$\varphi \in \Omega^0_M(E)$.

$$d(\omega \wedge \eta) = d\omega \wedge \eta + (-1)^k \omega \wedge d\eta$$

$$d_\Theta \varphi = d\varphi + \dot{\rho}(\Theta)\varphi.$$

$$d^2_\Theta \varphi = d_\Theta\left(d\varphi + \dot{\rho}(\Theta)\varphi\right)$$

$$= d_\Theta(d\varphi) + d_\Theta\left(\dot{\rho}(\Theta)\varphi\right)$$

$$= \left(\underset{\overset{\shortparallel}{0}.}{d^2\varphi} + \dot{\rho}(\Theta)\wedge d\varphi\right) + \left(d\left(\dot{\rho}(\Theta)\varphi\right) + \dot{\rho}(\Theta)\wedge\left(\dot{\rho}(\Theta)\varphi\right)\right)$$

Components of $\dot{\rho}(\Theta)\wedge d\varphi$ are $\dot{\rho}(\Theta)^i_j \wedge d\varphi^j$

Components of $d(\dot{\rho}(\Theta)\varphi)$ are

$$d\left(\dot{\rho}(\Theta)\varphi\right)^i = d\left(\dot{\rho}(\Theta)^i_j\, \varphi^j\right)$$

$$= d\left(\dot{\rho}(\Theta)^i_j\right)\varphi^j - \dot{\rho}(\Theta)^i_j \wedge d\varphi^j$$

Components of $\dot{\rho}(\Theta) \wedge (\dot{\rho}(\Theta)\varphi)$ are

$$\dot{\rho}(\Theta)^i_j \wedge \dot{\rho}(\Theta)^j_k \varphi^k$$

So the i^{th} component of $d_\Theta^2 \varphi$ is

$$d\left(\dot{\rho}(\Theta)^i_j\right)\varphi^j + \dot{\rho}(\Theta)^i_j \wedge \dot{\rho}(\Theta)^j_k \varphi^k$$

Want $d_\Theta^2 \varphi = \varphi \dot{\rho}(\Omega)$

$$\dot{\rho}(\Omega) = \dot{\rho}\left(d\Theta + \tfrac{1}{2}[\Theta \wedge \Theta]\right)$$

$$= \dot{\rho}(d\Theta) + \tfrac{1}{2}\dot{\rho}[\Theta \wedge \Theta]$$

$$\dot{\rho}(\Theta) \wedge \dot{\rho}(\Theta) \overset{?}{=} \tfrac{1}{2}\dot{\rho}[\Theta \wedge \Theta]$$

For $X, Y \in \mathfrak{g}$,

$$\left(\dot{\rho}(\Theta) \wedge \dot{\rho}(\Theta)\right)^i_j = \dot{\rho}(\Theta)^i_k \wedge \dot{\rho}(\Theta)^k_j$$

$$\dot{\rho}[\Theta \wedge \Theta] = [\dot{\rho}(\Theta) \wedge \dot{\rho}(\Theta)]$$

$$[\dot{\rho}(\Theta) \wedge \dot{\rho}(\Theta)](X, Y)$$

$$= [\dot{\rho}(\Theta)(X), \dot{\rho}(\Theta)(Y)]$$

$$= \dot{\rho}(X)\dot{\rho}(Y) - \dot{\rho}(Y)\dot{\rho}(X).$$

$$\dot{\rho}(\Theta) = \begin{pmatrix} \omega^1_1 & \omega^1_2 \\ \omega^2_1 & \omega^2_2 \end{pmatrix}^2 = \begin{pmatrix} \omega^1_2 \wedge \omega^2_1 & \omega^1_1 \wedge \omega^1_2 + \omega^1_2 \wedge \omega^2_2 \\ \omega^2_1 \wedge \omega^1_1 + \omega^2_2 \wedge \omega^2_1 & \omega^2_1 \wedge \omega^1_2 \end{pmatrix}$$

"$\dot{\rho}(\Theta) \wedge \dot{\rho}(\Theta)$"

