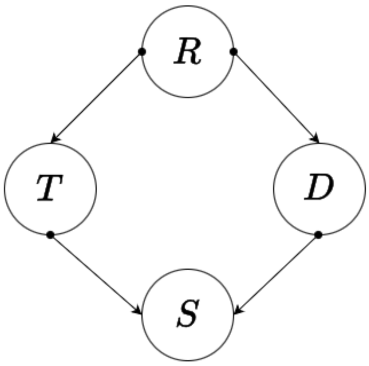
CS461 Section #:	
Name:	
NetID:	

CS461 Quiz Four

0. True / False questions.

- In a Bayesian network, a direct edge from A to B indicates a causal and effect relationship between A and B. (True / **False**)
- A joint density has a unique Bayesian network representation. (True / **False**)
- Given a Bayesian network (structure and conditional probability tables), we can quantify any probability of interest within the defined model. (**True** / False)
- When A and B are conditionally independent given C, we can say A and B do not have a direct causal and effect relationship. (**True** / False)
- In the EM algorithm, the parameters updated at each iteration are guaranteed to monotonically increase the log-likelihood of the observed data. (**True** / False)
- K-means is a discriminative algorithm. (True / **False**)

1. Choose all true statements.



1 $T \perp\!\!\!\perp D$ (False)

2 $T \perp\!\!\!\perp D | R$ (True)

3 $T \perp\!\!\!\perp D | S$ (False)

4 $T \perp\!\!\!\perp D | R, S$ (False)

5 $R \perp\!\!\!\perp S$ (False)

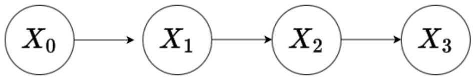
6 $R \perp\!\!\!\perp S | T$ (False)

7 $R \perp\!\!\!\perp S | D$ (False)

8 $R \perp\!\!\!\perp S | T, D$ (True)

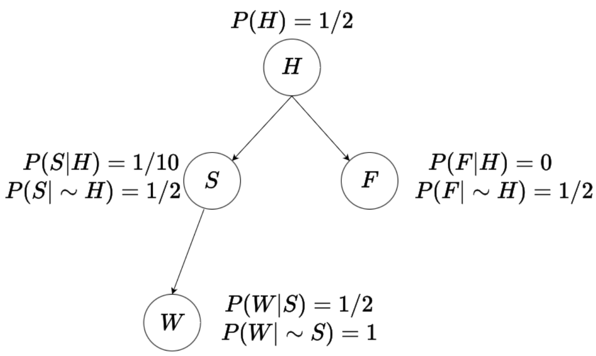
2. (25) Given the following Bayesian network, compute conditional density $P[X_2|X1=+,X3=-]$.

$$\boldsymbol{P[X_0] = 1/3}$$
$$\boldsymbol{P[X_{i+1} = +|x_i = +] = 2/3}$$
$$\boldsymbol{P[X_{i+1} = +|x_i = -] = 1/4}$$



$$\begin{aligned}
P[X_2|X1=+,X3=-] &= \alpha P[X_1=+,X_2,X3=-] \\
&= \alpha P[X_1=+]P[X_2|X_1=+]P[X_3=-|X_2] \\
&= \alpha \begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix} \begin{bmatrix} 1/3 \\ 3/4 \end{bmatrix} \\
&= \alpha \begin{bmatrix} 2/9 \\ 3/12 \end{bmatrix} \qquad\qquad \alpha = 108/51 \\
&= \begin{bmatrix} 24/51 \\ 27/51 \end{bmatrix}
\end{aligned}$$

3. (25) Given the following Bayesian network what is P(W,F)?



$$P(w+, F+) = \sum_{H,S} P(H) \cdot P(S \mid H) \cdot P(F+ \mid H) \cdot P(\overset{+}{w} \mid S)$$

$$= \sum_{H} \underbrace{P(H)}_{f_4(H)} \cdot \underbrace{P(F+ \mid H)}_{f_5(H)} \cdot \underbrace{\sum_{S} \cdot \underbrace{P(S \mid H)}_{f_1(S,H)} \cdot \underbrace{P(w+ \mid S)}_{f_2(S)}}_{f_3(H)}$$

① $f_1(S,H) =$

S \ H	+	−
+	$1/10$	$1/2$
−	$9/10$	$1/2$

② $f_2(S)$

	S
+	$1/2$
−	1

③ $f_3(H) = \frac{1}{2} \cdot \begin{bmatrix} 1/10 \\ 1/2 \end{bmatrix} + 1 \begin{bmatrix} 9/10 \\ 1/2 \end{bmatrix} = \begin{bmatrix} 19/20 \\ 3/4 \end{bmatrix}$

④ $f_4(H) = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$ ⑤ $f_5(H) = \begin{bmatrix} 0 \\ +1/2 \end{bmatrix}$

⑥ $P(w+, F+) = \cancel{19/20 \cdot 1/2 \cdot 0} + 3/4 \cdot 1/2 \cdot 1/2$

$= 3/16.$