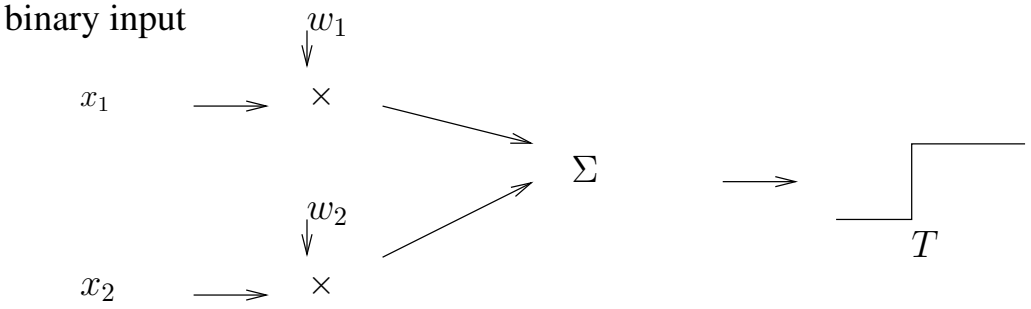
Neural Networks

ANN

- Long a curiosity
- 2012 paper, Hinton, image classification, 1000 categories, 60 million parameters
- neurones : axone, terminaison de l'axone, noyau, dendrites



What are we modeling?

1. all or none
2. cumulative influence
3. synaptic weight
4. (not) refractory period *(période réfractaire)*
5. (not) axonal bifurcation
6. (not) time patterns

So we have a model of a neuron (a collection of weights and thresholds).
But what about collections of neurons?

$$\text{inputs} \rightarrow (w, t) \rightarrow \text{outputs}$$

So we want $z = f(x, w, t)$. Training means adjusting w, t.
An ANN is a function approximator.

We want some desired function, $d = g(x)$.

Too simple example: perceptron

Perceptron is a supervised algorithm. Given inputs $D = \{(x_j, d_j)\}$, want classifier f.

We'll call $x_{j,i}$ the value of feature i in training datum j. We set $x_{j,0} \equiv 1$.

Perceptron algorithm:

$$\text{At each output, } f(x, w) = \begin{cases} 1 & \text{if } w \cdot x + b > 0 \\ 0 & \text{otherwise} \end{cases}$$

- Initialise weights randomly.
- $\forall x_j \in$ inputs, compute output: $y_j(t) = f(w(t) \cdot x_j)$
- $w_i(t+1) = w_i(t) + r \cdot (d_j - y_j(t))x_{j,i}$ for all features i

The weight are updated at the last step after each training sample.

Converges if separable.

Talk about convergence, about Seymor and Pappert.

Perceptron and MNIST

What other classifiers do we know?

- How would we do this with logistic regression?
- How would we do this with CARTs ?

Talk about OvO, OvA.

This just introduces voting...

Training a single neuron

Let's start with a performance function $P = \|d - z\|$.
Why don't we like that?

So instead use $P = \|d - z\|^2 = (d - z)^2$.

(Draw w_1-w_2 plot with isoclines. Show four candidate steps. Mention 60 million parameters in Hinton's model. Talk about exponential explosion with number of weights.)

So we can follow the gradient:

$$\frac{\partial P}{w_1} \quad , \quad \frac{\partial P}{w_2}$$

$$\Delta w = r\left(\frac{\partial P}{w_1}\mathbf{i} + \frac{\partial P}{w_2}\mathbf{j}\right)$$

What goes wrong?

Computer scientists were stuck on this for a quarter century. Then Paul Werbos showed in his 1974 PhD dissertation how to train neural networks using back-propagation of errors.

Two problems:

1. Thresholds are annoying. We don't want $z = f(x, w, T)$ but $z = f(x, w)$.

So let's add an extra weight w_0 and input $x_0 = -1$. Then set $w_0 \equiv T$.

(Show that this moves the threshold step to 0. So we don't have to pay attention to it anymore.)

2. We're using a step function, which isn't continuous.

Let's smooth it out. Use sigmoid function.

$$\beta = \frac{1}{1 + e^{-\alpha}}$$

(Walk through how to graph this.)

