Assignment 2

CS20BTECH11047

Download all python codes from

https://github.com/JeevanIITH/AI1102/blob/main/assignment2/assignment2.py

and latex codes from

https://github.com/JeevanIITH/AI1102/blob/main/assignment2/assignment2.tex

Section

Let X and Y be two continuous random variables with the joint probability density function

$$f(x,y) = \begin{cases} 2 & 0 < x+y < 1, x > 0, y > 0 \\ 0 & \text{elsewhere} \end{cases} \tag{0.0.1}$$

Then $P\left(X+Y<\frac{1}{2}\right)$ is

1) $\frac{1}{4}$
2) $\frac{1}{2}$
3) $\frac{3}{4}$
4) 1

Solution

Given X and Y be two continuous random variables with the joint probability density function

$$f(x,y) = \begin{cases} 2 & 0 < x+y < 1, x > 0, y > 0 \\ 0 & \text{elsewhere} \end{cases} \tag{0.0.2}$$

we know that

$$P((x,y) \in A) = \int\int_A f(x,y)\,dxdy \quad A \in \mathbb{R}^2 \tag{0.0.3}$$

from given information
for positive x and y

$$0 < x+y < \frac{1}{2} \Rightarrow 0 < x < \frac{1}{2} - y \tag{0.0.4}$$

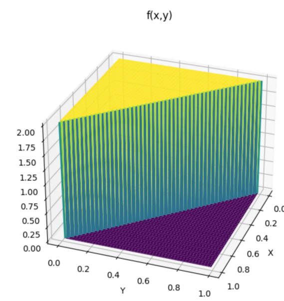

Fig. 1: $f(x,y)$

so using eq(0.0.3)

$$P\left(x+y<\frac{1}{2}\right) = \int_0^{\frac{1}{2}}\int_0^{\frac{1}{2}-y} f(x,y)dxdy \tag{0.0.5}$$

$$= \int_0^{\frac{1}{2}}\int_0^{\frac{1}{2}-y} 2 \quad dxdy = \int_0^{\frac{1}{2}}\left(2x \quad \Big|_0^{\frac{1}{2}-y}\right)dy \tag{0.0.6}$$

$$= \int_0^{\frac{1}{2}} 2\left(\frac{1}{2}-y\right) \quad dy = 2\left(\frac{1}{2}y - \frac{y^2}{2}\right)\Big|_0^{\frac{1}{2}} \tag{0.0.7}$$

$$= \left(\frac{1}{2}-\frac{1}{4}\right) = \frac{1}{4} \tag{0.0.8}$$

Therefore

$$P\left(x+y<\frac{1}{2}\right) = \frac{1}{4} \tag{0.0.9}$$

The volume under the graph which contains the region $X+Y<\frac{1}{2}$ gives us,

$P\left(x+y<\frac{1}{2}\right)$ = Area of the base triangle is $\frac{1}{2}.height.base = \frac{1}{2}.\frac{1}{2}.\frac{1}{2}$

volume = Area . height = $\frac{1}{8}.2 = \frac{1}{4}$

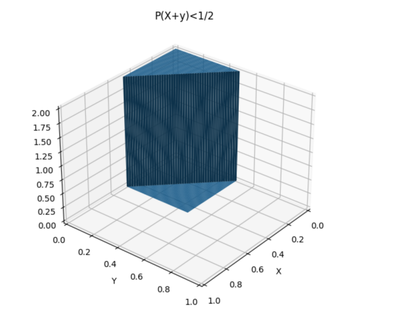

Fig. 2: $P\left(x+y<\frac{1}{2}\right)$