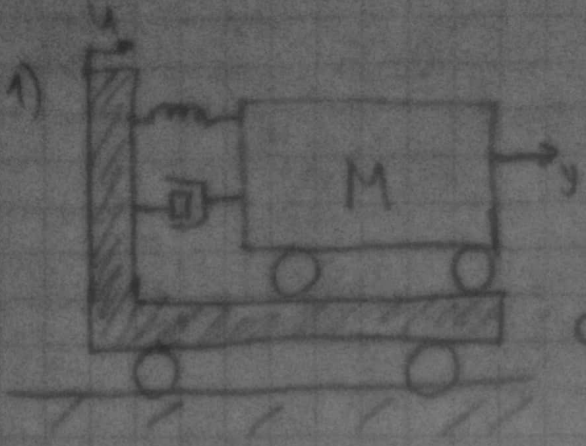
1)



Carro sin masa

$$\frac{m\,d^2y}{dt} = -b\left(\frac{dy}{dt} - \frac{du}{dt}\right) - K(y-u) = \frac{m\,d^2y}{dt} + b\frac{dy}{dt} + Ky = b\frac{du}{dt} + Ku$$

Transformada de la place.

$$(ms^2 + bs + K)\,Y(s) = (bs + K)\,U(s)$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{bs + K}{ms^2 + bs + K}$$ Función de transferencia.

Despejamos para espacios de estados.

$$\ddot{y} + \frac{b}{m}\dot{y} + \frac{K}{m}y = \frac{b}{m}\dot{u} + \frac{K}{m}u$$

$$a_1 = \frac{b}{m}, \quad a_2 = \frac{K}{m} \quad b_0 = 0 \quad b_1 = \frac{b}{m} \quad b_2 = \frac{K}{m}$$

$$\ddot{y} + a_1\dot{y} + a_2 y = b_0\ddot{u} + b_1\dot{u} + b_2 u$$

$$\beta_0 = b_0 = 0\,; \quad \beta_1 = b_1 - a_1\beta_0 = \frac{b}{m}$$

$$\beta_1 = b_2 - a_1\beta_1 - a_2\beta_0 = \frac{K}{m} - \left(\frac{b}{m}\right)^2$$

$$x_1 = y - \beta_0 u = y$$

$$x_2 = \dot{x}_1 - \beta_1 u = \dot{x}_1 - \frac{b}{m}u$$

$$\dot{x}_2 = -a_2x_1 - a_1x_2 + \beta_2 u = -\frac{K}{m}x_1 - \frac{b}{m}x = \left[\frac{K}{m} - \left(\frac{b}{m}\right)^2\right]u$$

