COSC 222 Data Structure

Sorting – Part 2

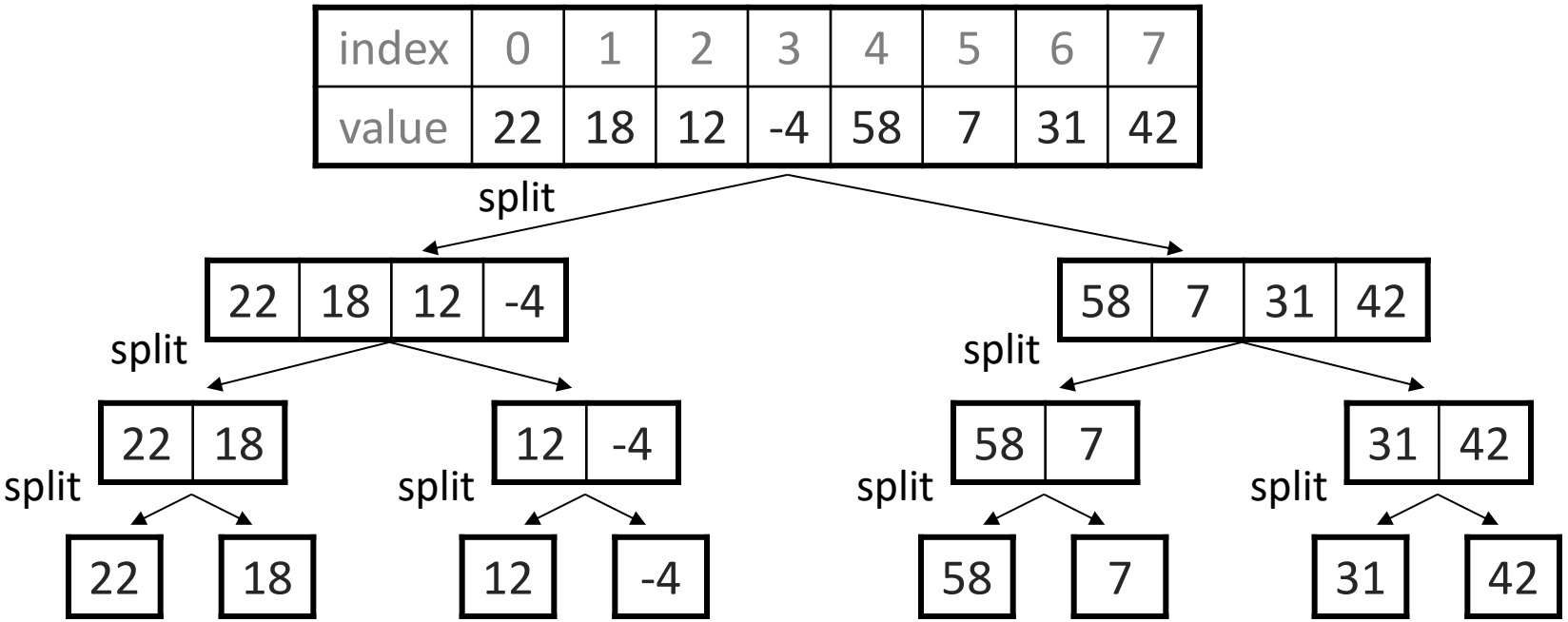
Merge sort

- **merge sort**: Repeatedly divides the data in half, sorts each half, and combines the sorted halves into a sorted whole.
  - invented by John von Neumann in 1945

