

Assignment 1

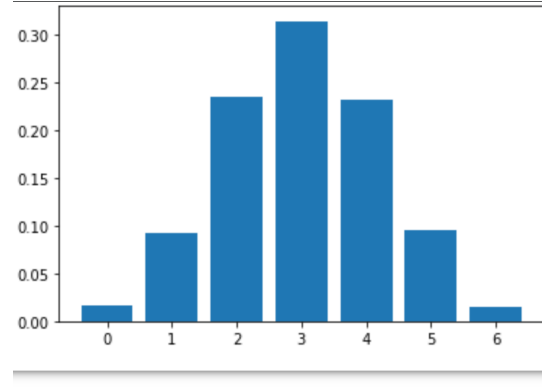
CS20BTECH11047

Download all python codes from

https://github.com/Jeevansammeswar/Assignment_1/blob/main/assignment_1/codes/assignment_1.py

and latex-tikz codes from

https://github.com/Jeevansammeswar/Assignment_1/blob/main/assignment_1/assignment_1.tex



QUESTION

A die is thrown 6 times. If 'getting an odd number' is a success, what is the probability of

- 1) 5 successes?
- 2) at least 5 successes?
- 3) at most 5 successes?

SOLUTION

Let $X_i \in (0, 1)$ where $X_i = 1$ represents successful throw of the dice for the i^{th} throw

$$Pr(X_i = 1) = p = \frac{3}{6} = \frac{1}{2} \quad (0.0.1)$$

Let X represents the random variable function of number of successful throws out of n throws

$$X = \sum_{i=1}^n X_i \quad (0.0.2)$$

Using Binomial distribution, the probability distribution is

$$Pr(X = r) = \binom{n}{r} p^r (1-p)^{n-r} \quad (0.0.3)$$

X	X=0	X=1	X=2	X=3	X=4	X=5	X=6
Pr	1/64	6/64	15/64	20/64	15/64	6/64	1/64

- 1) probability of getting 5 successes

$$Pr(X = r) = \binom{n}{r} p^r (1-p)^{n-r} \quad (0.0.4)$$

for $n=6, r=5$

$$Pr(X = 5) = \binom{6}{5} \left(\frac{1}{2}\right)^5 \left(1 - \frac{1}{2}\right)^{6-5} \quad (0.0.5)$$

$$Pr(X = 5) = \frac{3}{32} \quad (0.0.6)$$

- 2) probability of getting at least 5 successes

$$Pr(X \geq r) = \sum_{k=r}^n \binom{n}{k} p^k (1-p)^{n-k} \quad (0.0.7)$$

for $n=6, r=5$

$$Pr(X \geq 5) = \sum_{k=5}^6 \binom{6}{k} \left(\frac{1}{2}\right)^k \left(1 - \frac{1}{2}\right)^{6-k} \quad (0.0.8)$$

$$= \binom{6}{5} \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right) + \binom{6}{6} \left(\frac{1}{2}\right)^6 (1) \quad (0.0.9)$$

$$= \frac{7}{64} \quad (0.0.10)$$

- 3) Probability of getting at most 5 successes.

$$Pr(X \leq r) = \sum_{i=0}^r Pr(X = i) \quad (0.0.11)$$

for $n=6, r=5$

$$Pr(X \leq 5) = \sum_{i=0}^5 Pr(X = i) \quad (0.0.12)$$

$$= \frac{1}{64} + \frac{6}{64} + \frac{15}{64} + \frac{20}{64} + \frac{15}{64} + \frac{6}{64} \quad (0.0.13)$$

$$= \frac{63}{64} \quad (0.0.14)$$

or

Sum of the probabilities of all possible cases is 1

$$\sum_{i=0}^n Pr(X = i) = 1 \quad (0.0.15)$$

$$Pr(X \leq r) = \sum_{i=0}^r Pr(X = i) \quad (0.0.16)$$

from (0.0.10)

$$\sum_{i=0}^r Pr(X = i) = 1 - \sum_{i=r+1}^n Pr(X = i) \quad (0.0.17)$$

$$Pr(X \leq r) = 1 - \sum_{i=r+1}^n Pr(X = i) \quad (0.0.18)$$

for $n=6, r=5$

$$Pr(X \leq 5) = 1 - \sum_{i=5+1}^6 Pr(X = i) \quad (0.0.19)$$

$$= 1 - Pr(X = 6) \quad (0.0.20)$$

Using (0.0.3)

$$Pr(X = 6) = \binom{6}{6} (1/2)^6 (1 - 1/2)^{6-6} = (1/2)^6 \quad (0.0.21)$$

therefore

$$Pr(X \leq 5) = 1 - (1/2)^6 = \frac{63}{64} \quad (0.0.22)$$