
Decision Tree Notes

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1 Information Entropy

On this page I will note what I have learned throughout all the research I made towards better understanding of decision trees

Information Entropy : It is a concept from information theory. It tells how much Information there is in an event. In general, the more uncertain or random the event is, the more Information it will contain. More clearly stated, information is a decrease of uncertainty or entropy

from https://simple.wikipedia.org/wiki/Information_entropy

Basically, it is the probability of the information that will come out of a system, which will produce more and who is going to produce less informations.
i.e letters generated by a machine that produces a sequence of letters.

Example and formulas took from <https://www.khanacademy.org/computing/computer-science/informationtheory/moderninfotheory/v/information-entropy>

Example from Khan Academy : machine 1 has

- 50% chance to produce the letter **A**
- 12.5% chance to produce the letter **B**
- 12.5% chance to produce the letter **C**
- 25% chance to produce the letter **D**

Machine 2 has

- 25% chance to produce the letter **A**
- 25% chance to produce the letter **B**
- 25% chance to produce the letter **C**
- 25% chance to produce the letter **D**

The multiplier to the letter is found by finding the level at which the letter is in the tree of probability.

For machine one :

$$\#bounces = p_a \times 1 + p_b \times 3 + p_c \times 3 + p_d \times 2$$

$$\begin{aligned}\#bounces &= 0.50 \times 1 + 0.125 \times 3 + 0.125 \times 3 + 0.25 \times 2 \\ \#bounces &= 1.75 \text{ bounces to reach an answer}\end{aligned}$$

For machine two :

$$\begin{aligned}\#bounces &= p_a \times 1 + p_b \times 3 + p_c \times 3 + p_d \times 2 \\ \#bounces &= 0.25 \times 2 + 0.25 \times 2 + 0.25 \times 2 + 0.25 \times 2 \\ \#bounces &= 2 \text{ bounces to reach an answer}\end{aligned}$$

So if we have to guess 200 values from the machine.

- We would have to ask 175 times machine 1
- We would have to ask 200 times machine 2

Which means machine 1 will produce less information. It also means that there less **incertainty** with *machine 1*. This is called **entropy**

*Entropy is the value H
Probability is represented by p*

$$H = \sum_{i=1}^n p_i \times \#bounces_i$$

$$\begin{aligned}\#bounces &= \log_2(\#outcomes) \\ \#outcomes &= \frac{1}{p} \\ \#bounces &= \log_2\left(\frac{1}{p}\right)\end{aligned}$$

$$H = \sum_{i=1}^n p_i \times \log_2 \frac{1}{p_i}$$