ML Week

Linear Regression

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23–24 novembre 2016

Linear models

Problem: $\{(x_i, y_i)\}$.

Given x, predict $\hat{y}$.

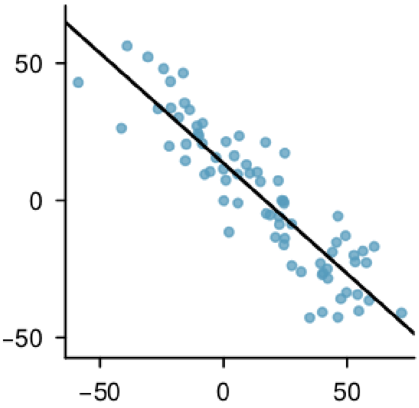
Linear models

x: **explanatory** or **predictor** variable.

y: **response** variable.

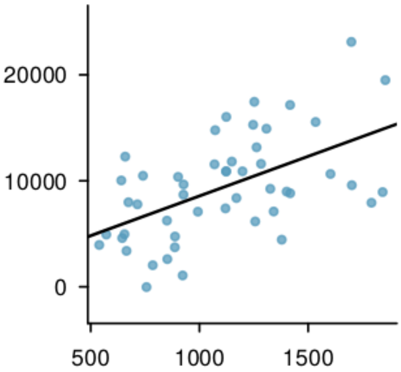
For some reason, we believe a linear model is a good idea.

Example:

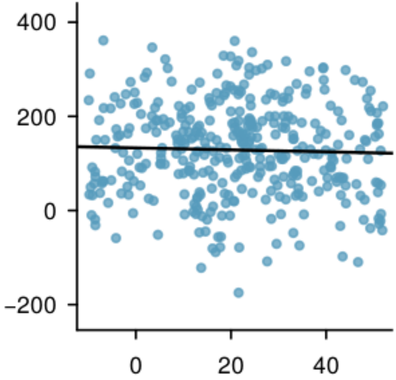


Linear models

Example:

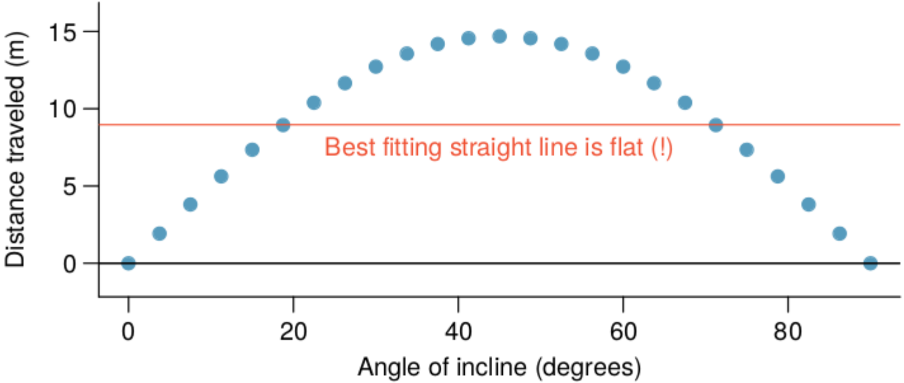


Example:



Linear models

Example:



Residuals

What's left over.

$$\text{data} = \text{fit} + \text{residual}$$

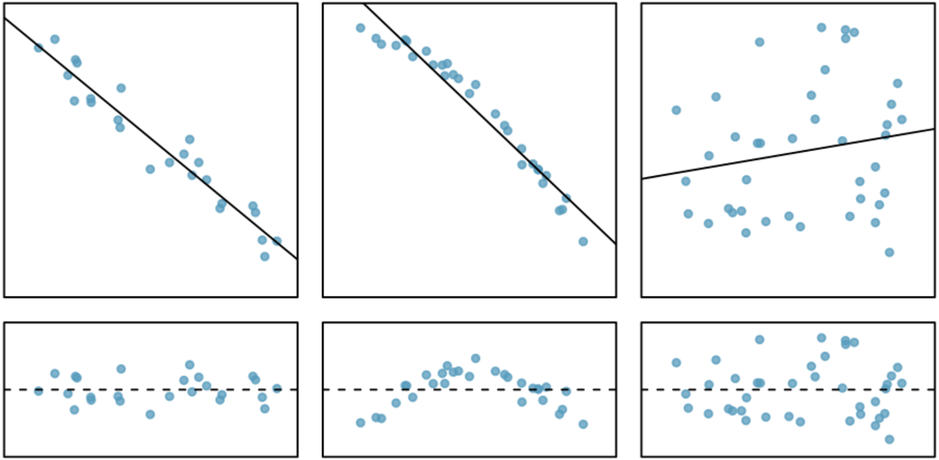
Residuals

What's left over.

