Chapter 425

Two-Sample T-Tests Allowing Unequal Variance (Enter Difference)

Introduction

This procedure provides sample size and power calculations for one- or two-sided two-sample t-tests when no assumption of equal variances for the two population is made. This is commonly known as the Aspin-Welch test, Welch's t-test (Welch, 1937), or the Satterthwaite method. There are two PASS procedures for this test. In this procedure, the assumed difference between means is specified by entering the difference directly. If you wish to enter the means for the two groups and let the software calculate the difference, you can use the Two-Sample T-Tests Allowing Unequal Variance (Enter Means) procedure.

The design corresponding to this test procedure is sometimes referred to as a *parallel-groups* design. This design is used in situations such as the comparison of the income level of two regions, the nitrogen content of two lakes, or the effectiveness of two drugs.

There are several statistical tests available for the comparison of the center of two populations. This procedure is specific to Aspin-Welch-Satterthwaite test. You can examine the sections below to identify whether the assumptions and test statistic you intend to use in your study match those of this procedure, or if one of the other PASS procedures may be more suited to your situation.

Other PASS Procedures for Comparing Two Means or Medians

Procedures in PASS are primarily built upon the testing methods, test statistic, and test assumptions that will be used when the analysis of the data is performed. You should check to identify that the test procedure described below in the Test Procedure section matches your intended procedure. If your assumptions or testing method are different, you may wish to use one of the other two-sample procedures available in PASS. These procedures are Two-Sample T-Tests Assuming Equal Variance, Two-Sample Z-Tests Assuming Equal Variance, Two-Sample Z-Tests Allowing Unequal Variance, and the nonparametric Mann-Whitney-Wilcoxon (also known as the Mann-Whitney U or Wilcoxon rank-sum test) procedure. The methods, statistics, and assumptions for those procedures are described in the associated chapters.

If you wish to show that the mean of one population is larger (or smaller) than the mean of another population by a specified amount, you should use one of the clinical superiority procedures for comparing means. Non-inferiority, equivalence, and confidence interval procedures are also available.

Test Assumptions

When running an Aspin-Welch-Satterthwaite t-test, the basic assumption is that the distributions of the two populations are normal. If that assumption is not likely to be met, another testing procedure, such as a non-parametric procedure could be used, and the corresponding procedure in PASS should be used for sample size or power calculations.

Test Procedure

If we assume that μ_1 and μ_2 represent the means of the two populations of interest, and that $\delta = \mu_1 - \mu_2$, the null hypothesis for comparing the two means is $H_0 : \delta = 0$. The alternative hypothesis can be any one of

$$H_1 : \delta \neq 0$$

$$H_1 : \delta > 0$$

$$H_1 : \delta < 0$$

depending upon the desire of the researcher or the protocol instructions. A suitable Type I error probability (α) is chosen for the test, the data is collected, and a *t*-statistic is generated using the formula:

$$t = \frac{\overline{x}_1 - \overline{x}_2}{\sqrt{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2} \right)}}$$

This *t*-statistic follows a *t* distribution approximately, with estimated degrees of freedom

$$\frac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2} \right)^2}{\dfrac{1}{n_1 - 1}\left(\dfrac{s_1^2}{n_1} \right)^2 + \dfrac{1}{n_2 - 1}\left(\dfrac{s_2^2}{n_2} \right)^2}$$

The null hypothesis is rejected in favor of the alternative if,

for $H_1 : \delta \neq 0$,

$$t < t_{\alpha/2} \text{ or } t > t_{1-\alpha/2},$$

for $H_1 : \delta > 0$,

$$t > t_{1-\alpha},$$

or, for $H_1 : \delta < 0$,

$$t < t_{\alpha}.$$

Comparing the *t*-statistic to the cut-off *t*-value (as shown here) is equivalent to comparing the *p*-value to α.

Power Calculation

This section describes the procedure for computing the power from n_1 and n_2, α, the assumed μ_1 and μ_2, and the assumed standard deviations, σ_1 and σ_2. Two good references for these general methods are Julious (2010) and Chow, Shao, and Wang (2008), although these texts do not specifically cover the Aspin-Welch-Satterthwaite t-test methods.

