QF527200: Robust and Stochastic Portfolio Optimization

Final Project Topic: Integrating All Weather Strategy with Hybrid Risk Measures in Portfolio Optimization

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Abstract

Traditional *All Weather portfolios* emphasize risk diversification across asset classes and frequently adopt volatility-based risk parity to achieve balanced allocations. However, volatility as a standalone metric may be insufficient to capture asymmetric or tail-heavy risk profiles, particularly during turbulent market conditions. In this study, we propose an extension to the All Weather strategy by introducing a *Hybrid Equal Risk Contribution (Hybrid ERC)* framework that blends two risk measures—mean absolute deviation (MAD) and conditional value-at-risk (CVaR)—using a convex combination governed by a mixing parameter $\alpha \in [0, 1]$. To ensure practical solvability, we demonstrate that the objective function is approximately convex, allowing optimization via Sequential Least Squares Programming (SLSQP). Furthermore, we implement both fixed and volatility-driven dynamic adjustments to α, enabling regime-sensitive responses to market fluctuations. Empirical tests on U.S. ETF portfolios from 2007 to 2025 reveal that the Hybrid ERC strategy achieves stable risk-adjusted returns, with Sharpe ratios consistently outperforming traditional equal-weight or single-risk-minimization approaches. These results suggest that the Hybrid ERC framework offers a flexible and stable alternative for multi-asset portfolio construction.

1 Introduction

The All Weather strategy, introduced by Bridgewater Associates, advocates for risk diversification across asset classes to enhance portfolio resilience under varying economic regimes [1]. Rather than allocating capital in fixed proportions, this philosophy underpins risk parity (RP) portfolios—particularly Equal Risk Contribution (ERC)—where each asset contributes equally to overall portfolio risk.

Traditional ERC implementations predominantly rely on volatility or variance as risk metrics. While effective in stable markets, such symmetric measures may fail to capture real-world distributional characteristics, including skewness, kurtosis, and heavy tails. To address these limitations, alternative risk measures have been proposed. The Mean Absolute Deviation (MAD), introduced by Konno and Yamazaki [2], provides a robust dispersion metric less sensitive to outliers. Conditional Value-at-Risk (CVaR), formalized by Rockafellar and Uryasev [3], captures expected losses beyond the Value-at-Risk threshold, offering direct insight into downside risk.

These risk measures have been incorporated individually into ERC frameworks. Bruni et al. [4] examine MAD-based ERC as a robust alternative to volatility-based models, while Roncalli [5] proposes CVaR-based ERC models for improved tail risk control. However, these implementations typically assume a fixed risk measure across the investment horizon. Although hybrid risk measures have been discussed in the mean-risk optimization literature [6], their application within ERC—especially in a dynamic, regime-sensitive context—remains underexplored.

This study proposes a **Hybrid Equal Risk Contribution (Hybrid ERC)** model that integrates MAD and CVaR via a convex combination governed by a mixing parameter $\alpha \in [0, 1]$. This setup allows the model to flexibly interpolate between mid-range risk (MAD) and tail risk (CVaR) sensitivity. To improve adaptability, we further allow α to evolve dynamically based on rolling realized volatility, enabling the portfolio to respond to shifting market regimes by adjusting its risk contribution profile.

We empirically evaluate the Hybrid ERC model using diversified exchange-traded fund (ETF) portfolios across multiple asset classes. Our analysis includes both U.S. and Taiwanese markets over extended backtesting periods, benchmarking against strategies such as Equal Weight (EW), Min-MAD, Min-CVaR, and fixed-risk ERC variants. By incorporating both static and dynamic α formulations, we explore how hybrid risk modeling enhances portfolio resilience and adaptability in dynamic environments.

