$$J = \frac{1}{2} w^T w - \sum_{i=1}^{n} \alpha_i \left(y_i (w^T x_i + b) - 1 + \xi_i\right) + C\sum_{i=1}^{n} \xi_i - \sum_{i=1}^{n} \beta_i \xi_i$$

$$\frac{\partial J}{\partial w} = w - \sum_{i=1}^{n} \alpha_i y_i x_i = 0 \qquad \sum_{i=1}^{n} \alpha_i y_i = 0$$

$$w = \sum_{i=1}^{n} \alpha_i y_i x_i$$

$$J = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_j \alpha_i y_i y_j x_j^T x_i - \sum_{i=1}^{n} \alpha_i y_i \sum_{j=1}^{n} a_j y_j x_j^T x_i - \cancelto{0}{b \sum_{i=1}^{n} \alpha_i y_i}$$

$$+ \sum_{i=1}^{n} \alpha_i$$

$$\cancel{- \sum_{i=1}^{n} \alpha_i \xi_i}$$

$$\cancel{+ C \sum_{i=1}^{n} \xi_i} \quad 0$$

$$\cancel{- \sum_{i=1}^{n} \beta_i \xi_i}$$

$$- \sum_{i=1}^{n} (\alpha_i + \beta_i) \xi_i$$

$$C = \alpha_i + \beta_i$$

$$\Rightarrow J = -\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_j \alpha_i y_i y_j x_j^T x_i + \sum_{i=1}^{n} \alpha_i \quad \#$$