Project 4: PMF with KNN and Kernel Ridge Post-processing

In this project, we explore probabilistic matrix factorization for recommender system. The goal is to match consumers with most appropriate products. We use two different methods for post-processing: KNN and Kernel Ridge Regression. We combine the prediction results from PMF and post-processing by using linear regression.

Step 1: Load Data and Train-test Split

Import modules and packages.

In [1]:
```
import pandas as pd
import numpy as np
from sklearn.model_selection import train_test_split
from sklearn.metrics import mean_squared_error
from math import sqrt
import seaborn as sns
import matplotlib.pylab as plt
import time
import PMF_model
%matplotlib inline
```

Load data and do train_test_split. Here we split again the train set into train set and validation set.

In [13]:
```
data = pd.read_csv('ratings.csv')

# train_test_split
train, test = train_test_split(data, test_size = 0.2, stratify = data['userId'
])

# Comment out the line below not to include validation set
train, validation = train_test_split(train, test_size = 0.1, stratify = train[
'userId'])

train_data = np.array(train.iloc[:,:-1])
test_data = np.array(test.iloc[:,:-1])

# Comment out the line below not to include validation set
validation_data = np.array(validation.iloc[:,:-1])

num_users = max(data['userId'].values)
num_items = max(data['movieId'].values)
```

Step 2: Matrix Factorization and Parameter Tuning

PMF from module PMF_model perform gradien descent to do probabilistic matrix factorization. The algorithm consider case that there are new users and movies adding to the dataset used to train. That is, the dimension of the matrix R (n*m), U (f*n), V (f*m) is dynamic.

Define function for grid search cross validation. The function takes in lists for sigma, sigma_u, sigma_v and latent_size each and return lists of sets of parameters and corresponding test_rmse.

In [14]:
```
# grid search for PMF
def GridSearch(sigma, sigma_u, sigma_v, latent_size):
    from sklearn.model_selection import KFold
    from statistics import mean
    params = []
    test_rmse = []

    # performing cv with k=3 for each set of parameters
    kf = KFold(n_splits=3, shuffle=True, random_state=1)
    for s in sigma:
        for su in sigma_u:
            for sv in sigma_v:
                for ls in latent_size:
                    print("Training with sigma={:f}, sigma_u={:f}, sigma_v={:f}, latent_size={:d}".format(s, su, sv, ls))
                    params.append([s, su, sv, ls])
                    model=PMF_model.PMF(m = num_items, n=num_users, sigma=s, sigma_u=su, sigma_v=sv, latent_size=ls)
                    rmse_list = []

                    # KFold
                    cnt=1
                    for train_index, test_index in kf.split(train_data):
                        print(" Training fold {:d}".format(cnt))
                        train_k = train_data[train_index, :]
                        test_k = train_data[test_index, :]
                        U, V, validation_rmse = model.fit(train_data=train_k, validation_data=validation_data)
                        preds = model.predict(data=test_k)

                        # get rmse for the given fold
                        rmse = sqrt(mean_squared_error(test_k[:, 2],preds))
                        rmse_list.append(rmse)
                        cnt+=1

                    # Get average rmse for all folds for the given set of parameters
                    test_rmse.append(mean(rmse_list))
                    print(" rmse:{:f}\n".format(mean(rmse_list)))
    return params, test_rmse
```

From testing different values, we observed that the sigmas' are the parameters that affects RMSE the most. So we tuned them first. To aim for lambdas 0.01, 0.001, and 0.0001, we did grid search with sigma fixed and only changed sigma_u and sigma_v. We repeated this process with different sigmas.