2 Problem Formulation

Let $R \in \mathbb{R}^{T\times N}$ denote the matrix of daily returns for N assets over T trading days, where each row $r_t \in \mathbb{R}^N$ represents the observed return vector for all assets at time t derived from historical market data. We define a hybrid risk measure that combines the Mean Absolute Deviation (MAD) and Conditional Value-at-Risk (CVaR) as follows:

$$\mathcal{R}(w;\alpha) = \alpha \cdot \mathrm{MAD}(w) + (1-\alpha)\cdot \mathrm{CVaR}_\beta(w), \tag{1}$$

where $w \in \mathbb{R}^N$ is the portfolio weight vector, $\alpha \in [0,1]$ is the mixing parameter controlling the contribution of MAD versus CVaR, and $\beta \in (0,1)$ is the CVaR confidence level, typically set to 0.95.

2.1 Risk Measure Definitions

2.1.1 Mean Absolute Deviation (MAD) [2]

The Mean Absolute Deviation (MAD) of a portfolio quantifies the average absolute deviation of portfolio returns from their mean, providing a dispersion-based risk metric less sensitive to extreme outliers than variance:

$$\mathrm{MAD}(w) = \frac{1}{T}\sum_{t=1}^{T}\left|r_t^\top w - \mu^\top w\right|, \tag{2}$$

where $r_t \in \mathbb{R}^N$ denotes the return vector at time t, $w \in \mathbb{R}^N$ is the portfolio weight vector, and $\mu = \frac{1}{T}\sum_{t=1}^{T} r_t$ is the sample mean of returns. T denotes the total number of time periods.

2.1.2 Conditional Value-at-Risk (CVaR) [3]

The Conditional Value-at-Risk (CVaR), also known as Expected Shortfall, measures the average loss in the worst-case $(1-\beta)$ tail of the return distribution. The portfolio-level CVaR at confidence level $\beta \in (0,1)$ is defined as:

$$\mathrm{CVaR}_\beta(w) = \min_{\eta\in\mathbb{R}}\left\{\eta + \frac{1}{(1-\beta)T}\sum_{t=1}^{T}\max\left(-r_t^\top w - \eta, 0\right)\right\}, \tag{3}$$

where η is an auxiliary variable representing the VaR (Value-at-Risk) threshold, and $r_t^\top w$ is the portfolio return at time t. The expression computes the expected loss conditional on exceeding η, averaged over T time periods.

2.2 Marginal and Total Risk Contributions

The concept of marginal and total risk contribution has been formalized in the context of risk budgeting and risk parity, as presented by Roncalli [7].

$$\mathrm{MRC}_i(w) = \frac{\partial \mathcal{R}(w;\alpha)}{\partial w_i} = \alpha \cdot \frac{\partial \mathrm{MAD}(w)}{\partial w_i} + (1-\alpha) \cdot \frac{\partial \mathrm{CVaR}_\beta(w)}{\partial w_i}. \tag{4}$$

MAD gradient:

$$\frac{\partial \mathrm{MAD}(w)}{\partial w_i} = \frac{1}{T} \sum_{t=1}^{T} \mathrm{sign}(r_t^\top w - \mu^\top w) \cdot (r_{t,i} - \mu_i),$$

CVaR gradient:

$$\frac{\partial \mathrm{CVaR}_\beta(w)}{\partial w_i} = \frac{1}{(1-\beta)T} \sum_{t \in T_\beta} (-r_{t,i}),$$

where $T_\beta = \{t \mid -r_t^\top w > \mathrm{VaR}_\beta(w)\}$ is the tail index set.

$$\mathrm{TRC}_i(w) = w_i \cdot \mathrm{MRC}_i(w). \tag{5}$$

To equalize contributions:

$$\mathrm{TRC}_i(w) = \mathrm{TRC}_j(w), \quad \forall i,j \in \{1,\dots,N\}.$$

2.3 Optimization Problem

We minimize the total squared deviation of individual TRCs from their mean:

$$\min_{w \in \mathbb{R}^N} \quad \sum_{i=1}^{N} \left(\mathrm{TRC}_i(w) - \bar{\mathrm{TRC}}(w)\right)^2, \quad \text{where} \quad \bar{\mathrm{TRC}}(w) = \frac{1}{N}\sum_{i=1}^{N} \mathrm{TRC}_i(w), \tag{6}$$

$$\text{subject to} \quad \sum_{i=1}^{N} w_i = 1, \tag{7}$$

$$0 \le w_i \le 0.2, \quad \forall i \in \{1,\dots,N\}. \tag{8}$$

We impose long-only constraints ($w_i \geq 0$) and a maximum position cap ($w_i \leq 0.2$) to prevent excessive concentration in low-risk assets or shorting high-risk components, which may otherwise occur under equal risk contribution conditions.

