Example 1:

$$G(s) = \frac{k}{\tau s + 1} = \frac{b}{s+a}$$

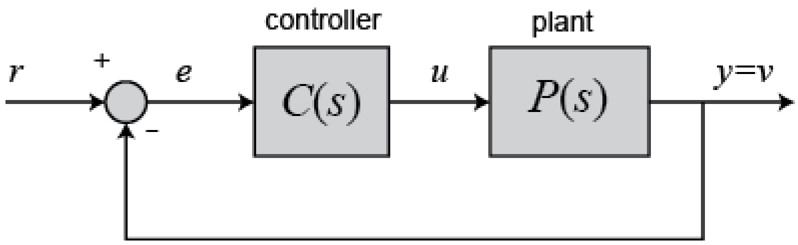

Fig 1: System with a PI controller

Aim: To find the PI parameters via state space formulation.

$$u(t) = K_I \int e.dt + K_C.e(t)$$

$$= [K_I \quad K_C].\begin{bmatrix} \int e.dt \\ e(t) \end{bmatrix}$$

$Let \int e.dt = x1,$

$\dot{x}1 = x2 = e;$

$= [K_I \quad K_C].\begin{bmatrix} x1 \\ x2 \end{bmatrix}$

$X = A\dot{X} + BU$

$\dot{x}1 = x2$

$\dot{x}2 = \dot{e}$

Here, X=2x1

V=1x1

Error e,

$$e = r - y$$
$$= -\frac{b}{s+a}u$$
$$\dot{e} + ae = -bu$$
$$=> \dot{x}2 = -ax2 - bu$$

Hence,

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -a \end{bmatrix}, B = \begin{bmatrix} 0 \\ -b \end{bmatrix}$$

$$U = -KX$$
$$\dot{X} = AX + BU$$
$$\dot{X} = AX - BKX$$

Now, find the eigen values of the closed loop system

$$|\mathrm{SI} - A + BK| \ --\rightarrow (1)$$

Desired closed loop transfer function $= \frac{1}{\lambda S+1}$

Desired closed loop poles --> $\lambda S + 1 -\rightarrow (2)$

Coeff comparison is not possible.
Eqn (1) is second order whereas eqn (2) first order.

Hence closed loop T.F. is restated as

$$\frac{\tau s + 1}{(\lambda S + 1).(\tau s + 1)}$$

Desired closed loop poles --> $(\tau s + 1).(\lambda S + 1) -\rightarrow (3)$

Compare coeff of (1) and (3) and find k

<u>Experiment</u> 2:

$$G(s) = \frac{1}{s}$$

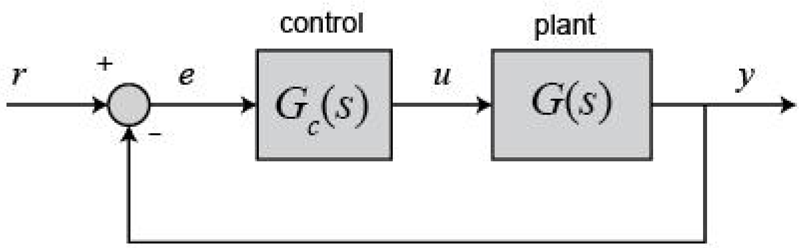

Fig 2: System with a P controller

Aim: To find the proportional controller

$$x(t) = K_C.e(t)$$
$$e(t) = x1(t)$$

$\dot{x} = -u$

$A = 0, \qquad B = -1$

$$G_{C1} = \frac{1}{\lambda S + 1}$$

Error e

$e = r - y$

$e = -\frac{1}{s}u$

$\dot{e} = -u$

Now,

$|\mathrm{SI} - A + BK| = \lambda S + 1$

Find K.

Example 3:

$$G(s) = \frac{k}{(\tau_1 S + 1).(\tau_2 s + 1)}$$

$$G_{C1} = \frac{1}{\lambda S + 1}$$

Restated specifications:

$$G_{C1} = \frac{(\tau_1 S + 1).(\tau_2 s + 1)}{(\lambda S + 1).(\tau_1 S + 1).(\tau_2 s + 1)}$$

Find PID

$$u = K_I \int e.dt + K_C.e(t) + K_d \frac{de}{dt}$$

$$= [K_I \quad K_C \quad K_d].\begin{bmatrix} \int e.dt \\ e(t) \\ \frac{de}{dt} \end{bmatrix}$$

Let $\int e.dt = x1$,

Hence, $\dot{x1} = x2 = e$

$\dot{x2} = x3 = \frac{de}{dt}$

Hence u,

$$u = [K_I \quad K_C \quad K_d].\begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix}$$

Error e,

$e = r - y$

$e = -\frac{k}{(\tau_1 S + 1).(\tau_2 s + 1)}u$

$\ddot{e} = f(x1, x2, x3) + (\ )u$

Where,
A = 3x3 B= 3x1

$$|SI - A + BK| = (\lambda S + 1).(\tau_1 S + 1).(\tau_2 s + 1)$$

Find K

Inference:
1) PI/PID/P controller tuning procedure via state space formulation is not requires for the above said specifications.
2) IMC based tuning or synthesis method may be used. But tuning procedure is simple.
3) Resultant parameters are same in all methods.
4) State space formulation based PI/PID tuning is the best option. When the specifications are performance index based one.