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ME- 4203

ACTIVITY 1

I. Solve for the Laplace transform of the following:

1. $t[3-e^{-3t}+5\sin 2t]=F(s)$

a. $3\mathcal{L}\{1\}=3[\frac{1}{s}]=3/s$

b. $-\mathcal{L}\{e^{-3t}\}$; $a=3$ $=-\frac{1}{s+3}$

c. $5\mathcal{L}\{\sin 2t\}$; $w=2$ $=5[\frac{2}{s^2+2^2}]=\frac{10}{s^2+4}$

$\boxed{F(s)=\frac{3}{s}-\frac{1}{s+3}+\frac{10}{s^2+4}}$

2. $t[3+12t+42t^3-3e^{2t}]=F(s)$

a. $3\mathcal{L}\{1\}=3[\frac{1}{s}]$

$=\frac{3}{s}$

b. $12\mathcal{L}\{t\}=12[\frac{1}{s^2}]$

$=\frac{12}{s^2}$

c. $42\mathcal{L}\{t^3\}=42[\frac{3!}{s^{3+1}}]$

$=42[\frac{6}{s^4}]$

$=252/s^4$

d. $-3\mathcal{L}\{e^{2t}\}$; $a=2$

$=-3[\frac{1}{s-2}]$

$=\frac{-3}{s-2}$

$\boxed{F(s)=\frac{3}{s}+\frac{12}{s^2}+\frac{252}{s^4}-\frac{3}{s-2}}$

3. $t[(t+1)(t+2)]=F(s)$

$\mathcal{L}\{(t^2+2t+t+2)\}=F(s)$

$\mathcal{L}\{(t^2+3t+2)\}=F(s)$

a. $\mathcal{L}\{t^2\}$; $n=2$

$=\frac{2!}{s^{2+1}}$

$=\frac{2}{s^3}$

b. $3\mathcal{L}\{t\}=3[\frac{1}{s^2}]$

$=\frac{3}{s^2}$

c. $2\mathcal{L}\{1\}=2[\frac{1}{s}]$

$=\frac{2}{s}$

$\boxed{F(s)=\frac{2}{s^3}+\frac{3}{s^2}+\frac{2}{s}}$

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ASSIGNMENT 1

II. Solve for the Inverse Laplace Transform of the following

1. $\mathcal{L}^{-1}\left[\frac{8-3s+s^2}{s^3}\right]=f(t)$

$\mathcal{L}^{-1}\left\{\frac{8}{s^3}-\frac{3s}{s^3}+\frac{s^2}{s^3}\right\}=f(t)$

a. $8\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\};\ n=2 = 8\mathcal{L}^{-1}\left\{\frac{2!/2!}{s^2+1}\right\}=\frac{8}{2!}\mathcal{L}^{-1}\left\{\frac{2!}{s^{2+1}}\right\}=\frac{8}{2}t^2=4t^2$

b. $-3\mathcal{L}^{-1}\left\{\frac{s}{s^3}\right\}=-3\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}=-3t$

c. $\mathcal{L}^{-1}\left\{\frac{s^2}{s^3}\right\}=\mathcal{L}^{-1}\left\{\frac{1}{s}\right\}=1$

$\boxed{f(t)=4t^2-3t+1}$

2. $\mathcal{L}^{-1}\left[\frac{5}{s-2}-\frac{4s}{s^2+9}\right]=f(t)$

a. $5\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\};\ a=-2 = 5\left[e^{2t}\right]=5e^{2t}$

b. $-4\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\}=-4\mathcal{L}^{-1}\left\{\frac{s}{s^2+3^2}\right\};\ w=3 = -4\cos 3t$

$\boxed{f(t)=5e^{2t}-4\cos 3t}$

3. $\mathcal{L}^{-1}\left[\frac{7}{s^2+6}\right]=f(t)$

$=7\mathcal{L}^{-1}\left\{\frac{1}{s^2+6}\right\}=7\mathcal{L}^{-1}\left\{\frac{\sqrt{6}/\sqrt{6}}{s^2+\sqrt{6}^2}\right\}$

$=\frac{7}{\sqrt{6}}\mathcal{L}^{-1}\left\{\frac{\sqrt{6}}{s^2+(\sqrt{6})^2}\right\};\ w=\sqrt{6}$

$=\frac{7}{\sqrt{6}}\left[\sin\sqrt{6}t\right]\cdot\frac{\sqrt{6}}{\sqrt{6}}=\frac{7\sqrt{6}}{6}\sin 6t$

