

# Count Attacks against Searchable Symmetric Encryption

## Case study I

$d1 = \{w1, w2, w3, w4, w6\}$

$d2 = \{w1, w2, w3, w6\},$

$d3 = \{w1, w2, w4, w6\},$

$d4 = \{w1, w4, w6\},$

$d5 = \{w4, w5, w6\}, d6 = \{w4, w6\},$

**Step 1:** create inverted index of the breached data and create co-occurrence matrix for the breached dataset.

Breached data:

keyword	Document id						frequency
w1	d1	d2	d3	d4			4
w2	d1	d2	d3				3
w3	d1	d2					2
w4	d1		d3	d4	d5	d6	5
w5					d5		1
w6	d1	d2	d3	d4	d5	d6	6

Keyword Co-occurrence matrix:

	w1	w2	w3	w4	w5	w6
w1	4	3	2	3	0	4
w2	3	3	2	2	0	3
w3	2	2	2	1	0	2
w4	3	2	1	5	1	5
w5	0	0	0	1	1	1
w6	4	3	2	5	1	6

**Step 2:** Construct the co-occurrence matrix for the query response from the server.

Server response:

Query	eid						frequency
q1	e1	e2	e3		e5	e6	5
q2			e3	e4			2
q3	e1	e2	e3	e4			4
q4		e2	e3	e4			3
q5	e1	e2	e3	e4	e5	e6	6
q6						e6	1

Query Co-occurrence matrix:

	q1	q2	q3	q4	q5	q6
q1	5	1	3	2	5	1
q2	1	2	2	2	2	0
q3	3	2	4	3	4	0
q4	2	2	3	3	3	0
q5	5	2	4	3	6	1
q6	1	0	0	0	1	1

**Step3:** Create the mapping knowledge K, which includes the mapping between query tokens and keywords that share a unique frequency of occurrences.

Unique length: {q3: w1}, {q4: w2}, {q2: w3}, {q1: w4}, {q6: w5}, {q5: w6}

Since, all the unique keyword mapping established for query and keyword is recovered. The count attack is possible.

## Case study II

$d1 = \{w1, w2, w3, w4, w6\},$

$d2 = \{w2, w3, w6\},$

$d3 = \{w1, w3, w4, w6\},$

$d4 = \{w1, w4, w6\},$

$d5 = \{w4, w5, w6\},$

$d6 = \{w5\},$

**Step 1:** create inverted index of the breached data and create co-occurrence matrix for the breached dataset.

Breached data:

keyword	Document id						frequency
w1	d1		d3	d4			3
w2	d1	d2					2
w3	d1	d2	d3				3
w4	d1		d3	d4	d5		4
w5					d5	d6	2
w6	d1	d2	d3	d4	d5		5

Keyword Co-occurrence matrix:

	w1	w2	w3	w4	w5	w6
w1	3	1	2	3	0	3
w2	1	2	2	1	0	2
w3	2	2	3	2	0	3
w4	3	1	2	4	1	4
w5	0	0	0	1	2	1
w6	3	2	3	4	1	5

**Step 2:** Construct the co-occurrence matrix for the query response from the server.

Server response:

Query	eid						frequency
q1		e2	e3	e4	e5	e6	5
q2		e2	e3		e5	e6	4
q3	e1		e3				2
q4				e4	e5		2
q5		e2			e5	e6	3
q6				e4	e5	e6	3

Query Co-occurrence matrix:

	q1	q2	q3	q4	q5	q6
q1	5	4	1	2	3	3
q2	4	4	1	1	3	2
q3	1	1	2	0	0	0
q4	2	1	0	2	1	2
q5	3	3	0	1	3	2
q6	3	2	0	2	2	3

**Step3:** Create the mapping knowledge K, which includes the mapping between query tokens and keywords that share a unique frequency of occurrences.

Unique length: {q1: w6}, {q2: w4}

No unique length: {q3, q4: w2, w5}, {q5, q6: w1, w3}

**Step4:** Examine the co-occurrence matrix to identify potential candidate keywords for the remaining query tokens and their occurrences.

For {q3, q4: w2, w5} and using unique length: {q1: w6},

$(q3, q1) = 1$   $(w2, w6) = 2$  -> not possible

$(q3, q1) = 1$   $(w5, w6) = 1$  -> possible

$(q4, q1) = 2$   $(w2, w6) = 2$  -> possible

$(q4, q1) = 2$   $(w5, w6) = 1$  -> not possible

Extracted data: {q3: w5}, {q4: w2}

For {q3, q4: w2, w5} and using unique length: {q2: w4}

$(q3, q2) = 1$   $(w2, w4) = 1$  -> possible

$(q3, q2) = 1$   $(w5, w4) = 1$  -> possible

$(q4, q2) = 1$   $(w2, w4) = 1$  -> possible

$(q4, q2) = 1$   $(w5, w4) = 1$  -> possible

Extracted data: {q3: w2, w5}, {q4: w2, w5} -> still not possible

For {q5, q6: w1, w3} and using unique length: {q2: w4}

$(q5, q2) = 3$   $(w1, w4) = 3$  -> possible

$(q5, q2) = 3$   $(w3, w4) = 2$  -> not possible

$(q6, q2) = 2$   $(w1, w4) = 3$  -> not possible

$(q6, q2) = 2$   $(w3, w4) = 2$  -> possible

Extracted data: {q5: w1}, {q6: w3}

Since, all the unique keyword mapping established for query and keyword is recovered. The count attack is possible.

