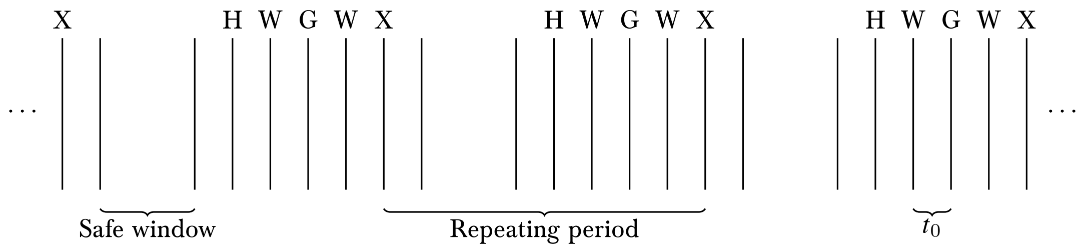
Periodic Batching

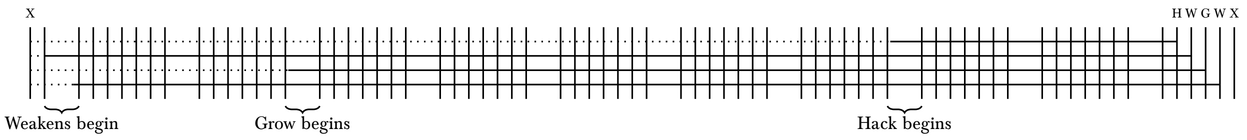
A fundamental problem in scheduling HWGW batching is to ensure scripts never start at a raised security level. Here, we solve the problem with a batch schedule that is *periodic*: by leaving a carefully calculated safe window between each HWGW, we can ensure that all hacks, grows, and weakens start in that window and so begin at minimum security. This periodic schedule can then be repeated as long as it remains valid – in theory forever, but in practice until the hacking level increases, though we describe a way of mitigating this limitation at the end.

First, we define a timing model, which tells us the structure of the batches we want to achieve. As usual, we need to leave some time buffer between events to work with Javascript concurrency timing; we call that time t_0. The timing model we will use is the following:

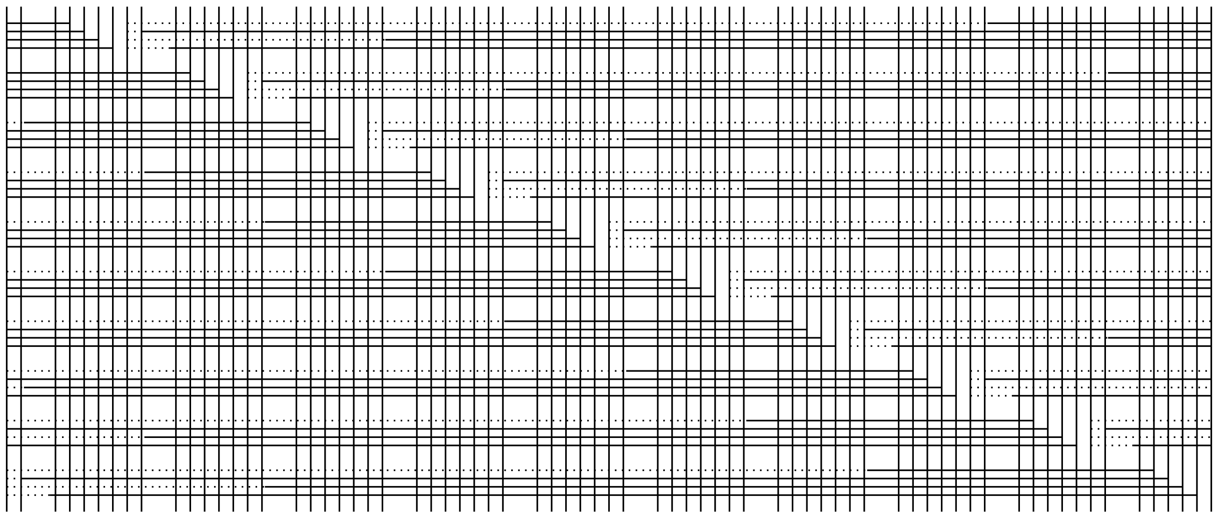


H, G, and W respectively denote when hacks, grows, and weakens end, and X denotes a point where a batch manager script both checks the previous batch completed properly and also launches scripts for the next batch. Different timing windows are possible: for example, you may want to run HGW batching, or have your manager script run at a different point, or use different buffer times between different events. Ideally, these notes should let you work through the calculations required to calculate a schedule for any timing model.

