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ASSIGNMENT 1

I. SOLVE FOR THE LAPLACE TRANSFORM OF THE FOLLOWING:

1.) $F(s) = \mathcal{L}\left[3 - e^{-3t} + 5\sin 2t\right]$

$F(s) = 3\mathcal{L}(1) - \mathcal{L}(e^{-3t}) + 5\mathcal{L}(\sin 2t)$

$F(s) = \frac{3}{s} - \frac{1}{s+3} + \frac{5(2)}{s^2+4}$

$$\boxed{F(s) = \frac{3}{s} - \frac{1}{s+3} + \frac{10}{s^2+4}}$$

2.) $F(s) = \mathcal{L}\left[3 + 12t + 42t^3 - 3e^{2t}\right]$

$F(s) = 3\mathcal{L}(1) + 12\mathcal{L}(t) + 42\mathcal{L}(t^3) - 3\mathcal{L}(e^{2t})$

$F(s) = \frac{3}{s} + \frac{12}{s^2} + \frac{42(3!)}{s^{3+1}} - \frac{3}{s-2}$

$$\boxed{F(s) = \frac{3}{s} + \frac{12}{s^2} + \frac{252}{s^4} - \frac{3}{s-2}}$$

3.) $F(s) = \mathcal{L}\left[(t+1)(t+2)\right]$

$F(s) = \mathcal{L}\left[t^2 + 3t + 2\right]$

$F(s) = \mathcal{L}(t^2) + 3\mathcal{L}(t) + 2\mathcal{L}(1)$

$F(s) = \frac{2!}{s^{2+1}} + \frac{3}{s^2} + \frac{2}{s}$

$$\boxed{F(s) = \frac{2}{s^3} + \frac{3}{s^2} + \frac{2}{s}}$$

II. SOLVE FOR THE INVERSE LAPLACE TRANSFORM OF THE FOLLOWING:

1.) $\mathcal{L}^{-1}\left[\frac{8 - 3s + s^2}{s^3}\right] = F(t)$

$f(t) = 8\mathcal{L}^{-1}\left(\frac{1}{s^3}\right) - 3\mathcal{L}^{-1}\left(\frac{s}{s^3}\right) + \mathcal{L}^{-1}\left(\frac{s^2}{s^3}\right)$

$$f(t) = \frac{8}{2!}\mathcal{L}^{-1}\left(\frac{2!}{s^{2+1}}\right) - 3t + 1$$

$$\boxed{f(t) = 4t^2 - 3t + 1}$$

2.) $f(t) = \mathcal{L}^{-1}\left[\frac{5}{s-2} - \frac{4s}{s^2+9}\right]$

$$f(t) = 5\mathcal{L}^{-1}\left(\frac{1}{s-2}\right) + 4\mathcal{L}^{-1}\left(\frac{s}{s^2+9}\right)$$

$$\boxed{f(t) = 5e^{2t} - 4\cos 3t}$$

3.) $f(t) = \mathcal{L}^{-1}\left[\frac{7}{s^2+6}\right]$

$$f(t) = \frac{7}{\sqrt{6}}\mathcal{L}^{-1}\left[\frac{\sqrt{6}}{s^2+\sqrt{6}^2}\right]$$

$$f(t) = \frac{7}{\sqrt{6}}\sin\sqrt{6} + \frac{\sqrt{6}}{\sqrt{6}}$$

$$\boxed{f(t) = \frac{7\sqrt{6}}{6}\sin\sqrt{6}\,t}$$

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ASSIGNMENT 2

1.) $F(s) = \dfrac{1}{s(s^2+2s+2)}$

$$\left[\frac{1}{s(s^2+2s+2)}\right] = \frac{A}{s} + \frac{Bs+C}{s^2+2s+2}$$

$$1 = A(s^2+2s+2) + s(Bs+C)$$

$$1 = A(s^2+2s+2) + Bs^2 + Cs$$

IF $s=0$; $1 = 2A$
$A = \frac{1}{2}$

SUBSTITUTING A:

$$\left[1 = \frac{s^2+2s+2}{2} + Bs^2 + Cs\right]2$$

$$2 = s^2+2s+2 + 2Bs^2 + 2Cs$$

$$2 = s^2(2B+1) + s(2C+2) + 2$$

$2B+1$ $2C+2$
$2B = -1$ $2C = -2$
$B = -\frac{1}{2}$ $C = -1$

$$\therefore \mathcal{L}^{-1} \frac{-\frac{1}{2}}{s} - \frac{\frac{1}{2}s+1}{s^2+2s+2}$$

$$\frac{1}{2}\mathcal{L}^{-1}\frac{1}{s} - \frac{1}{2}\mathcal{L}^{-1}\frac{s+2}{s^2+2s+2} \rightarrow \frac{1}{2}\mathcal{L}^{-1}\frac{1}{s} - \frac{1}{2}\mathcal{L}^{-1}\frac{s+2}{s^2+2s+1+1}$$

