STA237: Probability, Statistics and Data Analysis I

Section 0101 Lecture 1

Lijia Wang

Department of Statistical Sciences
University of Toronto

Outline

1. Course Introduction
2. Random experiment, sample space and event
3. Probability intuition

Course Introduction

Course Syllabus is Available on Quercus

- Instructors and Sections
- Course Description
- Textbooks
- Course Support
- Assessments

Introduction to Lecture 1

Lecture 1 coverage:

- Define probability in the context of course
- Events, Outcomes, and set operators (complements, unions, intersections)

Lecture 1 learning outcomes:

- Define key terminology: random experiment, sample space, events, probability
- Use set notation and Venn disatrams to represent events in a given sample space

Outline

Intuitive understanding of probability

Probability begins with some activity, process, or experiment whose outcome is uncertain. This can be as simple as throwing dice or as complicated as tomorrow's weather.

Definition (Random experiment)

We call the activity, process, or experiment whose outcome is uncertain a *random experiment*, or a *random trial*.

Sample space

Definition (Sample space)

Given such a "random experiment," the set of all possible outcomes is called the *sample space*. We will use the Greek capital letter Ω (omega) to represent the sample space.

Example:

Suppose a coin is tossed three times. Let H represent heads and T represent tails. The sample space is

is a set

sample point / element

outcome

$$\Omega = \{\mathrm{HHH}, \mathrm{HHT}, \mathrm{HTH}, \mathrm{HTT}, \mathrm{THH}, \mathrm{THT}, \mathrm{TTH}, \mathrm{TTT}\},$$

consisting of eight outcomes. The Greek lowercase omega ω will be used to denote these outcomes, the elements of Ω.

$\Omega = \{w_1, w_2, w_3\}$

Event

Definition (Event)

An *event* is a set of outcomes.

An outcome may not be a set

Remark: (An outcome is an element of the sample space) but the sample space and event must be sets.

- An event is a subset of the sample space Ω.
- Often, we refer to events by assigning them a capital letter near the beginning of the alphabet, such as event A.

Example:

The event of getting all heads in three coin tosses can be written as

$$A = \{ \text{ Three heads } \} = \{\underline{HHH}\}.$$

outcome

And clearly, $A \subseteq \Omega$.

Simple and compound event

Definition (Simple event)

We define the event that contains a single outcome as the *simple event*.

Example: $A = \{HHH\}$

Event A contains a single outcome, and is a simple event.

Definition (Compound event)

We define the event that includes multiple outcomes as the *compound event*.

Example:

The event of getting at least two tails is

$$B = \{\text{ At least two tails }\} = \{HTT, THT, TTH, TTT\}.$$

Sample space and event example

Example: Yolanda and Zach are running for president of the student association. One thousand students will be voting, and each voter will pick one of the two candidates. We will eventually ask questions like, What is the probability that Yolanda wins the election over Zach by at least 100 votes? But before actually finding this probability, first identify
(i) the sample space and
(ii) the event that Yolanda beats Zach by at least 100 votes.

Yolanda Zach 1000 voters

votes Yolanda receives

1. $\Omega = \{ (a, b) \mid a, b \in [0, 1000],\ a, b \in \mathbb{N},\ a+b = 1000 \}$

votes Zach receives

$= \{ (0, 1000),\ (1, 999),\ \ldots\ (1000, 0) \}$

2. $A = \{ (a, b) \mid a, b \in [0, 1000],\ a, b \in \mathbb{N},\ a+b = 1000,$
$a \geq b + 100 \}$

Outline

Probability Intuition

What does it mean to say that the probability that event A occurs or the probability of A is equal to x?

- From a formal, purely mathematical point of view, a probability is a number between 0 and 1 that satisfies certain properties, which we will describe later.
- From a practical, empirical point of view, a probability matches up with our intuition of the likelihood or "chance" that an event occurs.
    - An event that has probability 0 "never" happens.
    - An event that has probability 1 is "certain" to happen.

Example: In repeated coin flips, a fair coin comes up heads about half the time, and the probability of heads is equal to one-half.

Relative frequency interpretation of probability

Definition (Relative frequency interpretation of probability)

Let A be an event associated with some random experiment. Imagine we conduct the experiment over and over, infinitely often, and keep track of how often A occurs. The <u>proportion of event A occurring</u> is the *probability of A*, written as $P(A)$.

Remark:

- This is the relative frequency interpretation of probability, which says that the probability of an event is equal to its relative frequency in a large number of trials.

Example on the relative frequency interpretation of probability

Example: When the weather forecaster tells us that tomorrow there is a 20% chance of rain, we understand it means that:

- if we could repeat today's conditions - the air pressure, temperature, wind speed, etc. - over and over again, then 20% of the resulting "tomorrows" will result in rain.

Limitations on the relative frequency interpretation of probability

There are definite limitations to constructing a rigorous mathematical theory out of this intuitive and empirical view of probability.

- One cannot actually repeat an experiment infinitely many times.
- To define probability carefully, we need to take a formal, axiomatic, mathematical approach.

Nevertheless, the relative frequency viewpoint will still be useful in order to gain an intuitive understanding.