Assignment 1

CS20BTECH11047

QUESTION

A die is thrown 6 times. If 'getting an odd number' is a success, what is the probability of

1) 5 successes?
2) at least 5 successes?
3) at most 5 successes?

SOLUTION

Let $X_i \in (0, 1)$ where $X_i = 1$ represents successful thrown of the dice for the ith thrown
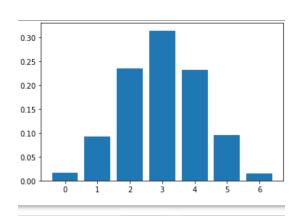
$$Pr(X_i = 1) = p = \frac{3}{6} = \frac{1}{2} \qquad (0.0.1)$$

Let X represents the random variable function of number of successful throws out of n throws

$$X = \sum_{i=1}^{n} X_i \qquad (0.0.2)$$

Using Binomial distribution ,the probability distribution is

$$Pr(X = r) = \binom{n}{r} p^r (1 - p)^{n-r} \qquad (0.0.3)$$

X	X=0	X=1	X=2	X=3	X=4	X=5	X=6
Pr	1/64	6/64	15/64	20/64	15/64	6/64	1/64

1) probability of getting 5 successes

$$Pr(X = r) = \binom{n}{r} p^r (1 - p)^{n-r} \qquad (0.0.4)$$

for n=6,r=5

$$Pr(X = 5) = \frac{3}{32} \qquad (0.0.5)$$

2) probability of getting at least 5 successes

$$Pr(X \geq r) = \sum_{k=r}^{n} \binom{n}{k} p^k (1 - p)^{n-k} \qquad (0.0.6)$$

for n=6,r=5

$$Pr(X \geq 5) = \frac{7}{64} \qquad (0.0.7)$$

3) Probability of getting at most 5 successes. Using CDF

$$F_X(r) = Pr(X \leq r) = \sum_{i=0}^{r} Pr(X = i) \qquad (0.0.8)$$

$$F_X(r) = \begin{cases} \dfrac{1}{64}, & r = 0 \\[2mm] \dfrac{7}{64}, & r = 1 \\[2mm] \dfrac{22}{64}, & r = 2 \\[2mm] \dfrac{42}{64}, & r = 3 \\[2mm] \dfrac{57}{64}, & r = 4 \\[2mm] \dfrac{63}{64}, & r = 5 \\[2mm] \dfrac{64}{64}, & r = 6 \end{cases}$$

from above

$$F_X(5) = \frac{63}{64} \qquad (0.0.9)$$

** or **

Sum of the probabilities of all possible cases is 1

$$\sum_{i=0}^{n} Pr(X = i) = 1 \qquad (0.0.10)$$

$$Pr(X \le r) = \sum_{i=0}^{r} Pr(X = i) \qquad (0.0.11)$$

from (0.0.10)

$$\sum_{i=0}^{r} Pr(X = i) = 1 - \sum_{i=r+1}^{n} Pr(X = i) \qquad (0.0.12)$$

$$Pr(X \le r) = 1 - \sum_{i=r+1}^{n} Pr(X = i) \qquad (0.0.13)$$

for n=6,r=5

$$Pr(X \le 5) = 1 - \sum_{i=5+1}^{6} Pr(X = i) \qquad (0.0.14)$$

$$= 1 - Pr(X = 6) \qquad (0.0.15)$$

Using (0.0.3)

$$Pr(X = 6) = (1/2)^6 \qquad (0.0.16)$$

therefore

$$Pr(X \le 5) = \frac{63}{64} \qquad (0.0.17)$$