In [15]:
```
sigma=[0.001]
sigma_u=[0.1, 0.03, 0.01]
sigma_v=[0.1, 0.03, 0.01]
latent_size=[10]

params, test_rmse = GridSearch(sigma, sigma_u, sigma_v, latent_size)
best_idx = test_rmse.index(min(test_rmse))
print("Best parameters: {}".format(params[best_idx]))
print("Best rmse:{}".format(min(test_rmse)))
best_rmse = min(test_rmse)
best_params = params[best_idx]
```

In [16]:
```
sigma=[0.01]
sigma_u=[1.0, 0.3, 0.1]
sigma_v=[1.0, 0.3, 0.1]
latent_size=[10]

params, test_rmse = GridSearch(sigma, sigma_u, sigma_v, latent_size)
best_idx = test_rmse.index(min(test_rmse))
print("Best parameters: {}".format(params[best_idx]))
print("Best rmse:{}".format(min(test_rmse)))
if min(test_rmse)<best_rmse:
    best_rmse = min(test_rmse)
    best_params = params[best_idx]
```

In [17]:
```
sigma=[1.0]
sigma_u=[10.0, 3.0, 1.0]
sigma_v=[10.0, 3.0, 1.0]
latent_size=[10]

params, test_rmse = GridSearch(sigma, sigma_u, sigma_v, latent_size)
best_idx = test_rmse.index(min(test_rmse))
print("Best parameters: {}".format(params[best_idx]))
print("Best rmse:{}".format(min(test_rmse)))
if min(test_rmse)<best_rmse:
    best_rmse = min(test_rmse)
    best_params = params[best_idx]
```

From the above grid search, we concluded that smaller sigmas work better. Then we performed cross validation for a different latent size for the smaller sigmas.

In [18]:
```
sigma=[best_params[0]]
sigma_u=[best_params[1]]
sigma_v=[best_params[2]]
latent_size=[20]

params, test_rmse = GridSearch(sigma, sigma_u, sigma_v, latent_size)
best_idx = test_rmse.index(min(test_rmse))
if min(test_rmse)<best_rmse:
    best_rmse = min(test_rmse)
    best_params = params[best_idx]
print("Best parameters: {}".format(best_params))
print("Best rmse:{}".format(best_rmse))
```

Finally, train the model with the whole training data, using the best parameters(sigma=0.001, sigma_u=0.03, sigma_v=0.03, latent_size=10) found from grid search.

In [19]:
```
# construct model
sigma = [best_params[0]]
sigma_u = [best_params[1]]
sigma_v = [best_params[2]]
latent_size = [best_params[3]]

num_iter = 2000
model = PMF_model.PMF(m = num_items, n = num_users, sigma = sigma, sigma_u = s
igma_u, sigma_v = sigma_v, lr=0.001, latent_size=latent_size)
print('PMF(sigma={:f}, sigma_u={:f}, sigma_v={:f}, latent_size={:d})'.format(s
igma, sigma_u, sigma_v, latent_size))
print('\ntraining model...')
U, V, validation_rmse = model.fit(train_data=train_data, validation_data=valid
ation_data)

train_preds = model.predict(data=train_data)
train['pmf_preds'] = train_preds
train_rmse = sqrt(mean_squared_error(train_data[:, 2],train_preds))
print('train rmse:{:f}'.format(train_rmse))

print('\ntesting model...')
preds = model.predict(data=test_data)
test['pmf_preds']=preds
test_rmse = sqrt(mean_squared_error(test_data[:, 2],preds))
print('test rmse:{:f}'.format(test_rmse))

# np.save('U.npy', U)
# np.save('V.npy', V)
```

```
PMF(sigma=0.001000, sigma_u=0.030000, sigma_v=0.030000, latent_size=10)

training model...
training iteration: 0, loss: 484617.041984, validation_rmse: 3.659554)
training iteration: 100, loss: 21772.462191, validation_rmse: 1.162506
convergence at iterations: 139
train rmse:0.699220

testing model...
test rmse:1.153616
```