$$\frac{1}{2}\mathcal{L}^{-1}\frac{1}{s} - \frac{1}{2}\mathcal{L}^{-1}\frac{(s+1)+1}{(s+1)^2+1}$$ WHERE $a = 1$, $\omega = 1$

$$f(t) = \frac{1}{2} - \frac{1}{2}e^{-t} \cdot \cos t + \sin t$$

$$\boxed{f(t) = \frac{1}{2} - \frac{1}{2}e^{-t}(\cos t + \sin t)}$$

2.) $F(s) = \dfrac{5(s+2)}{s^2(s+1)(s+3)}$

$$5\mathcal{L}^{-1} \frac{s+2}{s^2(s+1)(s+3)}$$

$$\left[\frac{s+2}{s^2(s+1)(s+3)} = \frac{A}{s+1} + \frac{B}{s+3} + \frac{C}{s} + \frac{D}{s^2}\right] s^2(s+1)(s+3)$$

$$s+2 = As^3(s+3) + Bs^2(s+1) + Cs(s+1)(s+3) + D(s+1)(s+3)$$

IF $s=-3$

$-1 = B(-3)^2(-3+1)$

$-1 = 9B(-2)$

$-1 = -18B$

IF $s=-1$

$1 = A(-1)^2(-1+3)$

$1 = 2A$

$A = \dfrac{1}{2}$

IF $s=0$

$2 = D(0+1)(0+3)$

$2 = 3D$

$D = \dfrac{2}{3}$

IF $s = 1$

$3 = 4A + 2B + C(2)(4) + D(2)(4)$

$3 = 2 + \dfrac{1}{9} + 8C + \dfrac{16}{3}$

$C = -\dfrac{5}{9}$

$$5\left[\mathcal{L}^{-1}\left(\frac{1/2}{s+1} + \frac{1/18}{s+3} + \frac{-5/9}{s} + \frac{2/3}{s^2}\right)\right]$$

$$5\left[\frac{1}{2}\mathcal{L}^{-1}\frac{1}{s+1} + \frac{1}{18}\mathcal{L}^{-1}\frac{1}{s+3} - \frac{5}{9}\mathcal{L}^{-1}\frac{1}{s} + \frac{2}{3}\mathcal{L}^{-1}\frac{1}{s^2}\right]$$

WHERE $a=1$ AND $a=3$

$$5\left(\frac{1}{2}e^{-t} + \frac{1}{18}e^{-3t} - \frac{5}{9} + \frac{2}{3}t\right)$$

$$\boxed{f(t) = \frac{10}{3}t - \frac{25}{9} + \frac{5}{2}e^{-t} + \frac{5}{18}e^{-3t}}$$

3.) $F(s) = \dfrac{s^4+2s^3+3s^2+4s+5}{s(s+1)}$

$$\mathcal{L}^{-1}\; s^2+s+s+2+\frac{2s+5}{s(s+1)}$$

$$\begin{array}{r|l} & s^2+s+2 \\ \hline s^2+s & s^4+2s^3+3s^2+4s+5 \\ & \underline{s^4+s^3} \\ & s^3+3s^2 \\ & \underline{s^3+s^2} \\ & 2s^2+4s \\ & \underline{2s^2+4s} \\ & 2s+5 \end{array}$$

$$\frac{d^2f}{dt^2}+\frac{df}{df}+2\delta(t) \longrightarrow \text{EQ1}$$

$$\mathcal{L}^{-1}\;\frac{2s+5}{s^2+5} \longrightarrow \mathcal{L}^{-1}\;\frac{2s+5}{s(s+1)}$$

$$\mathcal{L}^{-1}\;\frac{(2s+2)+3}{s(s+1)}$$

$$2\mathcal{L}^{-1}\;\frac{(s+1)}{s(s+1)} + 3\mathcal{L}^{-1}\;\frac{1}{s(s+1)}$$

$$2\mathcal{L}^{-1}\;\frac{1}{s} + 3\mathcal{L}^{-1}\;\frac{1}{s(s+1)}$$

$$\left[\frac{1}{s(s+1)} = \frac{A}{s}+\frac{B}{s+1}\right] s(s+1)$$

$$1 = A(s+1)+Bs$$

IF $s=-1$
$1=-1B$
$B=-1$

IF $s=0$
$1=1A$
$A=1$

$$3\left[\mathcal{L}^{-1}\;\frac{1}{s} - \mathcal{L}^{-1}\;\frac{1}{s+1}\right] \quad \text{WHERE } a=1$$

$$3-3e^{-t} \longrightarrow 2+3-3e^{-t}$$

$$5-3e^{-t} \longrightarrow \text{EQ 2}$$

SUBSTITUTING EQUATION 1 AND 2 :

$$\boxed{f(t) = \frac{d^2f}{dt^2}+\frac{df}{dt}+2\delta(t)+5-3e^{-t}}$$