Before moving on those calculations, first we show two sketches to help visualise the timing model we're using here. First, we show an example of a HWGW batch where each script overlaps with 10 different periods:

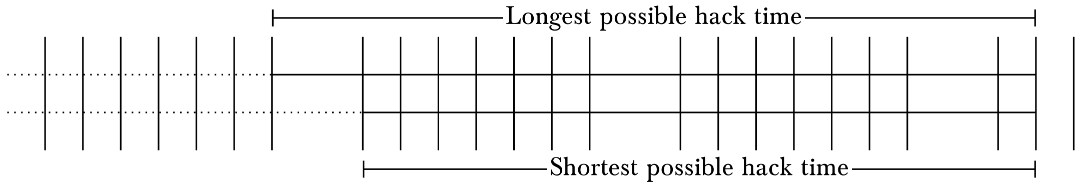


The dotted lines denotes the time each script is sleeping, and the solid lines denote the actual duration of the hack, grow, or weaken. Since each script overlaps with 10 periods, we can run 10 batches simultaneously; all 10 simultaneous batches are depicted below, each offset by a different number of periods:

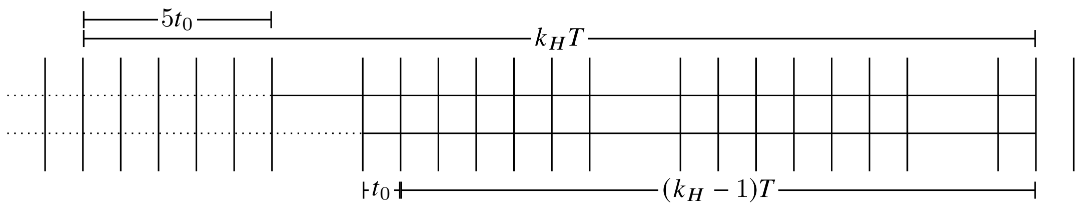


Our goal is to calculate two values: the period, T, which is the time taken for the process to go through one cycle, and the depth, k_W, which is the number of batches we can run simultaneously. (The reason for calling the depth 'k_W' will be clear shortly.) In particular, we want T to be as small as possible, to maximise the rate of HWGW batching.

To do so, first imagine we have some fixed schedule, and we can vary the hack time however we wish. If we want our hack to start in the particular safe window depicted above, what possible hack times could we have? Clearly, the longest possible hack time has the hack starting at the beginning of the window, and the shortest hack time starts at the end of the window:



Of course, the hack time does not have to overlap exactly three periods. We define k_H as the number of periods the hack overlaps; every hack has to occupy at least one period, so we must have $k_H \geq 1$. Given a particular k_H, period T, and buffer t_0, we can bound the hack time in terms of those values:



Thus, denoting the hack time as T_H, the hack time must be bounded by:

$$(k_H - 1)T + t_0 \leq T_H \leq k_H T - 5t_0 \tag{1}$$

In other words, if we choose k_H and T such that the above bounds hold, we are guaranteed to have the start of the hack fall in a safe window.

Similar analysis of grow gives rise to a parameter $k_G \geq 1$ which is used to bound the grow time, T_G:

$$(k_G - 1)T + 3t_0 \leq T_G \leq k_G T - 3t_0 \tag{2}$$

With the weakens, we must be a little careful since there's two of them; in order for both weakens to lie within the safe window, we require the bounds:

$$(k_W - 1)T + 4t_0 \leq T_W \leq k_W T - 4t_0 \tag{3}$$

Since the weaken time is always the longest, it is k_W which determines how many periods the entire batch overlaps, and thus how many batches can run concurrently; hence k_W is equivalent to the depth.

Thus, our objective is, given times T_H, T_G, T_W, and t_0, determine k_H, k_G, and k_W such that the above bounds are satisfied with the minimum possible choice of T.

