame.

The presented simulation results provide a detailed analysis of the quadrotor's performance, highlighting a scenario that demonstrates the optimal outcome for both successful capture and successful escape. The quadrotor effectively adheres to the specified traits, showcasing its ability to navigate and respond strategically in given situations. These simulation outcomes not only affirm the reliability of the proposed controller capabilities but also underscore its ability to seamlessly transition to a predetermined return-to-base protocol in the event of an unsuccessful capture.

References

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