$$y = x_1.$$

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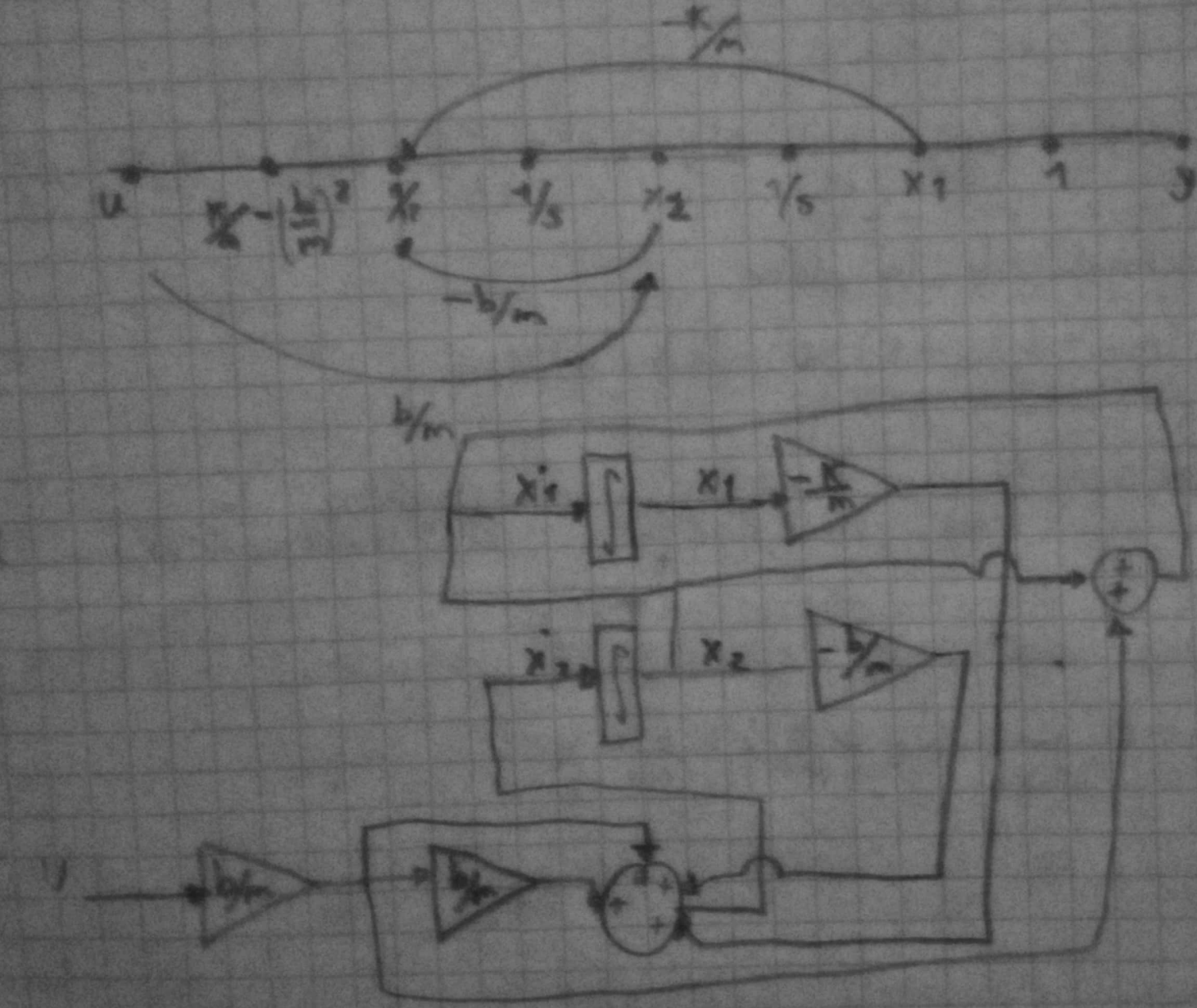
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{K}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{m} \\ \frac{K}{m} \cdot \left(\frac{1}{m}\right)^2 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



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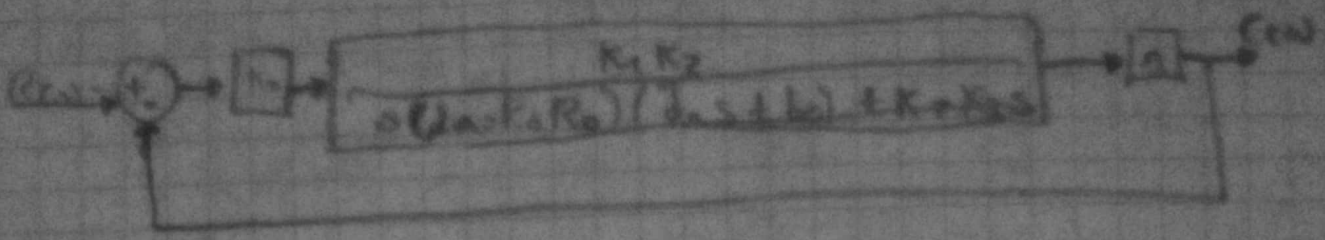
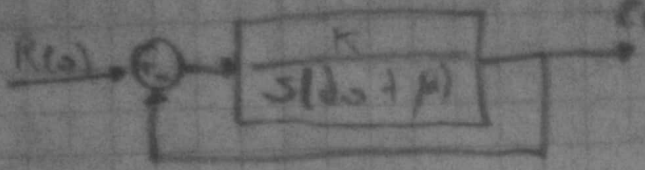
2) Servo motor