The figure below gives a visual representation for the calculation of power for a one-sided test.
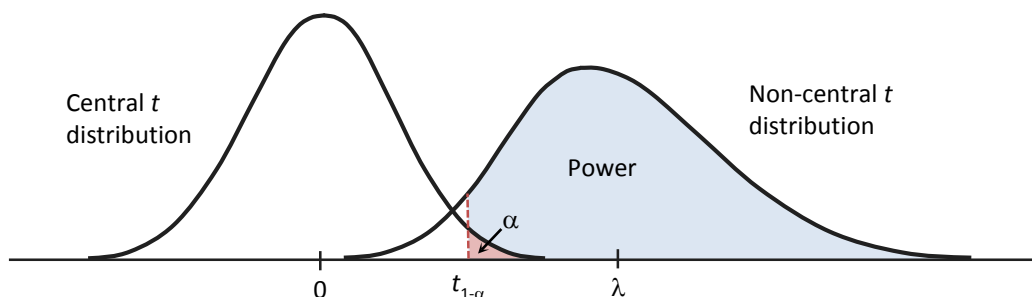
If we call the assumed difference between the means, $\delta = \mu_1 - \mu_2$, the steps for calculating the power are as follows:

1. Find $t_{1-\alpha}$ based on the central-t distribution with degrees of freedom,

$$df = \frac{\left(\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2} \right)^2}{\dfrac{1}{n_1 - 1}\left(\dfrac{\sigma_1^2}{n_1} \right)^2 + \dfrac{1}{n_2 - 1}\left(\dfrac{\sigma_2^2}{n_2} \right)^2} .$$

2. Calculate the non-centrality parameter:

$$\lambda = \frac{\delta}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} .$$

3. Calculate the power as the probability that the test statistic t is greater than $t_{1-\alpha}$ under the non-central-t distribution with non-centrality parameter λ :

$$Power = \Pr_{Non-central-t}\left(t > t_{1-\alpha} \mid df, \lambda \right).$$

The algorithms for calculating power for the opposite direction and the two-sided hypotheses are analogous to this method.

When solving for something other than power, PASS uses this same power calculation formulation, but performs a search to determine that parameter.

A Note on Specifying the Difference in Means

When the difference in means is specified in this procedure, it is the assumed difference in means for power or sample size calculations. It does not mean that the study will be powered to show that the mean difference is this amount, but rather that the design is powered to reject the null hypothesis of equal means if this were the true difference in means. If your purpose is to show that one mean is greater than another by a specific amount, you should use one of the clinical superiority procedures for comparing means.

A Note on Specifying the Standard Deviations

The sample size calculation for most statistical procedures is based on the choice of alpha, power, and an assumed difference in the primary parameters of interest – the difference in means in this procedure. An additional parameter that must be specified for means tests is the standard deviation. Here, we will briefly discuss some considerations for the choice of the standard deviation to enter.

If a number of previous studies of a similar nature are available, you can estimate the variance based on a weighted average of the variances, and then take the square root to give the projected standard deviation.

Perhaps more commonly, only a single pilot study is available, or it may be that no previous study is available. For both of these cases, the conservative approach is typically recommended. In PASS, there is a standard deviation estimator tool. This tool can be used to help select an appropriate value or range of values for the standard deviation.

If the standard deviation is not given directly from the previous study, it may be obtained from the standard error, percentiles, or the coefficient of variation. Once a standard deviation estimate is obtained, it may useful to then use the confidence limits tab to obtain a confidence interval for the standard deviation estimate. With regard to power and sample size, the upper confidence limit will then be a conservative estimate of the standard deviation. Or a range of values from the lower confidence limit to the upper confidence limit may be used to determine the effect of the standard deviation on the power or sample size requirement.

If there is no previous study available, a couple of rough estimation options can be considered. You may use the data tab of the standard deviation estimator to enter some values that represent typical values you expect to encounter. This tool will allow you to see the corresponding population or sample standard deviation. A second rough estimation technique is to base the estimate of the standard deviation on your estimate of the range of the population or the range of a data sample. A conservative divisor for the population range is 4. For example, if you are confident your population values range from 45 to 105, you would enter 60 for the Population Range, and, say, 4, for 'C'. The resulting standard deviation estimate would be 15.

If you are unsure about the value you should enter for the standard deviation, we recommend that you additionally examine a range of standard deviation values to see the effect that your choice has on power or sample size.

Procedure Options

This section describes the options that are specific to this procedure. These are located on the Design tab. For more information about the options of other tabs, go to the Procedure Window chapter.

Design Tab

The Design tab contains most of the parameters and options that you will be concerned with.

