Exp. no. 10

Interaction Measure

RGA (Relative Gain Array)

Software used:

MATLAB 2015.

Theory:

It is used to quantify the degree of directionality and the level of (two-way) interactions in MIMO systems are the condition number and the relative gain array (RGA), respectively. We £rst consider the condition number of a matrix which is de£ned as the ratio between the maximum and minimum singular values.

We follow Bristol (1966) here, and show that the RGA provides a measure of interactions. Let uj and yi denote a particular input–output pair for the multivariable plant G(s), and assume that our task is to use uj to control yi . Bristol argued that there will be two extreme cases:

• All other loops open: uk = 0, ∀k 6= j.

• All other loops closed with perfect control: yk = 0, ∀k 6= i

Derivation of RGA:

$$\lambda_{ij} = \frac{\begin{gathered}Gain\ btwn\ input\ j\ and\ output\ i\ with\ all\\ other\ loops\ open\end{gathered}}{\begin{gathered}Gain\ btwn\ input\ j\ and\ output\ i\ with\ all\\ other\ loops\ closed\end{gathered}}$$

$$\lambda_{11} = \frac{Gain\ btwn\ u_1\ and\ y_1}{Gain\ btwn\ u_1\ and\ y_1}\ ; u_2, y_3 = constant$$

$$y_1 = G_{11}u_1 + G_{12}u_2$$
$$y_2 = G_{21}u_1 + G_{22}u_2$$

Nr: $y_1 = G_{11}u_1$
Dr: $y_2 = 0$

$$-G_{21}u_1 = G_{22}u_2$$
$$-\frac{G_{21}u_1}{G_{22}} = u_2$$
$$y_1 = G_{11}u_1 + G_{12}\left(-\frac{G_{21}u_1}{G_{22}}\right)$$
$$y_1 = \left(G_{11} - \frac{G_{21}G_{12}}{G_{22}}\right)u_1$$
$$\lambda_{11} = \frac{G_{11}}{\left(G_{11} - \frac{G_{21}G_{12}}{G_{22}}\right)}$$

$$\lambda_{11} = \frac{G_{11}G_{22}}{G_{11}G_{22} - G_{21}G_{12}}$$

Similarly,

$$\lambda_{12} = \frac{-k_{12}k_{21}}{k_{11}k_{22} - k_{21}k_{12}}$$

$$\lambda_{12} = \frac{-k_{21}k_{12}}{k_{11}k_{22} - k_{21}k_{12}}$$

$$\lambda_{22} = \frac{k_{11}k_{22}}{k_{11}k_{22} - k_{21}k_{12}}$$

Now, $\lambda_{11} + \lambda_{12} = 1$

$\lambda_{12} + \lambda_{22} = 1$

$$RGA = \begin{bmatrix} \lambda_{11} & 1 - \lambda_{11} \\ 1 - \lambda_{11} & \lambda_{11} \end{bmatrix}$$

<u>Example 1:</u>

$$G(s) = \begin{bmatrix} \frac{2}{s+1} & \frac{1}{s+1} \\ \frac{1}{s+1} & \frac{2}{s+1} \end{bmatrix}$$

$$G(0) = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1.3333 & -0.3333 \\ -0.3333 & 1.3333 \end{bmatrix}$$

Use command RGA

$$k = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

MATLAB IMLEMENTATION:

```
k =

     2     1
     1     2

>> l = k'

l =

     2     1
     1     2

>> m = inv(l)

m =

    0.6667   -0.3333
   -0.3333    0.6667

>> rga = m .* k %element wise multiplication

rga =

    1.3333   -0.3333
   -0.3333    1.3333

>> %interaction between y1-u1 and y2-u2
```