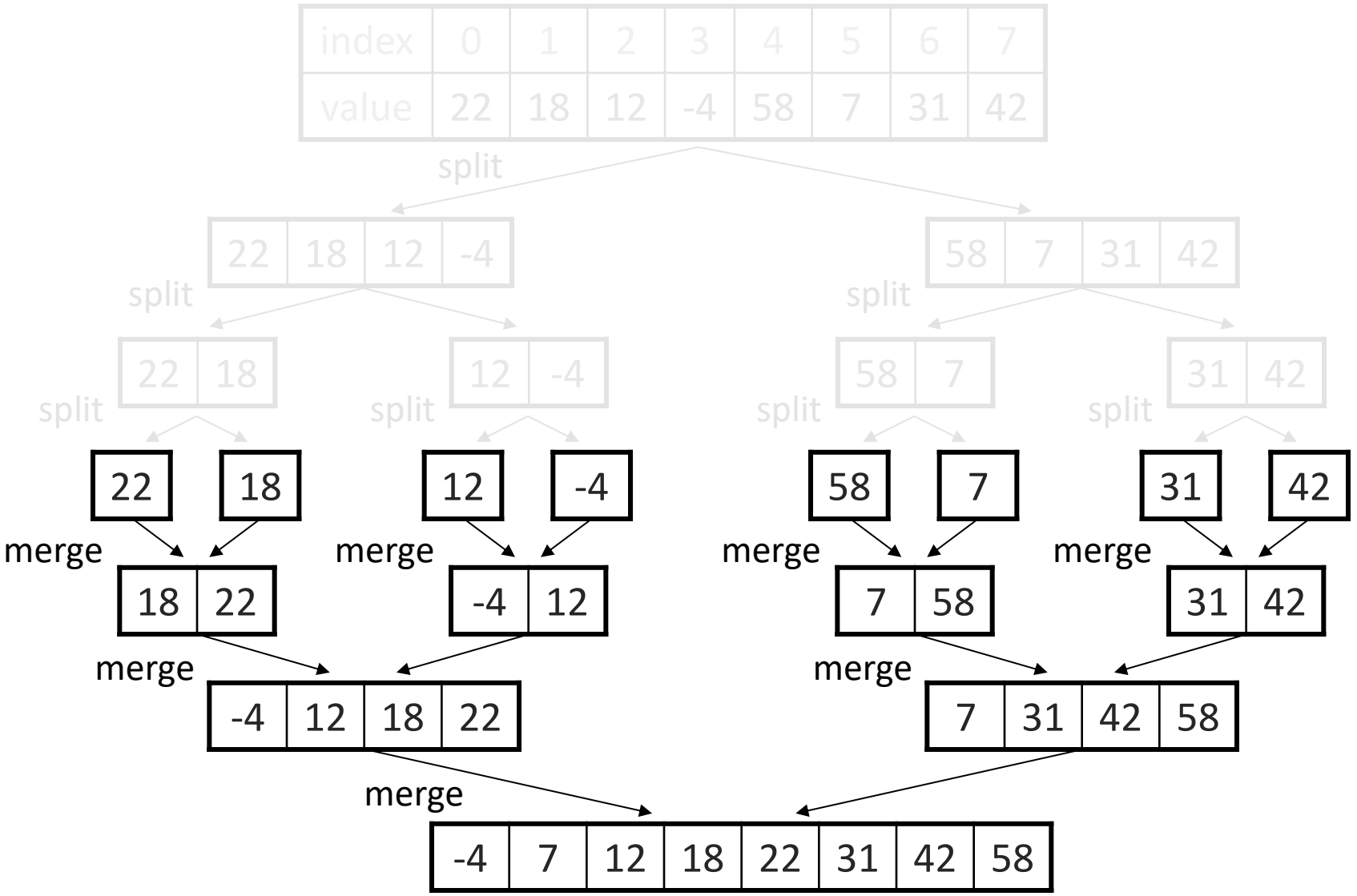
  The algorithm (divide and conquer):
  - Divide the list into two roughly equal halves.
  - Sort the left half.
  - Sort the right half.
  - Merge the two sorted halves into one sorted list.

- Often implemented recursively.

