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Assignment 2

CS20BTECH11047

Download all python codes from

https://github.com/JeevanIITH/AI1102/blob/main/assignment2/assignment2.py

and latex codes from

https://github.com/JeevanIITH/AI1102/blob/main/assignment2/assignment2.tex

SECTION

Let X and Y be two continuous random variables with the joint probability density function

$$f(x,y) = \begin{cases} 2 & 0 < x + y < 1, x > 0, y > 0 \\ 0 & \text{elsewhere} \end{cases}$$
 (0.0.1)

Then $P\left(X+Y<\frac{1}{2}\right)$ is

1) $\frac{1}{4}$

3) $\frac{3}{4}$

2) $\frac{1}{2}$

4) 1

Solution

Given X and Y be two continuous random variables with the joint probability density function

$$f(x,y) = \begin{cases} 2 & 0 < x + y < 1, x > 0, y > 0 \\ 0 & \text{elsewhere} \end{cases}$$
 (0.0.2)

we know that

$$P((x,y) \in A) = \int \int_{A} f(x,y) dxdy \quad A \in \mathbb{R}^{2}$$
(0.0.3)

from given information for positive *x* and *y*

$$0 < x + y < \frac{1}{2} \Rightarrow 0 < x < \frac{1}{2} - y$$
 (0.0.4)

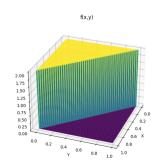


Fig. 1: f(x, y)

so using eq(0.0.3)

$$P\left(x+y<\frac{1}{2}\right) = \int_0^{\frac{1}{2}} \int_0^{\frac{1}{2}-y} f(x,y) dx dy \quad (0.0.5)$$

$$= \int_0^{\frac{1}{2}} \int_0^{\frac{1}{2}-y} 2 \quad dx dy = \int_0^{\frac{1}{2}} \left(2x \quad \Big|_0^{\frac{1}{2}-y}\right) dy \quad (0.0.6)$$

$$= \int_0^{\frac{1}{2}} 2\left(\frac{1}{2}-y\right) \quad dy = 2\left(\frac{1}{2}y - \frac{y^2}{2}\right) \Big|_0^{\frac{1}{2}} \quad (0.0.7)$$

$$= \left(\frac{1}{2} - \frac{1}{4}\right) = \frac{1}{4} \quad (0.0.8)$$

Therefore

$$P\left(x + y < \frac{1}{2}\right) = \frac{1}{4} \tag{0.0.9}$$

The volume under the graph which contains the region $X + Y < \frac{1}{2}$ gives us $P\left(x + y < \frac{1}{2}\right)$

$$P\left(x+y<\frac{1}{2}\right)=$$
 Area of the base . height Area of the base triangle is $\frac{1}{2}.height.base=\frac{1}{2}.\frac{1}{2}.\frac{1}{2}$ volume = Area . height = $\frac{1}{8}.2=\frac{1}{4}$

P(X+y) < 1/2

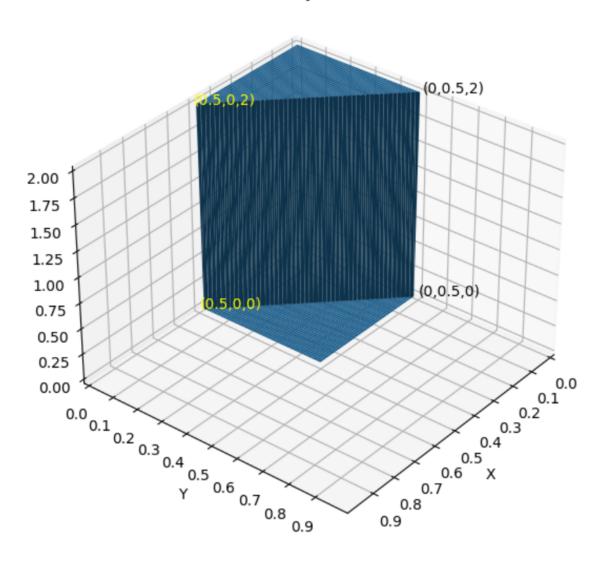


Fig. 2: $P(x + y < \frac{1}{2})$