$$y_i = \hat{y}_i + e_i$$

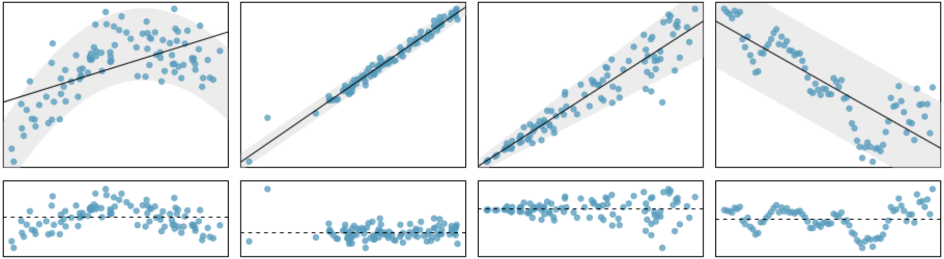
Residuals

What's left over.



Residuals

What's left over.



Residuals

What's left over.

Goal: small residuals.

$$\sum | e_i |$$

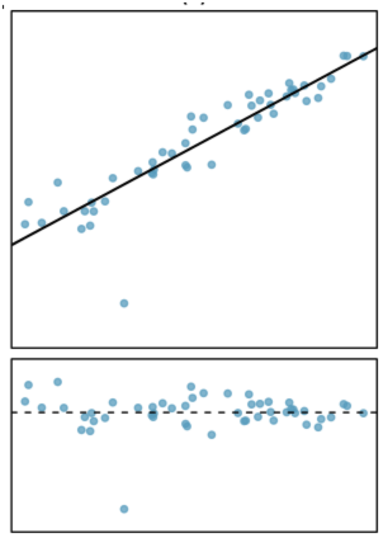
Residuals

What's left over.

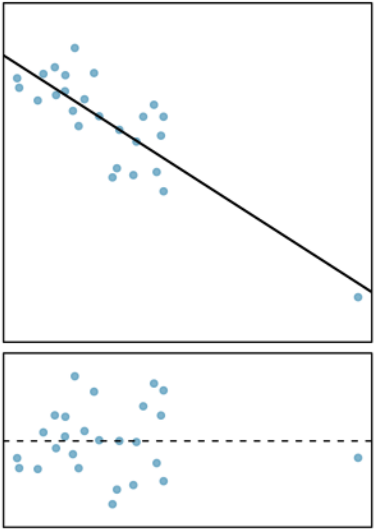
Goal: small residuals.

