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ASSIGNMENT 1

I. SOLVE FOR THE LAPLACE TRANSFORM OF THE FOLLOWING:

1. $\mathcal{L}[3-e^{-3t}+5\sin 2t] = F(s)$

$$F(s) = \mathcal{L}(3) - \mathcal{L}(e^{-3t}) + 5\mathcal{L}(\sin 2t)$$

$$F(s) = \frac{3}{s} - \frac{1}{s+3} + 5\left(\frac{2}{s^2+2^2}\right)$$

$$\boxed{F(s) = \frac{3}{s} - \frac{1}{s+3} + \frac{10}{s^2+4}}$$

2. $\mathcal{L}[3+12t+42t^3-3e^{2t}] = F(s)$

$$F(s) = \mathcal{L}(3)+\mathcal{L}(12t)+42\mathcal{L}(t^3)-3\mathcal{L}(e^{2t})$$

$$F(s) = \frac{3}{s}+\frac{12}{s^2}+42\left(\frac{3!}{s^{(3+1)}}\right)-3\left(\frac{1}{s-2}\right)$$

$$\boxed{F(s)=\frac{3}{s}+\frac{12}{s^2}+\frac{252}{s^4}-\frac{3}{s-2}}$$

3. $\mathcal{L}[(t+1)(t+2)] = F(s)$

$$F(s)=\mathcal{L}[t^2+3t+2]$$

$$F(s)=\mathcal{L}(t^2)+3\mathcal{L}(t)+\mathcal{L}(2)$$

$$F(s)=\left[\frac{2!}{s^{(2+1)}}\right]+\frac{3}{s^2}+\frac{2}{s}$$

$$\boxed{F(s)=\frac{2}{s^3}+\frac{3}{s^2}+\frac{2}{s}}$$

II. SOLVE FOR THE INVERSE LAPLACE TRANSFORM OF THE FOLLOWING:

1. $\mathcal{L}^{-1}\left[\frac{8-3s+s^2}{s^3}\right]=f(t)$

$$f(t)=\mathcal{L}^{-1}\left(\frac{8}{s^3}\right)-\mathcal{L}^{-1}\left(\frac{3s}{s^3}\right)+\mathcal{L}^{-1}\left(\frac{\cancel{s^2}\,1}{s^{\cancel{3}}}\right)$$

$$f(t)=\mathcal{L}^{-1}\left[4\left[\frac{2!}{s^{(2+1)}}\right]\right]-\mathcal{L}^{-1}\left(\frac{3\cancel{s}}{s^{\cancel{3}2}}\right)-\mathcal{L}^{-1}\left(\frac{1}{s}\right)$$

$$\boxed{f(t)=4t^2-3t+1}$$

2. $\mathcal{L}^{-1}\left[\frac{5}{s-2}-\frac{4s}{s^2+9}\right]=f(t)$

$$f(t)=5\mathcal{L}^{-1}\left(\frac{1}{s-2}\right)-4\mathcal{L}^{-1}\left(\frac{s}{s^2+3^2}\right)$$

$$\boxed{f(t)=5e^{2t}-4\cos 3t}$$

3. $\mathcal{L}^{-1}\left[\frac{7}{s^2+6}\right]=f(t)$

$$f(t)=7\mathcal{L}^{-1}\left[\frac{1}{s^2+(\sqrt{6})^2}\right]\left(\frac{\sqrt{6}}{\sqrt{6}}\right)$$

$$f(t)=\frac{7}{\sqrt{6}}\mathcal{L}^{-1}\left[\frac{\sqrt{6}}{s^2+(\sqrt{6})^2}\right]$$

$$f(t)=\frac{7}{\sqrt{6}}\sin\sqrt{6}\,t\left(\frac{\sqrt{6}}{\sqrt{6}}\right)$$

$$\boxed{f(t)=\frac{7\sqrt{6}}{6}\sin\sqrt{6}\,t}$$

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ASSIGNMENT 2

III. SOLVE FOR THE INVERSE LAPLACE TRANSFORM OF THE FOLLOWING:

1. $F(s) = \dfrac{1}{s(s^2+2s+2)}$

$$\frac{1}{s(s^2+2s+2)} = \frac{A}{s} + \frac{Bs+C}{s^2+2s+2}$$

$$1 = A(s^2+2s+2) + (Bs+C)s$$

$$1 = A(s^2+2s+2) + Bs^2 + Cs$$

if $s=0$; $1 = A(2)$

$$A = \tfrac{1}{2}$$

* SUBSTITUTE A

$$\left[1 = \frac{s^2+2s+2}{2} + Bs^2 + Cs\right]2$$

$$2 = s^2+2s+2 + 2Bs^2 + 2Cs$$

$$2 = s^2(2B+1) + s(2C+2) + 2$$

$2B+1=0$ $\quad$ $2C+2=0$

$2B=-1$ $\quad$ $2C=-2$

$B=-\frac{1}{2}$ $\quad$ $C=-1$

* SUBSTITUTE A, B, C

$$\mathcal{L}^{-1}[F(s)] = \mathcal{L}^{-1}\left[\frac{\frac{1}{2}}{s} + \frac{-\frac{1}{2}s-1}{s^2+2s+2}\right]$$