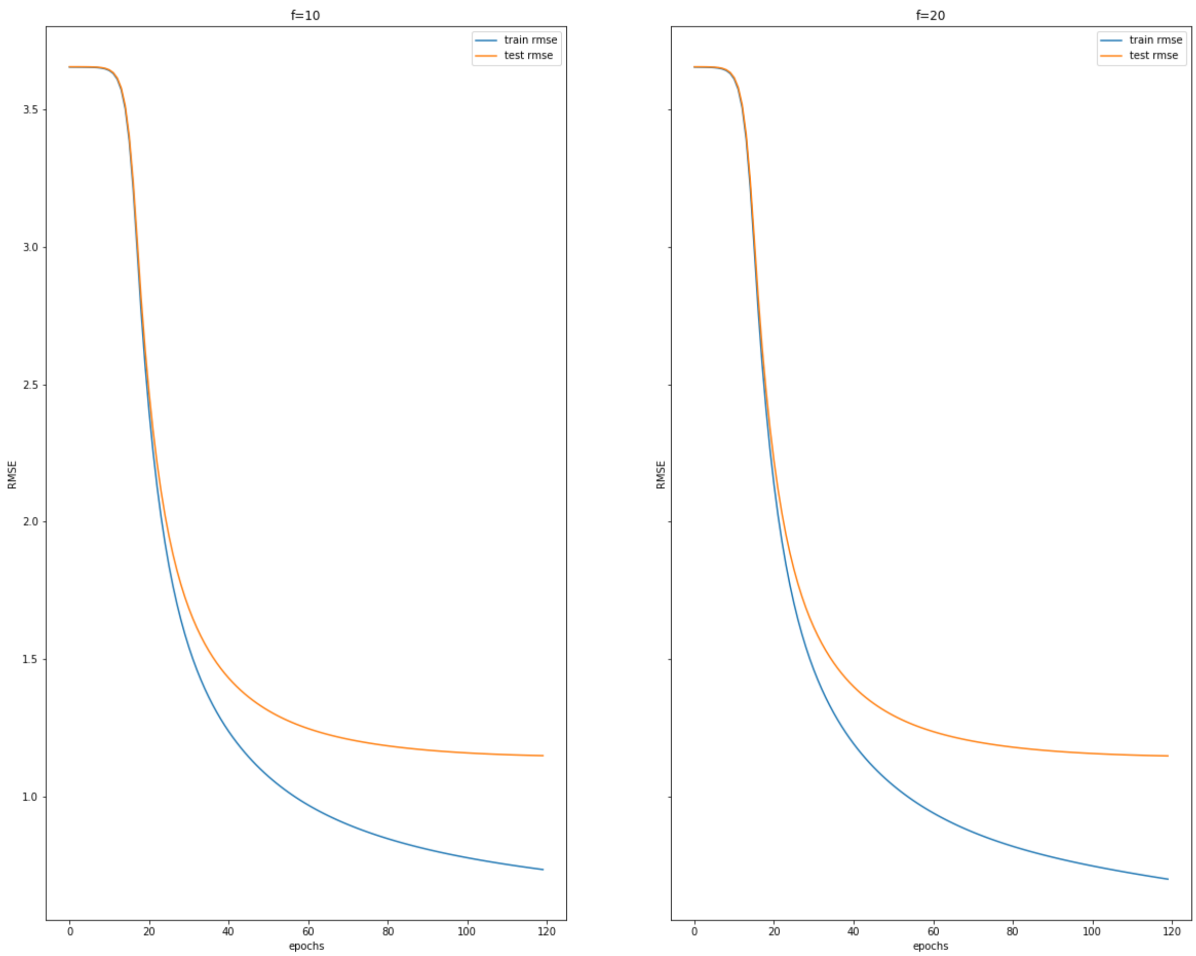
In [6]:
```
# U=np.load('U.npy')
# V=np.load('V.npy')
```

Training and testing RMSE by different dimensions of factors and epochs are visualized below. We can see that the gradient descent algorithm is working successfully as RMSE gets lower as we train more epochs. As mentioned above, the different number of features show only slight change in RMSE.