2.4 Non-Convexity and Motivation for Approximate Convexity

Although the hybrid risk measure $\mathcal{R}(w;\alpha)$ is convex in w for any fixed α, the full optimization problem becomes non-convex due to the structure of the total risk contribution terms:

$$\mathrm{TRC}_i(w) = w_i \cdot \frac{\partial \mathcal{R}(w;\alpha)}{\partial w_i}. \tag{9}$$

This formulation introduces multiplicative interactions between the portfolio weights and gradient components, resulting in a non-convex objective function. The empirical behavior of the marginal risk contribution (MRC) further highlights this issue. As illustrated in Figure 1,

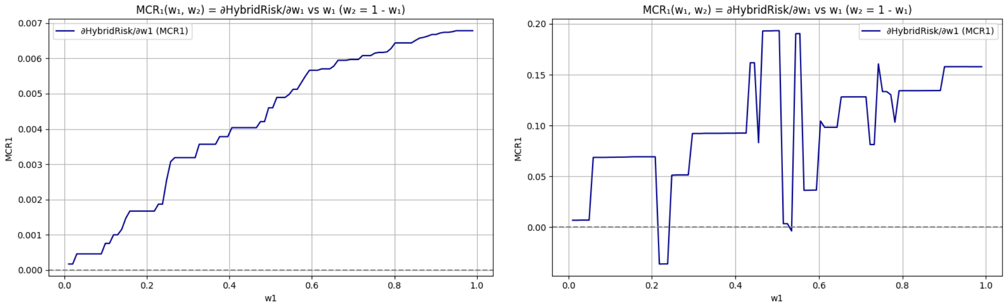

Figure 1: MRC behavior for $\alpha = 1$ (left: MAD) and $\alpha = 0$ (right: CVaR) during w1 is in [0,1]. Nonlinear and non-convex behavior observed in gradient shape.

the MRC curves can exhibit non-smooth and non-monotonic patterns, particularly under the CVaR component ($\alpha = 0$), further affirming the loss of global convexity.

Given this structural non-convexity, directly applying convex optimization techniques is theoretically invalid. However, in practice, many risk-parity-type objectives exhibit desirable local curvature properties near optimal solutions. To enable tractable optimization, we explore whether the objective exhibits *approximate convexity* in a local region around a solution w^*. This analysis is conducted via Taylor series expansion, which informs our use of Sequential Least Squares Programming (SLSQP) — a gradient-based solver that assumes local smoothness and approximate convexity.

2.5 Taylor-Based Approximate Convexity Justification

Step 1: First-Order and Second-Order Expansion. We consider a Taylor expansion of $f(w)$ at w^*:

$$f(w) \approx f(w^*) + \nabla f(w^*)^\top (w - w^*) + \frac{1}{2}(w - w^*)^\top \nabla^2 f(w^*)(w - w^*). \tag{10}$$

If the Hessian $H := \nabla^2 f(w^*)$ is positive semi-definite (i.e., $x^\top Hx \geq 0$ for all x), then $f(w)$ is locally convex around w^*. While H is not guaranteed to be PSD globally, we examine the structure of the components to argue its approximate convexity:

Step 2: Structural Argument from Risk Contributions. Note that each term of $f(w)$ is a squared difference:

$$g_i(w) := (\mathrm{TRC}_i(w) - \bar{\mathrm{TRC}}(w))^2.$$

Each $g_i(w)$ is differentiable and admits second derivatives almost everywhere. Around the solution w^*, we expect that $\mathrm{TRC}_i(w^*) \approx \bar{\mathrm{TRC}}(w^*)$ for all i, hence the gradient of g_i vanishes:

$$\nabla g_i(w^*) \approx 0.$$

Moreover, the second-order derivatives $\nabla^2 g_i(w^*)$ are positive semi-definite due to the quadratic form of g_i. Therefore, the Hessian of the total objective $f(w)$,

$$\nabla^2 f(w^*) = \sum_{i=1}^{N} \nabla^2 g_i(w^*),$$

is approximately positive semi-definite.