Keyword mapping:

Unique length: {q1: w6}, {q2: w4}, {q3: w5}, {q4: w2}, {q5: w1}, {q6: w3}

### Case study III

$d1 = \{w1, w2, w3, w6\},$

$d2 = \{w2, w3, w4, w6\},$

$d3 = \{w1, w2, w3, w4, w6\},$

$d4 = \{w1, w3, w4, w6\},$

$d5 = \{w5, w6\},$

$d6 = \{w5, w6\}$

**Step 1:** create inverted index of the breached data and create co-occurrence matrix for the breached dataset.

Breached data:

keyword	Document id						frequency
w1	d1		d3	d4			3
w2	d1	d2	d3				3
w3	d1	d2	d3	d4			4
w4		d2	d3	d4			3
w5					d5	d6	2
w6	d1	d2	d3	d4	d5	d6	6

Keyword Co-occurrence matrix:

	w1	w2	w3	w4	w5	w6
w1	3	2	3	2	0	3
w2	2	3	3	2	0	3
w3	3	3	4	3	0	4
w4	2	2	3	3	0	3
w5	0	0	0	0	2	2
w6	3	3	4	3	2	6

**Step2:** Construct the co-occurrence matrix for the query response from the server.

Server response:

Query	eid						frequency
q1	e1				e5	e6	3
q2	e1	e2				e6	3
q3			e3	e4			2
q4	e1	e2	e3	e4	e5	e6	6
q5		e2			e5	e6	3
q6	e1	e2			e5	e6	4

Query Co-occurrence matrix:

	q1	q2	q3	q4	q5	q6
q1	3	2	0	3	2	3
q2	2	3	0	3	2	3
q3	0	0	2	2	0	0
q4	3	3	2	6	3	4
q5	2	2	0	3	3	3
q6	3	3	0	4	3	4

**Step3:** Create the mapping knowledge K, which includes the mapping between query tokens and keywords that share a unique frequency of occurrences.

Unique length: {q4: w6}, {q6: w3}, {q3: w5}

No unique length: {q1, q2, q5: w1, w2, w4}

**Step4:** Examine the co-occurrence matrix to identify potential candidate keywords for the remaining query tokens and their occurrences.

For {q1, q2, q5: w1, w2, w4} and using unique length: {q4: w6}

(q1, q4) = 3 (w1, w6) = 3 -> possible

(q2, q4) = 3 (w2, w6) = 3 -> possible

(q5, q4) = 3 (w4, w6) = 3 -> possible

For {q1, q2, q5: w2, w1, w4} and using unique length: {q4: w6}

(q1, q4) = 3 (w2, w6) = 3 -> possible

(q2, q4) = 3 (w1, w6) = 3 -> possible

(q5, q4) = 3 (w4, w6) = 3 -> possible

For {q1, q2, q5: w4, w2, w4} and using unique length: {q4: w6}

(q1, q4) = 3 (w4, w6) = 3 -> possible

(q2, q4) = 3 (w2, w6) = 3 -> possible

(q5, q4) = 3 (w1, w6) = 3 -> possible

Extracted data: {q1, q2, q5: w1, w2, w4} -> still not possible to extract.

For {q1, q2, q5: w1, w2, w4} and using unique length: {q6: w3}

(q1, q6) = 3 (w1, w3) = 3 -> possible

(q1, q6) = 3 (w2, w3) = 3 -> possible

(q1, q6) = 3 (w4, w3) = 3 -> possible

(q2, q6) = 3 (w1, w3) = 3 -> possible

(q2, q6) = 3 (w2, w3) = 3 -> possible

(q2, q6) = 3 (w4, w3) = 3 -> possible

(q5, q6) = 3 (w1, w3) = 3 -> possible

(q5, q6) = 3 (w2, w3) = 3 -> possible

(q5, q6) = 3 (w4, w3) = 3 -> possible



Extracted data: {q1, q2, q5: w1, w2, w4} -> still not possible to extract.

For {q1, q2, q5: w1, w2, w4} and using unique length: {q3: w5}

$(q1, q3) = 0$   $(w1, w5) = 0$  -> possible

$(q1, q3) = 0$   $(w2, w5) = 0$  -> possible

$(q1, q3) = 0$   $(w4, w5) = 0$  -> possible

$(q2, q3) = 0$   $(w1, w5) = 0$  -> possible

$(q2, q3) = 0$   $(w2, w5) = 0$  -> possible

$(q2, q3) = 0$   $(w4, w5) = 0$  -> possible

$(q5, q3) = 0$   $(w1, w5) = 0$  -> possible

$(q5, q3) = 0$   $(w2, w5) = 0$  -> possible

$(q5, q3) = 0$   $(w4, w5) = 0$  -> possible

Extracted data: {q1, q2, q5: w1, w2, w4} -> still not possible to extract.

The following can't be mapped {q1, q2, q5: w1, w2, w4} since, the q1, q2, q5 and w1, w2, w4 share equal probability of combination.

The count attack is not possible for the case study 3, only part of the keyword can be extracted which is as follows:

Unique Length: {q4: w6}, {q6: w3}, {q3: w5}