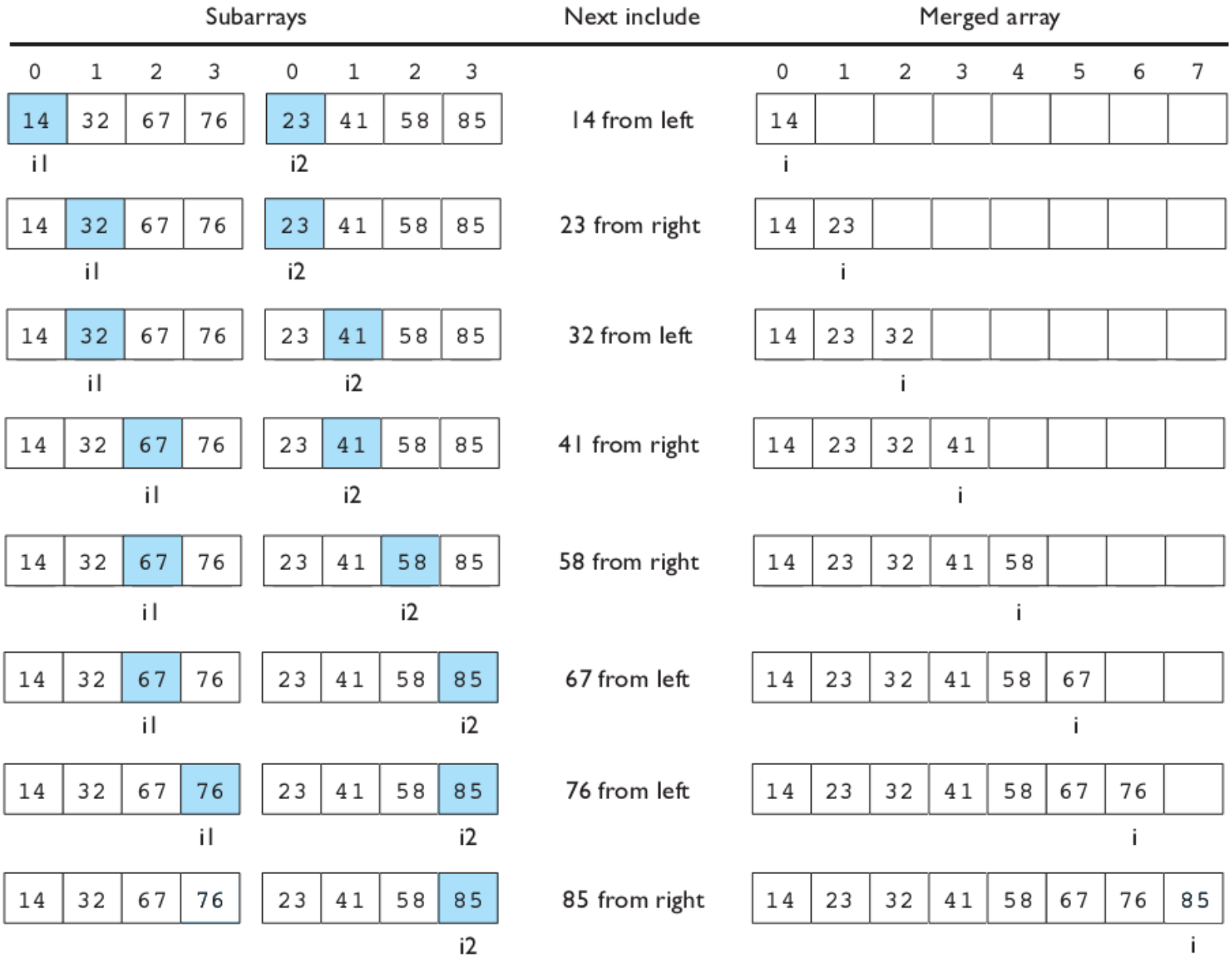
Merge sort example



Merge sort example



Merging sorted halves



Merge halves code

```
// Merges the left/right elements into a sorted result.
// Precondition: left/right are sorted
public static void merge(int[] result, int[] left,
                                       int[] right) {
    int i1 = 0;   // index into left array
    int i2 = 0;   // index into right array

    for (int i = 0; i < result.length; i++) {
        if (i2 >= right.length ||
           (i1 < left.length && left[i1] <= right[i2])) {
            result[i] = left[i1];    // take from left
            i1++;
        } else {
            result[i] = right[i2];   // take from right
            i2++;
        }
    }
}
```

