

# Homework 4

Price a lookback put with the binomial tree model. The payoff function of the lookback put is as follows.

$$\text{Payoff}_t = \max(S_{\max,t} - S_t, 0), \text{ where } S_{\max,t} = \max S_u, \text{ for } u = 0, \Delta t, 2\Delta t, \dots, t.$$

- Basic requirement (80 points):

(i) Implement the binomial tree model to price both European and American lookback puts.

(ii) Implement the Monte Carlo simulation to price European lookback puts.

(Inputs:  $S_t, r, q, \sigma, t, T, S_{\max,t}, n$ , number of simulations, number of repetitions. Outputs: Option values for both methods and 95% confidence level for Monte Carlo simulation.)

- Bonus 1 (5 points):

Based on the same binomial tree framework, devise and implement a quick way to determine the  $S_{\max}$  list for each node.

- Bonus 2 (10 points):

Implement the method in Cheuk and Vorst (1997) to price European and American lookback puts.

$u_t = \frac{S_{\max,t}}{S_t} = \text{on each node}$

$$\max(S_{\max,t} - S_t, 0) = S_t \max\left(\frac{S_{\max,t}}{S_t} - 1, 0\right) = S_t \max(u_t - 1, 0)$$

$$\text{EP} = S_0 \left[ \tilde{E}[e^{-rT} \max(u_T - 1, 0)] \right] \text{ or AP} = S_0 \left[ \tilde{E}[e^{-r\tau} \max(u_\tau - 1, 0)] \right]$$

※ Note that  $\tilde{P}_u$  and  $\tilde{P}_d$  are not exactly to be the branching probabilities  $P$  and  $1 - P$  in the standard binomial tree, respectively.

- Reference

Cheuk and Vorst (1997), "Currency lookback options and observation frequency: a binomial approach," *Journal of International Money and Finance* 16, pp. 173–187.