$$f(t) = \tfrac{1}{2}\mathcal{L}^{-1}\left(\tfrac{1}{s}\right) - \tfrac{1}{2}\mathcal{L}^{-1}\left(\frac{s+2}{s^2+2s+2}\right)$$

$$f(t) = \tfrac{1}{2}\mathcal{L}^{-1}\left(\tfrac{1}{s}\right) - \tfrac{1}{2}\mathcal{L}^{-1}\left[\frac{(s+1)+1}{(s+1)^2+1}\right]$$

$$\boxed{f(t) = \frac{1}{2} - \frac{1}{2}e^{-t}(\cos t + \sin t)}$$

2. $F(s) = \dfrac{5(s+2)}{s^2(s+1)(s+3)} = 5\left[\dfrac{s+2}{s^2(s+1)(s+3)}\right]$

$$\frac{s+2}{s^2(s+1)(s+3)} = \frac{A}{s+1} + \frac{B}{s+3} + \frac{C}{s} + \frac{D}{s^2}$$

$$s+2 = As^2(s+3) + Bs^2(s+1) + Cs(s+1)(s+3) + D(s+1)(s+3)$$

if $s=-1$; $1 = A(-1)^2(-1+3)$, $1 = 2A$, $A = \frac{1}{2}$

if $s=-3$; $-1 = B(-3)^2(-3+1)$, $-1 = -18B$, $B = \frac{1}{18}$

if $s=0$; $2 = D(1)(3)$, $2 = 3D$, $D = \frac{2}{3}$

if $s=1$ & SUBSTITUTE A, B, D

$$3 = 4A + 2B + C(2)(4) + D(2)(4)$$

$$3 = 2 + \frac{1}{9} + 8C + \frac{16}{3} \rightarrow C = -\frac{5}{9}$$

* SUBSTITUTE A, B, C, D

$$\mathcal{L}^{-1}[F(s)] = 5\mathcal{L}^{-1}\left[\frac{\frac{1}{2}}{s+1} + \frac{\frac{1}{18}}{s+3} + \frac{-\frac{5}{9}}{s} + \frac{\frac{2}{3}}{s^2}\right]$$

$$f(t) = \tfrac{1}{2}\mathcal{L}^{-1}\left(\frac{1}{s+1}\right) + \tfrac{1}{18}\mathcal{L}^{-1}\left(\frac{1}{s+3}\right) - \tfrac{5}{9}\mathcal{L}^{-1}\left(\frac{1}{s}\right) + \tfrac{2}{3}\mathcal{L}^{-1}\left(\frac{1}{s^2}\right)$$

$$f(t) = 5\left(\frac{1}{2}e^{-t} + \frac{1}{18}e^{-3t} - \frac{5}{9} + \frac{2}{3}t\right)$$

$$\boxed{f(t) = \frac{10}{3}t + \frac{5}{2}e^{-t} + \frac{5}{18}e^{-3t} - \frac{25}{9}}$$

3. $F(s) = \dfrac{s^4+2s^3+3s^2+4s+5}{s(s+1)} = \dfrac{s^4+2s^3+3s^2+4s+5}{s^2+s}$

$$
\begin{array}{r|l}
 & s^2 + s + 2 \\
\cline{2-2}
s^2 + s & s^4 + 2s^3 + 3s^2 + 4s + 5 \\
 & \underline{s^4 + s^3} \\
 & \quad s^3 + 3s^2 \\
 & \quad \underline{s^3 + s^2} \\
 & \qquad 2s^2 + 4s \\
 & \qquad \underline{2s^2 + 2s} \\
 & \qquad\qquad 2s + 5
\end{array}
$$

$$\mathcal{L}^{-1}[F(s)] = \mathcal{L}^{-1}\left[s^2 + s + 2 + \frac{2s+5}{s^2+s}\right]$$

$$\frac{d^2f}{dt^2} + \frac{df}{dt} + 2\delta(t) + \mathcal{L}^{-}\left[\frac{2s+5}{s(s+1)}\right]$$

$$\mathcal{L}^{-1}\left[\frac{(2s+2)+3}{s(s+1)}\right] = \mathcal{L}^{-1}\left(\frac{2s+2}{s(s+1)}\right) + 3\mathcal{L}^{-1}\left[\frac{1}{s(s+1)}\right]$$

$$2\mathcal{L}^{-1}\left[\frac{\cancel{s+1}}{s\cancel{(s+1)}}\right] + 3\mathcal{L}^{-1}\left[\frac{1}{s(s+1)}\right]$$

$$\frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$$

$$1 = A(s+1) + Bs$$

if $s = 0$; $1 = A$ if $s = -1$; $1 = -B$

$B = -1$

* SUBSTITUTE A , B

$$3\mathcal{L}^{-1}\left[\frac{1}{s} - \frac{1}{s+1}\right] = 3(1 - e^{-t}) = 3 - 3e^{-t}$$

$$\boxed{f(t) = \frac{d^2f}{dt^2} + \frac{df}{dt} + 2\delta(t) + 5 - 3e^{-t}}$$