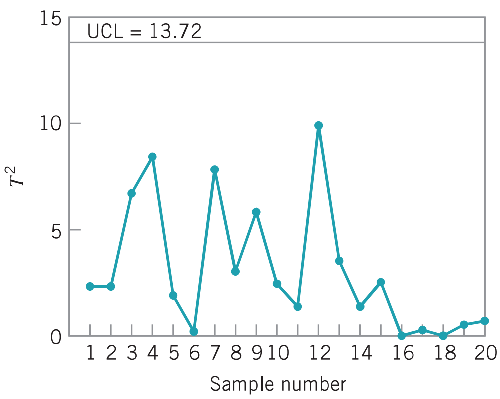
SOLUTION

The statistic he will use for process-control purposes is

$$T^2 = \frac{10}{(1.23)(0.83)-(0.79)^2}\left[0.83(\bar{x}_1-115.59)^2+1.23(\bar{x}_2-1.06)^2 -2(0.79)(\bar{x}_1-115.59)(\bar{x}_2-1.06)\right]$$



■ FIGURE 11.7 The Hotelling T^2 control chart for tensile strength and diameter, Example 11.1.

The data used in this analysis and the summary statistics are in Table 11.1, panels (a) and (b).

Figure 11.7 presents the Hotelling T^2 control chart for this example. We will consider this to be phase I, establishing statistical control in the preliminary samples, and calculate the upper control limit from equation (11.20). If $\alpha = 0.001$, then the UCL is

$$\begin{aligned}
\text{UCL} &= \frac{p(m-1)(n-1)}{mn-m-p+1}F_{\alpha,p,mn-m-p+1} \\
&= \frac{2(19)(9)}{20(10)-20-2+1}F_{0.001,2,20(10)-20-2+1} \\
&= \frac{342}{179}F_{0.001,2,179} \\
&= (1.91)7.18 \\
&= 13.72
\end{aligned}$$

This control limit is shown on the chart in Fig. 11.7. Notice that no points exceed this limit, so we would conclude that the process is in control. Phase II control limits could be calculated

■ **TABLE 11.1**
Data for Example 11.1

Sample Number k	(a) Means: Tensile Strength ($\bar{x}_{1k}$)	(a) Means: Diameter ($\bar{x}_{2k}$)	(b) Variances and Covariances: s^2_{1k}	s^2_{2k}	s_{12k}	(c) Control Chart Statistics: T^2_k	$\lvert S_k \rvert$
1	115.25	1.04	1.25	0.87	0.80	2.16	0.45
2	115.91	1.06	1.26	0.85	0.81	2.14	0.41
3	115.05	1.09	1.30	0.90	0.82	6.77	0.50
4	116.21	1.05	1.02	0.85	0.81	8.29	0.21
5	115.90	1.07	1.16	0.73	0.80	1.89	0.21
6	115.55	1.06	1.01	0.80	0.76	0.03	0.23
7	114.98	1.05	1.25	0.78	0.75	7.54	0.41
8	115.25	1.10	1.40	0.83	0.80	3.01	0.52
9	116.15	1.09	1.19	0.87	0.83	5.92	0.35
10	115.92	1.05	1.17	0.86	0.95	2.41	0.10
11	115.75	0.99	1.45	0.79	0.78	1.13	0.54
12	114.90	1.06	1.24	0.82	0.81	9.96	0.36
13	116.01	1.05	1.26	0.55	0.72	3.86	0.17
14	115.83	1.07	1.17	0.76	0.75	1.11	0.33
15	115.29	1.11	1.23	0.89	0.82	2.56	0.42
16	115.63	1.04	1.24	0.91	0.83	0.08	0.44
17	115.47	1.03	1.20	0.95	0.70	0.19	0.65
18	115.58	1.05	1.18	0.83	0.79	0.00	0.36
19	115.72	1.06	1.31	0.89	0.76	0.35	0.59
20	115.40	1.04	1.29	0.85	0.68	0.62	0.63
Averages	$\bar{\bar{x}}_1 = 115.59$	$\bar{\bar{x}}_2 = 1.06$	$\bar{s}^2_1 = 1.23$	$\bar{s}^2_2 = 0.83$	$\bar{s}_{12} = 0.79$		