Lec 2

$\frac{dx}{dt} = f(x, t)$ differential equation

Different kinds of differential equation

Linear :

- linear in x :

$\frac{dx}{dt} = a(t) \cdot x + b(t)$

Standard form

$p(t)x'(t) + q(t)x(t) = r(t)$

Reduced standard form

$x'(t) + p(t)x(t) = q(t)$

Example :

A cup of coffee cooling down

$C(t)$: temperature of coffee after t times

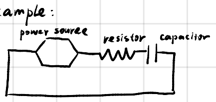
A : temperature of surrounding

$\frac{dc}{dt} = -k(C - A)$ $\quad$ k: certain constant

if temperature is higher than surrounding, negative, Otherwise positive
(to indicate cooling)

$\frac{1}{k} \cdot \frac{dc}{dt} + c(t) = A$

Example :



$V(t) = \underbrace{V_R(t)}_{\text{voltage of resistor}} + \underbrace{V_c(t)}_{\text{voltage of capacitor}}$

$V_R(t) = R \underset{\substack{\uparrow \\ \text{current in } t}}{I}(t)$

$V_c'(t) = \underset{\substack{\uparrow \\ \text{capacitance}}}{\frac{1}{c}} \cdot I(t)$

$V'(t) = V_R'(t) + V_c'(t)$

$V'(t) = R \cdot I'(t) + \frac{1}{c} \cdot I(t)$

Integrating Factor :

$\frac{dy}{dx} + P(x) \cdot y = q(x)$

Recall: $\frac{d}{dx}(u(x) \cdot v(x)) = u'(x)v(x) + u(x)v'(x)$

let : $\phi(x) = e^{\int p(x)\,dx}$

$\phi(x) \cdot y'(x) + \underbrace{\phi'(x)}_{= \phi(x) \cdot p(x)} \cdot y = \phi(x) \cdot q(x)$

$y(x) \cdot \phi(x) = \int \phi(x) \cdot q(x)\,dx$

$y(x) = \frac{1}{\phi(x)} \int \phi(x) \cdot q(x)\,dx$

$y(x) = e^{-\int p(x)dx} \cdot \int q(x) \cdot e^{\int p(x)dx}\,dx$

Example :

$\frac{dc}{dt} + kC(t) = kA$

$\phi(t) = e^{\int k\,dt}$

$= e^{kt + c}$

$\phi(t)\frac{dc}{dt} + \phi(t) \cdot k \cdot C(t) = k \cdot A \cdot \phi(t)$

$\phi(t)\frac{dc}{dt} + \phi'(t)C(t) = k \cdot A \cdot \phi(t)$

$\phi(t) \cdot c(t) = \int k \cdot A \cdot \phi(t)\,dt$

$c(t) = \frac{\int k \cdot A \cdot e^{kt+c}\,dt}{e^{kt+c}}$

$= [A \cdot e^{kt+c} + B] \cdot e^{-kt}$

$C(t) = A + B \cdot e^{-kt}$

$t = 0$, coffee $= 80k$, room temp $= 15k$

$C(0) = 15 + B \cdot e^{-k \cdot 0}$

$80 = 15 + B$

$B = 65$

$y' = y - x^2$

$y' - y = -x^2$

$\phi(x) = e^{\int -1 dx} = e^{-x}$

$\phi(x) \cdot y'(x) - \phi(x) \cdot y = \phi(x) \cdot -x^2$

$\phi(x) \cdot y'(x) + \phi'(x)\, y = \phi(x) \cdot -x^2$

$y \cdot \phi(x) = \int x^2 e^{-x}\,dx$