Exp. no. 9

Optimal Control

1. Study of various optimal control problems.
2. LQR concept.

1. Minimum control effort problem:
To transfer a system from an arbitrary initial state $X(t_0) = X_0$ to a specified target with a minimum expenditure of control effort.

$$\int_{t_0}^{t_t} u^2(t)dt$$

2.Tracking problem:
To maintain the system o/p as close as possible to desired i/p in the interval $[t_0, t_t]$

$$\int_{t_0}^{t_t} e^2(t)dt$$

3. Output regulatory problem:

$$\int_{t_0}^{t_t} y^2(t)dt$$

4. State Regulatory problem:

$$\int_{t_0}^{t_t} x^2(t)dt$$

5. Minimum Time Problem:
To transfer a system from an arbitrary initial state $X(t_0) = X_0$ to a specified target with a minimum time

$$\int_{t_0}^{t_t} dt$$

6. Terminal Control Problem:
To minimise the deviation of final state of the system from its desired value
Expression- $[X(t_t) - r(t_t)]^2$

<u>Derivation:</u>

<u>LQR</u>

<u>Exp 1</u>:

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \qquad \longrightarrow (1)$$
$$u = -R^{-1}B^T P$$
$$\dot{X}(t) = AX(t) + B.u(t)$$
$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$Q = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \qquad R = 2$$

From (1),

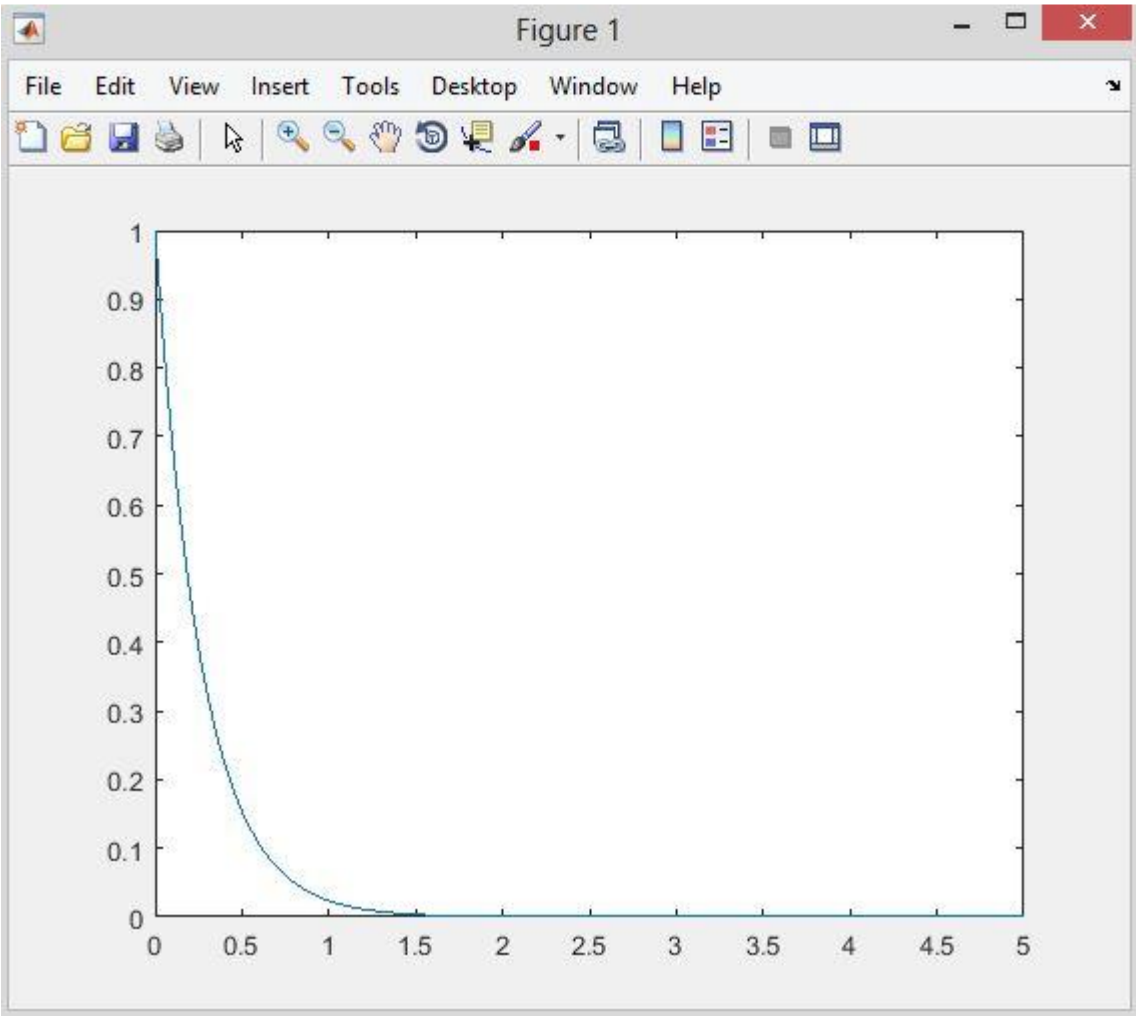
$$P = \begin{bmatrix} 2\sqrt{2} & 2 \\ 2 & 2\sqrt{20} \end{bmatrix}$$
$$u(t) = -R^{-1}.B^T.P.X(t)$$
$$\boxed{u(t) = -X_1(t) - \sqrt{2}X_2(t)}$$

<u>Exp 2</u>:

Matlab Implementation:

```
% tank level control
% A=1; R=0.5;FIN=0.5;
a=-2;
b=1;
c=1;
q=1;
r=0.01;
[k,p,e]=lqr(a,b,q,r)
sys=ss(a-b*k,eye(1),eye(1),eye(1));
t=0:.01:5;
x=initial(sys,1,t);
 plot(t,x);
 hold on
% q=1;r=0.1;state=0.1212;u=0.5347;k=1.7417;
% q=1;r=1;state=0.2188;u=0.01518;k=0.2361;
```

The ouput of this script is :



This output clearly shows how the flow is regulated and comes to zero.