Assignment 3

CS20BTECH11047

Download all python codes from

https://github.com/JeevanIITH/AI1102/blob/main/assignment3/assignment3.py

and latex codes from

https://github.com/JeevanIITH/AI1102/blob/main/assignment3/assignment3.tex

GATE EC 2012 Q.1

Two independent random variables X and Y are uniformly distributed in the interval [-1,1]. The probability that $\max(X, Y)$ is less than $\frac{1}{2}$ is

1) $\frac{3}{4}$
2) $\frac{9}{16}$
3) $\frac{1}{4}$
4) $\frac{2}{3}$

Solution

Since the random variable X is uniformly distributed in the interval [-1,1],let $f_X(x) = k$.Then

$$\int_{-1}^{1} f_X(x)dx = \int_{-1}^{1} kdx = 1 \tag{0.0.1}$$

$$\Rightarrow k = \frac{1}{2} \tag{0.0.2}$$

So

$$f_X(x) = \frac{1}{2} \quad , x \in (-1, 1) \tag{0.0.3}$$

Similarly

$$f_Y(y) = \frac{1}{2} \quad , y \in (-1, 1) \tag{0.0.4}$$

Now

$$\max(X, Y) < \frac{1}{2} \Rightarrow \left(X < \frac{1}{2}\right).\left(Y < \frac{1}{2}\right) \tag{0.0.5}$$

So

$$\Pr\left(\max(X, Y) < \frac{1}{2}\right) = \Pr\left(\left(X < \frac{1}{2}\right).\left(Y < \frac{1}{2}\right)\right) \tag{0.0.6}$$

Since X and Y are independent

$$\Pr\left(\left(X < \frac{1}{2}\right).\left(Y < \frac{1}{2}\right)\right) = \Pr\left(X < \frac{1}{2}\right) \times \Pr\left(Y < \frac{1}{2}\right) \tag{0.0.7}$$

Now using CDF

$$F_X(x) = \Pr(X < x) = \int_{-1}^{x} f_X(x)dx \tag{0.0.8}$$

$$= \int_{-1}^{x} \frac{1}{2}dx = \frac{1}{2}(x - (-1)) = \frac{1}{2}(x + 1) \tag{0.0.9}$$

similarly

$$F_Y(y) = \Pr(Y < y) = \int_{-1}^{y} f_Y(y)dy \tag{0.0.10}$$

$$= \int_{-1}^{y} \frac{1}{2}dy = \frac{1}{2}(y - (-1)) = \frac{1}{2}(y + 1) \tag{0.0.11}$$

Using (0.0.9) and (0.0.11) in (0.0.7) we get

$$\Pr\left(\max(X, Y) < \frac{1}{2}\right) = F_X\left(\frac{1}{2}\right) \times F_Y\left(\frac{1}{2}\right) \tag{0.0.12}$$

$$= \frac{1}{2}\left(\frac{1}{2} + 1\right) \times \frac{1}{2}\left(\frac{1}{2} + 1\right) = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16} \tag{0.0.13}$$

So option 2 is correct answer

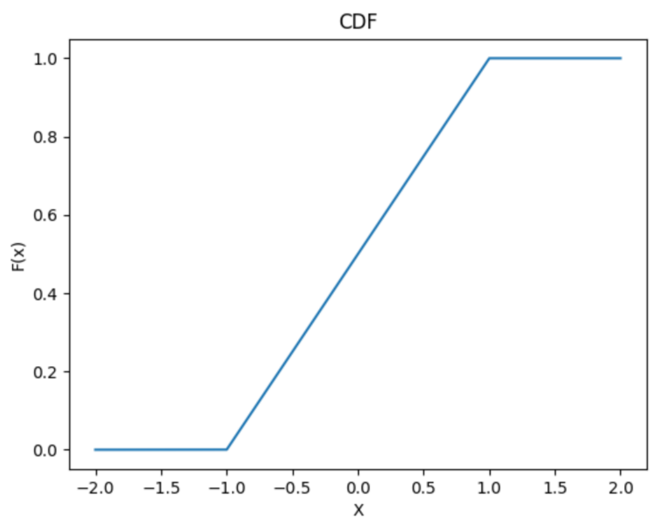

Fig. 1: $F_X(x)$