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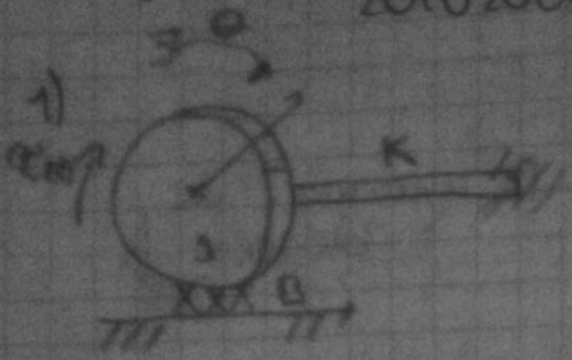
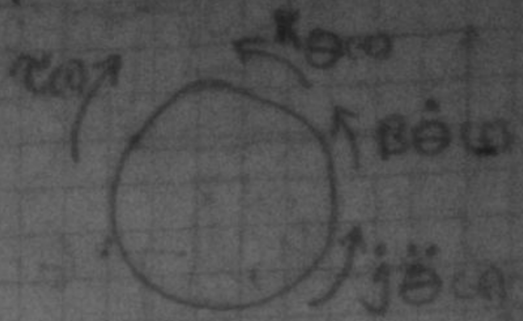
20182005021

1)





$$\tau(t) - K\theta(t) - B\dot{\theta}(t) - J\ddot{\theta}(t) = 0$$

$$\tau(t) = K\theta(t) + B\dot{\theta}(t) + J\ddot{\theta}(t)$$

$$\theta(s)(Js^2 + Bs + K) = \tau(s)$$

$$\frac{\theta(s)}{\tau(s)} = \frac{1}{Js^2 + Bs + K}$$ Funcion de transferencia

$q_1 = \theta$; $q_2 = \dot{q}_1 = \dot{\theta}$; $\dot{q}_2 = \ddot{\theta}$

$$\tau = Kq_1 + Bq_2 + J\dot{q}_2$$

$$\dot{q}_2 = \frac{\tau}{J} - \frac{Kq_1}{J} - \frac{Bq_2}{J}$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{K}{J} & -\frac{B}{J} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/J \end{bmatrix} \tau$$