Solve For

Solve For

This option specifies the parameter to be solved for from the other parameters. The parameters that may be selected are δ, $\sigma 1$, $\sigma 2$, *Alpha*, *Power*, and *Sample Size*. In most situations, you will likely select either *Power* or *Sample Size*.

The 'Solve For' parameter is the parameter that will be displayed on the vertical axis of any plots that are shown.

Test Direction

Alternative Hypothesis

Specify the alternative hypothesis of the test.

If you want to input means rather than differences, there is a similar procedure to this one where means are entered rather than differences.

The options with $\delta < 0$ and $\delta > 0$ are one-tailed tests. When you choose one of these, you should be sure that you enter a δ value that matches this alternative hypothesis direction.

Power and Alpha

Power

Power is the probability of rejecting the null hypothesis when it is false. Power is equal to 1 - Beta, so specifying power implicitly specifies beta. Beta is the probability obtaining a false negative with the statistical test. That is, it is the probability of accepting a false null hypothesis.

The valid range is 0 to 1. Different disciplines have different standards for setting power. The most common choice is 0.90, but 0.80 is also popular.

You can enter a single value, such as 0.90, or a series of values, such as .70 .80 .90, or .70 to .90 by .1.

When a series of values is entered, PASS will generate a separate calculation result for each value of the series.

Alpha

Alpha is the probability of obtaining a false positive with the statistical test. That is, it is the probability of rejecting a true null hypothesis. The null hypothesis is usually that the parameters of interest (means, proportions, etc.) are equal.

Since Alpha is a probability, it is bounded by 0 and 1. Commonly, it is between 0.001 and 0.10.

Alpha is often set to 0.05 for two-sided tests and to 0.025 for one-sided tests.

You can enter a single value, such as 0.05, or a series of values, such as .05 .10 .15, or .05 to .15 by .01.

When a series of values is entered, PASS will generate a separate calculation result for each value of the series.

Sample Size (When Solving for Sample Size)

Group Allocation

Select the option that describes the constraints on *N1* or *N2* or both.

The options are

- **Equal (N1 = N2)**

  This selection is used when you wish to have equal sample sizes in each group. Since you are solving for both sample sizes at once, no additional sample size parameters need to be entered.

- **Enter N1, solve for N2**

  Select this option when you wish to fix *N1* at some value (or values), and then solve only for *N2*. Please note that for some values of *N1*, there may not be a value of *N2* that is large enough to obtain the desired power.

- **Enter N2, solve for N1**

  Select this option when you wish to fix *N2* at some value (or values), and then solve only for *N1*. Please note that for some values of *N2*, there may not be a value of *N1* that is large enough to obtain the desired power.

- **Enter R = N2/N1, solve for N1 and N2**

  For this choice, you set a value for the ratio of *N2* to *N1*, and then PASS determines the needed *N1* and *N2*, with this ratio, to obtain the desired power. An equivalent representation of the ratio, *R*, is

  $$N2 = R * N1.$$

- **Enter percentage in Group 1, solve for N1 and N2**

  For this choice, you set a value for the percentage of the total sample size that is in Group 1, and then PASS determines the needed *N1* and *N2* with this percentage to obtain the desired power.

N1 (Sample Size, Group 1)

This option is displayed if Group Allocation = "Enter N1, solve for N2"

N1 is the number of items or individuals sampled from the Group 1 population.

N1 must be ≥ 2. You can enter a single value or a series of values.

N2 (Sample Size, Group 2)

This option is displayed if Group Allocation = "Enter N2, solve for N1"

N2 is the number of items or individuals sampled from the Group 2 population.

N2 must be ≥ 2. You can enter a single value or a series of values.

R (Group Sample Size Ratio)

This option is displayed only if Group Allocation = "Enter R = N2/N1, solve for N1 and N2."

R is the ratio of *N2* to *N1*. That is,

$$R = N2 / N1.$$

Use this value to fix the ratio of *N2* to *N1* while solving for *N1* and *N2*. Only sample size combinations with this ratio are considered.

N2 is related to *N1* by the formula:

$$N2 = [R \times N1],$$

where the value *[Y]* is the next integer $\geq Y$.

For example, setting $R = 2.0$ results in a Group 2 sample size that is double the sample size in Group 1 (e.g., *N1* = 10 and *N2* = 20, or *N1* = 50 and *N2* = 100).

R must be greater than 0. If $R < 1$, then *N2* will be less than *N1*; if $R > 1$, then *N2* will be greater than *N1*. You can enter a single or a series of values.

