COMP2610 / COMP6261 Information Theory
Lecture 22: Final Exam Information & Revision

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Outline

Exam: Basic Information

- Tuesday 15/11/2022
- Start at 9:10 am
- Reading Period: 15 minutes
- Writing Period: 150 minutes
- Upload time: 15 minutes
- Permitted materials: Any material
- Online Exam - The paper will be available in Wattle - Write your answers on paper, scan and upload as a single file.
- This examination is worth 50% of the course mark. It will be marked out of 100 and then scaled. The exam is a hurdle assessment.
- You need to connect via the Exam Zoom link (will be made available) and should have your video on throughout the exam.

Exam: Basic Information - Cont.

- 6 Main Questions - all need written answers. The mark for each question is indicated next to the question.
- Answer all questions.
- Wherever applicable, you must explain and show all steps taken to arrive arrive at your answer. The clarity and precision of your explanations and answers will be taken into account when marking.
- For TRUE/FALSE problems, providing ONLY the correct TRUE/FALSE answer constitutes 1/3 of the mark for that question. You need to fully and correctly justify each TRUE/FALSE answer to get the full mark.

Exam: Basic Comments

- Basic probability
- Entropy, mutual information and inequalities (including the material from Assignment 1)
- Compression (source coding)
- Error correction (channel coding)

COMP2610 and COMP6261 will sit the exact same exam

- no “bonus” or separate questions

Deliberately variable hardness of questions reflecting its hurdle status.

Exam: Some Tips

Make sure you understand all tutorial and assignment questions. Also, go through the notes!

Expect some "conceptual" questions

- Not just a bunch of calculations
- e.g. "Is it possible to have a code with expected length less than entropy?"

There will be an ordering of the questions, but difficulty is a personal thing — what is hard for one person is easy for the other, and vice versa.

- Don't get stuck on a question for too long
- "Easy" questions can take time under exam conditions

Leave numbers in terms of fractions/logs if easier

- Need to provide a correct, unambiguous answer

Exam: Common Errors

- Not actually knowing the material
- Not labelling answers clearly (We mark questionwise)
- Not explaining what you are trying to do (so We can not give part
- marks) Trying to bluff your way to an answer — *It will not work.*
- Rushing; sloppy careless work
- Only answering part of the question
- Confusing and incoherent explanations
- Not even attempting some questions
- Answering a question different to that which was asked
- Writing before thinking
- Spending too much time on questions worth few marks and not enough on those worth many.

Outline

Probability Theory: Overview

Expect to be able to compute probabilities given either explicit distributions, or English descriptions

Be very comfortable with Bayes' rule!

- Always write down formally the random variables you are using
- Translating an English description into conditionals is tricky

Be familiar with maximum likelihood and Bayesian estimation of probabilities

- What are they?
- Why do we need them?
- How do we use them?

Understand Bernoulli and Binomial random variables

Probability Theory - I

- Different types of probability distribution
  - Joint distribution: $p(X, Y)$
  - Conditional distribution: $p(X|Y) = \dfrac{p(X, Y)}{p(Y)}$
  - Marginal distribution: $p(Y) = \sum_x p(X = x, Y)$
- Basic rules of probability
  - Sum rule (marginalisation)
  - Product rule
- Statistical (marginal and conditional) independence:

$$X \perp\!\!\!\perp Y \leftrightarrow p(X, Y) = p(X)p(Y)$$
$$X \perp\!\!\!\perp Y|Z \leftrightarrow p(X, Y|Z) = p(X|Z)p(Y|Z)$$

Probability Theory - II

- Bayes' theorem: posterior $\propto$ prior $\times$ likelihood:

$$
\begin{aligned}
p(X|Y) &= \frac{p(X)p(Y|X)}{p(Y)} \\
&= \frac{p(X)p(Y|X)}{\sum_x p(X=x, Y)}
\end{aligned}
$$

- Application of Bayes' theorem
  - Translating description to conditional probability
  - Natural frequency interpretation

Probability Theory - III

- Basic probability distributions
  - Bernoulli: for a single flip of a coin, will it land heads?
  - Binomial: out of N flips of a coin, how many land heads?
  - see lectures for other distributions

- Estimating probabilities from observations
  - The maximum likelihood principle

Outline

Entropy, Mutual Information and Inequalities: Overview

Be familiar with the various forms of entropy (marginal, conditional, ...) and mutual information

- Be prepared to explain their intuitive meaning
- Be comfortable with computing such quantities given either explicit distributions, or English descriptions

