# SPIN GEOMETRY CONFERENCE COURSE

JEFFREY JIANG

## Week 1

**Exercise 1.1.** Prove $SL_n(\mathbb{R})$ and $O(n)$ are manifolds

**Exercise 1.2.** What is the "shape" of $SL_2(\mathbb{R})$?

**Exercise 1.3.** Prove that
$$O(2) = \left\{ \begin{pmatrix} \cos\theta - \sin\theta \\ \sin\theta \cos\theta \end{pmatrix} \right\} \cup \left\{ \begin{pmatrix} \cos\theta \sin\theta \\ \sin\theta - \cos\theta \end{pmatrix} \right\}$$
The first set consists of rotations and the second set consists of reflections. Which rotations commute? Which reflections commute? Do reflections commute with reflections?

**Exercise 1.4.** Investigate $O(3)$. What is it's "shape?"

## Week 2

**Exercise 2.1.** What is the derivative of det?

**Exercise 2.2.** Explore the exponetial map $\mathfrak{sl}_2(\mathbb{R}) \to SL_2(\mathbb{R})$

**Exercise 2.3.** Prove that every element of $O(n)$ can be written as the composition of at most $n$ reflections about hyperplanes in $\mathbb{R}^n$.

TODO insert motivation of $A_n^{\pm}$

**Definition 2.4.** Define $A_n^{\pm}$ to be the unital algebra generated by $\mathbb{R}^n$ such that $\xi^2 = \pm 1$. and $\xi\eta =?\ \eta\xi$. Determine the sign of $\eta\xi$. Explore these algebras. Find $A\pm_1$, $A_2^{\pm}$ .... What are they isomorphic to? Can you identify $O(n)$ as a subgroup?