$$\theta = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

Espacios de estados

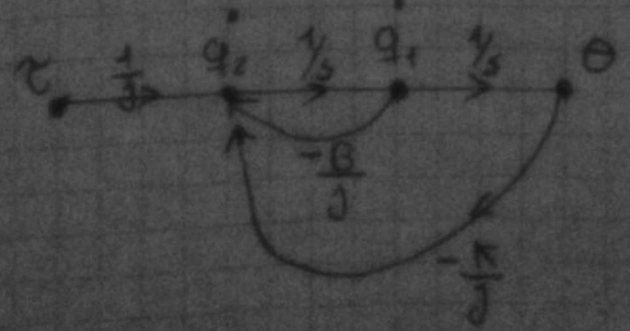


Diagrama de Flujo

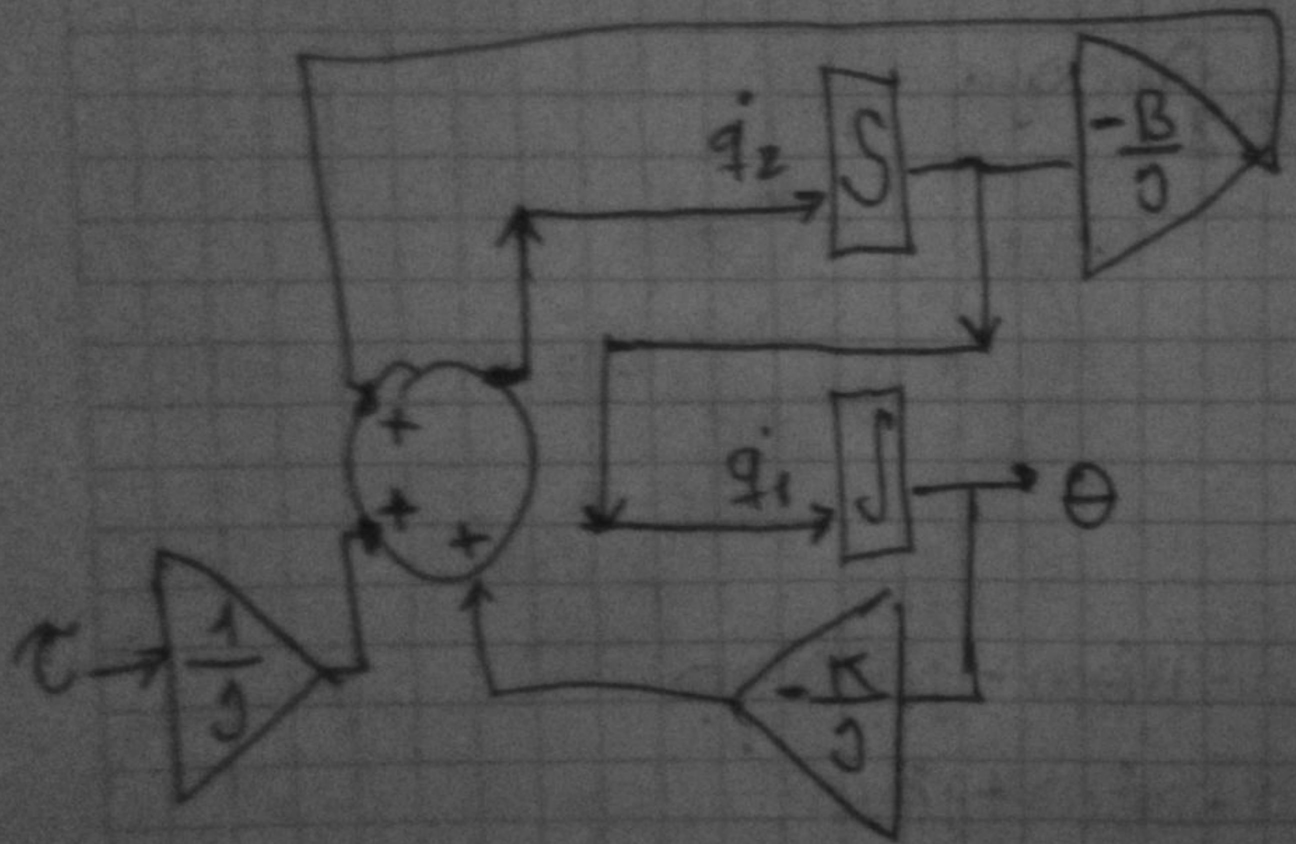
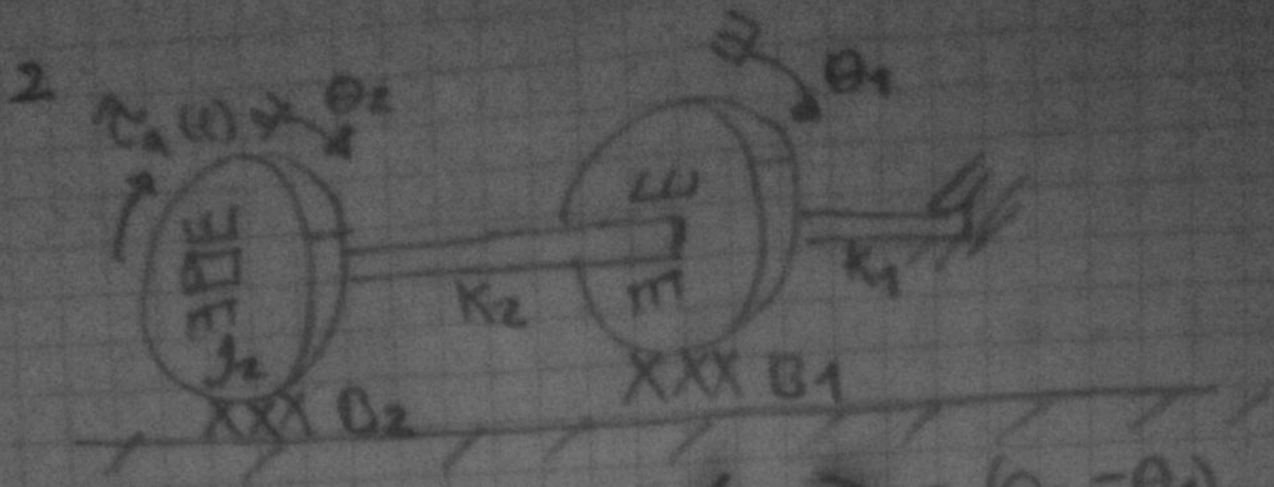
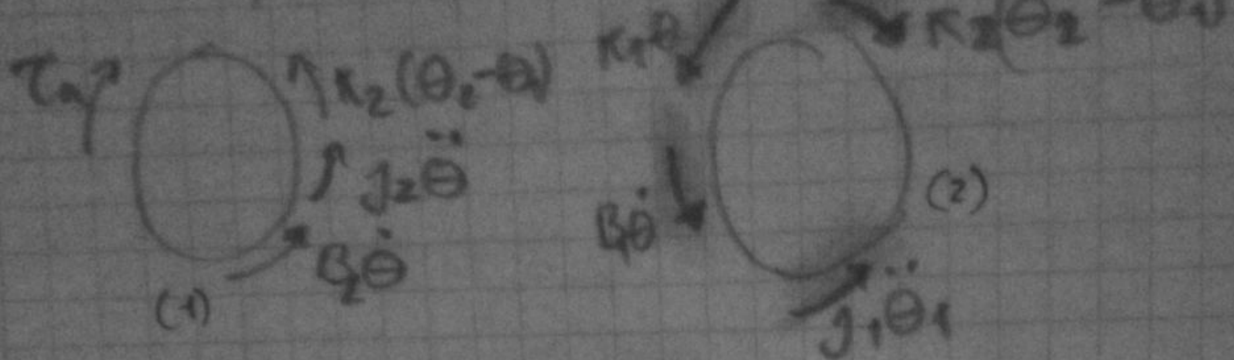