$$\sum e_i^2$$

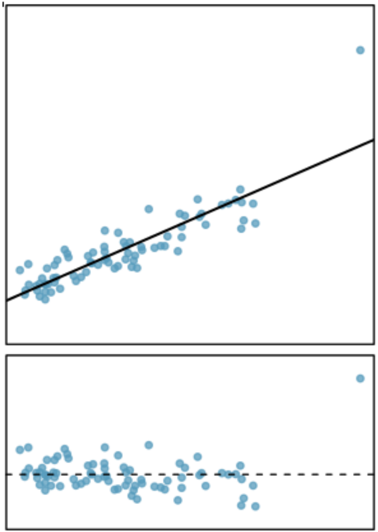
Outliers



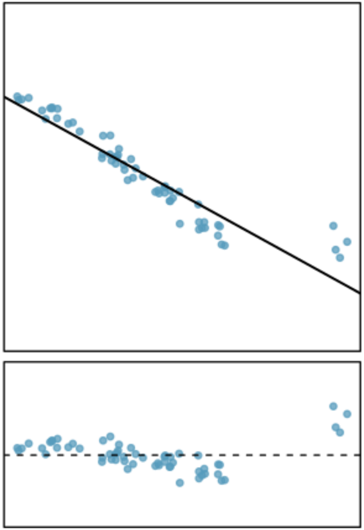
Outliers



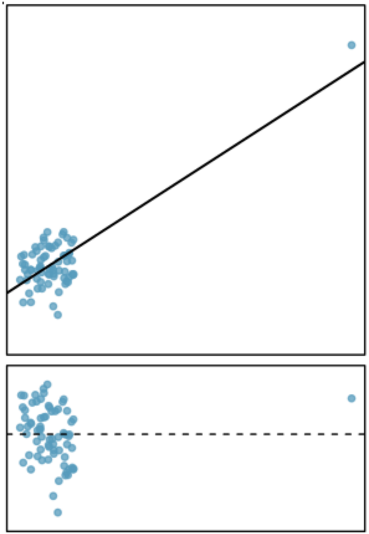
Outliers



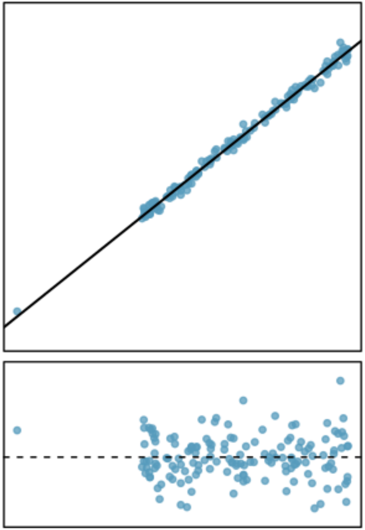
Outliers



Outliers



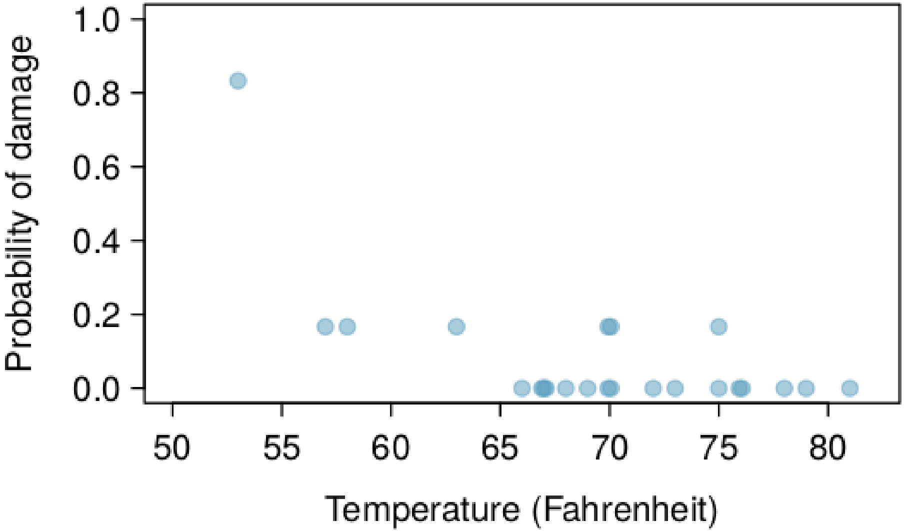
Outliers



Outliers

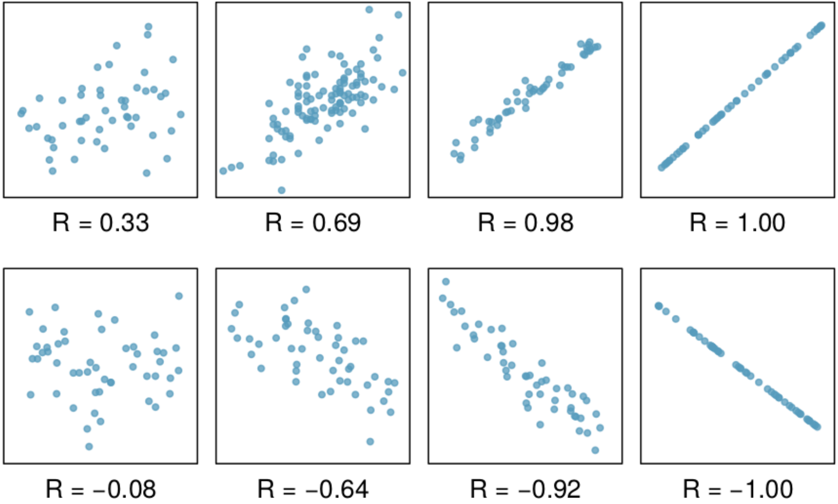
Don't ignore outliers.