$e = r - c$ $\quad T = K_2 i_a$ $\quad e_b = K_3 \frac{d\theta}{dt}$





$$L_a \frac{di_a}{dt} + R_a i_a + K_3 \frac{d\theta}{dt} = K_1 e_i$$

$$L_a \frac{di_a}{dt} + R_a i_a + e_b = e_a$$

$$J_o \frac{d^2\theta}{dt^2} + b_o \frac{d\theta}{dt} = T = K_2 i_a$$

$$\frac{\theta(s)}{\theta_i(s)} = \frac{K_1 K_2}{s(L_a s + R_a)(J_o s + b_o) + K_2 K_3 s}$$

$$C(s) = n\theta(s)$$
$$Q(s) = K_a [R(s) - C(s)] = K_a E(s)$$

$$G(s) = \frac{\frac{K_a K_1 K_2 n}{R_a}}{J_o s^2 + \left(b_o + \frac{K_2 K_3}{R_a}\right) s}$$

$$\frac{Y(s)}{X(s)} = \frac{\frac{K_a K_1 K_2 n}{R_a}}{J_o s^2 + \left(b_o + \frac{K_2 K_3}{R_a}\right) s}$$

$$\left(J_o s^2 + \left(b_o + \frac{K_2 K_3}{R_a}\right) s\right) Y(s) = \left(\frac{K_a K_1 K_2 n}{R_a}\right) X(s)$$

$$J_o \ddot{y} + \left(b_o + \frac{K_2 K_3}{R_a}\right) \dot{y} = \frac{K_2 K_1 K_a n}{R_a} u$$

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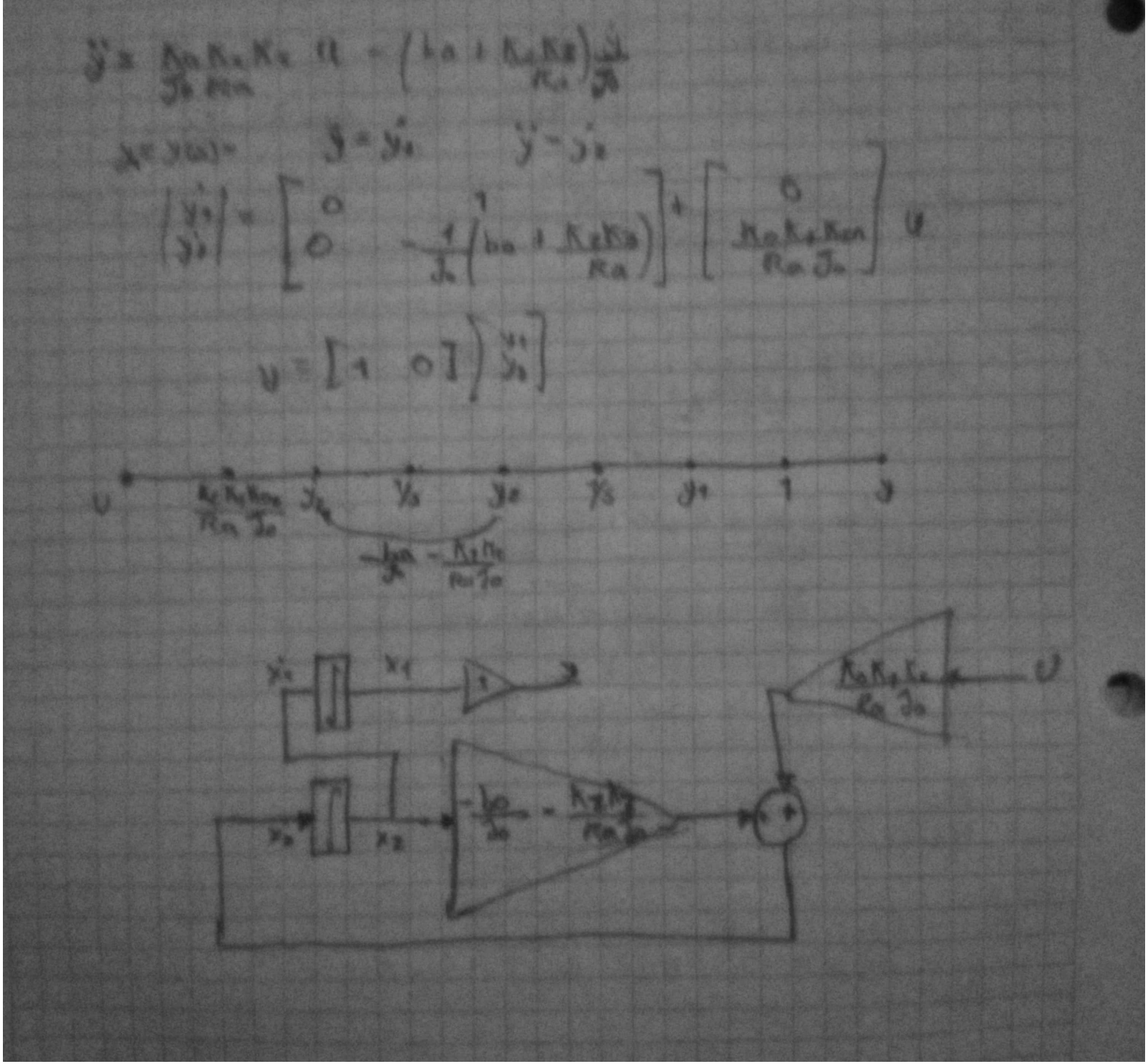