Diagrama de bloques

2





$$\tau_a - K_2(\theta_2-\theta_1) - B_2\dot\theta_2 - J_2\ddot\theta_2 = 0 \quad (1)$$

$$K_2(\theta_2-\theta_1) - K_1\theta_1 - B\dot\theta_1 - J_1\ddot\theta_1 = 0 \quad (2)$$

$$\tau_a = K_2(\theta_2-\theta_1) + B_2\dot\theta_2 + J_2\ddot\theta_2 \quad (1)$$

$$0 = (K_1+K_2)\theta_1 + B_1\dot\theta_1 + J_1\ddot\theta_1 - K_2\theta_2 \quad (2)$$

Despejamos θ_1 de la ecuacion (1)

$$K_2\theta_1 + \tau_a = K_2\theta_2 + B_2\dot\theta_2 + J_2\ddot\theta_2$$

$$\theta_1 = \theta_2 + \frac{B_2}{K_2}\dot\theta_2 + \frac{J_2}{K_2}\ddot\theta - \frac{\tau_a}{K_2} \quad (3)$$

Reemplazo (3) en (2)

$$(K_1+K_2)\left(\theta_2 + \frac{B_2}{K_2}\dot\theta_2 + \frac{J_2}{K_2}\ddot\theta_2 - \frac{\tau_a}{K_2}\right) \quad \text{Termino 1}$$

$$B_1\left(\dot\theta_2 + \frac{B_2}{K_2}\ddot\theta_2 + \frac{J_2}{K_2}\dddot\theta_2\right) \quad \text{termino 2}$$

$$J_1\left(\ddot\theta_2 + \frac{B_2}{K_2}\dddot\theta_2 + \frac{J_2}{K_2}\ddddot\theta_2\right) \quad \text{Termino 3}$$

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reemplazo (3) en (2)

$$(K_1+K_2)\left(\theta_2 + \frac{B_2}{K_2}\dot{\theta}_2 + \frac{J_2}{K_2}\ddot{\theta}_2 - \frac{\tau_a}{K_2}\right) + \ldots$$

$$B_1\left(\dot{\theta}_2 + \frac{B_2}{K_2}\ddot{\theta}_2 + \frac{J_2}{K_2}\dddot{\theta}_2 - \frac{\dot{\tau}_a}{K_2}\right) + \ldots$$

$$J_1\left(\ddot{\theta}_2 + \frac{B_2}{K_2}\dddot{\theta}_2 + \frac{J_2}{K_2}\ddddot{\theta}_2 - \frac{\ddot{\tau}_a}{K_2}\right) - K_2\theta_2 = 0$$

