Parameter Estimation & Hypothesis Testing

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Ques 1 Let $x_1, x_2 \ldots$ be random sample of size n taken from normal population with parameters : mean = θ_1 & Variance = θ_2. Find max likelihood estimates of these 2 parameters

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}$$

$x_1, x_2 \ldots x_n$ → sample of size n

$$L(x_1, x_2 \ldots x_n) = f(x_1) \times f(x_2) \ldots f(x_n)$$

$$\Rightarrow \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x_1-\mu)^2/2\sigma^2} \times \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x_2-\mu)^2/2\sigma^2} \times \ldots$$

taking ln both sides.

$$\ln(L) = \frac{-n}{2} \ln(2\pi\sigma^2) + \sum_{i=1}^{n} \frac{(x_i-\mu)^2}{2\sigma^2} \quad - (1)$$

taking partial derivative wrt. μ, we get.

$$\frac{\partial \ln(L)}{\partial \mu} = 0 + \sum_{i=1}^{n} -\frac{2(x_i-\mu)}{2\sigma^2} = 0$$

$$\sum_{i=1}^{n} x_i - \mu = 0$$

$$n\bar{x} - n\mu = 0$$

$$\bar{x} = \mu$$

Hence $\theta_1 = \bar{x}$ ie. sample mean

taking partial derivative of (1) wrt. σ^2, we get apsara

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$$\frac{\partial \ln L}{\partial \sigma^2} = \frac{n}{2\sigma^2} + \sum_{i=1}^{n} -\frac{(x_i-\mu)^2}{2\sigma^2} = 0$$

$$n + \sum_{i=1}^{n} -\frac{(x_i-\mu)^2}{\sigma^2} = 0$$

$$\sigma^2 = \frac{1}{n}\left(\sum_{i=1}^{n}(x_i-\mu)^2\right)$$

Hence $\theta_2 = \frac{1}{n}\left(\sum_{i=1}^{n}(x_i-\mu)^2\right)$

Ques 2 Let $X_1, X_2 \ldots X_n$ be random sample from $B(m,\theta)$ distribution, where $\theta \in \Theta = (0,1)$ is unknown & m is a known +ve integer. Compute value of θ using MLE.

Binomial distr. $\rightarrow {}^{m}C_{x_i}\,\theta^{x_i}(1-\theta)^{m-x_i}$

$$L = \prod_{i=1}^{n} {}^{m}C_{x_i}\,\theta^{x_i}(1-\theta)^{m-x_i}$$

taking log both sides

$$\log L = \sum_{i=1}^{n}\left[\log {}^{m}C_{x_i}\right] + \left[(\log\theta)\sum_{i=1}^{n} x_i\right] + \left[(\log(1-\theta))\sum_{i=1}^{n}(m-x_i)\right]$$

differentiate w.r.t. θ

$$\frac{d\log L}{d\theta} = 0$$

$$\frac{1}{\theta}\Sigma x_i - \frac{1}{1-\theta}\Sigma(m-x_i) = 0$$

$$\frac{1}{\theta(1-\theta)}\Sigma x_i = \frac{n^2}{1-\theta}$$

$$\theta = \frac{\Sigma x_i}{n^2}$$

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