Control Engineering Laboratory.
Experiment No. 4
Non-Linear Control (Simple Tank Application) and Feedforward Controller.

Name: Pragyaditya Das.
Roll no.: 110113062.

<u>Date:</u>
<u>Signature:</u>

Software used:

MATLAB R2015a.

Theory:

In this experiment we analysed and designed a non-linear controller and a feed-forward controller. We will discuss the theory and mathematics behind both of them in detail below:

Non-Linear Control for Tank Application:

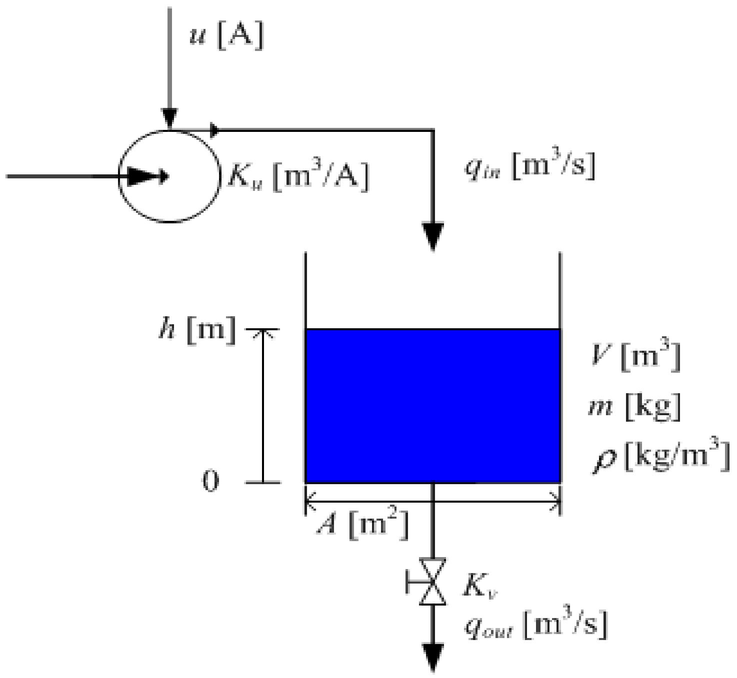
Nonlinear control theory is the area of control theory which deals with systems that are nonlinear, time-variant, or both. Control theory is an interdisciplinary branch of engineering and mathematics that is concerned with the behaviour of dynamical systems with inputs, and how to modify the output by changes in the input using feedback. The system to be controlled is called the "plant". In order to make the output of a system follow a desired reference signal a controller is designed which compares the output of the plant to the desired output, and provides feedback to the plant to modify the output to bring it closer to the desired output.

Nonlinear control theory covers a wider class of systems, as compared to linear control systems, that do not obey the superposition principle. It applies to more real-world systems, because all real control systems are nonlinear. These systems are often governed by nonlinear differential equations. The mathematical techniques which have been developed to handle them are more rigorous and much less general, often applying only to narrow categories of systems. These include limit cycle theory, Poincaré maps, Liapunov stability theory, and describing functions. If only solutions near a stable point are of interest, nonlinear systems can often be linearized by approximating them by a linear system obtained by expanding the nonlinear solution in a series, and then linear techniques can be used. Nonlinear systems are often analyzed using numerical methods on computers, for example by simulating their operation using a simulation language. Even if the plant is linear, a nonlinear controller can often have attractive features such as simpler implementation, faster speed, more accuracy, or reduced control energy, which justify the more difficult design procedure.