Simplificando ecuacion

$$(K_1+K_2)\theta_2 + \frac{K_1 B_2}{K_2}\dot{\theta}_2 + B_2\dot{\theta}_2 + \frac{K_1}{K_2}J_2\ddot{\theta}_2 + J_2\ddot{\theta}_2 - \tau_a\frac{K_1}{K_2} - \tau_a \ldots +$$

$$\ldots + B_1\dot{\theta}_2 + \frac{B_1B_2}{K_2}\ddot{\theta}_2 + \frac{J_2}{K_2}B_1\dddot{\theta}_2 - \dot{\tau}_a\frac{B_1}{K_2} + J_1\ddot{\theta}_2 + J_1\frac{B_2}{K_2}\dddot{\theta}_2 + \ldots$$

$$\ldots + \frac{J_1J_2}{K_2}\ddddot{\theta}_2 - J_1\frac{\ddot{\tau}_a}{K_2} = 0$$

Factorizando (Todo se dividio por K_2)

$$\tau_a(K_1+K_2+B_1\dot{\theta} + J_1\ddot{\theta}_2) = K_1K_2\theta_2 + (B_1K_2+B_2K_1+B_2K_2)\dot{\theta}_2 + \ldots$$

$$\ldots + (J_2K_2+J_2K_1+J_2K_2+B_1B_2)\ddot{\theta}_2 + (J_1B_2+J_2B_1)\dddot{\theta}_2 + J_1J_2\ddddot{\theta}_2$$

Se realiza transformada de LaPlace

$$\tau_a(s)(K_1+K_2+B_1s+J_1s^2)$$

$$\theta_2(s)\left(\begin{array}{l} J_1J_2s^4 + (J_1B_2+J_2B_1)s^3 + (J_1K_2+J_2K_1+J_2K_2+B_1B_2)s^2 + \ldots \\ \ldots + (B_1K_2+B_2K_1+B_2K_2)s + (K_1K_2) \end{array}\right)$$

$$\frac{\theta_2(s)}{\tau_a(s)} = \frac{J_1s^2 + B_1s + K_1 + K_2}{\left(\begin{array}{l} J_1J_2s^4 + (J_1B_2+J_2B_1)s^3 + (J_1K_2+J_2K_1+J_2K_2+B_1B_2)s^2 + (B_1K_2 \ldots + \\ \ldots + B_2K_1+B_2K_2)s + K_1K_2 \end{array}\right)}$$

Funcion de transferencia

$$\theta_2 = \theta$$

$$q_1 = \theta \; ; \; \dot{q}_1 = q_2 = \dot{\theta} \; ; \; \dot{q}_2 = q_3 = \ddot{\theta} \; ; \; \dot{q}_3 = q_4 = \dddot{\theta} \; ; \; \dot{q}_4 = \ddddot{\theta}$$

$$\dot{q}_4 = \tau_a\left(\frac{K_1+K_2+B_1 q_2+J_1 q_3}{J_1 J_2}\right) + \left(\frac{J_1B_2+J_2B_1}{J_1J_2}\right) q_4 + \left(\frac{J_1K_2+J_2K_1+J_1K_2+B_1B_2}{J_1J_2}\right) q_3 \ldots +$$

$$\ldots + \left(\frac{B_1K_1+B_2K_1+B_2K_2}{J_1J_2}\right) q_2 + \left(\frac{K_1K_2}{J_1J_2}\right) q_1$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \left(\frac{K_1K_2}{J_1J_2}\right) & \left(\frac{B_1K_1+B_2K_1+B_2K_2}{J_1J_2}\right) & \left(\frac{J_1K_2+J_2K_1+J_2K_2+B_1B_2}{J_1J_2}\right) & \left(\frac{J_1B_2+J_2B_1}{J_1J_2}\right) \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

$$+ \begin{bmatrix} \frac{K_1+K_2}{J_1J_2} \\ \frac{B_1}{J_1J_2} \\ 1/J_2 \\ 0 \end{bmatrix} \tau_a$$