Outliers

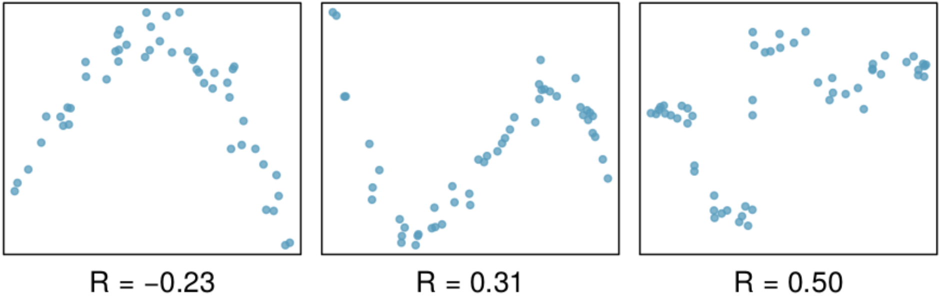


Probability of damage
1.0
0.8
0.6
0.4
0.2
0.0
50
55
60
65
70
75
80
Temperature (Fahrenheit)

Correlation

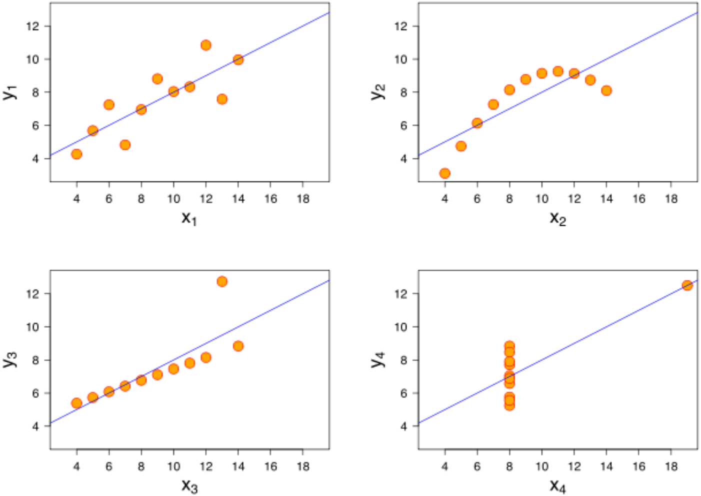


Correlation

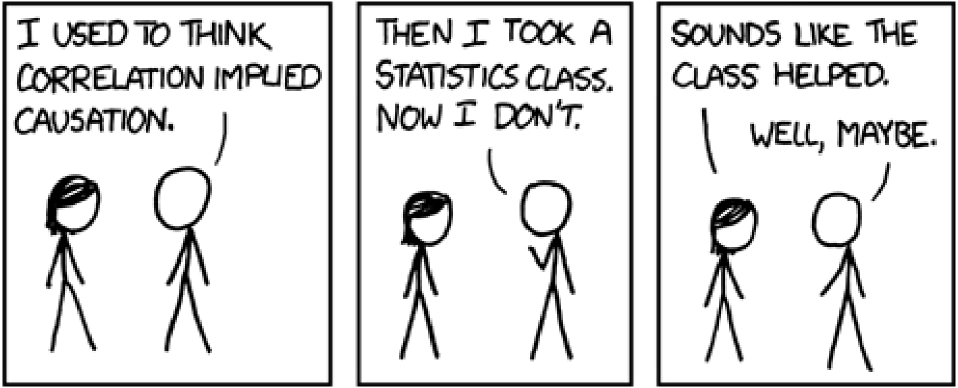


Correlation

Anscombe's Quartet



Correlation does not imply causation



https://xkcd.com/552/

Hypothesis (model)