Understand definition and meaning of typical sequences, AEP

Apply Markov and Chebyshev to specific problems

Entropy

- Entropy, joint entropy and conditional entropy:

$$H(X) = \sum_x p(x) \underbrace{\log \frac{1}{p(x)}}_{h(x)}$$

$$H(X, Y) = \sum_{x,y} p(x, y) \log \frac{1}{p(x, y)}$$

$$H(Y|X) = \sum_x p(x) \underbrace{\sum_y p(y|x) \log \frac{1}{p(y|x)}}_{H(Y|X=x)}$$

- Intuitive meaning
  - Inherent / conditional uncertainty in a random variable
  - Relation to average-length code
  - Relation to minimum number of binary questions

KL and Mutual information

- Definition and properties of KL divergence

$$D_{\mathrm{KL}}(p\|q) = \sum_{x} p(x) \log \frac{p(x)}{q(x)}.$$

- Definition and understanding of mutual information:

$$\begin{aligned} I(X;Y) &= D_{\mathrm{KL}}\big(p(X,Y)\|p(X)p(Y)\big) \\ &= \sum_{x\in\mathcal{X}} \sum_{y\in\mathcal{Y}} p(x,y) \log \frac{p(x,y)}{p(x)p(y)} \end{aligned}$$

  - Relation to statistical independence:

$$X \perp\!\!\!\perp Y \leftrightarrow I(X;Y) = 0$$

  - Relation to entropy:

$$I(X;Y) = H(X) - H(X|Y) = H(Y) - H(Y|X)$$

Inequalities

- Statement and applications of Jensen's inequality: for convex f, and any rv X,
$$f(\mathbb{E}[X]) \leq \mathbb{E}[f(X)].$$
- Statement and applications of Markov's inequality: for nonnegative rv X,
$$p(X \geq a) \leq \frac{\mathbb{E}[X]}{a}.$$
- Statement and applications of Chebyshev's inequality: for any rv X with finite expectation,
$$p(|X - \mathbb{E}[X]| \geq a) \leq \frac{\mathbb{V}[X]}{a^2}.$$

Typicality and AEP

- Definition and computation of typical set:

$$T_{N\beta} = \left\{ x : \left| -\frac{1}{N} \log P(x) - H(X) \right| < \beta \right\}$$

- Statement of AEP: if $x_1, \ldots, x_N$ are iid with distribution P,

$$-\frac{1}{N} \log P(x_1, \ldots, x_N) \to H(X)$$

Outline

Source Coding: Overview

Review basic notions of (extended) ensembles, δ-sufficient subsets

Be prepared to apply SCT for block codes

Be prepared to compute Shannon-Fano-Elias/Huffman/Arithmetic

- Arithmetic only for simpler cases

Be very comfortable with basic properties of prefix codes (definition, expected length, Kraft's inequality, ...)

Source Coding

Uniform Lossy Codes

- Extended Ensembles X^N
- Smallest δ-Sufficient Sets S_δ
- Source Coding Theorem I

$$\frac{1}{N} H_\delta \left(X^N\right) \to H(X)$$

Stream Codes

- Interval Coding (Shannon-Fano-Elias)
- Arithmetic Coding

Variable-Length Lossless Codes

- Prefix & Uniquely Decodable
- Expected Code Length $L(C, X)$
- Kraft's Inequality

$$\sum_i 2^{-\ell_i} \leq 1 \iff \text{prefix/U.D.}$$

- Shannon Codes $\ell_i = \left\lceil \log \frac{1}{p_i} \right\rceil$
- Source Coding Theorem II

$$H(X) \leq L(C, X) \leq H(X) + 1$$

- Huffman Codes

Noisy-Channel Coding

Noisy Channels $X \stackrel{Q}{\to} Y$

- Transition Matrix Q
  $Q_{i,j} = P(Y = y_i | X = x_j)$
- Binary Symmetric Channel

$$\begin{bmatrix} 1-f & f \\ f & 1-f \end{bmatrix}$$

- Z Channel

$$\begin{bmatrix} 1 & f \\ 0 & 1-f \end{bmatrix}$$

- Noisy Typewriter Channel
- Channel Capacity

$$C = \max_{\mathbf{p}_X} I(X; Y)$$

Noisy Channel Codes

- (N, K) Block Codes
- Repetition Code
- Block Code Rate $R = \frac{K}{N}$
- Probability of Block Error

$$p_B = P(\mathbf{s}_{in} \neq \mathbf{s}_{out})$$

- Noisy-Channel Coding Theorem
  Arbitrarily good codes with rate R if and only if $R \leq C$

Channel Coding: Overview

Understand notion of rate for a block code

Recall steps to computing channel capacity (easier to do with $H(Y)$ and $H(Y|X)$ rather than $H(X)$ and $H(X|Y)$)

Be prepared to apply NCCT

Final Thoughts

- Explain what you are trying to do
- I am particularly looking for understanding of the material
- Your job: make my job easier! Where explanations are required of you, the *quality* of your explanation matters!

Thanks & Good Luck!