In [21]:
```
# Get RMSE
num_iter = 120
model_10 = PMF_model.PMF(m = num_items, n = num_users, sigma = sigma, sigma_u
= sigma_u, sigma_v = sigma_v, lr=0.001, latent_size=10)
_, _, train_rmse_10, test_rmse_10 = model_10.fit(train_data=train_data, test_d
ata = test_data)

model_20 = PMF_model.PMF(m = num_items, n = num_users, sigma = sigma, sigma_u
= sigma_u, sigma_v = sigma_v, lr=0.001, latent_size=20)
_, _, train_rmse_20, test_rmse_20 = model_20.fit(train_data=train_data, test_d
ata = test_data)

# np.save('train_rmse_10.npy', np.array(train_rmse_10))
# np.save('test_rmse_10.npy', np.array(test_rmse_10))
# np.save('train_rmse_20.npy', np.array(train_rmse_20))
# np.save('test_rmse_20.npy', np.array(test_rmse_20))
```

In [22]:
```
# train_rmse_10 = np.load('train_rmse_10.npy')
# test_rmse_10 = np.load('test_rmse_10.npy')
# train_rmse_20 = np.load('train_rmse_20.npy')
# test_rmse_20 = np.load('test_rmse_20.npy')
```

In [41]:
```
# Visualize RMSE
fig, ax = plt.subplots(1,2,figsize=(20,16), sharey=True)
sns.lineplot(x=range(120), y=train_rmse_10, ax=ax[0], label='train rmse')
sns.lineplot(x=range(120), y=test_rmse_10, ax=ax[0], label='test rmse')
ax[0].set_title('f=10')
ax[0].set_xlabel('epochs')
ax[0].set_ylabel('RMSE')
ax[0].legend()

sns.lineplot(x=range(120), y=train_rmse_20, ax=ax[1], label='train rmse')
sns.lineplot(x=range(120), y=test_rmse_20, ax=ax[1], label='test rmse')
ax[1].set_title('f=20')
ax[1].set_xlabel('epochs')
ax[1].set_ylabel('RMSE')
ax[1].legend();
```



Step 3: Post-processing

Postporcessing is performed to improve accuracy.

1) KNN with K=1

We define similarity between movies i1 and i2 as the cosine similarity between vectors V_i1 and V_i2. Then we use the average rating of most similar movie as prediction. If there is no ratings for the closest movie, we set the prediction to 0.

In [8]:
```
# Post processing using K Nearest Neighbors with K=1
from sklearn.neighbors import NearestNeighbors
start = time.time()
model_knn = NearestNeighbors(metric='cosine', algorithm='brute', n_neighbors=1
, n_jobs=-1)
model_knn.fit(V.T)
dist, ind = model_knn.kneighbors(V.T,2)
nearest_movie = ind[:,1]+1
end = time.time()

knn_train_time = end-start
```

In [25]:
```
# Average rating of the nearest neighbor (0 if no ratings)
movie_rates = data.groupby(['movieId'])['rating'].mean()
start = time.time()
test_nn = nearest_movie[test['movieId']-1]
test['knn_preds'] = list(movie_rates[test_nn].fillna(0))
end = time.time()

train_nn = nearest_movie[train['movieId']-1]
train['knn_preds'] = list(movie_rates[train_nn].fillna(0))

knn_test_time = end-start
```

In [9]:
```
# compute test and train RMSE of KNN prediction
knn_test_rmse = sqrt(np.array(mean_squared_error(test['rating'],test['knn_pred
s'])))
knn_train_rmse = sqrt(np.array(mean_squared_error(train['rating'],train['knn_p
reds'])))
```

2) Kernel Ridge Regression

Define y as user specific ratings. X consists of normalized vector of factors for movies rated by the user in each row. Using y and X, we solve Kernel Ridge Regression.