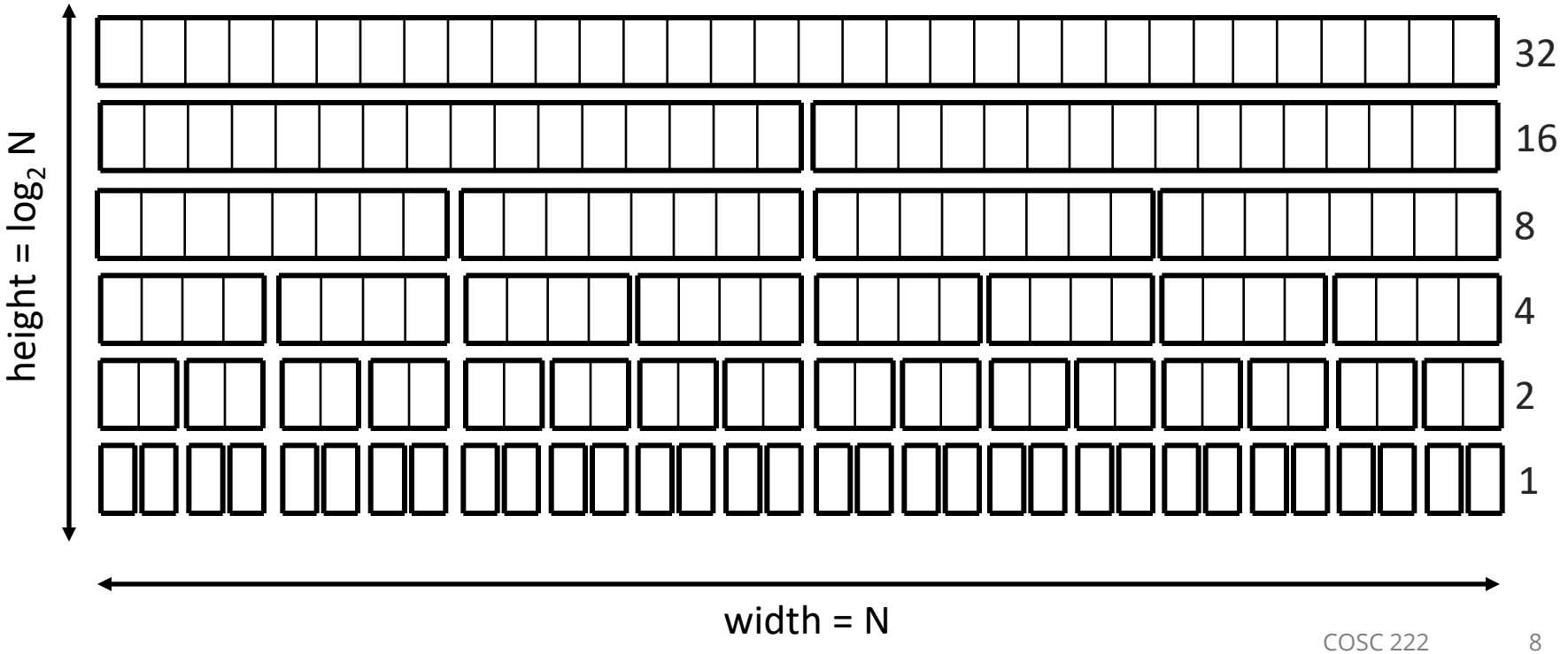
Merge sort code

```
// Rearranges the elements of a into sorted order using
// the merge sort algorithm.
public static void mergeSort(int[] a) {
    if (a.length >= 2) {
        // split array into two halves
        int[] left  = Arrays.copyOfRange(a, 0, a.length/2);
        int[] right = Arrays.copyOfRange(a, a.length/2,
                                         a.length);
        // recursively sort the two halves
        mergeSort(left);
        mergeSort(right);

        // merge the sorted halves into a sorted whole
        merge(a, left, right);
    }
}
```

More runtime intuition

- Merge sort performs O(*N*) operations on each level.
  - Each level splits the array in 2, so there are $\log_2 N$ levels.
  - Product of these = $N * \log_2 N$ = O(N log N).
  - Example: N = 32. Performs ~ $\log_2 32$ = 5 levels of N operations each:



Quick sort

- **quick sort**: Orders a list of values by partitioning the list around one element called a *pivot*, then sorting each partition.
  - invented by British computer scientist C.A.R. Hoare in 1960

- Quick sort is another divide and conquer algorithm:
  - Choose one element in the list to be the pivot.
  - *Divide* the elements so that all elements less than the pivot are to its left and all greater (or equal) are to its right.
  - *Conquer* by applying quick sort (recursively) to both partitions.

