Assignment 3

CS20BTECH11047

Download all python codes from

https://github.com/JeevanIITH/AI1102/blob/main/ assignment3/assignment3.py

and latex codes from

https://github.com/JeevanIITH/AI1102/blob/main/ assignment3/assignment3.tex

GATE EC 2012 Q.1

Two independent random variables X and Y are uniformly distributed in the interval [-1,1]. The probability that $\max[X, Y]$ is less than $\frac{1}{2}$ is

1)
$$\frac{3}{4}$$

2)
$$\frac{9}{16}$$
 3) $\frac{1}{4}$

3)
$$\frac{1}{4}$$

4)
$$\frac{2}{3}$$

SOLUTION

Since the random variable *X* is uniformly distributed in the interval [-1,1], let $f_X(x) = k$. Then

$$\int_{-1}^{1} f_X(x)dx = \int_{-1}^{1} kdx = 1$$
 (0.0.1)

$$\Rightarrow k = \frac{1}{2} \tag{0.0.2}$$

So

$$f_X(x) = \frac{1}{2}$$
 , $x \in (-1, 1)$ (0.0.3)

Similarly

$$f_Y(y) = \frac{1}{2}$$
 , $y \in (-1, 1)$ (0.0.4)

Now

$$\max[X, Y] < \frac{1}{2} \Rightarrow \left(X < \frac{1}{2}\right) \cdot \left(Y < \frac{1}{2}\right)$$
(0.0.5)

So

$$\Pr\left(\max[X,Y] < \frac{1}{2}\right) = \Pr\left(\left(X < \frac{1}{2}\right) \cdot \left(Y < \frac{1}{2}\right)\right) \tag{0.0.6}$$

Since *X* and *Y* are independent

$$\Pr\left(\left(X < \frac{1}{2}\right).\left(Y < \frac{1}{2}\right)\right) = \Pr\left(X < \frac{1}{2}\right).\Pr\left(Y < \frac{1}{2}\right)$$
(0.0.7)

Now using CDF

$$F_X(x) = \Pr(X < x) = \int_{-1}^{x} f_X(x) dx$$
 (0.0.8)

$$= \int_{-1}^{x} \frac{1}{2} dx = \frac{1}{2} (x - (-1)) = \frac{1}{2} (x + 1)$$
 (0.0.9)

similarly

$$F_Y(y) = \Pr(Y < y) = \int_{-1}^{y} f_Y(y) dy$$
 (0.0.10)

$$= \int_{-1}^{y} \frac{1}{2} dy = \frac{1}{2} (y - (-1)) = \frac{1}{2} (y + 1) \qquad (0.0.11)$$

Using (0.0.11) and (0.0.9) in (0.0.7), we get

$$\Pr\left(\max[X,Y] < \frac{1}{2}\right) = \frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$$
 (0.0.12)

So option 2 is correct answer

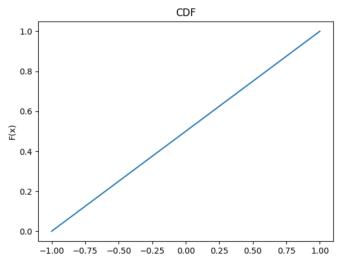


Fig. 1: $F_X(x)$