Step 3: Approximate Convexity Condition. Hence, within a sufficiently small neighborhood around w^*, the function $f(w)$ satisfies:

$$f(w) \geq f(w^*) + \nabla f(w^*)^\top (w - w^*),$$

which is the first-order condition for convexity. The approximation error depends on the degree of smoothness of $\mathcal{R}(w;\alpha)$ and regularity of $\mathrm{TRC}_i(w)$ near w^*.

Conclusion. Although the objective function $f(w)$ is not globally convex, its local structure around feasible optima is sufficiently smooth and exhibits approximate positive curvature, largely due to the quadratic form of TRC-based squared deviations. This local behavior justifies the use of gradient-based optimization methods that assume local convexity. In particular, we adopt the Sequential Least Squares Programming (SLSQP) algorithm, which effectively handles smooth, constrained nonlinear objectives; details are provided in Appendix A.1.

2.6 Dynamic Adjustment of α

To enhance adaptability under different market regimes, we allow the mixing parameter α to vary dynamically over time based on recent volatility. This enables the portfolio to shift its emphasis between MAD (during stable markets) and CVaR (during turbulent periods).

Let σ_t denote the rolling realized volatility over the past w days. The time-varying α_t is computed as:

$$\alpha_t = \mathrm{SMA}_k \left(\alpha_{\max} - (\alpha_{\max} - \alpha_{\min}) \cdot \frac{\sigma_t - \min(\sigma)}{\max(\sigma) - \min(\sigma)} \right), \tag{11}$$

where $\mathrm{SMA}_k(\cdot)$ denotes a k-day moving average for smoothing, and $\alpha_{\min}, \alpha_{\max} \in [0,1]$ (e.g., 0.1 and 0.9).

3 Empirical Study

This empirical study investigates the performance of various risk-based portfolio construction methods, with a focus on the proposed Hybrid ERC framework, which integrates Mean Absolute Deviation (MAD) and Conditional Value-at-Risk (CVaR) into a unified risk parity model. Two horizons are considered: a short-term window (2020–2025) and a long-term window (2007–2025), both using diversified ETF portfolios.

3.1 Dataset and Setup

The dataset comprises daily returns of 11 ETFs—SPY, IWM, EEM, TLT, LQD, HYG, SHY, GLD, DBC, VNQ, and QQQ—spanning equities, bonds, commodities, and real estate. These assets are commonly used in All Weather strategies. Parameters are set as follows:

- Short-term backtest: 2020/01/01 to 2025/05/31
- Long-term backtest: 2007/01/01 to 2025/05/31
- CVaR confidence level: 95%
- Initial portfolio value: \$1
- Comparison focus: fixed $\alpha = 0.5$ vs. dynamic α

3.2 Benchmarked Strategies

The following strategies are evaluated:

- **Hybrid ERC (Dynamic α)**: Combines MAD and CVaR using a volatility-sensitive, time-varying α.
- **Hybrid ERC** ($\alpha = 0.5$): A fixed-weight hybrid baseline equally weighting MAD and CVaR.
- **MAD-ERC** ($\alpha = 1.0$) and **CVaR-ERC** ($\alpha = 0.0$): ERC portfolios based solely on MAD or CVaR.
- **Min-MAD** and **Min-CVaR**: Traditional risk minimization portfolios.
- **Equal Weight**: Naïve $1/N$ allocation.

All strategies are implemented under a rolling-window framework. Both MAD and CVaR are estimated using a 60-day rolling window, which ensures timely adaptation to recent market conditions.