Percent in Group 1

This option is displayed only if Group Allocation = "Enter percentage in Group 1, solve for N1 and N2."

Use this value to fix the percentage of the total sample size allocated to Group 1 while solving for *N1* and *N2*. Only sample size combinations with this Group 1 percentage are considered. Small variations from the specified percentage may occur due to the discrete nature of sample sizes.

The Percent in Group 1 must be greater than 0 and less than 100. You can enter a single or a series of values.

Sample Size (When Not Solving for Sample Size)

Group Allocation

Select the option that describes how individuals in the study will be allocated to Group 1 and to Group 2.

The options are

- **Equal (N1 = N2)**

  This selection is used when you wish to have equal sample sizes in each group. A single per group sample size will be entered.

- **Enter N1 and N2 individually**

  This choice permits you to enter different values for *N1* and *N2*.

- **Enter N1 and R, where N2 = R * N1**

  Choose this option to specify a value (or values) for *N1*, and obtain *N2* as a ratio (multiple) of *N1*.

- **Enter total sample size and percentage in Group 1**

  Choose this option to specify a value (or values) for the total sample size (*N*), obtain *N1* as a percentage of *N*, and then *N2* as *N - N1*.

Sample Size Per Group

This option is displayed only if Group Allocation = "Equal (N1 = N2)."

The Sample Size Per Group is the number of items or individuals sampled from each of the Group 1 and Group 2 populations. Since the sample sizes are the same in each group, this value is the value for *N1*, and also the value for *N2*.

The Sample Size Per Group must be ≥ 2. You can enter a single value or a series of values.

N1 (Sample Size, Group 1)

*This option is displayed if Group Allocation = "Enter N1 and N2 individually" or "Enter N1 and R, where N2 = R * N1."*

N1 is the number of items or individuals sampled from the Group 1 population.

N1 must be ≥ 2. You can enter a single value or a series of values.

N2 (Sample Size, Group 2)

This option is displayed only if Group Allocation = "Enter N1 and N2 individually."

N2 is the number of items or individuals sampled from the Group 2 population.

N2 must be ≥ 2. You can enter a single value or a series of values.

R (Group Sample Size Ratio)

*This option is displayed only if Group Allocation = "Enter N1 and R, where N2 = R * N1."*

R is the ratio of *N2* to *N1*. That is,

$$R = N2/N1$$

Use this value to obtain *N2* as a multiple (or proportion) of *N1*.

N2 is calculated from *N1* using the formula:

$$N2=[R \ x \ N1],$$

where the value *[Y]* is the next integer ≥ *Y*.

For example, setting *R* = 2.0 results in a Group 2 sample size that is double the sample size in Group 1.

R must be greater than 0. If *R* < 1, then *N2* will be less than N1; if *R* > 1, then *N2* will be greater than *N1*. You can enter a single value or a series of values.

Total Sample Size (N)

This option is displayed only if Group Allocation = "Enter total sample size and percentage in Group 1."

This is the total sample size, or the sum of the two group sample sizes. This value, along with the percentage of the total sample size in Group 1, implicitly defines *N1* and *N2*.

The total sample size must be greater than one, but practically, must be greater than 3, since each group sample size needs to be at least 2.

You can enter a single value or a series of values.

Percent in Group 1

This option is displayed only if Group Allocation = "Enter total sample size and percentage in Group 1."

This value fixes the percentage of the total sample size allocated to Group 1. Small variations from the specified percentage may occur due to the discrete nature of sample sizes.

The Percent in Group 1 must be greater than 0 and less than 100. You can enter a single value or a series of values.

Difference in Means

δ

Enter a value for the assumed difference between the means of Groups 1 and 2. This difference is the difference at which the design is powered to reject equal means.

If you want to input means rather than differences, there is a similar procedure to this one where means are entered rather than differences.

δ can be any non-zero value (positive or negative), but it should be consistent with the choice of the alternative hypothesis.

You can enter a single value, such as 10, or a series of values, such as 10 20 30, or 5 to 50 by 5.

When a series of values is entered, PASS will generate a separate calculation result for each value of the series.

Standard Deviations

σ1 (Standard Deviation 1)

The standard deviation entered here is the assumed standard deviation for the Group 1 population.

σ1 must be a positive number.

You can enter a single value, such as 5, or a series of values, such as 1 3 5 7 9, or 1 to 10 by 1.

