

Assignment 2

CS20BTECH11047

Download all python codes from

<https://github.com/JeevanIITH/AI1102/blob/main/assignment2/assignment2.py>

and latex codes from

<https://github.com/JeevanIITH/AI1102/blob/main/assignment2/assignment2.tex>

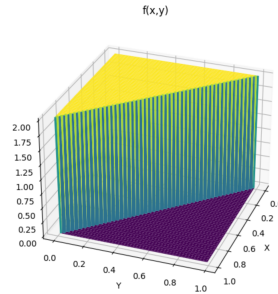


Fig. 1: $f(x, y)$

SECTION

Let X and Y be two continuous random variables with the joint probability density function

$$f(x, y) = \begin{cases} 2 & 0 < x + y < 1, x > 0, y > 0 \\ 0 & \text{elsewhere} \end{cases} \quad (0.0.1)$$

Then $P\left(X + Y < \frac{1}{2}\right)$ is

- | | |
|------------------|------------------|
| 1) $\frac{1}{4}$ | 3) $\frac{3}{4}$ |
| 2) $\frac{1}{2}$ | 4) 1 |

SOLUTION

Given X and Y be two continuous random variables with the joint probability density function

$$f(x, y) = \begin{cases} 2 & 0 < x + y < 1, x > 0, y > 0 \\ 0 & \text{elsewhere} \end{cases} \quad (0.0.2)$$

we know that

$$P((x, y) \in A) = \int \int_A f(x, y) dx dy \quad A \in \mathbb{R}^2 \quad (0.0.3)$$

from given information
for positive x and y

$$0 < x + y < \frac{1}{2} \Rightarrow 0 < x < \frac{1}{2} - y \quad (0.0.4)$$

so using eq(0.0.3)

$$P\left(x + y < \frac{1}{2}\right) = \int_0^{\frac{1}{2}} \int_0^{\frac{1}{2}-y} f(x, y) dx dy \quad (0.0.5)$$

$$= \int_0^{\frac{1}{2}} \int_0^{\frac{1}{2}-y} 2 dx dy = \int_0^{\frac{1}{2}} \left(2x \Big|_0^{\frac{1}{2}-y}\right) dy \quad (0.0.6)$$

$$= \int_0^{\frac{1}{2}} 2\left(\frac{1}{2} - y\right) dy = 2\left(\frac{1}{2}y - \frac{y^2}{2}\right) \Big|_0^{\frac{1}{2}} \quad (0.0.7)$$

$$= \left(\frac{1}{2} - \frac{1}{4}\right) = \frac{1}{4} \quad (0.0.8)$$

Therefore

$$P\left(x + y < \frac{1}{2}\right) = \frac{1}{4} \quad (0.0.9)$$

volume under the graph which contains the region

$$X + Y < \frac{1}{2} \quad \text{gives us} \quad P\left(x + y < \frac{1}{2}\right) \quad (0.0.10)$$

$$P\left(x + y < \frac{1}{2}\right) = \text{Area of the base} \cdot \text{height} \quad (0.0.11)$$

Area of the base triangle is

$$\frac{1}{2} \cdot \text{height} \cdot \text{base} = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \quad (0.0.12)$$

$$\text{volume} = \text{Area} \cdot \text{height} = \frac{1}{8} \cdot 2 = \frac{1}{4} \quad (0.0.13)$$

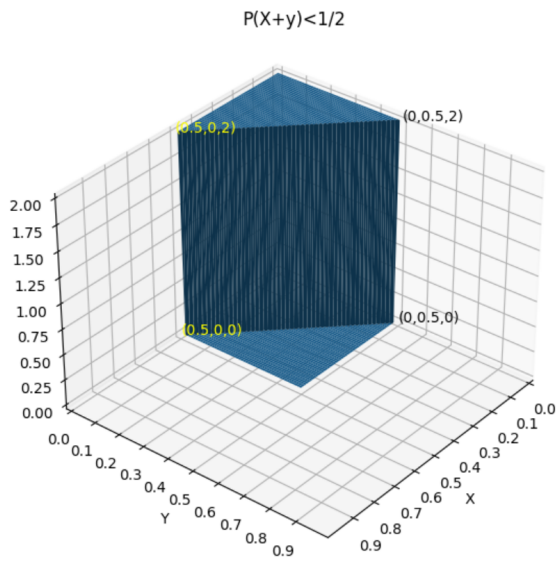


Fig. 2: $P\left(x + y < \frac{1}{2}\right)$