ML Week

Introduction

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23–24 novembre 2016

About Machine Learning

The literature is overwhelmingly in English

Disclaimer: Time is short

Definition

Supervised

Unsupervised

Reinforcement

Curse of Dimensionality

Machine learning is not magic

Machine learning is mathematics

Mostly, it's these maths:

- Probability
- Statistics
- Linear algebra
- Optimisation theory
- Differential calculus

Probability

Addition rule: independent events

$$\Pr(A \cup B) = \Pr(A) + \Pr(B)$$

Addition rule: dependent events

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

Multiplication rule: independent events

$$\Pr(A \cap B) = \Pr(A)\Pr(B)$$

Multiplication rule: dependent events

$$\Pr(A \cap B) = \Pr(A \mid B) \Pr(B)$$

Conditional probability

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

Conditional probability

$$\cup_i A_i = A \quad \wedge \quad A_i \cap A_j = \varnothing \implies$$

$$P(A_1 \mid B) = \frac{\Pr(B \mid A_1)\Pr(A_1)}{\sum_i \Pr(B \mid A_1)\Pr(A_1) + \cdots + \Pr(B \mid A_k)\Pr(A_k)}$$

Statistics

What is Statistics

1. Identify a question or problem.
2. Collect relevant data on the topic.
3. Analyze the data.
4. Form a conclusion.

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Sadly, sometimes people forget 1.

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4. Form a conclusion.

Statistics is about making 2–4 efficient, rigorous, and meaningful.

OpenIntro Statistics, 2nd edition, D. Diez, C. Barr, M. Çetinkaya-Rundel, 2013.

What is data science?

(Exercise: Is this the same question as the last slide?)

1. Define the question of interest
2. Get the data
3. Clean the data
4. Explore the data
5. Fit statistical models
6. Communicate the results
7. Make your analysis reproducible

What is data science?

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What the public thinks.

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Where we spend most of our time.

What is data science?

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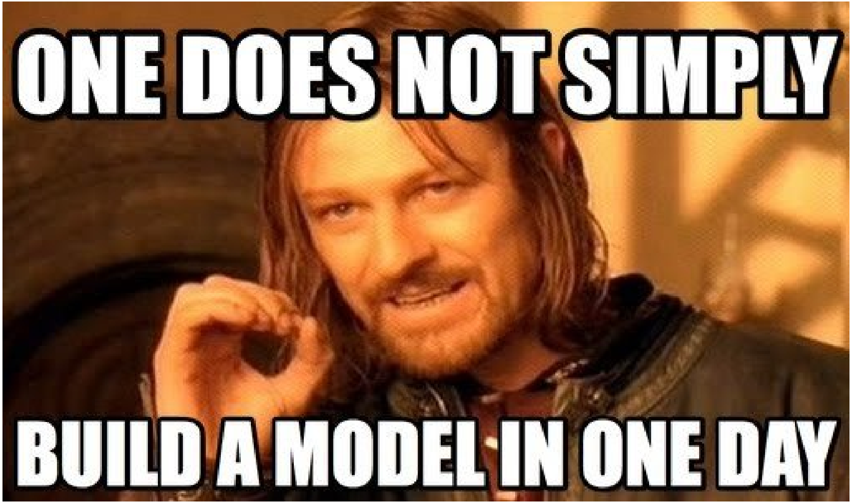
The easiest part to forget.

What is data science?

http://simplystatistics.org/2015/03/17/
data-science-done-well-looks-easy-and-that-is-a-big-
problem-for-data-scientists/

What is data science?



