Sep 3 - Class 1 - Random Experiment, Sample Space, Events, Probability

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Definitions:

- ***Random Experiments***: An activity, process, or expriemnt that has uncertain outcomes or results.
  - Examples: flip a coin, roll a dice, waiting time for a bus

- ***Sample Space***: A set of all possible outcomes of the expriement:

$$\Omega = \{\omega_1, \omega_2, ...\}$$

  - In which "Ω" is the all possible outcome list, and "ω_1..." is each outcome.
  - Example: The sameple space for flipping three coins is:

$$\Omega = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

  - Remark: It is possible to have infinite sample spaces (stop tossing when getting a tail or the bus waiting time scenario):

$$\Omega = \{T, HT, HHT, HHHT, HHHHT, HHHHHT, ...\}$$

$$\Omega = \{t : t \geq 0\} = [0, \text{inf})$$

  - This first sample space shows a set of discrete elements while the second is continuous.

- ***Events***: Collection (subset) of outcomes contained in the sample space Ω.
  - **Simple Event**: Event consists exactly one outcome.
  - **Compound Event**: Event consists more than one outcomes.
  - Example 1: Tossing three coins...
    - Event 1: Getting all heads $A = \{HHH\}$ is a *simple* event
    - Event 2: Getting at least two tails

      $B = \{HHT, HTH, HTT, THH, THT, TTH, TTT\}$

      is a *compound* event
  - Example 2: At least three coin flips to get a tail
    - $C = \{HHT, HHHT, HHHHT...\}$ is a *compound* event
  - Example 3: The bus arrives within the next 5 minutes
    - $D = \{t \in \mathbb{R} : 0 \leq t \leq 5\}$ is a *compound* event

- ***Randomness***: Individual outcomes are uncertain, but there is structure to how often outcomes occur in a very large number of repetitions.
- ***Probability***: Refers to the study of randomness and uncertainty
  - From a mathematical point of view, a probability is a number between 0 and 1 that satisfies certain axioms.
  - From an empirical point of view, the probability of an event describes the likelihood, or chance, that it occurs.
  - **Notation:** $P(A) =$ "The probability of event A occuring"
  - $P(A) = 0:$ Event A never happens
  - $P(A) = 1:$ Event A is certain to happen