For the Hybrid ERC (Dynamic α) strategy, α is updated every 60 trading days based on normalized volatility within the same rolling window. A 20-day simple moving average is applied to smooth the dynamic α values. At each update point, a line search is conducted to identify the optimal α that minimizes the ERC objective.

3.3 Short-Term Performance (2020–2025)

During the short-term horizon, we observe notable differences across strategies in both cumulative return and drawdown behavior. Figures 2 and 3 show that while the Equal Weight strategy yields the highest terminal value, it also experiences the largest drawdowns and greatest volatility. In contrast, Hybrid ERC ($\alpha = 0.5$) delivers the most favorable risk-adjusted return, evidenced by its superior Sharpe ratio and smoother equity curve. The dynamic-α variant exhibits a modest sacrifice in cumulative return but benefits from noticeably lower drawdown during volatile periods, such as in 2022.

Compared to the Min-MAD and Min-CVaR strategies, Hybrid ERC strategies demonstrate more balanced performance—avoiding excessive conservativeness while still mitigating downside risks. ERC-MAD and ERC-CVaR offer improvements over their minimization counterparts but still underperform the hybrid structure in terms of Sharpe ratio. This highlights the benefit of blending mid-range and tail-risk measures.

Figure 4 further validates these observations. Hybrid ERC ($\alpha = 0.5$) achieves the highest Sharpe ratio, while the dynamic strategy provides a risk-buffering effect during high-volatility windows. As shown in Figure 5, the dynamic α value declines during turbulent episodes, shifting emphasis toward CVaR, and increases during calmer periods, favoring MAD. This volatility-sensitive adjustment improves robustness without drastically compromising return.

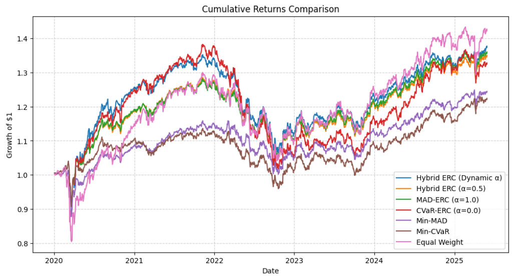

Figure 2: Cumulative returns (2020–2025): Hybrid ERC variants vs. benchmarks.

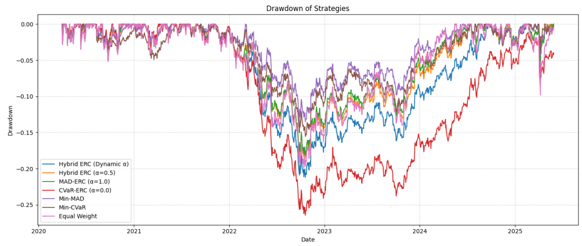

Figure 3: Drawdown comparison (2020–2025) across strategies.

	Annual Return	Volatility	Sharpe	Max Drawdown
Hybrid ERC (Dynamic α)	0.0661	0.0895	0.7371	0.2119
Hybrid ERC (α=0.5)	0.0617	0.0829	0.7436	0.1852
MAD-ERC (α=1.0)	0.0630	0.0819	0.7686	0.1817
CVaR-ERC (α=0.0)	0.0595	0.0948	0.6264	0.2643
Min-MAD	0.0435	0.0759	0.5710	0.1361
Min-CVaR	0.0404	0.0759	0.5310	0.1554
Equal Weight	0.0739	0.1264	0.5837	0.2144

Figure 4: Performance metrics (2020–2025): Annualized return, volatility, Sharpe ratio, and max drawdown.

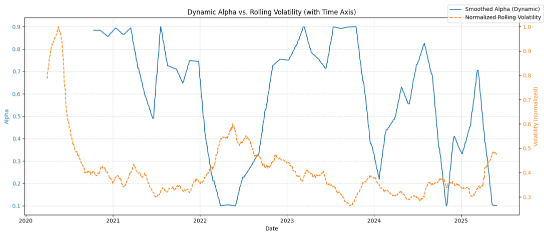

Figure 5: Dynamic α vs. normalized rolling volatility (2020–2025).