To actually calculate this, first note that the hack, grow, and weaken times are all defined to be directly related to each other:

$$T_G = \frac{16}{5} T_H \qquad T_W = 4T_H$$

This lets us write down direct relations between k_H, k_G, and k_W. Taking half of eq. 1 and eq. 3 each:

$$4\big[(k_H - 1)T + t_0\big] \leq T_W \qquad T_W \leq k_W T - 4t_0$$

Combining the two inequalities and rearranging:

$$4k_H T - 4T + 4t_0 \leq k_W T - 4t_0$$

$$k_H + \frac{2t_0}{T} \leq \frac{1}{4} k_W + 1$$

But clearly $k_H \le k_H + \frac{2t_0}{T}$. Thus, we can drop that term and simply write:

$$k_H \le \frac{1}{4}k_W + 1$$

And since k_H must be an integer, we can tighten this to:

$$k_H \le \left\lfloor \frac{1}{4}k_W + 1 \right\rfloor$$

Taking the other halves of eq. 1 and eq. 3 gives:

$$(k_W - 1)T + 4t_0 \le T_W \qquad T_W \le 4\left[k_H T - 5t_0\right]$$

And by the same process:

$$\frac{1}{4}\left(k_W - 1\right) + \frac{6t_0}{T} \le k_H$$
$$\left\lceil \frac{1}{4}\left(k_W - 1\right) \right\rceil \le k_H$$

Putting the two inequalities together, and also doing the same with k_H/k_G and k_G/k_W, we get:

$$\left\lceil \frac{1}{4}\left(k_W - 1\right) \right\rceil \le k_H \le \left\lfloor 1 + \frac{1}{4}k_W \right\rfloor \tag{4}$$

$$\left\lceil \frac{5}{16}\left(k_G - 1\right) \right\rceil \le k_H \le \left\lfloor 1 + \frac{5}{16}k_G \right\rfloor \tag{5}$$

$$\left\lceil \frac{4}{5}\left(k_W - 1\right) \right\rceil \le k_G \le \left\lfloor 1 + \frac{4}{5}k_W \right\rfloor \tag{6}$$

Finally, consider the left-hand side of eq. 3. Rearranging it gives:

$$k_W \le 1 + \frac{T_W - 4t_0}{T}$$

However, also from eq. 3:

$$(k_W - 1)T + 4t_0 \le k_W T - 4t_0$$
$$8t_0 \le T$$

Thus, T must be at least $8t_0$. This gives us an upper bound on k_W:

$$k_W \le 1 + \frac{T_W - 4t_0}{8t_0}$$

Since k_W must be an integer, and since we already know $k_W \ge 1$ we can bound k_W as:

$$1 \le k_W \le \left\lfloor 1 + \frac{T_W - 4t_0}{8t_0} \right\rfloor$$

This gives us everything we need to navigate our choices of k_H, k_G, and k_W. First, we choose k_W from within the bound above, starting from the upper bound since a larger k_W means a smaller T, which means more profitable batching. For each choice of k_W, we can choose a k_G from the bounds in eq. 6, again starting from the upper bound. Finally, we can choose a k_H that satisfies both eq. 4 and eq. 5.

For each choice of k_H, k_G, and k_W, we then need to check if there is a range of T which is valid. If there isn't, we move on to the next choice of $k_H/k_G/k_W$. If there is, since a T calculated for smaller ks will always be larger, we can end our search as soon as we get any valid T and be sure that it will be the minimum possible period.

To get the range of possible T, we rearrange eqs. 1–3 to get:

$$\frac{T_H + 5t_0}{k_H} \le T \le \frac{T_H - t_0}{k_H - 1} \tag{7}$$

$$\frac{T_G + 3t_0}{k_G} \le T \le \frac{T_G - 3t_0}{k_G - 1} \tag{8}$$

$$\frac{T_W + 4t_0}{k_W} \le T \le \frac{T_W - 4t_0}{k_W - 1} \tag{9}$$

Thus, given our choice of k_H, k_G, and k_W, we get bounds on T of:

$$T^{\min} = \max\left[\frac{T_H + 5t_0}{k_H}, \frac{T_G + 3t_0}{k_G}, \frac{T_W + 4t_0}{k_W}\right]$$

$$T^{\max} = \min\left[\frac{T_H - t_0}{k_H - 1}, \frac{T_G - 3t_0}{k_G - 1}, \frac{T_W - 4t_0}{k_W - 1}\right]$$

If $T^{\min} \leq T^{\max}$, the choice of k_H, k_G, and k_W is valid, and we take $T^{\min}$ as the period.