$$h_\theta(x) = \theta_0 + \theta_1 x$$

Cost function

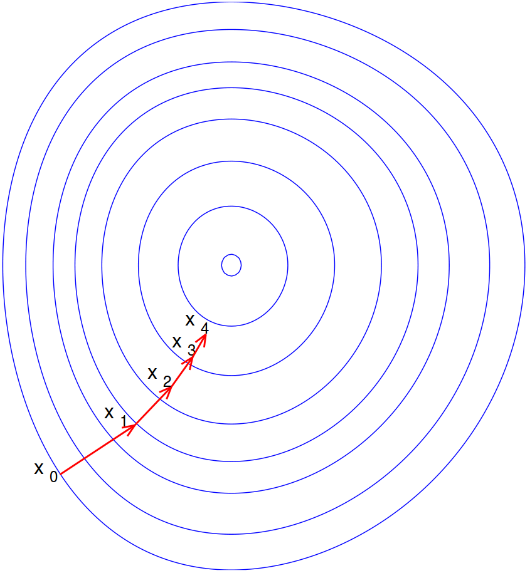
$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x_i) - y_i)^2$$

Gradient descent

$$\begin{cases} \theta_0 & \leftarrow \theta_0 - \alpha \dfrac{\partial}{\partial \theta_0} J(\theta_0, \theta_1) \\ \theta_1 & \leftarrow \theta_1 - \alpha \dfrac{\partial}{\partial \theta_1} J(\theta_0, \theta_1) \end{cases}$$

Gradient descent

$$\begin{cases} \theta_0 & \leftarrow \theta_0 - \dfrac{\alpha}{m}\displaystyle\sum_{i=1}^{m}(h_\theta(x_i) - y_i) \\ \theta_1 & \leftarrow \theta_1 - \dfrac{\alpha}{m}\displaystyle\sum_{i=1}^{m}(h_\theta(x_i) - y_i) \end{cases}$$


x 0
x 1
x 2
x 3
x 4

Hypothesis again

$$
\begin{aligned}
h_\theta(x) &= \theta_0 + \theta_1 x_1 \\
&= \theta_0 + \sum_{i=1}^{1} \theta_i x_i \\
&= [\theta_0, \theta_1] \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} \\
&= \theta^T x
\end{aligned}
$$

Hypothesis (multiple regression)

$$
\begin{aligned}
h_\theta(x) &= \theta_0 + \sum_{i=1}^{n} \theta_i x_i \\
&= [\theta_0, \cdots, \theta_n] \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_n \end{bmatrix} \\
&= \theta^T x
\end{aligned}
$$

Hypothesis (multiple regression)

$$
\begin{aligned}
h_\theta(x) &= \theta^T x \\
&= \theta^T x^{(1)}
\end{aligned}
$$

Hypothesis (multiple regression)

$$X = \begin{bmatrix} | & | & \cdots & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & \cdots & | \end{bmatrix} = \begin{bmatrix} x_0^{(1)} & x_0^{(2)} & \cdots & x_0^{(m)} \\ x_1^{(1)} & x_1^{(2)} & \cdots & x_1^{(m)} \\ \vdots & \vdots & \ddots & \vdots \\ x_n^{(1)} & x_n^{(2)} & \cdots & x_n^{(m)} \end{bmatrix}$$

Hypothesis (multiple regression)

$$
\begin{aligned}
h_\theta(X) &= \theta^T X \\
&= \left[h_0(x^{(1)}), h_0(x^{(2)}), \cdots, h_0(x^{(m)})\right] \\
&= \theta^T X
\end{aligned}
$$

Hypothesis (multiple regression)

or $X\theta$ if row vectors...

Cost function (multiple regression)

$$
\begin{aligned}
J(\theta) &= \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2 \\
&= \frac{1}{2m} (X\theta - Y)^T (X\theta - Y)
\end{aligned}
$$

Gradient descent (multiple regression)

$$\theta_j \leftarrow \theta_j - \frac{\alpha}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) \cdot x_j^{(i)}$$

$$\text{for } j = 1, \cdots, n$$

Gradient descent (multiple regression)

$$\theta \leftarrow \theta - \nabla J(\theta)$$

$$\text{where } \nabla = \begin{bmatrix} \frac{\partial}{\partial \theta_0} \\ \frac{\partial}{\partial \theta_1} \\ \vdots \\ \frac{\partial}{\partial \theta_n} \end{bmatrix}$$

Questions?

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