Anecdote

Some properties of anecdote:

- is data
- haphazardly collected
- is generally not representative
- sometimes result of selective retention
- does not accumulate to be representative
- might be true (by chance)
- is ok to use as hypothesis, but be clear that hypothesis is anecdote

Study Types

- Observational
- Experimental

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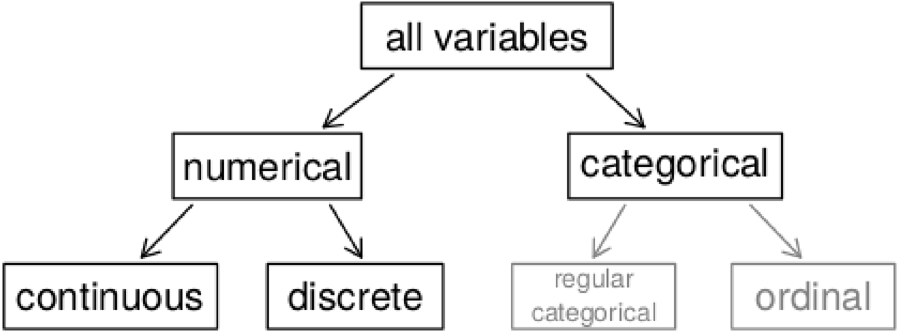
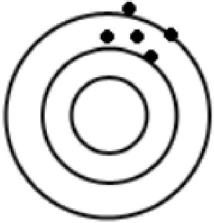
- Observational
- Experimental

What can go wrong?

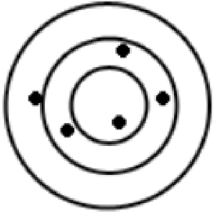
- Forgetting that association $\neq$ causation
- Not random
- Confounding variables

Variable types

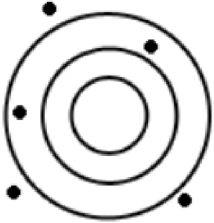




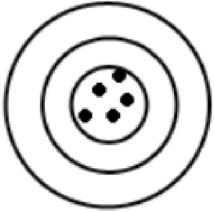
High bias, low variance



Low bias, high variance



High bias, high variance



Low bias, low variance

Mean

- Weighted and unweighted
- Centroid to physicists

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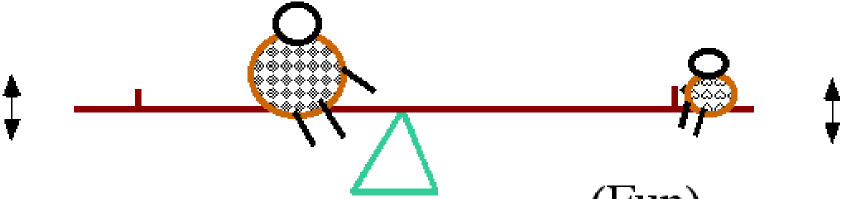

(Fun)

Mean

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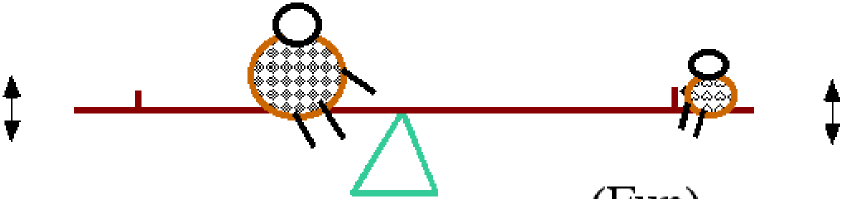

(Fun)

$$\mu = E(X) = \sum w_i x_i = \mathbf{w} \cdot \mathbf{x}$$

http://telescopes.stardate.org/images/research/teeter-totter/TT4.gif

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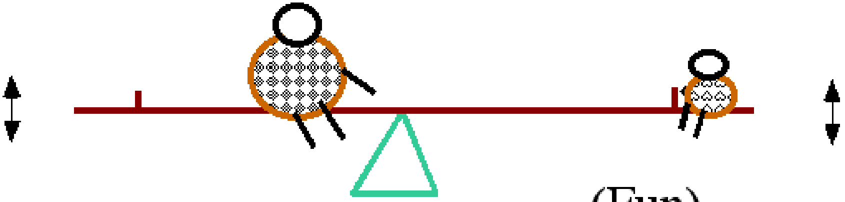