When a series of values is entered, PASS will generate a separate calculation result for each value of the series.

Press the small 'σ' button to the right to obtain calculation options for estimating the standard deviation.

σ2 (Standard Deviation 2)

The standard deviation entered here is the assumed standard deviation for the Group 2 population.

σ2 must be a positive number.

You can enter a single value, such as 5, or a series of values, such as 1 3 5 7 9, or 1 to 10 by 1.

When a series of values is entered, PASS will generate a separate calculation result for each value of the series.

Press the small 'σ' button to the right to obtain calculation options for estimating the standard deviation.

Example 1 – Finding the Sample Size

Researchers wish to compare two types of local anesthesia to determine whether there is a difference in time to loss of pain. Subjects will be randomized to treatment, the treatment will be administered, and the time to loss of pain measured. The researchers would like to generate a sample size for the study with 90% power to reject the null hypothesis of equal loss-of-pain time if the true difference is at least 2 minutes. How many participants are needed to achieve 90% power at significance levels of 0.01 and 0.05?

Past experiments of this type have had standard deviations in the range of 1 to 5 minutes for Group 1, and at about 2.5 for Group 2. It is unknown which treatment has lower time to loss of pain, so a two-sided test will be used.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Two-Sample T-Tests Allowing Unequal Variance (Enter Difference)** procedure window by expanding **Means**, then **Two Independent Means**, then clicking on **Test (Inequality)**, and then clicking on **Two-Sample T-Tests Allowing Unequal Variance (Enter Difference)**. You may then make the appropriate entries as listed below, or open **Example 1** by going to the **File** menu and choosing **Open Example Template**.

Option	**Value**
Design Tab	
Solve For	**Sample Size**
Alternative Hypothesis	**Ha: δ ≠ 0**
Power..	**0.90**
Alpha..	**0.01 0.05**
Group Allocation	**Equal (N1 = N2)**
δ	**2**
σ1	**1 to 5 by 1**
σ2	**2.5**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results for Two-Sample T-Test Allowing Unequal Variance
Alternative Hypothesis: δ ≠ 0

Target Power	Actual Power	N1	N2	N	δ	σ1	σ2	Alpha
0.90	0.90538	30	30	60	2.0	1.0	2.5	0.010
0.90	0.90607	21	21	42	2.0	1.0	2.5	0.050
0.90	0.90085	40	40	80	2.0	2.0	2.5	0.010
0.90	0.90032	28	28	56	2.0	2.0	2.5	0.050
0.90	0.90315	59	59	118	2.0	3.0	2.5	0.010
0.90	0.90637	42	42	84	2.0	3.0	2.5	0.050
0.90	0.90097	85	85	170	2.0	4.0	2.5	0.010
0.90	0.90187	60	60	120	2.0	4.0	2.5	0.050
0.90	0.90140	119	119	238	2.0	5.0	2.5	0.010
0.90	0.90202	84	84	168	2.0	5.0	2.5	0.050

References
Julious, S. A. 2010. Sample Sizes for Clinical Trials. Chapman & Hall/CRC. Boca Raton, FL.
Chow, S.-C., Shao, J., and Wang, H. 2008. Sample Size Calculations in Clinical Research (Second Edition).
    Chapman & Hall/CRC. Boca Raton, FL.
Machin, D., Campbell, M., Fayers, P., and Pinol, A. 1997. Sample Size Tables for Clinical Studies, 2nd
    Edition. Blackwell Science. Malden, MA.
Zar, Jerrold H. 1984. Biostatistical Analysis (Second Edition). Prentice-Hall. Englewood Cliffs, New Jersey.

Report Definitions
Target Power is the desired power value (or values) entered in the procedure. Power is the probability of
    rejecting a false null hypothesis.
Actual Power is the power obtained in this scenario. Because N1 and N2 are discrete, this value is often
    (slightly) larger than the target power.
N1 and N2 are the number of items sampled from each population.
N is the total sample size, N1 + N2.
δ is the difference between population means at which power and sample size calculations are made.
σ1 and σ2 are the assumed population standard deviations for groups 1 and 2, respectively.
Alpha is the probability of rejecting a true null hypothesis.

Summary Statements
Group sample sizes of 30 and 30 achieve 90.538% power to reject the null hypothesis of equal
means when the population mean difference is 2.0 with standard deviations of 1.0 for group 1
and 2.5 for group 2, and with a significance level (alpha) of 0.010 using a two-sided
two-sample unequal-variance t-test.