In [27]:
```
# Post processing using Kernel Ridge Regression
from sklearn.kernel_ridge import KernelRidge
from sklearn.preprocessing import normalize
# For user specific ratings
most_rated = data.groupby(['movieId']).count().sort_values(by = 'rating', asce
nding=False)
krr_test_time = 0
krr_train_time = 0

for userId in range(1,num_users+1):
    # get user specific data
    start = time.time()
    user_spec = train.loc[train['userId'] == userId]
    end = time.time()
    krr_train_time += end-start

    start = time.time()
    user_spec_test = test.loc[test['userId'] == userId]
    end = time.time()
    krr_test_time += end-start

    if (len(user_spec)!=0) and (len(user_spec_test)!=0):
        # use 500 most rated movies if the user rated more than 500 movies
        start = time.time()
        if(len(user_spec)>500):
            user_spec['num_rates']=list(most_rated.loc[user_spec['movieId'],'r
ating'])
            user_spec=user_spec.sort_values(by='num_rates', ascending=False)[:
500]

        # get X and y for Kernel Ridge Regression
        X_train = normalize(V.T[user_spec['movieId']-1,:], axis=0)
        y_train = user_spec['rating']

        # Construct and train model
        model_kr =  KernelRidge(kernel='rbf', alpha=0.5)
        cls = model_kr.fit(X_train,y_train)
        end = time.time()
        krr_train_time += end-start

        # get predictions
        start = time.time()
        X_test = normalize(V.T[user_spec_test['movieId']-1,:], axis=0)
        y_preds = cls.predict(X_test)
        end = time.time()
        krr_test_time += end-start

        train_preds = cls.predict(X_train)
        test.loc[list(user_spec_test.index),'krr_preds'] = y_preds
        train.loc[list(user_spec.index),'krr_preds'] = train_preds
        if len((user_spec>500)): train['krr_preds'].fillna(0, inplace=True)
```

In [10]:
```
# compute test and train RMSE of Kerner Ridge Regression prediction
krr_test_rmse = sqrt(np.array(mean_squared_error(test['rating'],test['krr_pred
s'])))
krr_train_rmse = sqrt(np.array(mean_squared_error(train['rating'],train['krr_p
reds'])))
```

In [11]:
```
comp_time = pd.DataFrame(data = {'KNN':[knn_test_time, knn_train_time], 'KRR':
[krr_test_time, krr_train_time]}, index = ['test_time', 'train_time'])
```

Step 4: Evaluation

Using linear regression, we combine the predictions from PMF and each post-processed predictions. Then we use RMSE to evaluate the combined models.

1) KNN with K=1

In [31]:
```
# Linear regression to combine PMF and KNN
from sklearn.linear_model import LinearRegression
lm_knn = LinearRegression(fit_intercept=False)
lm_knn.fit(X = np.vstack([train['knn_preds'], train['pmf_preds']]).T, y = trai
n['rating'])
knn_beta1, knn_beta2 = lm_knn.coef_
```

In [14]:
```
# betas for linear regression combining PMF and KNN
print('beta for knn perdictions:{}'.format(knn_beta1))
print('beta for pmf predictions: {}'.format(knn_beta2))
```

```
beta for knn perdictions:-0.002726992838388865
beta for pmf predictions: 1.0094466331127983
```

In [15]:
```
# compute test and train rmse of KNN processed results
test['knn_processed_preds'] = knn_beta1*test['knn_preds'] + knn_beta2*test['pm
f_preds']
pmf_knn_test_rmse = sqrt(mean_squared_error(test['rating'], test['knn_processe
d_preds']))
```

In [16]:
```
# compute train rmse of KNN processed results
train['knn_processed_preds'] = knn_beta1*train['knn_preds'] + knn_beta2*train[
'pmf_preds']
pmf_knn_train_rmse = sqrt(mean_squared_error(train['rating'], train['knn_proce
ssed_preds']))
```