$$\theta_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

Espacios de Estados

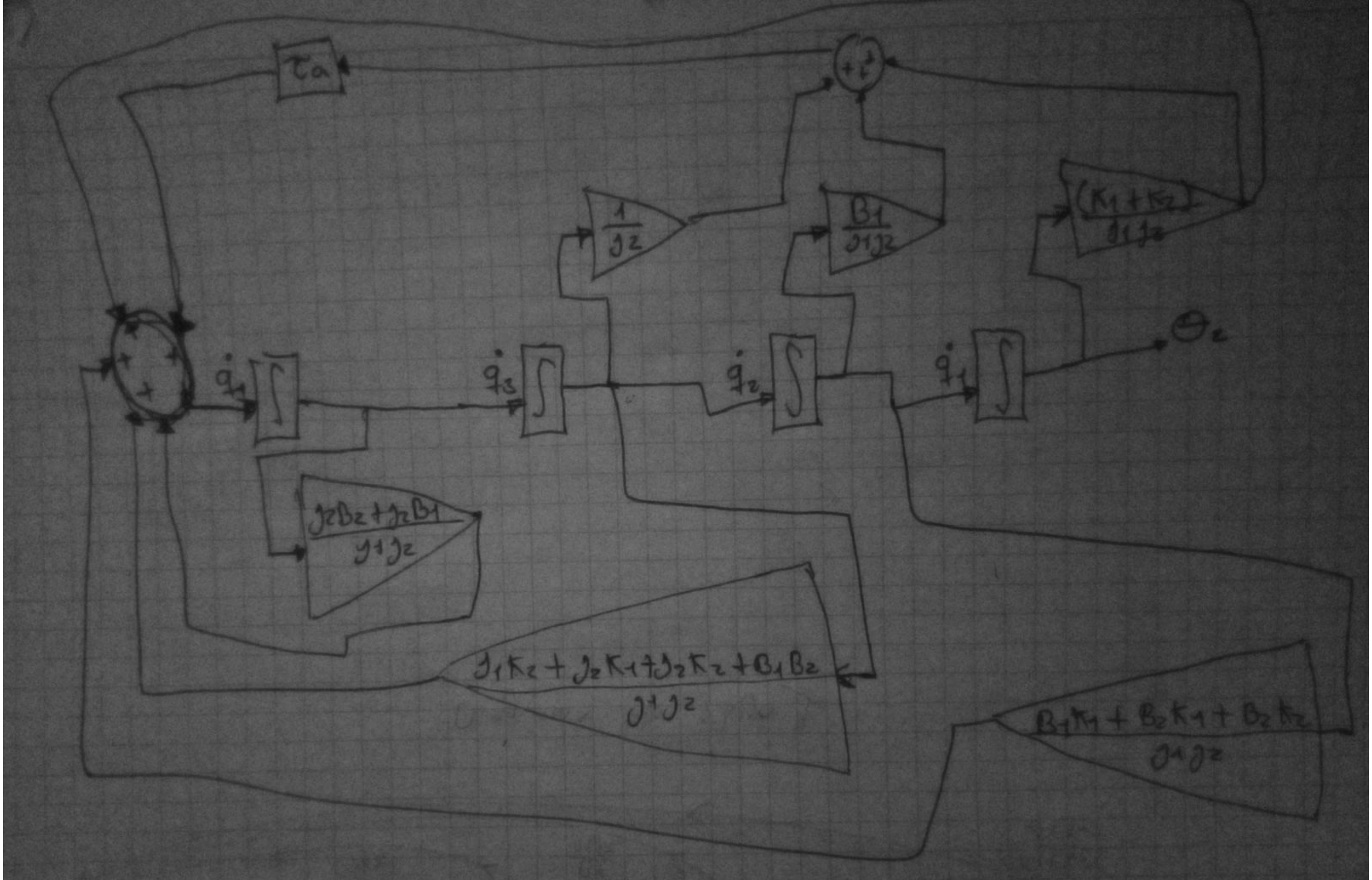


Diagrama de Bloques.