These reports show the values of each of the parameters, one scenario per row.
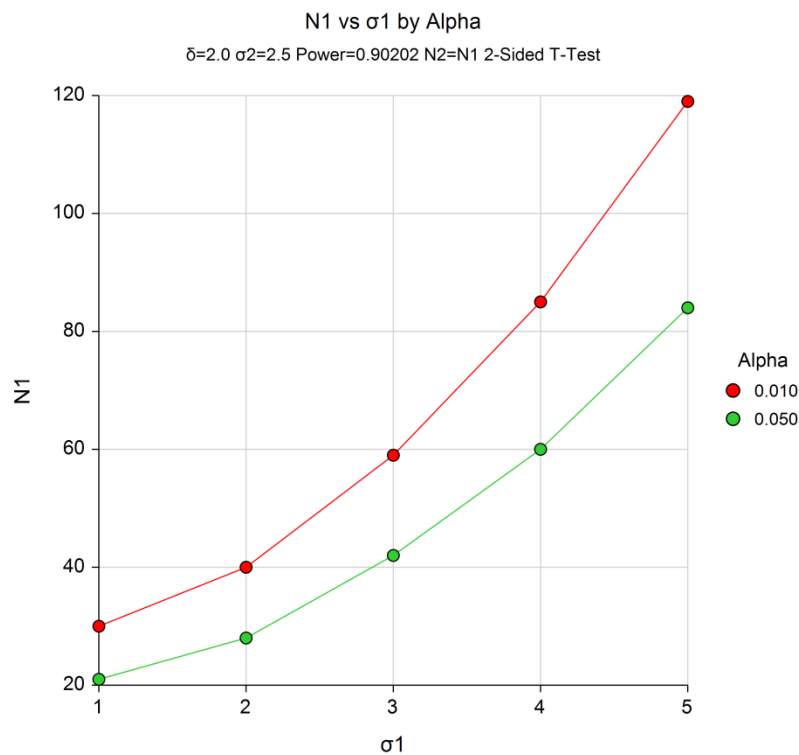
Plots Section
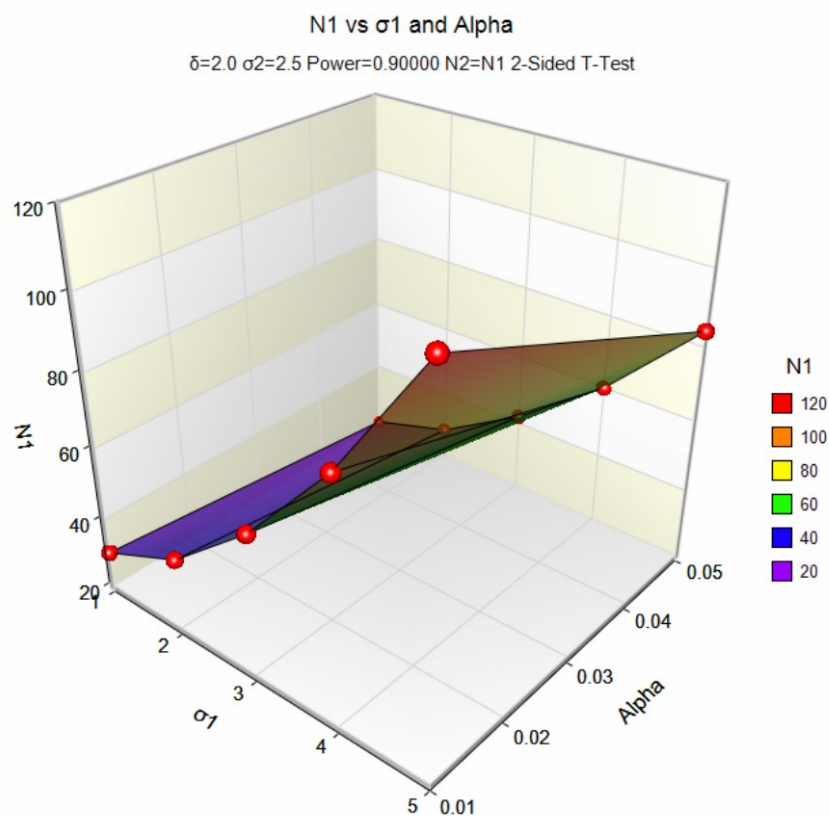
These plots show the relationship between the standard deviation for Group 1 and sample size for the two alpha levels.

