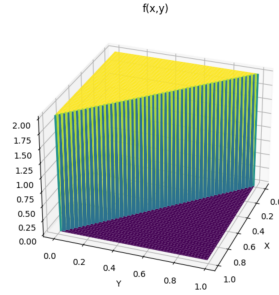# Assignment 2

CS20BTECH11047

Fig. 1: $f(x, y)$

SECTION

Let X and Y be two continuous random variables with the joint probability density function

$$f(x, y) = \begin{cases} 2 & 0 < x + y < 1, x > 0, y > 0 \\ 0 & \text{elsewhere} \end{cases}$$

(0.0.1)

Then $P\left(X + Y < \dfrac{1}{2}\right)$ is

1) $\dfrac{1}{4}$ 3) $\dfrac{3}{4}$

2) $\dfrac{1}{2}$ 4) 1

SOLUTION

Given X and Y be two continuous random variables with the joint probability density function

$$f(x, y) = \begin{cases} 2 & 0 < x + y < 1, x > 0, y > 0 \\ 0 & \text{elsewhere} \end{cases}$$

(0.0.2)

we know that

$$P((x, y) \in A) = \int \int_A f(x, y)\, dxdy \quad A \in \mathbb{R}^2$$

(0.0.3)

from given information
for positive x and y

$$0 < x + y < \frac{1}{2} \Rightarrow 0 < x < \frac{1}{2} - y$$

(0.0.4)

so using eq(0.0.3)

$$P\left(x + y < \frac{1}{2}\right) = \int_0^{\frac{1}{2}} \int_0^{\frac{1}{2}-y} f(x, y) dxdy \quad (0.0.5)$$

$$= \int_0^{\frac{1}{2}} \int_0^{\frac{1}{2}-y} 2 \quad dxdy = \int_0^{\frac{1}{2}} \left(2x \quad \Big|_0^{\frac{1}{2}-y}\right) dy \quad (0.0.6)$$

$$= \int_0^{\frac{1}{2}} 2\left(\frac{1}{2} - y\right) \quad dy = 2\left(\frac{1}{2}y - \frac{y^2}{2}\right)\Big|_0^{\frac{1}{2}} \quad (0.0.7)$$

$$= \left(\frac{1}{2} - \frac{1}{4}\right) = \frac{1}{4} \quad (0.0.8)$$

Therefore

$$P\left(x + y < \frac{1}{2}\right) = \frac{1}{4}$$

(0.0.9)

The volume under the graph which contains the region $X + Y < \dfrac{1}{2}$ gives us $P\left(x + y < \dfrac{1}{2}\right)$

$$P\left(x + y < \frac{1}{2}\right) = \text{Area of the base . height}$$

Area of the base triangle is $\frac{1}{2}.height.base = \frac{1}{2}.\frac{1}{2}.\frac{1}{2}$

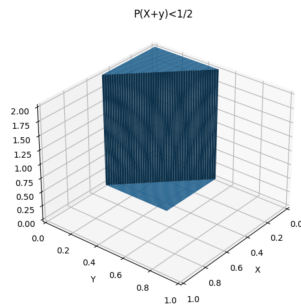$\text{volume} = \text{Area . height} = \frac{1}{8}.2 = \frac{1}{4}$

P(X+y)<1/2

Fig. 2: $P\left(x + y < \frac{1}{2}\right)$