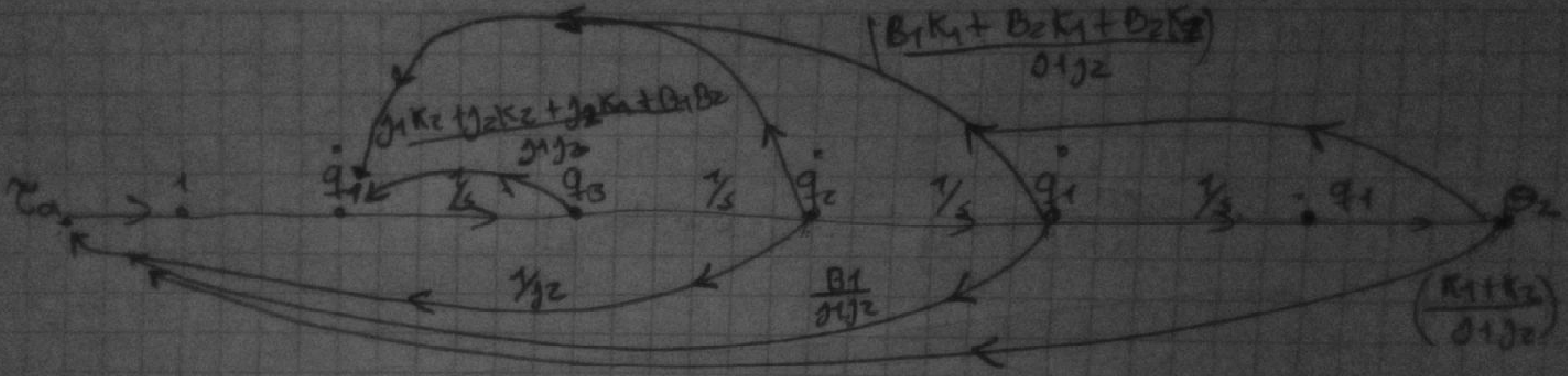


Diagrama de Flujo

3. Si $K_1 = 0$

Función de transferencia

$$\frac{\Theta_2(s)}{T_a(s)} = \frac{J_1 s^2 + B_1 s + K_2}{J_1 J_2 s^4 + (J_1 B_2 + J_2 B_1)s^3 + (J_1 K_2 + J_2 K_2 + B_1 B_2)s^2 + \dots + (B_1 K_2 + B_2 K_2)s + K_2}$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \left(\frac{K_2}{J_1 J_2}\right) & \left(\frac{\text{[redacted]} + B_2 K_2}{J_1 J_2}\right) & \left(\frac{J_1 K_2 + J_2 K_2 + B_1 B_2}{J_1 J_2}\right) & \left(\frac{J_1 B_2 + J_2 B_1}{J_1 J_2}\right) \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

$$+ \begin{bmatrix} \frac{K_2}{J_1 J_2} \\ \frac{B_1}{J_1 J_2} \\ 1/J_2 \\ 0 \end{bmatrix} \tau_a$$