Example 2 – Finding the Power

Suppose a new corn fertilizer is to be compared to a current fertilizer. The researchers need only show that there is difference in yield with the new fertilizer. They would like to consider the effect of the number of plots used on the power of the test if the improvement in yield is at least 10 lbs.

Researchers plan to use a one-sided two-sample Aspin-Welch-Satterthwaite t-test with alpha equal to 0.05. Previous studies indicate the standard deviation for plot yield to be 24 lbs. for Group 1, and between 20 and 30 lbs for Group 2. The plot group sizes of interest are 10 to 100 plots per group.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Two-Sample T-Tests Allowing Unequal Variance (Enter Difference)** procedure window by expanding **Means**, then **Two Independent Means**, then clicking on **Test (Inequality)**, and then clicking on **Two-Sample T-Tests Allowing Unequal Variance (Enter Difference)**. You may then make the appropriate entries as listed below, or open **Example 2** by going to the **File** menu and choosing **Open Example Template**.

<u>Option</u>	<u>Value</u>
Design Tab	
Solve For	**Power**
Alternative Hypothesis	**Ha: δ > 0**
Alpha	**0.05**
Group Allocation	**Equal (N1 = N2)**
Sample Size Per Group	**10 to 100 by 10**
δ	**10**
σ1	**24**
σ2	**20 25 30**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results for Two-Sample T-Test Allowing Unequal Variance
Alternative Hypothesis: δ > 0

Power	N1	N2	N	δ	σ1	σ2	Alpha
0.25087	10	10	20	10.0	24.0	20.0	0.050
0.40528	20	20	40	10.0	24.0	20.0	0.050
0.53474	30	30	60	10.0	24.0	20.0	0.050
0.64110	40	40	80	10.0	24.0	20.0	0.050
0.72653	50	50	100	10.0	24.0	20.0	0.050
0.79385	60	60	120	10.0	24.0	20.0	0.050
0.84605	70	70	140	10.0	24.0	20.0	0.050
0.88598	80	80	160	10.0	24.0	20.0	0.050
0.91618	90	90	180	10.0	24.0	20.0	0.050
0.93880	100	100	200	10.0	24.0	20.0	0.050
0.22168	10	10	20	10.0	24.0	25.0	0.050
0.35293	20	20	40	10.0	24.0	25.0	0.050
0.46698	30	30	60	10.0	24.0	25.0	0.050
0.56523	40	40	80	10.0	24.0	25.0	0.050
0.64855	50	50	100	10.0	24.0	25.0	0.050
0.71817	60	60	120	10.0	24.0	25.0	0.050

(Numeric Results - continued)

0.77562	70	70	140	10.0	24.0	25.0	0.050
0.82251	80	80	160	10.0	24.0	25.0	0.050
0.86041	90	90	180	10.0	24.0	25.0	0.050
0.89080	100	100	200	10.0	24.0	25.0	0.050
0.19657	10	10	20	10.0	24.0	30.0	0.050
0.30765	20	20	40	10.0	24.0	30.0	0.050
0.40647	30	30	60	10.0	24.0	30.0	0.050
0.49456	40	40	80	10.0	24.0	30.0	0.050
0.57234	50	50	100	10.0	24.0	30.0	0.050
0.64028	60	60	120	10.0	24.0	30.0	0.050
0.69905	70	70	140	10.0	24.0	30.0	0.050
0.74946	80	80	160	10.0	24.0	30.0	0.050
0.79235	90	90	180	10.0	24.0	30.0	0.050
0.82861	100	100	200	10.0	24.0	30.0	0.050

These reports show the values of each of the parameters, one scenario per row.