- Runtime: O(N log N) average, O(N^2) worst case.

Choosing a "pivot"

- The algorithm will work correctly no matter which element you choose as the pivot.
  - A simple implementation can just use the first element.

- But for efficiency, it is better if the pivot divides up the array into roughly equal partitions.

Partitioning an array

- Swap the pivot to the last array slot, temporarily.
- Repeat until done partitioning (until *i*,*j* meet):
  - Starting from *i* = 0, find an element a[*i*] ≥ pivot.
  - Starting from *j* = *N*-1, find an element a[*j*] ≤ pivot.
  - These elements are out of order, so swap a[*i*] and a[*j*].
- Swap the pivot back to index *i* to place it between the partitions.

index	0	1	2	3	4	5	6	7	8	9
value	6	1	4	9	0	3	5	2	7	8

8 i j 6

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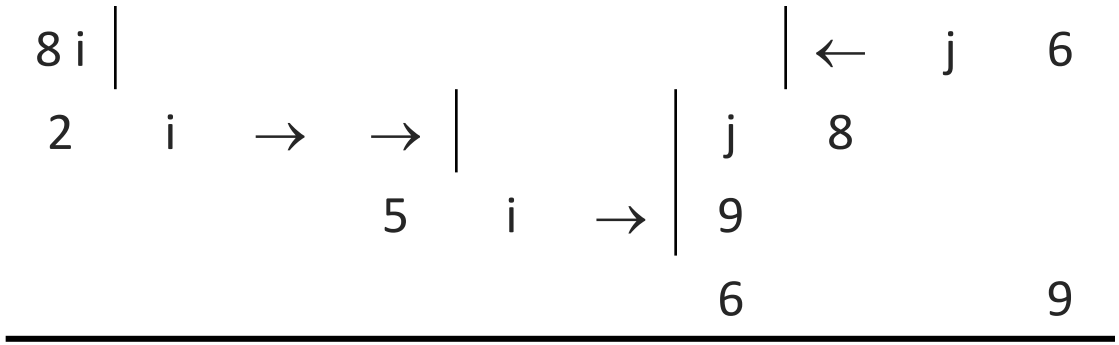

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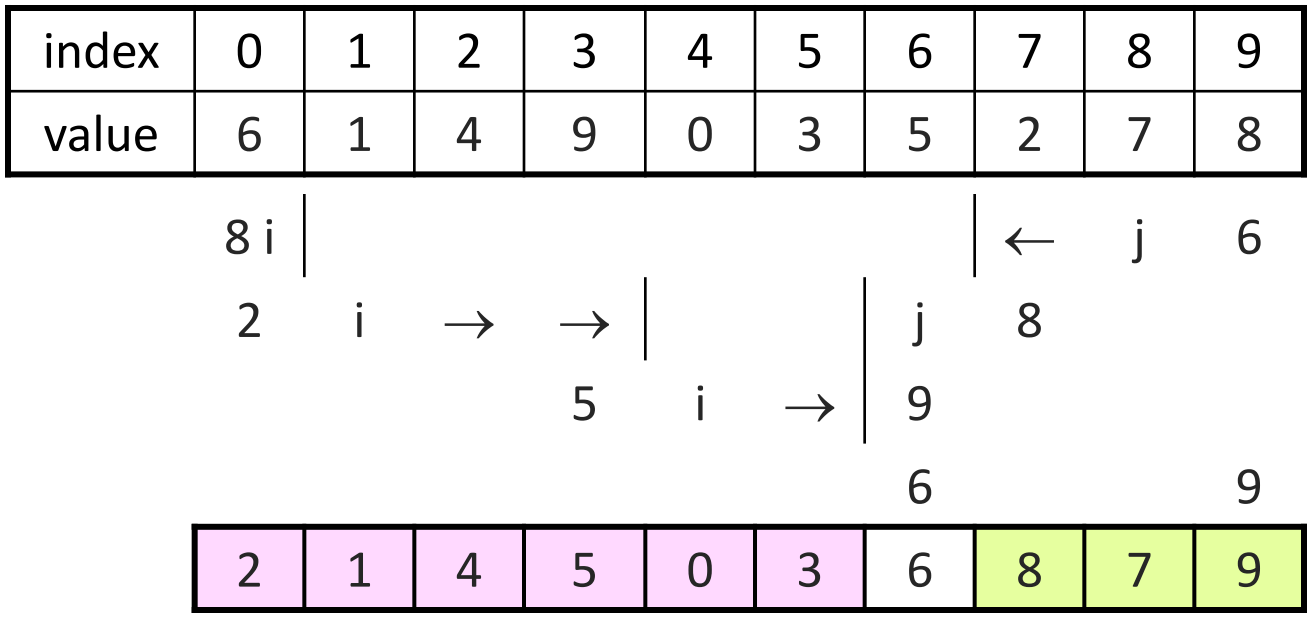

