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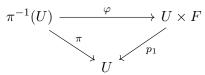
Principal Bundles

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Why Fiber Bundles?

Suppose we want to study some space M, which for our purposes is a smooth manifold. One way to study M is to study functions $M \to F$ for some fixed target space F, e.g. a manifold, \mathbb{R} , or a vector space. This is a perfectly good method of studying M, but is sometimes not enough. More often then not, we want to study "function" from M into a vector space that varies over the base manifold M. For example, a vector field $X \in \mathfrak{X}(M)$ is not really a map $M \to \mathbb{R}^n$, it is an assignment to each $p \in M$ a vector in T_pM . In this way, we are led to the study of a smoothly parameterized family of vector spaces – the tangent bundle TM. This leads us to define a fiber bundle.

Definition 0.1. Let M and F be a smooth manifolds. A **fiber bundle** over M with model fiber F is the data of a smooth manifold E with a smooth surjective map $\pi: E \to M$ such that for each $p \in M$, there exists an open set U and a diffeomorphism $\varphi: \pi^{-1}(U) \to U \times F$ such that



where $p_1: U \times F \to U$ is projection onto the first factor.