Assignment 1

CS20BTECH11047

Download all python codes from

https://github.com/Jeevansammeswar/Assignment_1/blob/main/assignment_1/codes/assignment_1.py

and latex-tikz codes from

https://github.com/Jeevansammeswar/Assignment_1/blob/main/assignment_1/assignment_1.tex

Question

A die is thrown 6 times. If 'getting an odd number' is a success, what is the probability of

1) 5 successes?
2) at least 5 successes?
3) at most 5 successes?

Solution

Let $X_i \in (0, 1)$ where $X_i = 1$ represents successful thrown of the dice for the i^{th} thrown

$$Pr(X_i = 1) = p = \frac{3}{6} = \frac{1}{2} \tag{0.0.1}$$

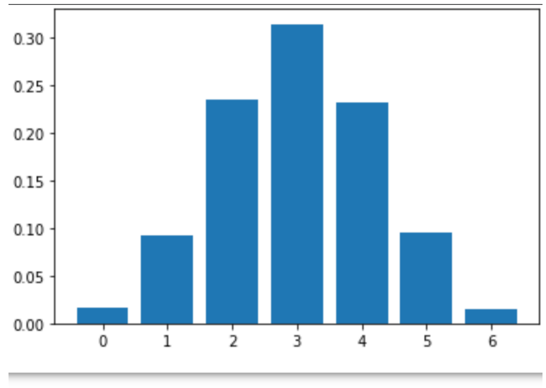
Let X represents the random variable function of number of successful throws out of n throws

$$X = \sum_{i=1}^{n} X_i \tag{0.0.2}$$

Using Binomial distribution ,the probability distribution is

$$Pr(X = r) = \binom{n}{r} p^r (1 - p)^{n-r} \tag{0.0.3}$$

X	X=0	X=1	X=2	X=3	X=4	X=5	X=6
Pr	1/64	6/64	15/64	20/64	15/64	6/64	1/64



1) probability of getting 5 successes

$$Pr(X = r) = \binom{n}{r} p^r (1 - p)^{n-r} \tag{0.0.4}$$

for n=6,r=5

$$Pr(X = 5) = \binom{6}{5} (\frac{1}{2})^5 (1 - \frac{1}{2})^{6-5} \tag{0.0.5}$$

$$Pr(X = 5) = \frac{3}{32} \tag{0.0.6}$$

2) probability of getting at least 5 successes

$$Pr(X \geq r) = \sum_{k=r}^{n} \binom{n}{k} p^k (1 - p)^{n-k} \tag{0.0.7}$$

for n=6,r=5

$$Pr(X \geq 5) = \sum_{k=5}^{6} \binom{6}{k} (1/2)^k (1 - 1/2)^{6-k} \tag{0.0.8}$$

$$= \binom{6}{5} (1/2)^5 (1/2) + \binom{6}{6} (1/2)^6 (1) \tag{0.0.9}$$

$$= \frac{7}{64} \tag{0.0.10}$$

3) Probability of getting at most 5 successes. Using CDF

$$F(r) = Pr(X \leq r) = \sum_{i=0}^{r} Pr(X = i) \tag{0.0.11}$$

$$F(r) = \begin{cases} \frac{1}{64}, r = 0 \\ \frac{7}{64}, r = 1 \\ \frac{22}{64}, r = 2 \\ \frac{42}{64}, r = 3 \\ \frac{57}{64}, r = 4 \\ \frac{63}{64}, r = 5 \\ \frac{64}{64}, r = 6 \end{cases}$$

$$F(r) = Pr(X \leq r) = \sum_{i=0}^{r} Pr(X = i) \quad (0.0.12)$$

for n=6,r=5

$$F(5) = Pr(X \leq 5) = \sum_{i=0}^{5} Pr(X = i) \quad (0.0.13)$$

$$= \frac{1}{64} + \frac{6}{64} + \frac{15}{64} + \frac{20}{64} + \frac{15}{64} + \frac{6}{64} \quad (0.0.14)$$

$$= \frac{63}{64} \quad (0.0.15)$$

or

Sum of the probabilities of all possible cases is 1

$$\sum_{i=0}^{n} Pr(X = i) = 1 \quad (0.0.16)$$

$$Pr(X \leq r) = \sum_{i=0}^{r} Pr(X = i) \quad (0.0.17)$$

from (0.0.10)

$$\sum_{i=0}^{r} Pr(X = i) = 1 - \sum_{i=r+1}^{n} Pr(X = i) \quad (0.0.18)$$

$$Pr(X \leq r) = 1 - \sum_{i=r+1}^{n} Pr(X = i) \quad (0.0.19)$$

for n=6,r=5

$$Pr(X \leq 5) = 1 - \sum_{i=5+1}^{6} Pr(X = i) \quad (0.0.20)$$

$$= 1 - Pr(X = 6) \quad (0.0.21)$$

Using (0.0.3)

$$Pr(X = 6) = \binom{6}{6}(1/2)^6(1 - 1/2)^{6-6} = (1/2)^6 \quad (0.0.22)$$

therefore

$$Pr(X \leq 5) = 1 - (1/2)^6 = \frac{63}{64} \quad (0.0.23)$$