HW4
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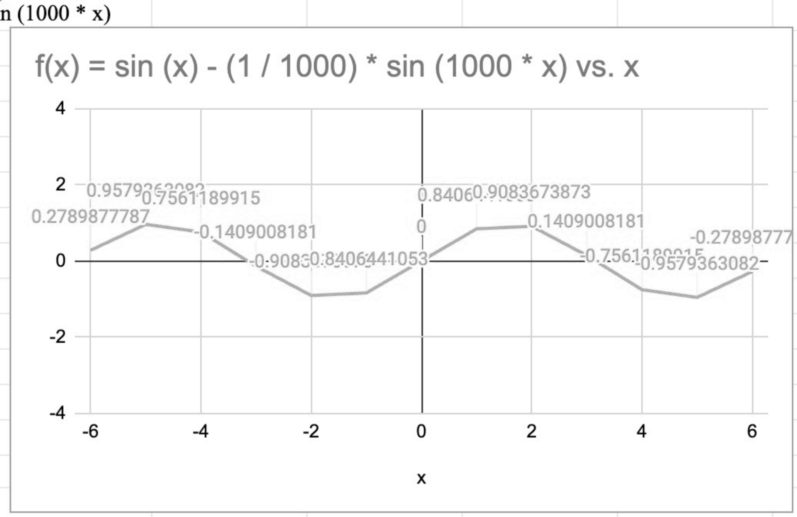
COUSE: CALCULUS-I

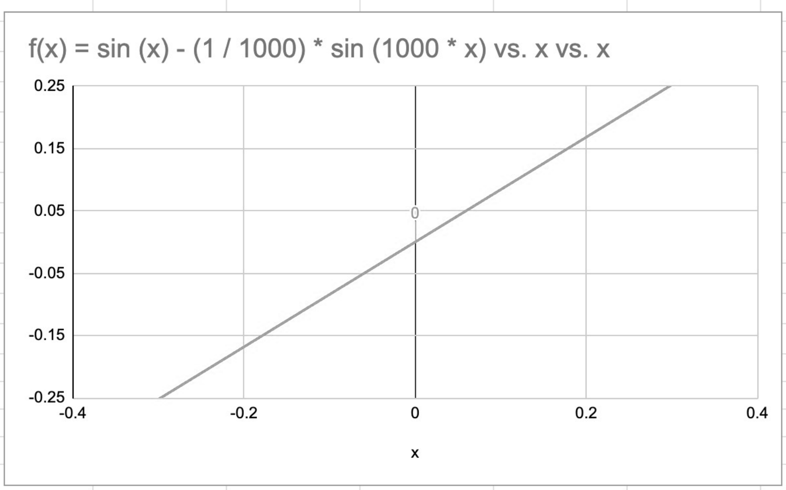
#Solution-1:

a)We can see that, the slope of the graph appears to have at the origin is: -0.0509 x – 0.070.
slope=y⇒-0.0509x - 0.070⇒R^2=0.0421

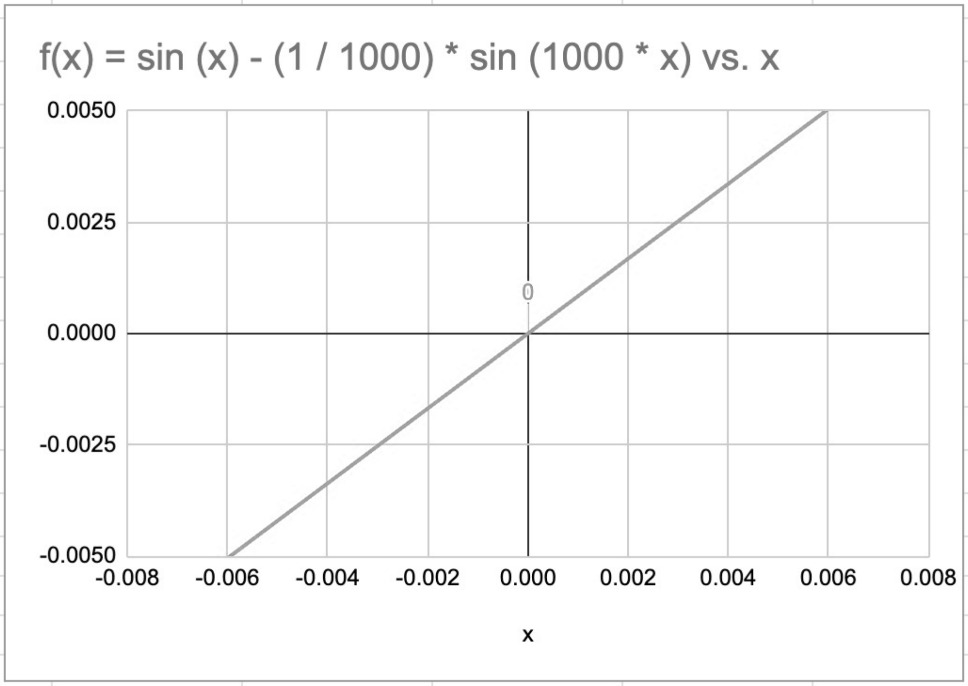
	#Solution-1
x	f(x) = sin (x) - (1 / 1000) * sin (1000 * x)
-6	0.2789877787
-5	0.9579363082
-4	0.7561189915
-3	-0.1409008181
-2	-0.9083673873
-1	-0.8406441053
0	0
1	0.8406441053
2	0.9083673873
3	0.1409008181
4	-0.7561189915
5	-0.9579363082
6	-0.2789877787



