COMP2610/COMP6261 - Information Theory

Tutorial 11: Noisy Channel Coding

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1. Consider a channel with inputs X = {a,b,c}, outputs Y = {a,b,c,d}, and transition matrix

$$Q = \begin{bmatrix} 0.5 & 0 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 0.5 \end{bmatrix}.$$

   (a) Assuming $p_X = (0.25, 0.25, 0.5)$, what is the mutual information $I(X;Y)$ between the input and output of the channel?

   (b) Assuming $p_X = (0.25, 0.25, 0.5)$, what is the average probability of error of the channel?

   (c) Calvin claims that he has constructed a block code for Q with rate 0.01 bits per transmission and maximal block error probability 1%. Is his claim possible? Justify your answer.

   (d) Hobbes claims that he has constructed a block code for Q with rate 100 bits per transmission and maximal block error probability 1%. Is his claim possible? Justify your answer.

2. *Noisy Coding (Exercise 10.12 in MacKay)*

   (a) A binary erasure channel with input $x \in \{0,1\}$ and output $y \in \{0,?,1\}$ has transition matrix

$$Q_E = \begin{bmatrix} 1-q & 0 \\ q & q \\ 0 & 1-q \end{bmatrix}.$$

   Find the mutual information $I(X;Y)$ between the input and output for a general distribution $\mathbf{p}_X = (p_0, p_1)$ over inputs. Show that the capacity of this channel is $C_E = 1 - q$ bits.

3. Let X be an ensemble with A_X = {a,b,c} and probabilities $\mathbf{p}$ = (0.5,0.25,0.25). Write out the alphabet and probabilities for the extended ensemble X^2.

4. Consider a binary erasure channel with input $x \in \{0,1\}$, and $p_X = (0.6, 0.4)$, and transition matrix of

$$\begin{bmatrix} \frac{2}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{3}{4} \end{bmatrix}.$$

An input sequence of length n = 10 is drawn and it is send through the channel. The resulting input and output sequences are 0001111000 and 0011110001, respectively.

   (a) Is the drawn input sequence typical in p_X ? Justify your answer.

   (b) Is the drawn input and output sequences jointly typical ? Justify your answer.