Homework 1
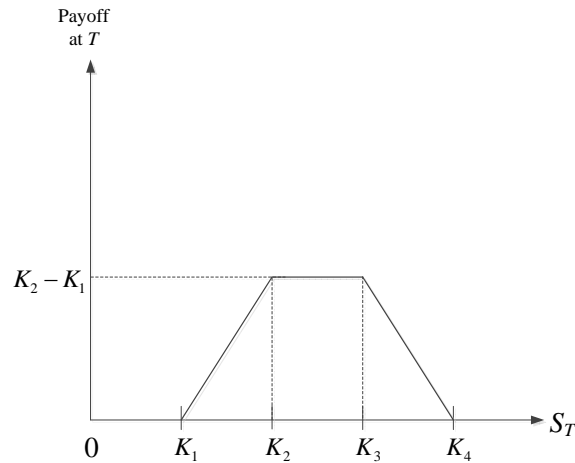
Derive the closed-form formula for an option with the following payoff function:

- Basic requirement (80 points):

  (i) Derive the closed-form formula using the martingale pricing method by hands.

  (ii) Based on the formula you derive, implement a program to price this option.

  (Inputs: S_0, r, q, σ, T, K_1, K_2, K_3, K_4. Output: Option value.)

- Bonus (10 points):

  Employ the Monte Carlo simulation to price this option.

  Based on $\ln S_T \sim N(\ln S_0 + (r - q - \sigma^2/2)T, \sigma^2 T)$, draw 10,000 random samples for S_T to compute an option price. Repeat the above step 20 times to obtain the 95% confidence interval for the option value:

  [mean of 20 repetitions $- 2\times$(s.d. of 20 repetitions), mean of 20 repetitions $+ 2\times$(s.d. of 20 repetitions)].