Plots Section
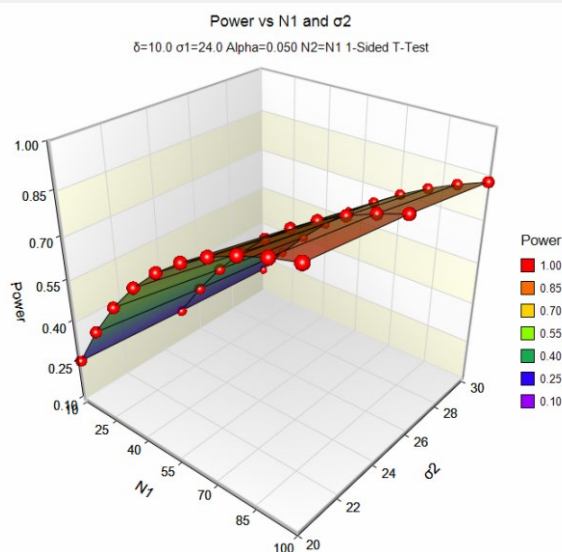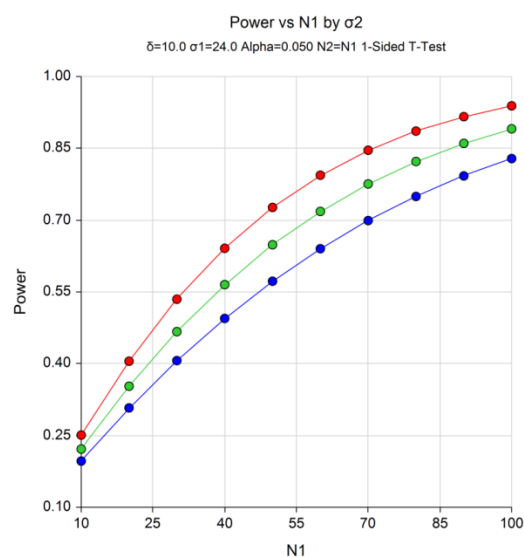
These plots show the relationship between the power and sample size for the three values of σ2.

Example 3 – Finding the Difference

In some cases it may be useful to determine how different the means of two populations would need to be to achieve the desired power with a specific constrained sample size.

Suppose, for example, that 80 subjects are available for a study to compare weight loss regimens. Researchers would like to determine which of the two regimens is better. The researchers would like to know how different the mean weight loss must be to have a study with 90% power when alpha is 0.05 and the standard deviation is assumed to be 6 lbs. for Group 1 and 8 lbs. for Group 2.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Two-Sample T-Tests Allowing Unequal Variance (Enter Difference)** procedure window by expanding **Means**, then **Two Independent Means**, then clicking on **Test (Inequality)**, and then clicking on **Two-Sample T-Tests Allowing Unequal Variance (Enter Difference)**. You may then make the appropriate entries as listed below, or open **Example 3** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	δ
Alternative Hypothesis	**Ha: $\delta \neq 0$**
Power...	**0.90**
Alpha..	**0.05**
Group Allocation	**Equal (N1 = N2)**
Sample Size Per Group............................	**40**
δ	**Search > 0**
$\sigma 1$	**6**
$\sigma 2$	**8**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results for Two-Sample T-Test Allowing Unequal Variance
Alternative Hypothesis: $\delta \neq 0$

Power	N1	N2	N	δ	$\sigma 1$	$\sigma 2$	Alpha
0.90000	40	40	80	5.195	6.0	8.0	0.050

If the true population mean weight loss for one group is 5.195 lbs. more or less than the other, the researchers would have 90% power to show a difference between the groups.

Example 4 – Validation of Sample Size using Simulation

We used the Tests for Two Means (Simulation) procedure to find the sample size needed for a two-sided test with alpha equal to 0.05, where the difference in means is 3.5, the standard deviation for Group 1 is 1 and the standard deviation for Group 2 is 3. The sample size result was 11 per group.

Setup

This section presents the values of each of the parameters needed to run this example. First, from the PASS Home window, load the **Two-Sample T-Tests Allowing Unequal Variance (Enter Difference)** procedure window by expanding **Means**, then **Two Independent Means**, then clicking on **Test (Inequality)**, and then clicking on **Two-Sample T-Tests Allowing Unequal Variance (Enter Difference)**. You may then make the appropriate entries as listed below, or open **Example 4** by going to the **File** menu and choosing **Open Example Template**.

Option	Value
Design Tab	
Solve For	**Sample Size**
Alternative Hypothesis	**Ha: δ ≠ 0**
Power	**0.90**
Alpha	**0.05**
Group Allocation	**Equal (N1 = N2)**
δ	**3.5**
σ1	**1**
σ2	**3**

Output

Click the Calculate button to perform the calculations and generate the following output.

Numeric Results

Numeric Results for Two-Sample T-Test Allowing Unequal Variance
Alternative Hypothesis: δ ≠ 0

Target Power	Actual Power	N1	N2	N	δ	σ1	σ2	Alpha
0.90	0.92066	11	11	22	3.5	1.0	3.0	0.050

The sample size of 11 per group matches the simulation result exactly.