We also give a code listing to actually implement this for given `hack_time`, `grow_time`, `weak_time`, and `t0`:

```
let period, depth;
const kW_max = Math.floor(1 + (weak_time - 4 * t0) / (8 * t0));
schedule: for (let kW = kW_max; kW >= 1; --kW) {
    const t_min_W = (weak_time + 4 * t0) / kW;
    const t_max_W = (weak_time - 4 * t0) / (kW - 1);

    const kG_min = Math.ceil(Math.max((kW - 1) * 0.8, 1));
    const kG_max = Math.floor(1 + kW * 0.8);

    for (let kG = kG_max; kG >= kG_min; --kG) {
        const t_min_G = (grow_time + 3 * t0) / kG
        const t_max_G = (grow_time - 3 * t0) / (kG - 1);

        const kH_min = Math.ceil(Math.max((kW - 1) * 0.25, (kG - 1) * 0.3125, 1));
        const kH_max = Math.floor(Math.min(1 + kW * 0.25, 1 + kG * 0.3125));

        for (let kH = kH_max; kH >= kH_min; --kH) {
            const t_min_H = (hack_time + 5 * t0) / kH;
            const t_max_H = (hack_time - 1 * t0) / (kH - 1);

            const t_min = Math.max(t_min_H, t_min_G, t_min_W);
            const t_max = Math.min(t_max_H, t_max_G, t_max_W);

            if (t_min <= t_max) {
                period = t_min;
                depth  = kW;
                break schedule;
            }
        }
    }
}
```

Unless the hack time is so short as to be less than t_0, this procedure will *always* generate a valid `period` and `depth` as $k_H = k_G = k_W = 1$ is always a solution to the inequalities. This solution corresponds to a depth of 1, i.e. sequential HWGW batching instead of overlapping batching, with a period of $T = T_W + 4t_0$.

Once a `period` and `depth` have been calculated, the delay that needs to be passed to each script is:

```
const hack_delay    = depth * period - 4 * t0 - hack_time;
const weak_delay_1  = depth * period - 3 * t0 - weak_time;
const grow_delay    = depth * period - 2 * t0 - grow_time;
const weak_delay_2  = depth * period - 1 * t0 - weak_time;
```

Finally, there are two useful modifications to the above code which can be considered. The first is to limit the maximum allowed depth to some upper bound. This makes the period longer, but having fewer overlapping batches reduces overall RAM use if RAM is limited, as well as the number of concurrent scripts which relieves stress on the Javascript scheduler. This upper bound can simply be applied to `kW_max`:

```
const kW_max = Math.min(Math.floor(1 + (weak_time - 4 * t0) / (8 * t0)), max_depth);
```

Secondly, the calculated schedule is only guaranteed to work for a single value of `hack_time`, `grow_time`, and `weak_time` – i.e. a single hacking level. However, this can be extended to multiple hacking levels by calculating `hack_time_max` for the current hacking level, and `hack_time_min` for the hacking level the schedule is to remain valid until (which requires `Formulas.exe`), and the same for grow and weaken times. These can then be used in the above code as:

```
const t_min_H = (hack_time_max + 5 * t0) / kH;
const t_max_H = (hack_time_min - 1 * t0) / (kH - 1);
```

and similar for grow and weaken. Additionally, `kW_max` should be calculated using `weak_time_min`. However, this comes with a disadvantage, which is that the required safe window needs to become a fair amount larger to accommodate the range of hack/grow/weaken times. This makes batching much less efficient. On the other hand, on starting batching, there is always a delay until the first batch lands; if scripts no longer need to be restarted for every hacking level up, then this delay occurs less often. Calculating schedules over multiple hacking levels is thus a tradeoff based on how quickly you expect to be gaining hacking levels while batching versus how efficient you want the batching to be.