$\boxed{f(t)=\frac{7\sqrt{6}}{6}\sin 6t}$

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ASSIGNMENT 2

1. $F(s) = \frac{1}{s(s^2+2s+2)}$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s(s^2+2s+2)}\right\}$$

$$= \left[\frac{1}{s(s^2+2s+2)} = \frac{A}{s} + \frac{Bs+C}{s^2+2s+2}\right] s(s^2+2s+2)$$

$$1 = A(s^2+2s+2) + (Bs+C)(s)$$

If $s=0$, then

$$1 = A(0^2+2(0)+2) + (B(0)+C)(0)$$

$$\frac{1}{2} = \frac{\cancel{2}A}{\cancel{2}} \; ; \; \boxed{A = \frac{1}{2}}$$

$$\cancel{2}\left[1 = \frac{1}{\cancel{2}}(s^2+2s+2) + s(Bs+C)\right]\cancel{2}$$

$$2 = s^2 + 2s + 2 + 2Bs^2 + 2Cs$$

$$2 = s^2(1+2B) + 2s(1+C) + 2$$

$1+2B = 0$ $\quad$ $1+C=0$

$\frac{\cancel{2}B}{\cancel{2}} = \frac{-1}{2}$ $\quad$ $\boxed{C=-1}$

$\boxed{B = -\frac{1}{2}}$

$$\mathcal{L}^{-1}\left\{\frac{1}{2}/s + \frac{-1/2\,s-1}{s^2+2s+2}\right\} = \mathcal{L}^{-1}\left\{{}^{1/2}/s - \frac{1/2\,s+1}{s^2+2s+2}\right\}$$

a. $\frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = \frac{1}{2}\cdot 1 = \frac{1}{2}$

b. $-\mathcal{L}^{-1}\left\{\frac{1/2\,s+1}{s^2+2s+2}\right\}$; $1 = \frac{2}{2}$ $= -\mathcal{L}^{-1}\left\{\frac{\frac{1}{2}s+2(\frac{1}{2})}{s^2+2s+2}\right\}$

$$= -\frac{1}{2}\mathcal{L}^{-1}\left\{\frac{s+2}{s^2+2s+2}\right\} ; \quad \frac{(s+a)+w}{(s+a)^2+w^2}$$

$$= -\frac{1}{2}\mathcal{L}^{-1}\left\{\frac{(s+1)+1}{(s^2+2s+2)+1}\right\} = -\frac{1}{2}\mathcal{L}^{-1}\left\{\frac{(s+1)+1}{(s+1)^2+1^2}\right\}$$

=

$a=1, \; w=1$

$$f(t) = \frac{1}{2} - \frac{1}{2}\left[e^{-t}\cos t + \sin t\right]$$

$$\boxed{f(t) = \frac{1}{2}\left(1 - e^{-t}\left[\cos t + \sin t\right]\right)}$$

2. $F(s)=\frac{5(s+2)}{s^2(s+1)(s+3)}$

$=5\mathcal{L}^{-1}\left[\left\{\frac{s+2}{s^2(s+1)(s+3)}\right\}=\frac{A}{3}+\frac{B}{s^2}+\frac{C}{s+1}+\frac{D}{s+3}\right]s^2(s+1)(s+3)$

$s+2=As(s+1)(s+3)+B(s+1)(s+3)+C(s^2)(s+3)+D(s^2)(s+1)$

If $s=0$

$0+2=\cancel{A(0)(0+1)(0+3)}+B(0+1)(0+3)+\cancel{C(0^2)(0+3)}+\cancel{D(0^2)(0+1)}$

$\frac{2}{3}=\frac{\cancel{3}B}{\cancel{3}}$; $B=2/3$

If $s=-1$

$-1+2=\cancel{A(-1)(-1+1)(-1+3)}+\cancel{B(-1+1)(-1+3)}+C(-1^2)(-1+3)+\cancel{D(-1^2)(-1+1)}$

$\frac{1}{2}=\frac{\cancel{2}C}{\cancel{2}}$; $C=1/2$

If $s=-3$

$-3+2=\cancel{A(-3)(-3+1)(-3+3)}+\cancel{B(-3+1)(-3+3)}+\cancel{C(-3^2)(-3+3)}+D(-3^2)(-3+1)$