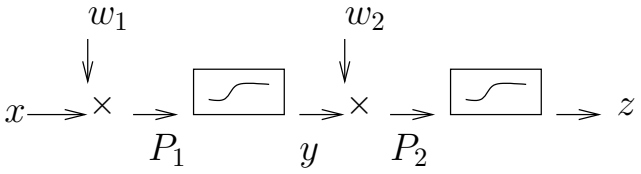
Summary: What have we done ?

- Built a neuron
- Described how to do gradient descent
- Modified our activation function to be compatible with gradient descent

Training two neurons

- 1 neuron isn't a net
- 2 neurons is a net

This is a simplest possible neural network.



Then performance function is $P = \frac{1}{2}(d-z)^2$

So we want to do gradient descent, this means computing partial derivatives.

$$\frac{\partial P}{\partial w_2} = \frac{\partial P}{\partial z}\frac{\partial z}{\partial w_2} = \frac{\partial P}{\partial z}\frac{\partial z}{\partial P_2}\frac{\partial P_2}{\partial w_2}$$

$$\frac{\partial P}{\partial w_1} = \frac{\partial P}{\partial z}\frac{\partial z}{\partial P_2}\frac{\partial P_2}{\partial y}\frac{\partial y}{\partial P_1}\frac{\partial P_1}{\partial w_1}$$

How do we compute these things?

$$\frac{\partial P_2}{\partial w_2} = y$$

$$\frac{\partial z}{\partial P_2} = z(1-z) \quad \text{(come back to this)}$$

$$\frac{\partial P}{\partial z} = d - z$$

How do we compute the derivative of the sigmoid function?

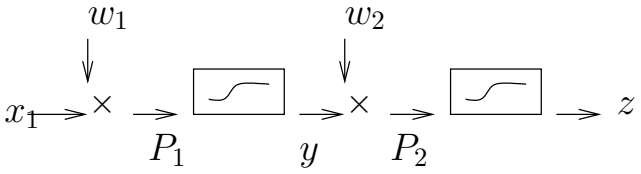
$$
\begin{aligned}
\frac{\partial \beta}{\partial \alpha} &= \frac{\partial}{\partial \alpha}\left(\frac{1}{1+e^{-\alpha}}\right) \\
&= \frac{\partial}{\partial \alpha}\left(1+e^{-\alpha}\right)^{-1} \\
&= -\left(1+e^{-\alpha}\right)^{-2} e^{-\alpha}(-1) \\
&= \frac{e^{-\alpha}}{1+e^{-\alpha}} \cdot \frac{1}{1+e^{-\alpha}} \\
&= \frac{\mathbf{1}+e^{-\alpha}-\mathbf{1}}{1+e^{-\alpha}} \cdot \frac{1}{1+e^{-\alpha}} \\
&= \left(\frac{1+e^{-\alpha}}{1+e^{-\alpha}} - \frac{1}{1+e^{-\alpha}}\right) \cdot \frac{1}{1+e^{-\alpha}} \\
&= (1-\beta)\beta
\end{aligned}
$$

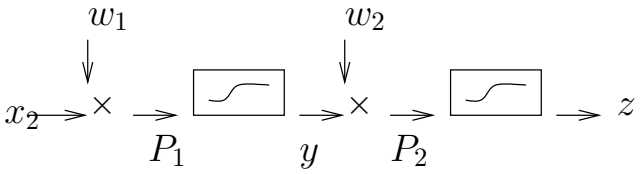
This is pretty cool: the derivative of the sigmoid function is independent of the input!

Note : at each phase, we only need to compute the last three factors.

Towards real life...

In real life, we have more than one of these going on at once. Sort of like this:





And then there are cross-overs with more weights, more multipliers. So we are again at risk of exponential blow-up: there are an exponential number of paths through the network.

But note that the dependencies are only by column, so lots of things are already computed.

In other words,

- Linear in depth
- Quadratic in width

Deep networks

We want to go from 2 parameters to millions.

- convolution
- pooling — max pooling
- kernel *(taking some neurons and running them across the image)*
- multiple kernels
- softmax
- dropout

We don't really know why any of this works.

Talk about auto-encoders.