(Fun)

$$\mu = E(X) = \sum \Pr(X = x_i) x_i$$

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Mean

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$$\mu = E(X) = \int x f(x)\,\mathrm{d}x$$

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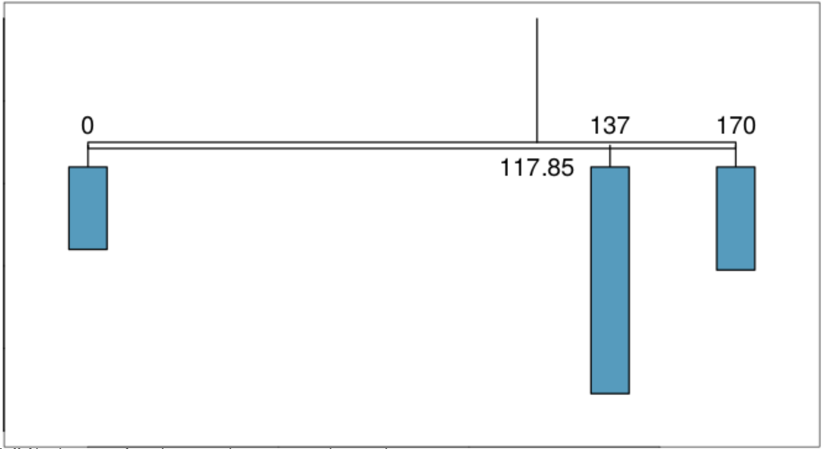

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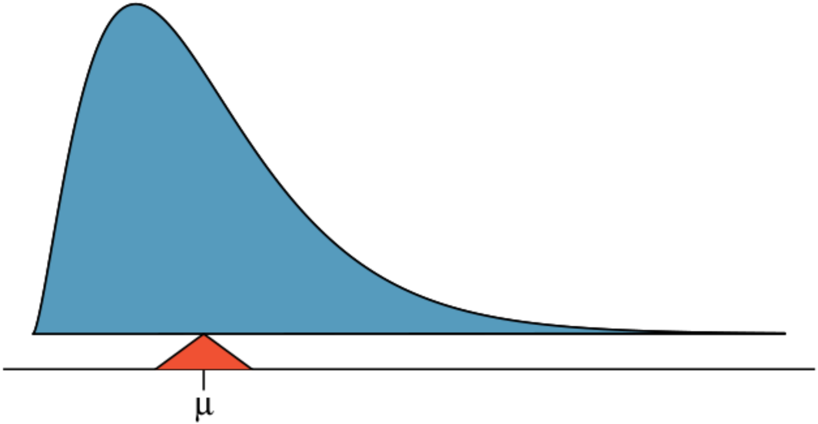


Population statistics

Deviation is distance from mean.

Population statistics

Variance is mean square of deviations

Population statistics

Standard deviation is square root of variance

Population statistics

$$s^2 = \frac{(\overline{x} - x_1)^2 + \cdots (\overline{x} - x_n)^2}{n - 1}$$

Population statistics

$$\sigma^2 = \frac{(\overline{x} - x_1)^2 + \cdots (\overline{x} - x_n)^2}{n}$$

Population statistics

$$\mathrm{Var}(X) = \sigma^2 = (\overline{x} - x_1)^2 \Pr(X = x_1) + \cdots (\overline{x} - x_n)^2 \Pr(X = x_n)$$

Words

Given a distribution X, the

probability distribution function (pdf) (continuous) or **probability mass function (pmf)** is the probability that the variate has value x:

$$\Pr(a \leqslant X \leqslant b)$$

or

$$\Pr(X = a)$$

Words

Given a distribution X, the

cumulative probability function (cdf) is the probability that the variate is less than x:

$$\Pr(X \leqslant x) = \int_{-\infty}^{x} \mathrm{pdf}(x)\,\mathrm{d}x \text{ or } \sum_{i \leqslant x} \Pr(X = x)$$