Now, we mathematically analyse the Simple Tank Model. Before we do that, we need to consider the following assumptions:

1. The liquid density is the same in the inlet, in the outlet, and in the tank.
2. The tank has straight, vertical walls.
3. The liquid mass and level are related through

The setup of the tank looks like this:



The mathematical analysis is as follows:

$$\frac{Accumulation\ of\ total\ mass}{time\ taken\ for\ the\ accumulation} = \frac{input\ mass - output\ mass}{time\ taken}.$$

We use, $mass = \rho V, \rho = density\ of\ the\ liquid\ and\ V = volume\ of\ the\ liquid.$

The difference in the height will be directly related to the input and the output fluid flow. This means, the $gradient$ of the fluid flow is equal to the difference in the $input\ and\ output\ flow\ rate.$

$$\frac{dAh}{dt} = F_i - F_o. \qquad (1)$$

$$\frac{dh}{dt} = F_i - b\sqrt{h}. \qquad (2)$$

$$b = constant. And\ we\ assumed\ A = 1m^2.$$

The block diagram showing this relation is just a block diagram relating the above equation.

Now, we do feedback linearization. For doing that we equate the equation 2 to a standard equation.
The mathematical analysis is as follows,

$$\frac{1}{\lambda s+1}=\frac{y}{r};\ r=set\ point, y=output.$$

=> $\lambda sy+y=r.$

=>$\lambda\frac{dh}{dt}+h(t)=h(sp).$

=>$\lambda\left(F_i-b\sqrt{h}\right)+h(t)=h(sp).$

=>$F_i=\frac{h(sp)}{\lambda}-\frac{h(t)}{\lambda}+b\ \sqrt{h}$

Now, when we substitute it in equation 2, we get,

$$\frac{dh}{dt}=\frac{h(sp)}{\lambda}-\frac{h(t)}{\lambda}+b\ \sqrt{h}-b\ \sqrt{h}.$$

$$\frac{dh}{dt}=\frac{h(sp)}{\lambda}-\frac{h(t)}{\lambda}.$$

This term is linear, hence there is no problem of a non-linear component in the feedback side.

Feed-forward controller design:

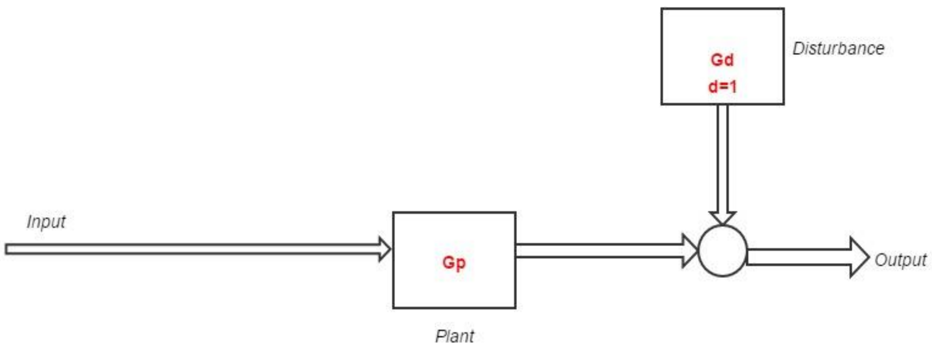
Combined feedforward plus feedback control can significantly improve performance over simple feedback control whenever there is a major disturbance that can be measured before it affects the process output. In the most ideal situation, feedforward control can entirely eliminate the effect of the measured disturbance on the process output. Even when there are modeling errors, feedforward control can often reduce the effect of the measured disturbance on the output better than that achievable by feedback control alone. However, the decision as to whether or not to use feedforward control depends on whether the degree of improvement in the response to the measured disturbance justifies the added costs of implementation and maintenance. The economic benefits of feedforward control can come from lower operating costs and/or increased salability of the product due to its more consistent quality.

Feedforward control is always used along with feedback control because a feedback control system is required to track setpoint changes and to suppress unmeasured disturbances that are always present in any real process.

A control system which has only feed-forward behaviour responds to its control signal in a pre-defined way without responding to how the load reacts; it is in contrast with a system that also has feedback, which adjusts the output to take account of how it affects the load, and how the load itself may vary unpredictably; the load is considered to belong to the external environment of the system.

In a feed-forward system, the control variable adjustment is not error-based. Instead it is based on knowledge about the process in the form of a mathematical model of the process and knowledge about or measurements of the process disturbances.