Evaluating on X, Y, we have that this is equal to

$$\begin{pmatrix} (\omega^1_2 \wedge \omega^2_1)(X, Y) & (\omega^1_1 \wedge \omega^1_2)(X, Y) + (\omega^1_2 \wedge \omega^2_2)(X, Y) \\ (\omega^2_1 \wedge \omega^1_1)(X, Y) + (\omega^2_2 \wedge \omega^2_1)(X, Y) & (\omega^2_1 \wedge \omega^1_2)(X, Y \end{pmatrix}$$

$$= \begin{pmatrix} \omega_2^1(X)\,\omega_1^2(Y) - \omega_1^2(X)\,\omega_2^1(Y) & \begin{array}{l} \omega_1^1(X)\,\omega_2^1(Y) - \omega_2^1(X)\,\omega_1^1(Y) \\ + \omega_1^1(X)\,\omega_2^2(Y) - \omega_2^2(X)\,\omega_2^1(Y) \end{array} \\ \\ \begin{array}{l} \omega_1^2(X)\,\omega_1^1(Y) - \omega_1^1(X)\,\omega_1^2(Y) \\ + \omega_2^2(X)\,\omega_1^2(Y) - \omega_1^2(Y)\,\omega_2^2(X) \end{array} & \omega_1^2(X)\,\omega_2^1(Y) - \omega_2^1(X)\,\omega_1^2(Y) \end{pmatrix}$$

$[\dot\rho(\Theta) \wedge \dot\rho(\Theta)]$? Action on X, Y is

$$\dot\rho(\Theta(X))\,\dot\rho(\Theta(Y)) - \dot\rho(\Theta(Y))\,\dot\rho(\Theta(X))$$

$$=$$

$$\begin{pmatrix} \omega_1^1(X) & \omega_2^1(X) \\ \omega_1^2(X) & \omega_2^2(X) \end{pmatrix} \begin{pmatrix} \omega_1^1(Y) & \omega_2^1(Y) \\ \omega_1^2(Y) & \omega_2^2(Y) \end{pmatrix} - \begin{pmatrix} \omega_1^1(Y) & \omega_2^1(Y) \\ \omega_1^2(Y) & \omega_2^2(Y) \end{pmatrix} \begin{pmatrix} \cdots \\ \cdots \end{pmatrix}$$

$$=$$

$$\begin{pmatrix} \omega_1^1(X)\,\omega_1^1(Y) + \omega_2^1(X)\,\omega_1^2(Y) & \omega_1^1(X)\,\omega_2^1(Y) + \omega_2^1(X)\,\omega_2^2(Y) \\ \\ \omega_1^2(X)\,\omega_1^1(Y) + \omega_2^2(X)\,\omega_1^2(Y) & \omega_1^2(X)\,\omega_2^1(Y) + \omega_2^2(X)\,\omega_2^2(Y) \end{pmatrix}$$

$$
-\begin{pmatrix} \omega^1_1(y)\omega^1_1(x) + \omega^1_2(y)\omega^2_1(x) & \omega^1_1(y)\omega^1_2(x) + \omega^1_2(y)\omega^2_2(x) \\ \omega^2_1(y)\omega^1_1(x) + \omega^2_2(y)\omega^2_1(x) & \omega^2_1(y)\omega^1_2(x) + \omega^2_2(y)\omega^2_2(x) \end{pmatrix}
$$

$$
= \begin{pmatrix} \omega^1_2(x)\omega^2_1(y) - \omega^2_1(x)\omega^1_2(y) & \begin{matrix} \omega^1_1(x)\omega^1_2(y) + \omega^1_2(x)\omega^2_2(y) \\ - \omega^1_1(y)\omega^1_2(x) - \omega^1_2(y)\omega^2_2(x) \end{matrix} \\ \begin{matrix} \omega^2_1(x)\omega^1_1(y) + \omega^2_2(x)\omega^2_1(y) \\ - \omega^2_1(y)\omega^1_1(x) + \omega^2_2(y)\omega^2_1(x) \end{matrix} & \omega^2_1(x)\omega^1_2(y) - \omega^2_1(y)\omega^1_2(x) \end{pmatrix}
$$

VS.

$$
= \begin{pmatrix} \omega^1_2(x)\omega^2_1(y) - \omega^2_1(x)\omega^1_2(y) & \begin{matrix} \omega^1_1(x)\omega^1_2(y) - \omega^1_2(x)\omega^1_1(y) \\ + \omega^1_1(x)\omega^2_2(y) - \omega^2_2(x)\omega^1_2(y) \end{matrix} \\ \begin{matrix} \omega^2_1(x)\omega^1_1(y) - \omega^1_1(x)\omega^2_1(y) \\ + \omega^2_2(x)\omega^2_1(y) - \omega^2_1(y)\omega^2_2(x) \end{matrix} & \omega^2_1(x)\omega^1_2(y) - \omega^1_2(x)\omega^2_1(y) \end{pmatrix}
$$