b)The values of x-axis and y-axis on the graph to match the viewing window [-0.4, 0.4] by [-0.25, 0.25]. We can observe that, the slope is still the same, which agrees with an answer from part(a)



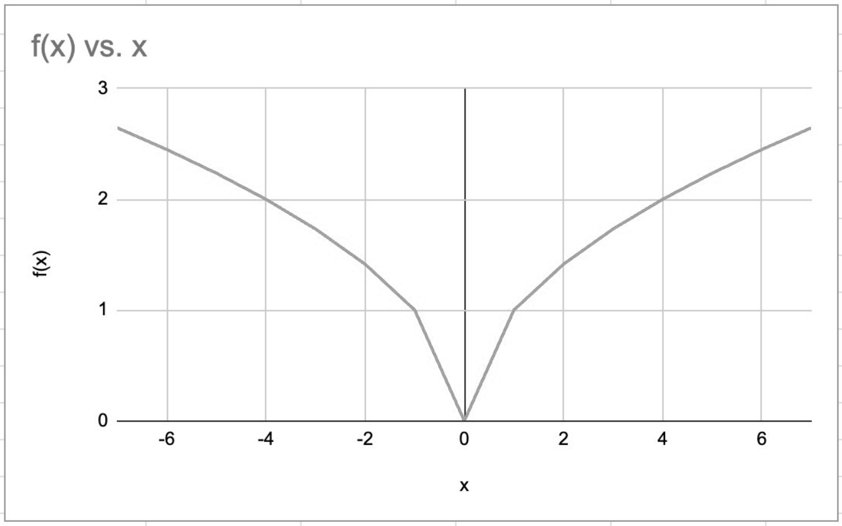
c)Now the graph gets much stronger. The slope of the curve at 0 which is f'(0) is - 0.0509. The slope remains the same.



#Solution-2:

Zooming at (-1,0) f is distinguishable at (-1,0) since it is smooth at (-1,0) and there are no sudden transitions. Zooming at origin f is NOT differentiable because of kink at origin. The slope suddenly shifts.

	#Solution-2:
x	f(x)
-7	2.645751311
-6	2.449489743
-5	2.236067977
-4	2
-3	1.732050808
-2	1.414213562
-1	1
0	0
1	1
2	1.414213562
3	1.732050808
4	2
5	2.236067977
6	2.449489743
7	2.645751311



#Solution-3:

a) Find f' — (4) and f' + (4) for the function

To find f'-(4). we substitute the value of 4 in the function.

We get,

f'(-4) = $_{h}\lim_{5-0(-4+} \overline{f(4+h_h)-f(4)}$[Here, if h<0, 4+h is also less than 4]

=== hh-1limlim→→ $00_{-}-\overset{\rightarrow}{}-hh$ $hh)-1$ since, from definition, f(4)=1/(5-x)=1]

To find f'+(4). we substitute the value of 4 in the function. We get, f'(+4)= lim→ =+ $h$lim→$5(41+0hh+)-f1(4+[hh$From$)-f(4)$ the[Here,definition,if

h>0,if4+xhisis>greateror equalthanto 44], we use 1/ (5-x)] h 0

= lim→ $+1-{}^{1}{}_{h}h-1$

h 0

= lim→ $+h\underline{1}(\underline{-}1\underline{1}-\underline{\pm}h\underline{h})$

