

Assignment 3

CS20BTECH11047

Download all python codes from

<https://github.com/JeevanIITH/AI1102/blob/main/assignment3/assignment3.py>

and latex codes from

<https://github.com/JeevanIITH/AI1102/blob/main/assignment3/assignment3.tex>

GATE EC 2012 Q.1

Two independent random variables X and Y are uniformly distributed in the interval $[-1,1]$. The probability that $\max[X, Y]$ is less than $\frac{1}{2}$ is

- 1) $\frac{3}{4}$ 2) $\frac{9}{16}$ 3) $\frac{1}{4}$ 4) $\frac{2}{3}$

SOLUTION

Since the random variable X is uniformly distributed in the interval $[-1,1]$, let $f_X(x) = k$. Then

$$\int_{-1}^1 f_X(x) dx = \int_{-1}^1 k dx = 1 \quad (0.0.1)$$

$$\Rightarrow k = \frac{1}{2} \quad (0.0.2)$$

So

$$f_X(x) = \frac{1}{2}, \quad x \in (-1, 1) \quad (0.0.3)$$

Similarly

$$f_Y(y) = \frac{1}{2}, \quad y \in (-1, 1) \quad (0.0.4)$$

Now

$$\max[X, Y] < \frac{1}{2} \Rightarrow \left(X < \frac{1}{2}\right) \cdot \left(Y < \frac{1}{2}\right) \quad (0.0.5)$$

So

$$\Pr\left(\max[X, Y] < \frac{1}{2}\right) = \Pr\left(\left(X < \frac{1}{2}\right) \cdot \left(Y < \frac{1}{2}\right)\right) \quad (0.0.6)$$

Since X and Y are independent

$$\Pr\left(\left(X < \frac{1}{2}\right) \cdot \left(Y < \frac{1}{2}\right)\right) = \Pr\left(X < \frac{1}{2}\right) \cdot \Pr\left(Y < \frac{1}{2}\right) \quad (0.0.7)$$

Now using CDF

$$F_X(x) = \Pr(X < x) = \int_{-1}^x f_X(x) dx \quad (0.0.8)$$

$$= \int_{-1}^x \frac{1}{2} dx = \frac{1}{2}(x - (-1)) = \frac{1}{2}(x + 1) \quad (0.0.9)$$

similarly

$$F_Y(y) = \Pr(Y < y) = \frac{1}{2}(y + 1) \quad (0.0.10)$$

Using (1.0.10) and (1.0.9) in (1.0.7), we get

$$\Pr\left(\max[X, Y] < \frac{1}{2}\right) = \frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16} \quad (0.0.11)$$

So option 2 is correct answer

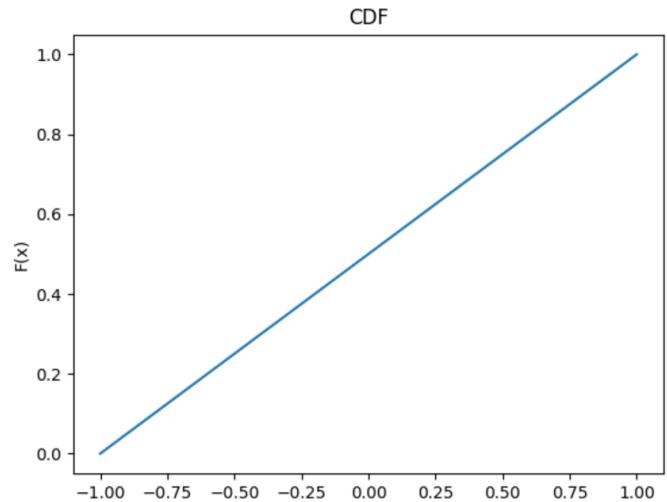


Fig. 1: $F_X(x)$