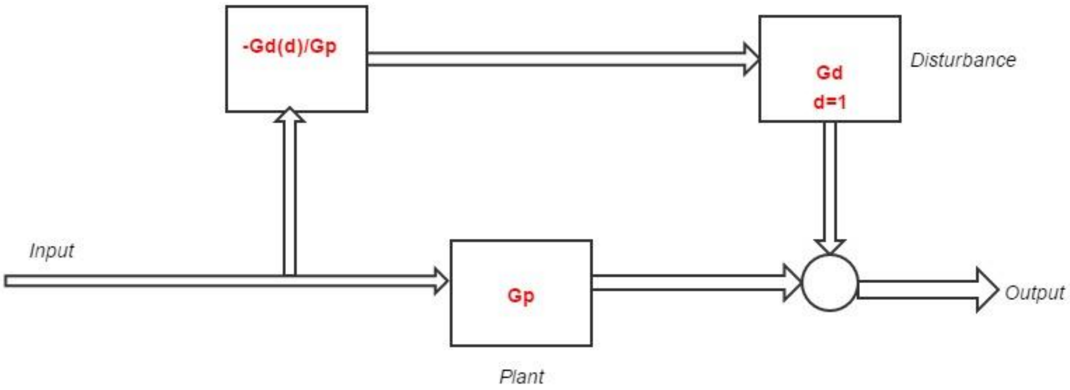
This is one of the disadvantages, because, if we do not have the knowledge about the system, we will not be able to design the controller properly.



Now, we mathematically determine the structure of the as follows:

$$y = G_{plant}u + G_{disturbance}.d$$

$$u = \frac{-G_d}{G_p}, \text{ d = 1.}$$



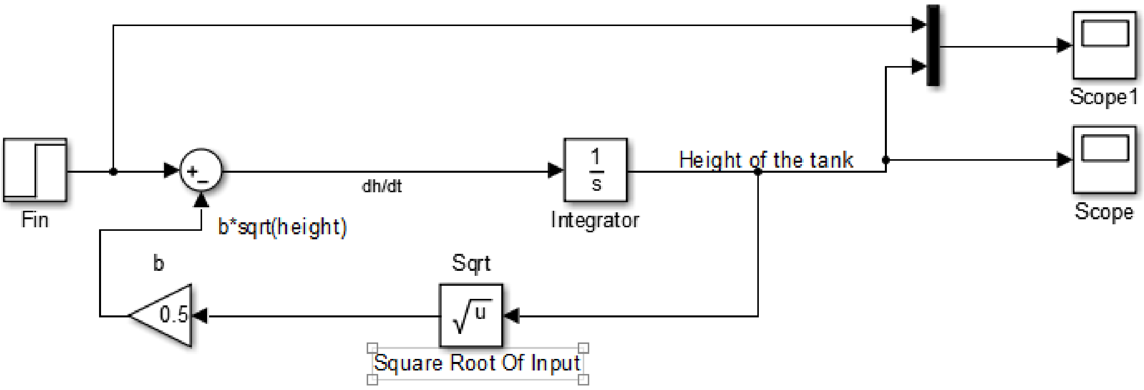
Simulation and results:

Procedure:

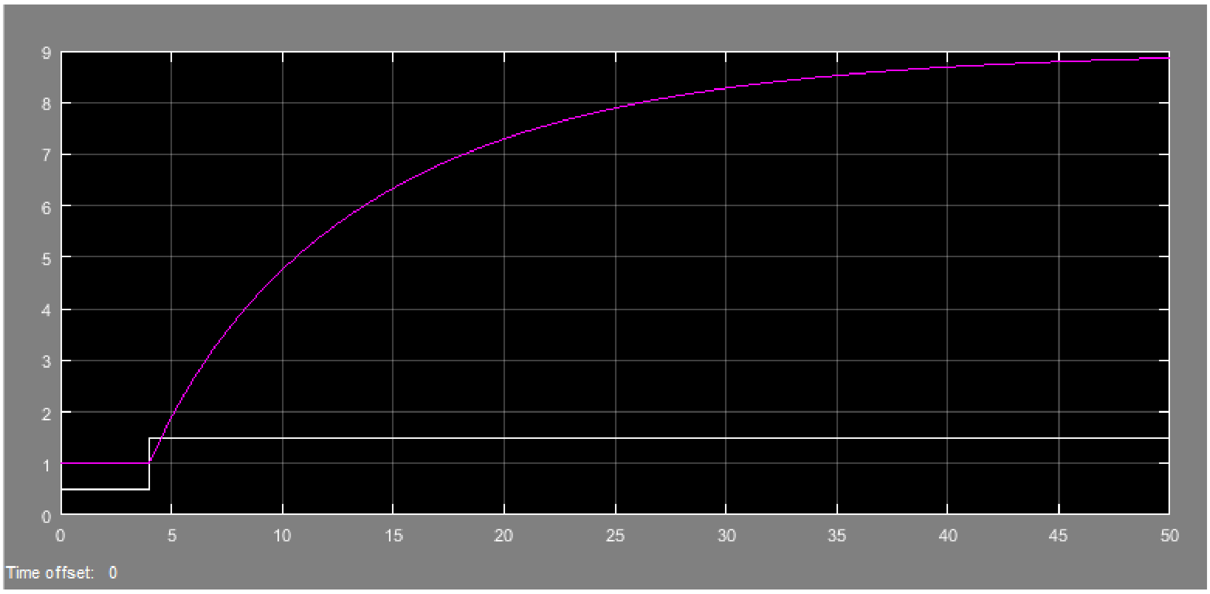
1. Determine the mathematical equation that governs the system.
2. Design the block diagram using SIMULINK.
3. Obtain the corresponding wave form and analyse it.

1. Non-linear control:

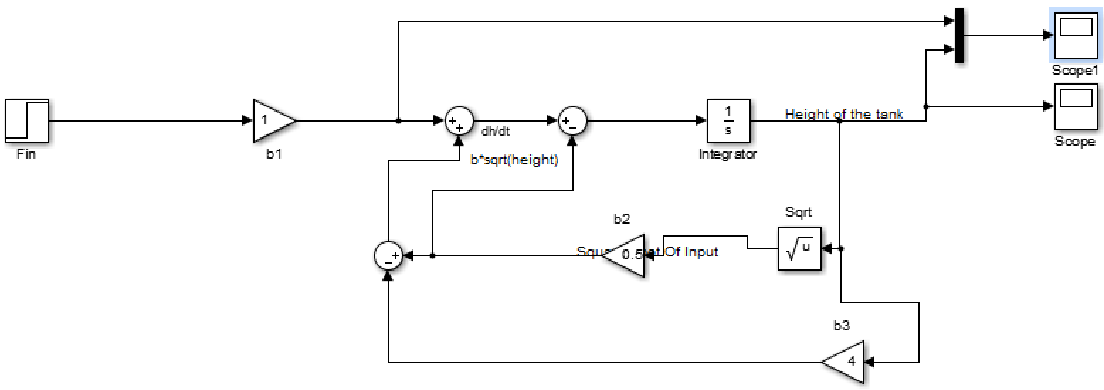
The basic block diagram of the system is as follows (Simple Tank):