3.4 Long-Term Performance (2007–2025)

Over the long-term horizon, similar patterns persist. Figures 6 and 7 indicate that although Equal Weight generates the highest cumulative return, it remains vulnerable to large drawdowns during market crises such as the 2008 global financial crisis, the 2020 COVID crash, and the 2022 rate-hike cycle. Hybrid ERC ($\alpha = 0.5$) again demonstrates a superior risk-return profile, with more stable growth and improved downside control.

Relative to Min-MAD and Min-CVaR, the hybrid structure avoids extreme conservativeness while still taming drawdowns. Compared to ERC-MAD and ERC-CVaR, the hybrid approaches yield more balanced allocations, reducing sensitivity to either mid-range fluctuations or tail events alone. This advantage becomes increasingly evident over longer cycles that include multiple stress periods.

As summarized in Figure 8, Hybrid ERC ($\alpha = 0.5$) retains the highest Sharpe ratio, while the dynamic-α version delivers consistent stability with lower tail risk exposure. Figure 9 shows that the volatility-responsive adjustment mechanism remains effective even over longer timeframes, smoothly shifting portfolio emphasis in accordance with rolling risk conditions.

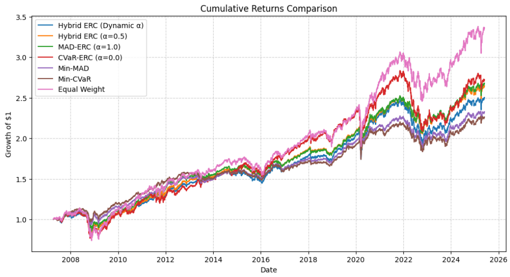

Figure 6: Cumulative returns (2007–2025): Long-run strategy comparison.

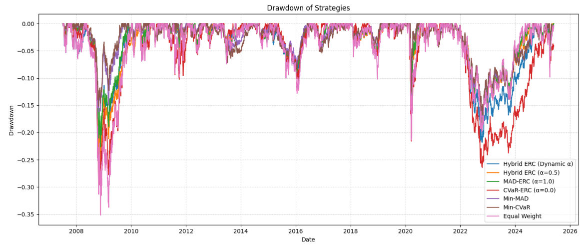

Figure 7: Drawdown trajectories (2007–2025) across strategies.

	Annual Return	Volatility	Sharpe	Max Drawdown
Hybrid ERC (Dynamic α)	0.0546	0.0807	0.6759	0.2415
Hybrid ERC (α=0.5)	0.0575	0.0775	0.7407	0.2562
MAD-ERC (α=1.0)	0.0581	0.0760	0.7632	0.2264
CVaR-ERC (α=0.0)	0.0608	0.0968	0.6266	0.2972
Min-MAD	0.0487	0.0634	0.7658	0.1720
Min-CVaR	0.0471	0.0627	0.7490	0.1674
Equal Weight	0.0745	0.1221	0.6093	0.3517

Figure 8: Performance metrics (2007–2025): Annualized return, volatility, Sharpe ratio, and max drawdown.

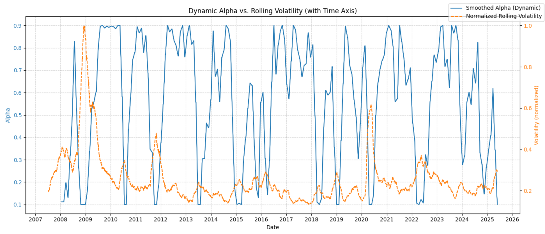

Figure 9: Dynamic α vs. normalized rolling volatility (2007–2025).

3.5 Summary of Results

Overall, empirical results suggest that the proposed Hybrid ERC model offers improved risk-adjusted performance compared to traditional risk-minimization and equal-weight strategies. The fixed $\alpha = 0.5$ version consistently delivers the highest Sharpe ratio, while the dynamic version offers enhanced drawdown control and robustness to regime shifts.