Words

Given a distribution X, the

percent point function (ppf) is the inverse of the cdf. Given a probability, what's x? Also called the **inverse distribution.**

Words

Given a distribution X, the

survival function (sf) is the probability that the variate takes a value greater than x:

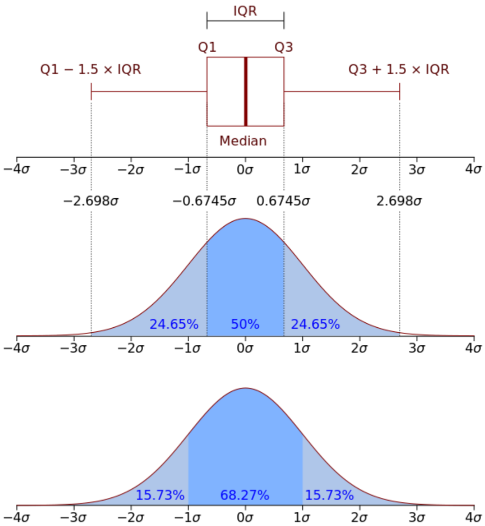
$$\mathrm{ss}(x) = \Pr(X > x) = 1 - \mathrm{cdf}(x)$$

Words

Given a distribution X, the

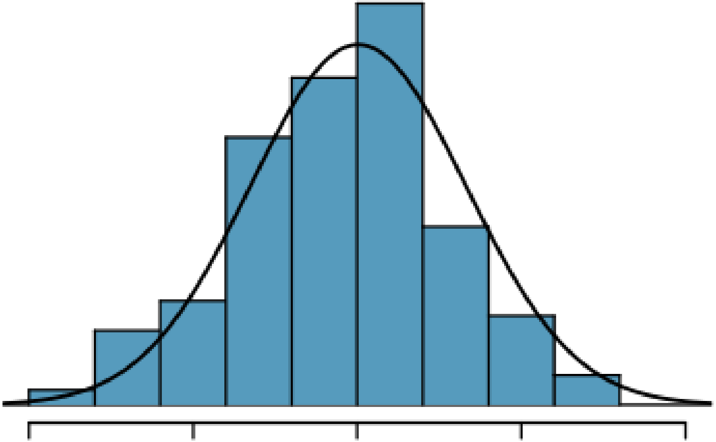
inverse survival function (isf) is the inverse of the survival function:

$$\text{isf}(\alpha) = \text{ppf}(1 - \alpha)$$


IQR
Q1
Q3
Q1 − 1.5 × IQR
Q3 + 1.5 × IQR
Median
−4σ −3σ −2σ −1σ 0σ 1σ 2σ 3σ 4σ
−2.698σ
−0.6745σ
0.6745σ
2.698σ
24.65%
50%
24.65%
−4σ −3σ −2σ −1σ 0σ 1σ 2σ 3σ 4σ
15.73%
68.27%
15.73%
−4σ −3σ −2σ −1σ 0σ 1σ 2σ 3σ 4σ

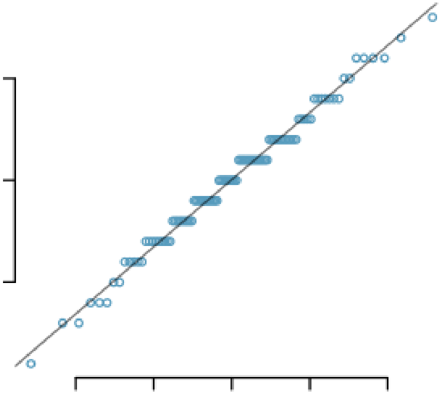
Evaluating Normal Approximations

Easy technique 1: visually compare to normal plot.



Evaluating Normal Approximations

Easy technique 2: normal probability plot.