Quick sort example

index	0	1	2	3	4	5	6	7	8	9	
value	**65**	23	81	43	92	39	57	16	75	32	choose pivot=65
	32	23	81	43	92	39	57	16	75	**65**	swap pivot (65) to end
	32	23	**16**	43	92	39	57	**81**	75	65	swap 81, 16
	32	23	16	43	**57**	39	**92**	81	75	65	swap 57, 92
	32	23	16	43	57	39	92	81	75	65	
	32	23	16	43	57	39	**65**	81	75	**92**	swap pivot back in

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	32	23	16	43	57	39	92	81	75	65	
	32	23	16	43	57	39	**65**	81	75	**92**	swap pivot back in

recursively quicksort each half

32	23	16	43	57	39	pivot=32
39	23	16	43	57	**32**	swap to end
16	23	**39**	43	57	32	swap 39, 16
16	23	**32**	43	57	**39**	swap 32 back in

...

81	75	92	pivot=81	
92	75	**81**	swap to end	
75	92	81	swap 92, 75	
75	**81**	**92**	swap 81 back in	

...

Quick sort code

```
public static void quickSort(int[] a) {
    quickSort(a, 0, a.length - 1);
}
private static void quickSort(int[] a, int min, int max) {
    if (min >= max) {  // base case; no need to sort
        return;
    }

    // choose pivot; we'll use the first element (might be bad!)
    int pivot = a[min];
    swap(a, min, max);      // move pivot to end

    // partition the two sides of the array
    int middle = partition(a, min, max - 1, pivot);

    swap(a, middle, max);   // restore pivot to proper location

    // recursively sort the left and right partitions
    quickSort(a, min, middle - 1);
    quickSort(a, middle + 1, max);
}
```

