

# Assignment 3

CS20BTECH11047

Download all python codes from

<https://github.com/JeevanIITH/AI1102/blob/main/assignment3/assignment3.py>

and latex codes from

<https://github.com/JeevanIITH/AI1102/blob/main/assignment3/assignment3.tex>

GATE EC 2012 Q.1

Two independent random variables  $X$  and  $Y$  are uniformly distributed in the interval  $[-1,1]$ . The probability that  $\max(X, Y)$  is less than  $\frac{1}{2}$  is

- 1)  $\frac{3}{4}$       2)  $\frac{9}{16}$       3)  $\frac{1}{4}$       4)  $\frac{2}{3}$

SOLUTION

Since the random variable  $X$  is uniformly distributed in the interval  $[-1,1]$ , let  $f_X(x) = k$ . Then

$$\int_{-1}^1 f_X(x) dx = \int_{-1}^1 k dx = 1 \quad (0.0.1)$$

$$\Rightarrow k = \frac{1}{2} \quad (0.0.2)$$

So

$$f_X(x) = \frac{1}{2}, \quad x \in (-1, 1) \quad (0.0.3)$$

Similarly

$$f_Y(y) = \frac{1}{2}, \quad y \in (-1, 1) \quad (0.0.4)$$

Now

$$\max(X, Y) < \frac{1}{2} \Rightarrow \left(X < \frac{1}{2}\right) \cdot \left(Y < \frac{1}{2}\right) \quad (0.0.5)$$

So

$$\Pr\left(\max(X, Y) < \frac{1}{2}\right) = \Pr\left(\left(X < \frac{1}{2}\right) \cdot \left(Y < \frac{1}{2}\right)\right) \quad (0.0.6)$$

Since  $X$  and  $Y$  are independent

$$\Pr\left(\left(X < \frac{1}{2}\right) \cdot \left(Y < \frac{1}{2}\right)\right) = \Pr\left(X < \frac{1}{2}\right) \times \Pr\left(Y < \frac{1}{2}\right) \quad (0.0.7)$$

Now using CDF

$$F_X(x) = \Pr(X < x) = \int_{-1}^x f_X(x) dx \quad (0.0.8)$$

$$= \int_{-1}^x \frac{1}{2} dx = \frac{1}{2}(x - (-1)) = \frac{1}{2}(x + 1) \quad (0.0.9)$$

so

$$F_X(x) = \begin{cases} 0 & x < -1 \\ \frac{1}{2}(x + 1) & -1 < x < 1 \\ 1 & x > 1 \end{cases} \quad (0.0.10)$$

similarly

$$F_Y(y) = \Pr(Y < y) = \int_{-1}^y f_Y(y) dy \quad (0.0.11)$$

$$= \int_{-1}^y \frac{1}{2} dy = \frac{1}{2}(y - (-1)) = \frac{1}{2}(y + 1) \quad (0.0.12)$$

so

$$F_Y(y) = \begin{cases} 0 & y < -1 \\ \frac{1}{2}(y + 1) & -1 < y < 1 \\ 1 & y > 1 \end{cases} \quad (0.0.13)$$

Using (0.0.10) and (0.0.13) in (0.0.7) we get

$$\Pr\left(\max(X, Y) < \frac{1}{2}\right) = F_X\left(\frac{1}{2}\right) \times F_Y\left(\frac{1}{2}\right) \quad (0.0.14)$$

$$= \frac{1}{2}\left(\frac{1}{2} + 1\right) \times \frac{1}{2}\left(\frac{1}{2} + 1\right) = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16} \quad (0.0.15)$$

So option 2 is correct answer

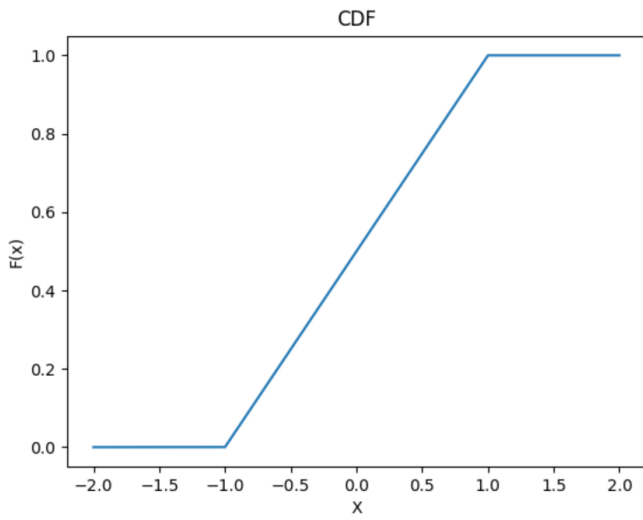


Fig. 1:  $F_X(x)$