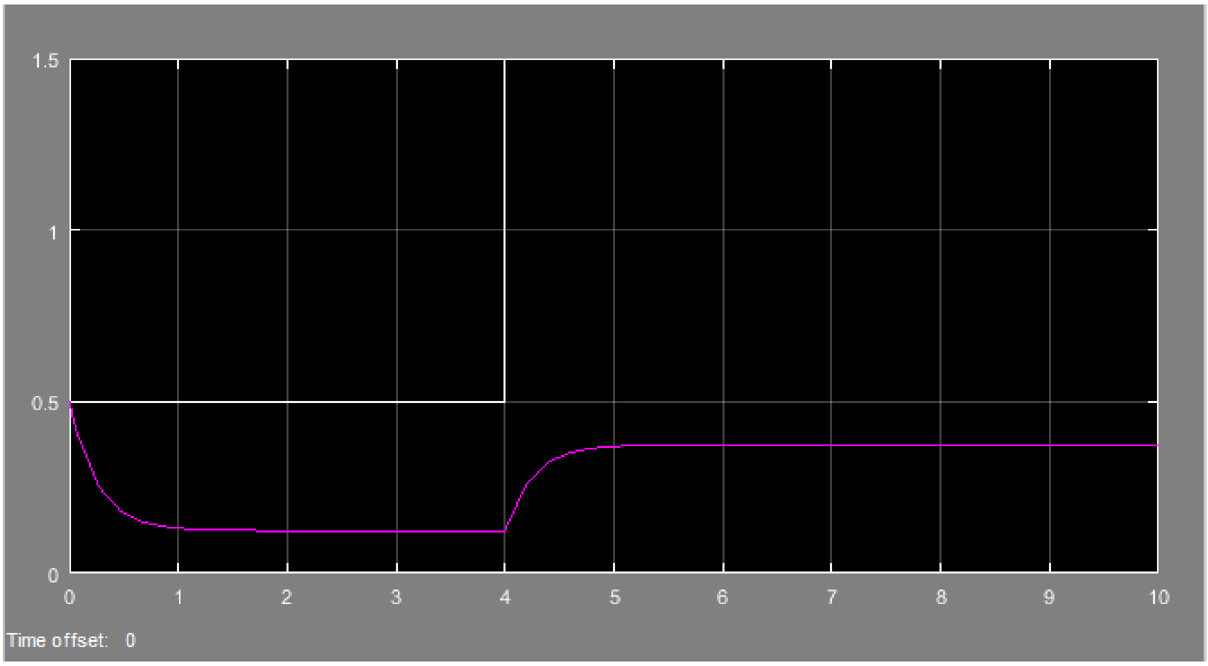
Now, the output at **scope1** as shown below:



We can see that the output is pretty ideal and has no problem as of such. But the introduction of the non-linear component in the feedback might create some problems when we scale it. Thus, here is the design where we have omitted the non-linear part.



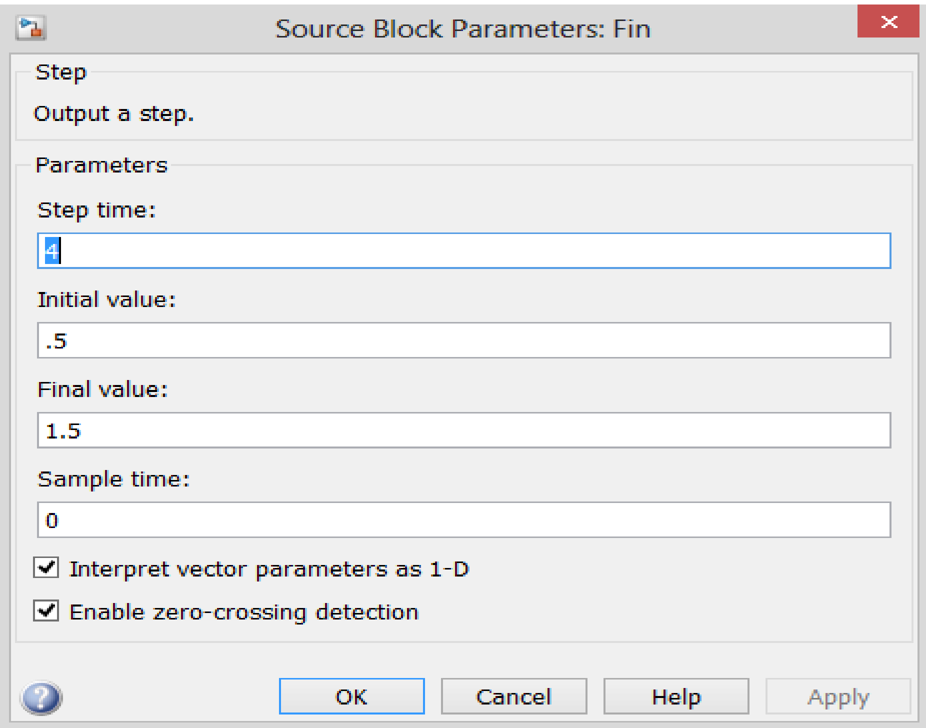
This will give an output that is as follows:



This response shows the input in **white(0.5 to 1.5)** and the output in **purple.** The response shows that the tank level will decrease and then as the input increases the level will increase and then settle at a given point. We can calculate the settling value using the value of lamba in the driving equation.

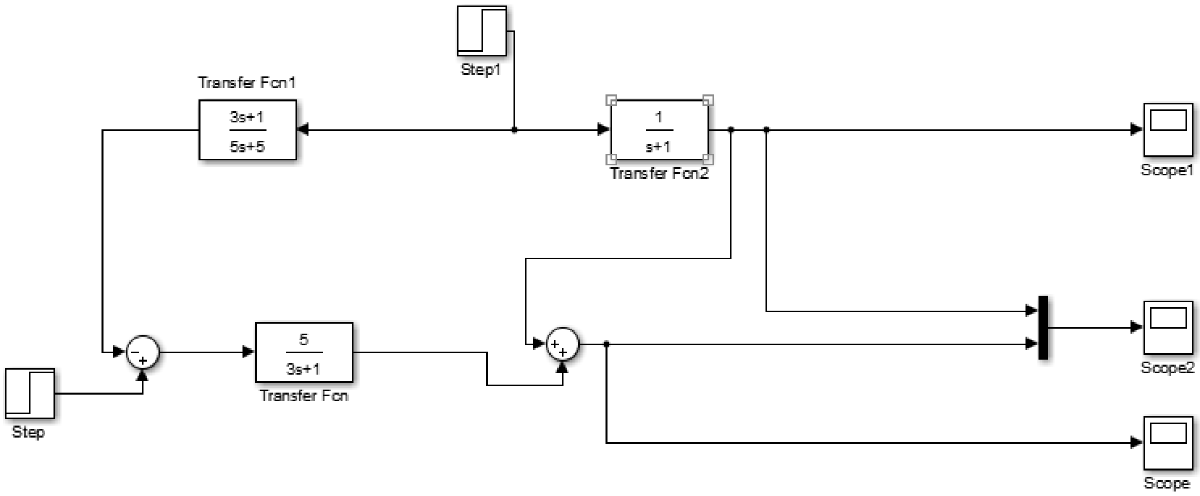
The integrator initial value is always set to **0.5,** for our application.

The input specifications are:



2. Feedforward Controller:

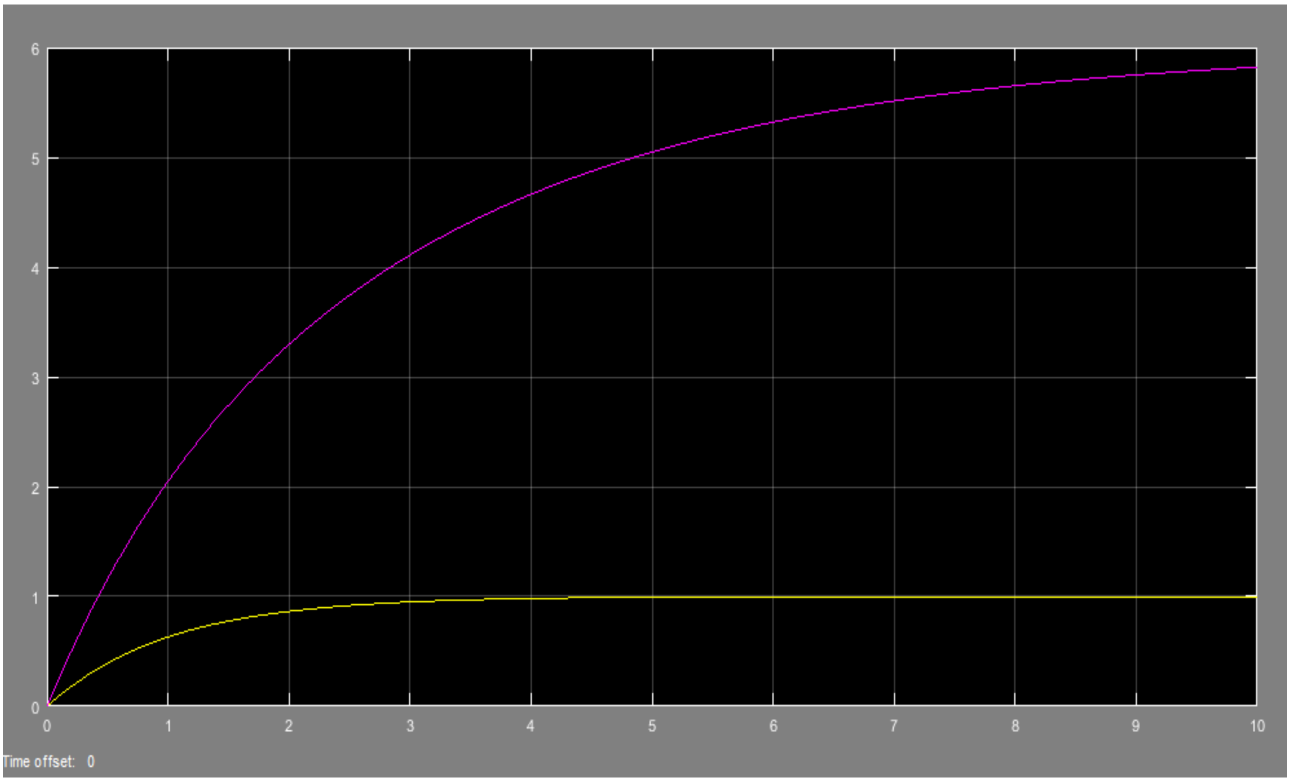
The basic block diagram of the feed-forward controller is as follows:



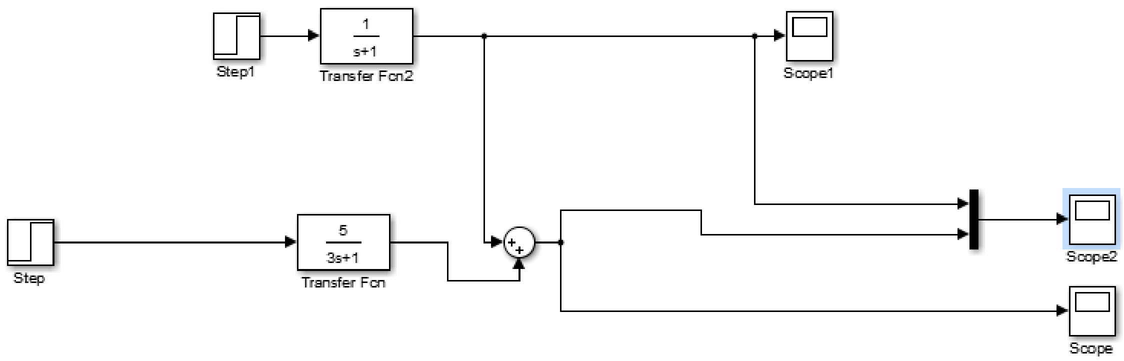
Here, G(plant) is the **Transfer Fcn,** and the G(disturbance) is **Transfer Fcn 2.** The Feed-forward controller transfer function is **Transfer Fcn 2.**