= = hhhlimlim→→$00++$ $((11--{}^{1}1h0))$

0

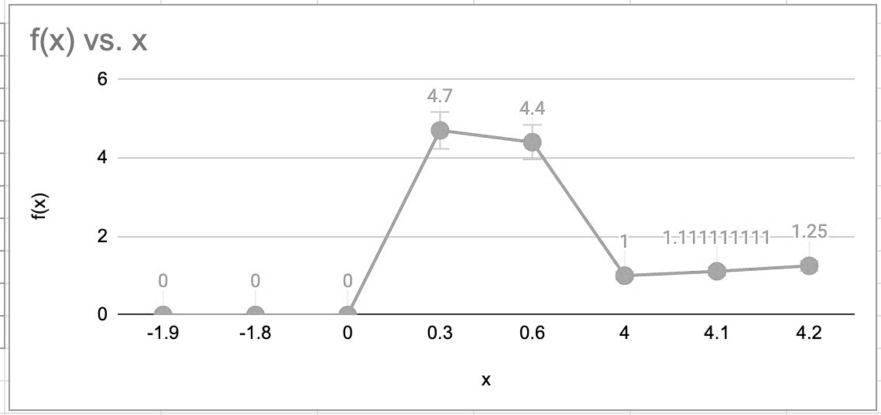
= = 1 $^{1}0)$

(1−

b)

#Solution-3

x	f(x)
-2	0
-1.9	0
-1.8	0
0	0
0.3	4.7
0.6	4.4
4	1
4.1	1.111111111
4.2	1.25



c) Where is f discontinuous?

The graph of f is discontinuous at x=0 and x=5.

d) Where is f not differentiable?

The graph of f is not differentiable

At x=0 and x=4 and x=5.

#Solution-4:

To show that $g'(x) = xf'(x) + f(x)$,

we'll use the definition of the derivative. The derivative of $g(x)$ with respect to x is defined as:

$g'(x)$ =$_x$lim→ $_a$ $\underline{g(x}$$_xthe\underline{)-}$$-\underline{ga(}$function$\underline{a)}$$g(x) = xf(x)$ into the above expression:

Let's substitute

gNow,$'(x)$ =let's$_x$lim→ a $\underline{xf(xx)-}$$-\underline{af}$$athe\underline{(a)}$expression:

simplify

$g'(x) =$ limlim$\to\to$ ^{a} $\underline{xf(x-(xa))-faf(x)(+xxx-)a+-[afafa(x()a-)f-(afa)(]a)}$

$g'(x) = x$

Next, we$_x$can$_a$rewrite the expression as:

$g'(x) =$ $\underline{(x-xa-)af(x)}$ $+$ $_x$lim$\to$ $_{a}$ $\underline{a[f(xx)--af(a)]}$

The first$_x$lim term$\to$ $_a$ in the above expression simplifies to $f(x)$ and the second term simplifies to $af'(a)$.

Therefore, we have:

$g'(x) = f(x) + af'(a)$

Since a is a constant, we can substitute a with x in the equation to get the desired result:

$g'(x) = xf'(x) + f(x)$

Thus, we have shown that $g'(x) = xf'(x) + f(x)$ using the definition of a derivative.

#Solution-5:

a) Boyle's Law can be expressed mathematically as: P . V = *k* where *k* is a constant for a given amount of gas at a constant temperature. To express V as a function of P, we rearrange the equation:

V = $\overline{P^{k}}$

Given that the pressure P = 50*kPa* when the volume V = 0.106m^3, we can find *k* :

k= P . V = 50*k*Pa . 0.106m^3 = 5.3*k*Pa . m^3

So, the function V(P) is:

V(P)= $5P.3$

(b) To find $\overline{dP^{dV}}$, , we take the derivative$\underline{5P.}$of$\underline{3}$V(P) :

V(P)=

The derivative is:

$\frac{}{dVdP} = \frac{}{-}$

5

$P.$

3

2

When P = 50kPa:

$\underline{dV}dP = -dV(505.^3) \Rightarrow -2500\underline{5.3} \Rightarrow$ -0.00212m^3 /kPa

The derivative is the rate of change of the gas volume for pressure at constant temperature.dP

The value of this derivative means that the volume shrinks about 0.00212m3 per 1 kPa in

pressure for every 1 mm of rise in pressure. The units of the derivative are m3 /kPa.

#Solution-6:

a) Use a calculator to model tire life with a quadratic function of the pressure.

→ Here, the input to the function is the pressure and the output is the tire life.

The quadratic function will be: L(P) = a * P ^2 + b * P + c

Now we get,

L(P) = −0.2754P^2 + 19.7485P − 273.5523

b)Use the model to estimate dL/dP when P = 30 and P = 40. What is the meaning of the derivative? What are the units? What is the significance of the signs of the derivatives?

dL/dP = −0.5508P + 19.7485

so,

dL/dP(P =30)

= −0.5508 (30) + 19.7485

≈ 3.22 lb/in^2

dL/dP(P=40)

= −0.5508 (40) + 19.7485

≈ −2.28 lb/in^2

the derivative is the percentage change in tire life as a function of the pressure. The units are thousand miles/ (lb/in2). (Just like any other derivative, the values are units of output from the original function divided by units of input to the original function. At P = 30 the derivative is positive (i.e, tire life is going up) at P = 40 the derivative is negative (i.e, tire life is going down).