Also known as a quantile-quantile plot.

$$\overline{x} \neq \mu$$

Inference Concepts

Running mean. Sequence of partial sums (divided by number in sum).

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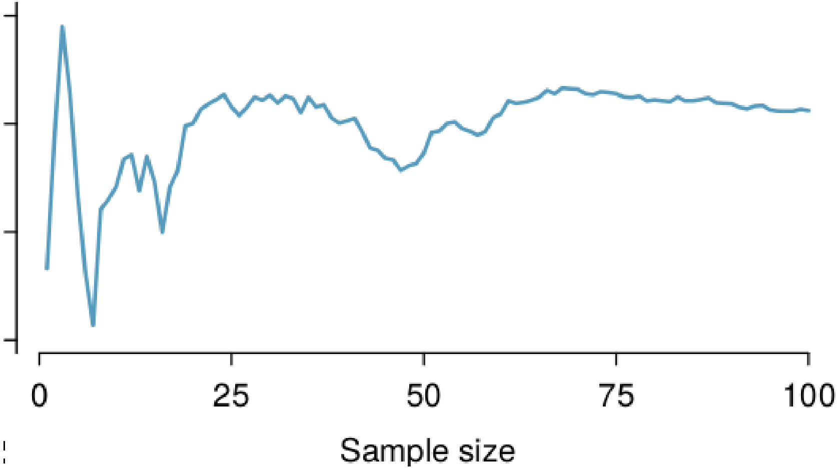

Inference Concepts

Running mean. Sequence of partial sums (divided by number in sum).

Sampling variation. Change of $\overline{x}$ from one sample to the next.

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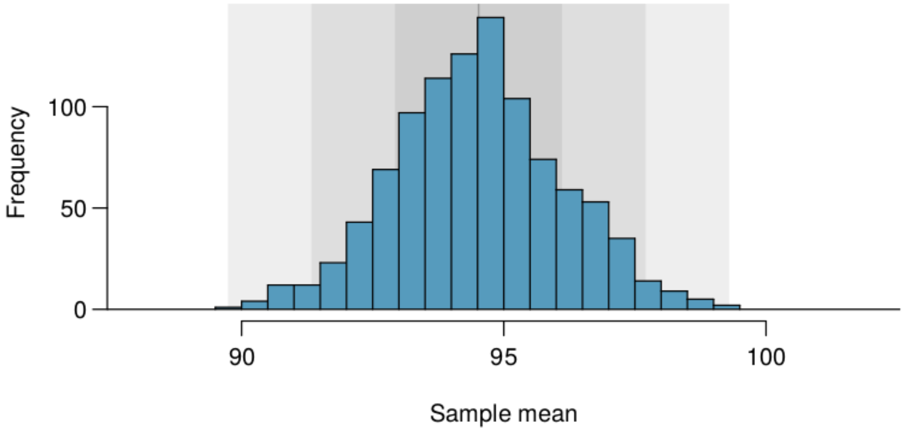
Running mean. Sequence of partial sums (divided by number in sum).

Sampling variation. Change of $\overline{x}$ from one sample to the next.

Sampling distribution. The distribution of possible point samples of a fixed size from a given population.

Sampling distribution



Confidence intervals

Sample *n* points, choose an interval around the sample mean.

A 95% confidence interval means if we sample repeatedly, about 95% of the samples will contain the population mean.

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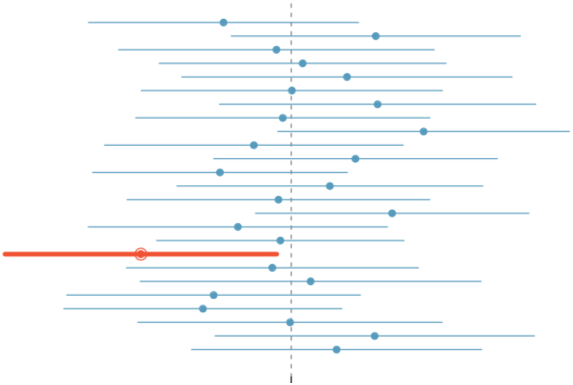