2) Kernel Ridge Regression

In [34]:
```
# Linear regression to combine PMF and KRR
from sklearn.linear_model import LinearRegression
lm_krr = LinearRegression(fit_intercept=False)
lm_krr.fit(X = np.vstack([train['krr_preds'], train['pmf_preds']]).T, y = trai
n['rating'])
krr_beta1, krr_beta2 = lm_krr.coef_
```

In [19]:
```
# betas for linear regression combining PMF and KRR
print('beta for krr perdictions:{}'.format(krr_beta1))
print('beta for pmf predictions: {}'.format(krr_beta2))
```

```
beta for krr perdictions:0.020029670373081978
beta for pmf predictions: 0.9892725124376046
```

In [20]:
```
# compute test rmse of KRR processed results
test['krr_processed_preds'] = krr_beta1*test['krr_preds'] + krr_beta2*test['pm
f_preds']
pmf_krr_test_rmse = sqrt(mean_squared_error(test['rating'], test['krr_processe
d_preds']))
```

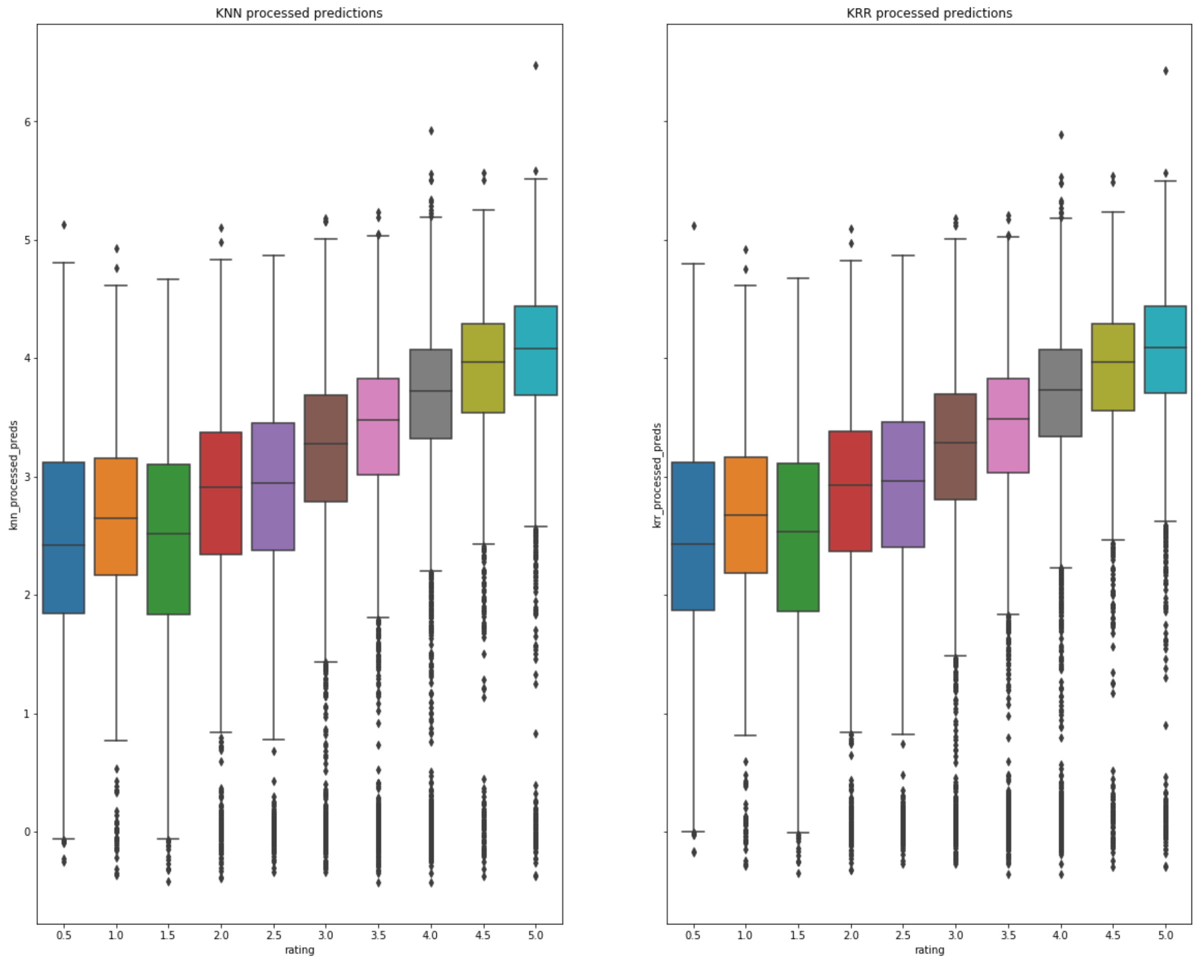
In [21]:
```
# train rmse of KRR processed results
train['krr_processed_preds'] = krr_beta1*train['krr_preds'] + krr_beta2*train[
'pmf_preds']
pmf_krr_train_rmse = sqrt(mean_squared_error(train['rating'], train['krr_proce
ssed_preds']))
```