$\frac{-1}{8}=\frac{-18D}{-18}$; $D=\frac{-1}{8}$

$s+2=A(s^3+4s^2+3s)+B(s^2+4s+3)+C(s^3+3s^2)+D(s^3+s^2)$

SUBSTITUTE B,C,D

$s+2=A(s^3+4s^2+3s)+2/3(s^2+4s+3)+1/2(s^3+3s^2)+1/18(s^3+s^2)$

$s+2=As^3+4As^2+3As+2/3s^2+8/3s+2+1/2s^3+3/2s^2+1/18s^3+1/18s^2$

$s+2=(As^3+1/2s^3+1/18s^3)+(4As^2+2/3s^2+3/2s^2+1/18s^2)+(3As+8/3s)+2$

$s+2=s^3(A+5/9)+s^2(4A+20/9)+s(3A+8/3)+2$

$s+2-2=s=s^3(A+5/9)+s^2(4A+20/9)+3(3A+8/3)$

$\frac{s}{s}=\frac{s(3A+8/3)}{s}$; $1=3A+8/3$; $1-8/3=3A$; $A=\frac{-5/3}{3}\rightarrow -5/9$

$\frac{-5/3}{3}=\frac{\cancel{3}A}{\cancel{3}}$

$5\mathcal{L}^{-1}\left\{\frac{-5/9}{3}+\frac{2/3}{s^2}+\frac{1/2}{s+1}+\frac{1/18}{s+3}\right\}$

a. $5\mathcal{L}^{-1}\left\{\frac{-5/9}{s}\right\}=5(-5/9)\mathcal{L}^{-1}\left\{\frac{1}{s}\right\}=5(-5/9)(1)=-25/9$

b. $5\mathcal{L}^{-1}\left\{\frac{2/3}{s^2}\right\}=5(2/3)\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}=5(2/3)t=\frac{10}{3}t$

c. $5\mathcal{L}^{-1}\left\{\frac{1/2}{s+1}\right\}=5(1/2)\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}$; $a=1=5(1/2)(e^{-t})=\frac{5}{2}e^{-t}$

d. $5\mathcal{L}^{-1}\left\{\frac{1/18}{s+3}\right\}=5(1/18)\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\}$; $a=3=5(1/18)(e^{-3t})=\frac{5}{18}e^{-3t}$

$\boxed{f(t)=\frac{-25}{9}+\frac{10}{3}t+\frac{5}{2}e^{-t}+\frac{5}{18}e^{-3t}}$

3. $F(s)=\frac{s^4+2s^3+3s^2+4s+5}{s(s+1)}$; $s(s+1)=s^2+3$

$$\begin{array}{r} s^2+s+2 \\ s^2+s\,\overline{)\,s^4+2s^3+3s^2+4s+5} \\ \underline{s^4+s^3} \\ s^3+3s^2 \\ \underline{s^3+s^2} \\ 2s^2+4s \\ \underline{2s^2+2s} \\ 2s+5 \end{array}$$

$F(s)=\mathcal{L}^{-1}\{s^2+s+2+\left(\frac{2s+5}{s(s+1)}\right)\}$

a. $\mathcal{L}^{-1}\{s^2\}=f''(t)$

b. $\mathcal{L}^{-1}\{s\}=f'(t)$

c. $2\mathcal{L}^{-1}\{1\}=2\delta$

d. $\mathcal{L}^{-1}\{\frac{2s+5}{s(s+1)}\}=F(s)$

$\left[\frac{2s+5}{s(s+1)}=\frac{A}{s}+\frac{B}{s+1}\right]s(s+1)$

$2s+5=A(s+1)+B(s)$

If $s=-1$

$2(-1)+5=\cancel{A(-1+1)}+B(-1)$

$\frac{3}{-1}=\frac{\cancel{-1}B}{\cancel{-1}}$; $B=-3$

If $=s=0$

$\cancel{2(0)}+5=A(0+1)+\cancel{B(0)}$

$5\quad A$

then

$\mathcal{L}^{-1}\{\frac{5}{s}+\frac{-3}{s+1}\}$

a. $5\mathcal{L}^{-1}\{\frac{1}{s}\}=5(1)=5$

b. $-3\{\frac{1}{s+1}\}$; $a=1=-3(e^{-t})=-3e^{-t}$

$f(t)=5-3e^{-t}$

$$\boxed{f(t)=f''(t)+f'(t)+2\delta+5-3e^{-t}}$$