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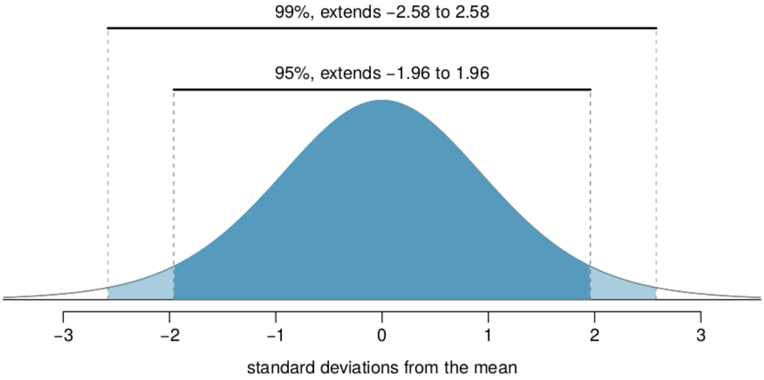

Linear Algebra

Linear algebra: basics

$$v = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} \in \mathbb{R}^n$$

Linear algebra: basics

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{bmatrix} = \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix}$$

$$= \begin{Bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{Bmatrix} \in \mathbb{R}^{n \times n}$$

Linear algebra: basics

$$u + v = \begin{pmatrix} u_1 + v_1 \\ u_2 + v_2 \\ \vdots \\ u_n + v_n \end{pmatrix}$$

Linear algebra: basics

$$\alpha v = \begin{pmatrix} \alpha v_1 \\ \alpha v_2 \\ \vdots \\ \alpha v_n \end{pmatrix} \qquad (\alpha \in \mathbb{R})$$

Linear algebra: basics

$$\| v \| = \sqrt{v_1^2 + \cdots + v_n^2}$$

Linear algebra: basics

$$u \cdot v = u_1 \cdot v_1 + \cdots + u_n \cdot v_n$$

$$= \| u \| \| v \| \cos \theta$$

Linear algebra: basics

$$C = A + B \iff c_{ij} = a_{ij} + b_{ij}$$

$$C = AB \iff c_{ij} = \sum_k a_{ik} b_{kj}$$

$$A = B^T \iff a_{ij} = b_{ji}$$

$$AA^{-1} = A^{-1}A = \mathrm{diag}(1)$$

Linear algebra: transformations

$$Ax = y \qquad f = T_A : \mathbb{R}^n \to \mathbb{R}^n$$

$$x = A^{-1}Ax = A^{-1}y \qquad f^{-1} = T_{A^{-1}} : \mathbb{R}^n \to \mathbb{R}^n$$

Linear algebra: transformations

B is a basis for V iff any of these conditions are met:

- B is a minimal generating set of V
- B is a maximal set of linearly independent vectors
- Every vector $v \in V$ can be expressed in a unique way as a sum of $b_i \in B$

(The conditions are equivalent.)

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Bases are not unique.

Linear algebra: transformations

Eigenvectors, eigenvalues:

$$Av = \lambda v$$

Linear algebra: transformations

Eigenvectors, eigenvalues:

$$Av = \lambda v$$

$$Av = \lambda 1 v \iff (A - \lambda 1)v = 0$$

Linear algebra: transformations

Eigenvectors, eigenvalues:

$$Av = \lambda v$$

Some matrices are diagonalisable. Then

$$A = Q \Lambda Q^{-1} \qquad \text{with } \Lambda = \begin{bmatrix} \lambda_1 & \cdots & 0 \\ \vdots & \ddots & 0 \\ 0 & 0 & \lambda_n \end{bmatrix}$$

$$\text{and } Q = \begin{bmatrix} | & & | \\ v_1 & \cdots & v_n \\ | & & | \end{bmatrix}$$

Questions?

ml-week.com/1