3) Motor DC con carga

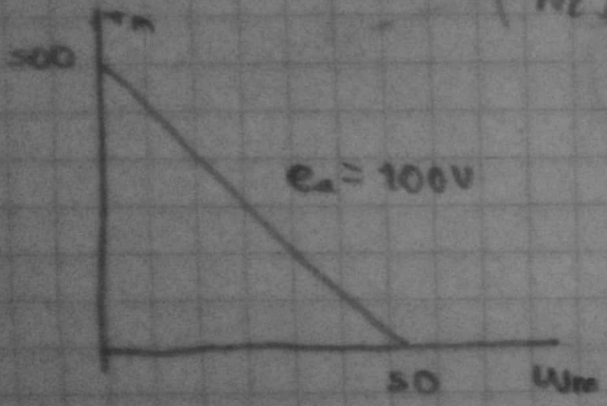
$T_{stall} = 500$ $\quad \omega_{no-load} = 50$ $\quad e_a = 100$

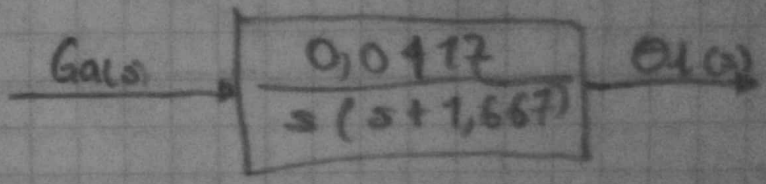
$K_2 = 800$ Nm/load $\quad N_2 = 1000$ $\quad D_m = 2$ Nm s/rad

$J_m = 5$ Kg m²

$$J_m = J_a + J_2 \left(\frac{N_1}{N_2}\right)^2 = 5 + 700\left(\frac{1}{100}\right)^2 = 12$$

$$D_m = D_a + D_1 \left(\frac{N_1}{N_2}\right)^2 = 2 + 800\left(\frac{1}{10}\right)^2 = 10$$





$$\frac{K_1}{K_2} = \frac{T_s J_a}{e_a} = \frac{500}{100} = 5$$

$$K_b = \frac{e_a}{\omega_{no-load}} = \frac{100}{50} = 2$$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{1/12}{s\left(s + \frac{1}{12}[10 + (5)(2)]\right)} = \frac{0,417}{s(s+1,667)} = \frac{\theta_m - N_1}{E_a(s)\ N_2}$$

$$\theta_m(s)\left(s^2 + 1,667s\right) = 0,0417\, E_a(s)$$

$$\ddot{y} + 1,667\dot{y} = 0,0417u$$

$$y_1 = y \qquad y_2 = \dot{y}$$

$$\dot{y}_1 = y_2$$

$$\ddot{y} = 0,0417u - 1,667\dot{y}$$

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -1,667 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0,0417 \end{bmatrix} u$$