We additionally conducted a robustness test using Taiwan's stock market. A portfolio consisting of the top 10 stocks by market capitalization in the Taiwan market over the same periods shows consistent conclusions: Hybrid ERC strategies continue to outperform on a Sharpe-adjusted basis, with dynamic α adaptations yielding improved drawdown resilience even under different market structures and regional dynamics.

4 Conclusion and Future Work

This study proposes a novel *Hybrid Equal Risk Contribution (Hybrid ERC)* framework that integrates Mean Absolute Deviation (MAD) and Conditional Value-at-Risk (CVaR) into a unified portfolio risk allocation model. Unlike traditional ERC models that rely solely on volatility or a single risk metric, our hybrid approach flexibly balances mid-range and tail risk via a convex combination governed by a dynamic mixing parameter α.

A key contribution of this work lies in identifying that, although the total risk contribution (TRC) objective is inherently non-convex due to multiplicative interactions between weights and marginal risk contributions, the Hybrid ERC formulation exhibits *approximate convexity* in practice. Through a second-order Taylor expansion around the optimal solution, we show that the objective's local curvature tends toward positive semi-definiteness, particularly under well-behaved MAD and CVaR components. This empirical observation enables the effective use of Sequential Least Squares Programming (SLSQP) for optimization, without requiring global convexity guarantees.

Empirical results across both short-term (2020–2025) and long-term (2007–2025) U.S. asset portfolios demonstrate the advantage of our method. The Hybrid ERC with fixed $\alpha = 0.5$ achieves the best Sharpe ratio among all benchmarks, including Min-MAD, Min-CVaR, and pure ERC variants. The dynamic α strategy, which adjusts according to realized volatility through a rolling-window line search mechanism, further enhances robustness by reducing drawdowns during turbulent regimes.

Future Work

Potential future research directions include:

- Extending the hybrid framework to incorporate higher-order moments, such as skewness and kurtosis, for more comprehensive risk modeling.
- Generalizing the approximate convexity approach to more complex hybrid risk objectives beyond MAD and CVaR.
- Investigating hybrid frameworks that combine risk parity with risk minimization principles, such as integrating ERC with minimum variance objectives.
- Analyzing the sensitivity of portfolio weights to the frequency of α updates, and identifying thresholds to avoid excessive turnover.
- Exploring nonlinear combinations of MAD and CVaR, recognizing their fundamentally distinct risk perspectives.

In conclusion, the Hybrid ERC model offers a practical and theoretically grounded advancement in risk-based portfolio construction, enabling more resilient and adaptive asset allocation in increasingly uncertain markets.

A Appendix

A.1 Optimization Algorithm: Sequential Least Squares Programming (SLSQP)

We adopt the Sequential Least Squares Programming (SLSQP) algorithm to solve the Hybrid ERC optimization problem, which involves a nonlinear objective function subject to both equality and bound constraints.

SLSQP belongs to the family of Sequential Quadratic Programming (SQP) methods, and at each iteration, it solves a quadratic programming (QP) subproblem derived from a second-order Taylor expansion of the objective function:

$$f(w + \Delta w) \approx f(w) + \nabla f(w)^\top \Delta w + \frac{1}{2}\Delta w^\top H \Delta w,$$

where H denotes the Hessian approximation.

The QP subproblem is then:

$$\min_{\Delta w} \quad \frac{1}{2}\Delta w^\top H \Delta w + \nabla f(w)^\top \Delta w \quad \text{s.t.} \quad h_i(w) + \nabla h_i(w)^\top \Delta w = 0, \quad g_j(w) + \nabla g_j(w)^\top \Delta w \geq 0.$$

Lagrangian multipliers are used to incorporate constraints, and line search is performed along the QP step:

$$w_{k+1} = w_k + \alpha_k \Delta w,$$

with convergence criteria such as:

$$\|\nabla f\| < 10^{-6}, \quad |f_{k+1} - f_k| < 10^{-8}.$$

This method is particularly suitable for locally smooth, approximately convex optimization problems such as our hybrid ERC formulation.

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