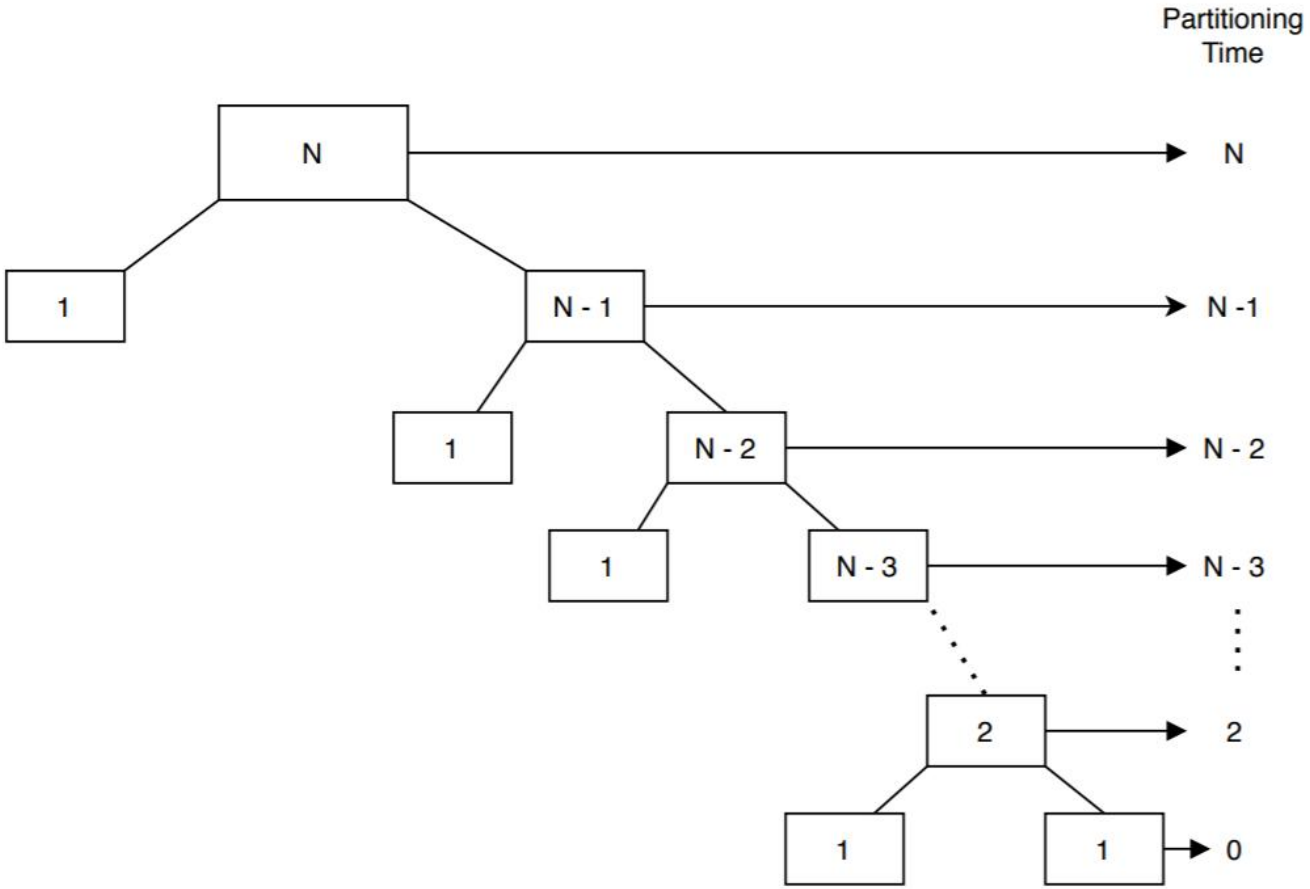
Partition code

```
// partitions a with elements < pivot on left and
// elements > pivot on right;
// returns index of element that should be swapped with pivot
private static int partition(int[] a, int i, int j, int pivot) {
    while (i <= j) {
        // move index markers i,j toward center
        // until we find a pair of out-of-order elements
        while (i <= j && a[i] < pivot) { i++; }
        while (i <= j && a[j] > pivot) { j--; }

        if (i <= j) {
            swap(a, i, j);
            i++;
            j--;
        }
    }
    return i;
}
```

Quick sort runtime

- Worst-case: What if the pivot is chosen poorly? What is the runtime? O(N^2)



Choosing a better pivot

- Choosing the first element as the pivot leads to very poor performance on certain inputs (ascending, descending)
  - does not partition the array into roughly-equal size chunks

- Alternative methods of picking a pivot:
  - *random:* Pick a random index from [*min* .. *max*]
  - *median-of-3:* look at left/middle/right elements and pick the one with the medium value of the three:

    `a[min]`, `a[(max+min)/2]`, and `a[max]`

    better performance than picking random numbers every time

    provides near-optimal runtime for almost all input orderings

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	8	18	91	-4	27	30	86	50	65	78	5	56	2	25	42	98	31

Summary

- **bubble sort**: swap adjacent pairs that are out of order
- **selection sort**: look for the smallest element, move to front
- **insertion sort**: build an increasingly large sorted front portion
- **merge sort**: recursively divide the array in half and sort it
- **quick sort**: recursively partition array based on a pivot

- other specialized sorting algorithms:
- **radix sort**: sort integers by last digit, then 2nd to last, then …

Questions?