Now, we see that the transfer function in disturbance side will settle at 1, and the transfer function at the plant will settle at 5. Thus, the **overall output must settle at 6, which will be the result if there was no feed-forward controller.**

The waveform without the feed-forward controller will be:

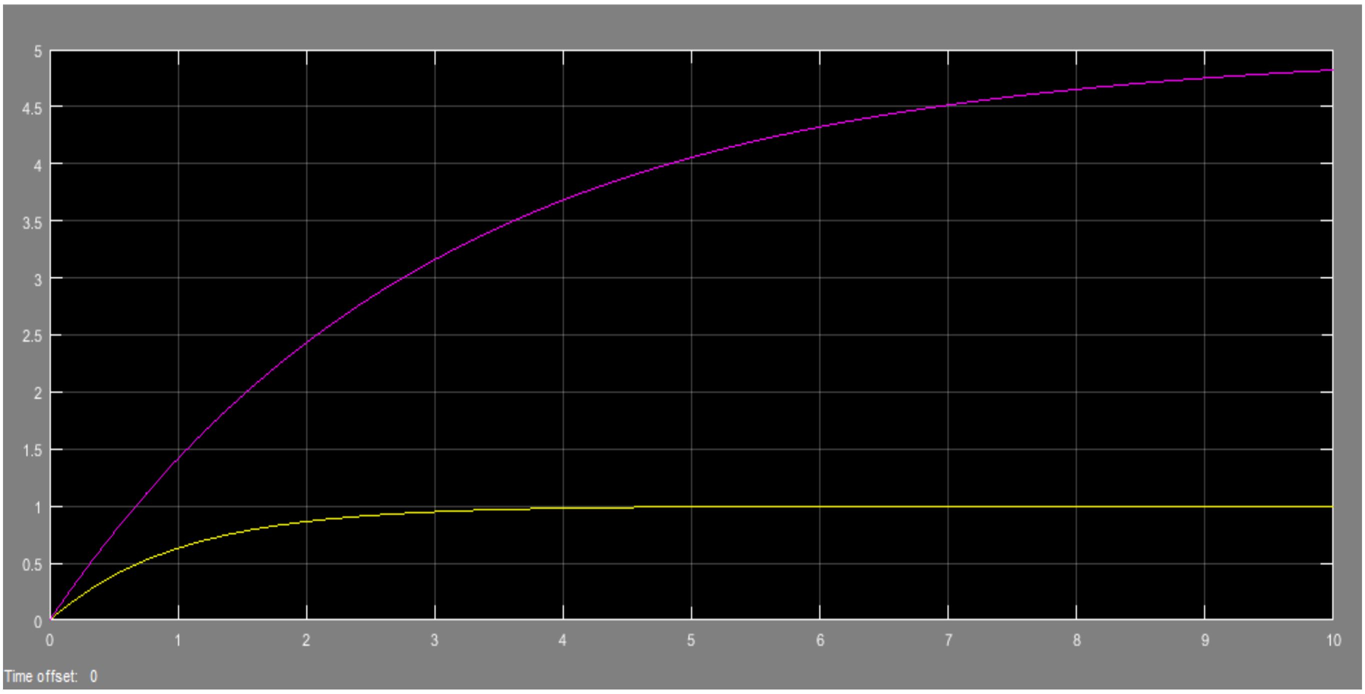


And the corresponding block diagram is:



As we see that there is no compensation and the output waveform settles at around 5.8.

Now we analyse the waveform that will be the output when the we use the feed-forward controller.



We see that the output waveform(the one with the larger magnitude) will settle at a value very close to the desired value.
Thus, our objective was successful.

Result:

Hence we designed and analysed the non-linear control of a simple tank application and a feed-forward controller.

Conclusion:

1. The Feed-forward controller depends upon our knowledge of the system. If we do not analyse and understand the system properly, the controller design will a failure.
2. The non-linear controller is very useful in simple controller applications. However, if we go to higher order systems, the analysis and design part will be very tedious.