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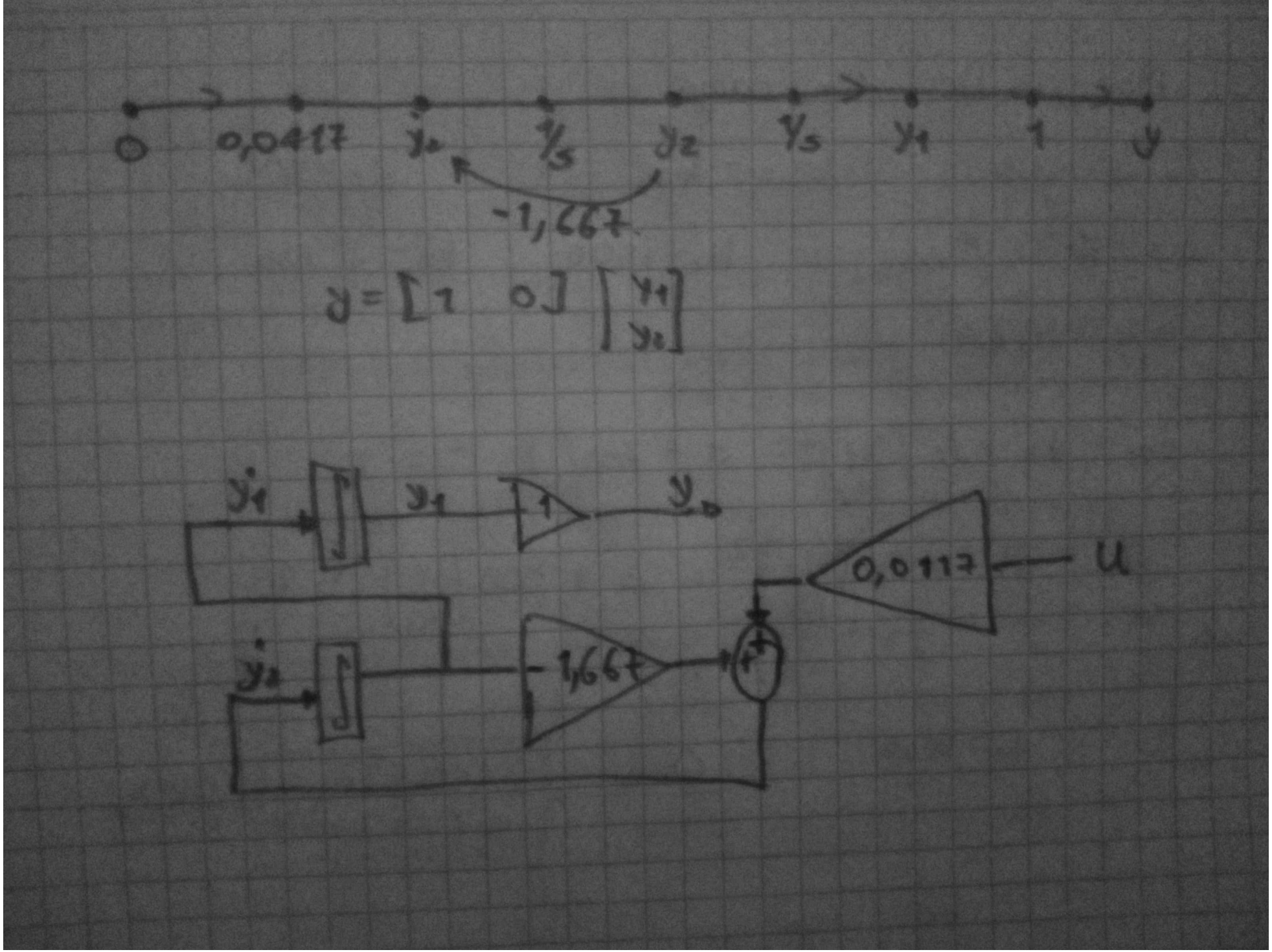

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