In [26]:
```
rmfe_df = pd.DataFrame(data = {'PMF':[pmf_test_rmse, pmf_train_rmse], 'KNN':[k
nn_test_rmse, knn_train_rmse], 'KRR':[krr_test_rmse, krr_train_rmse], 'PMF_KN
N':[pmf_knn_test_rmse, pmf_knn_train_rmse], 'PMF_KRR':[pmf_krr_test_rmse, pmf_
krr_train_rmse]}, index = ['test_rmse', 'train_rmse'])
```

In [37]:
```
# train.to_csv('train.csv')
# test.to_csv('test.csv')
```

3) Visualization in Boxplot

In [40]:
```
fig, ax = plt.subplots(1,2,figsize=(20,16), sharey=True)
sns.boxplot(x='rating', y='knn_processed_preds', data=test, ax = ax[0])
sns.boxplot(x='rating', y='krr_processed_preds', data=test, ax = ax[1])
ax[0].set_title('KNN processed predictions')
ax[1].set_title('KRR processed predictions');
```



The predictions are higher when the actual ratings are higher. The betas for KNN predictions and KRR predictions are very small, so knn_processed_preds and krr_processed_preds, which are linear regression of each with PMF predictions, consist mostly of PMF predictions. Accordingly, the box plots for the two are very similar, and knn_processed_preds and krr_processed_preds are highly correlated.

In [4]:
```
train[['knn_processed_preds','krr_processed_preds']].corr()
```

Out[4]:

	knn_processed_preds	krr_processed_preds
knn_processed_preds	1.000000	0.999557
krr_processed_preds	0.999557	1.000000

In [34]:
```
comp_time
```

Out[34]:

	KNN	KRR
test_time	0.054995	2.551214
train_time	944.052324	5.460061

In [37]:
```
rmse_df
```

Out[37]:

	PMF	KNN	KRR	PMF_KNN	PMF_KRR
test_rmse	1.153616	1.526282	0.932927	1.154128	1.142635
train_rmse	0.699220	1.349358	1.385064	0.698768	0.698402

From the results above, we can observe that KRR post-processed results are better off by 0.01 RMSE compared to PMF predictions. The test RMSE for the KRR predictions is lower than train RMSE, indicating that the model may be underfitting and has more room to improve. We used the same parameters as given in the paper for this model, but with parameter tuning we may get better results. On the other hand, KNN post-processed results are similar to PMF (worse off by 0.0005 RMSE). A possible explanation is that we use K=1 so the KNN predictions only depend on a single nearest item. This may have caused noised due to overfitting and have evened out the ensemble effect. Also, we are not given ratings for every movie in the first place, and more movies have been omitted as we set the KNN prediction to 0 for movies without ratings. Accordingly, almost half of the test prediction with KNN was set to 0. KNN may perform better with more data available and/or with larger K.