$$\theta_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

Espacios de estados

Diagrama de bloques

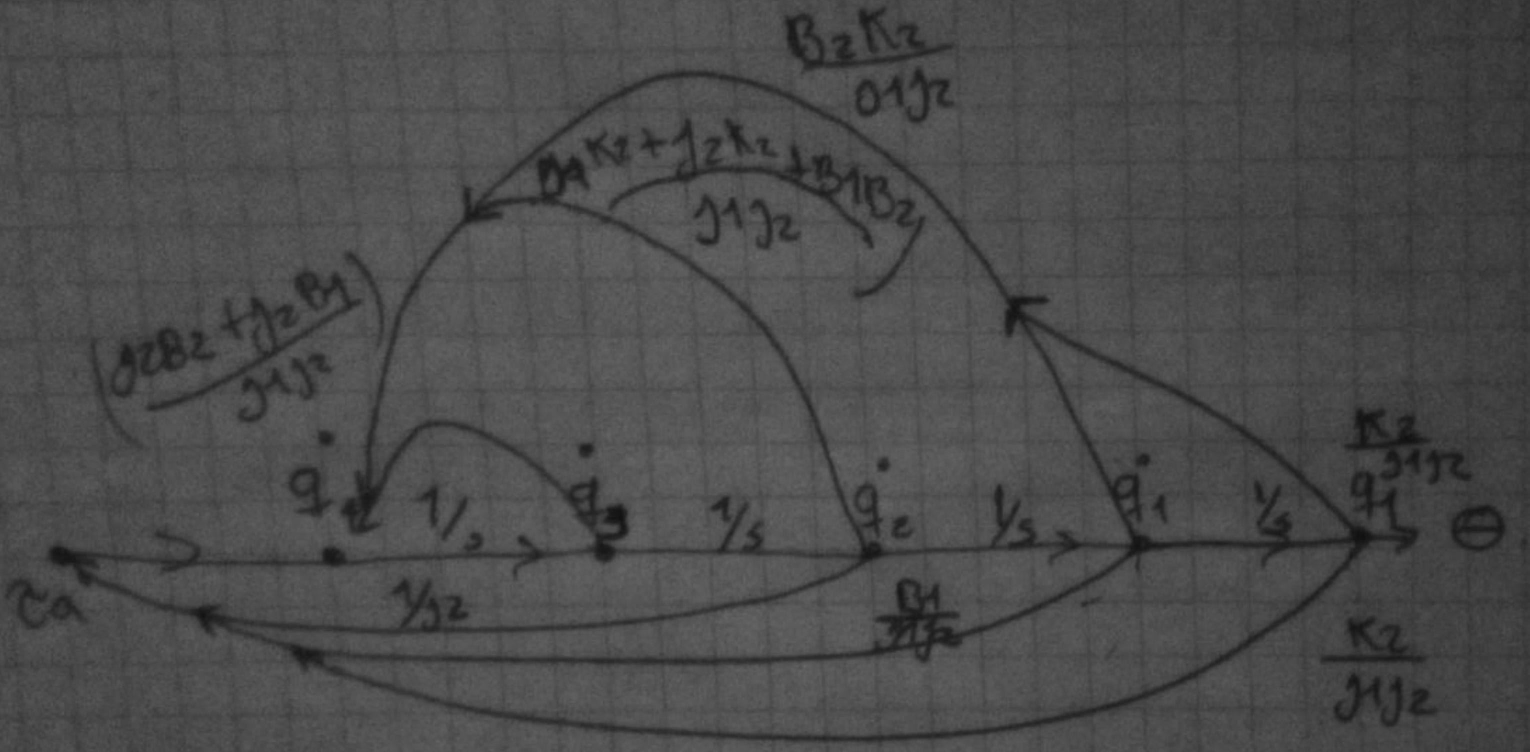
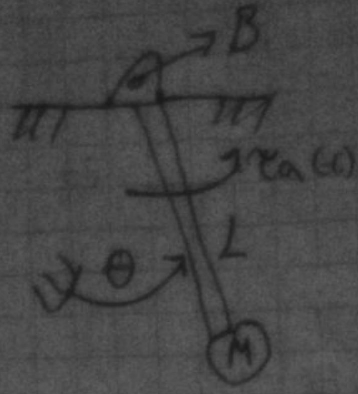


Diagrama de Flujo

4



$$\tau_a - MgL\,\text{sen}\,\theta = ML^2\ddot{\theta} + B\dot{\theta}$$

$$\frac{\tau_a}{ML^2} - \frac{g}{L}\,\text{sen}\,\theta = \ddot{\theta} + \frac{B}{ML^2}\dot{\theta}$$

$$\ddot{\theta} = \frac{\tau_a}{ML^2} - \frac{g}{L}\,\text{sen}\,\theta - \frac{B}{ML^2}\dot{\theta}$$

$$\frac{\Theta(s)}{\tau_a(s)} = \frac{\frac{1}{ML^2}}{\left(\frac{g}{L}\right)\left(\frac{2\pi}{s^2+(2\pi)^2}\right) + \frac{B}{ML^2}s} = G(s)$$

Funcion de transferencia si $\theta = \omega = \frac{2\pi}{T} =$

transformada de $\mathcal{L}[\text{sen}(at)] = \frac{a}{s^2+a^2}$

$$\frac{\Theta(s)}{\tau_a(s)} = G(s)\left(\frac{ML^2}{ML^2}\right) \quad \text{ó} \quad \frac{1}{\mathcal{L}[gML\,\text{sen}\,\theta] + Bs^2}$$

$\downarrow$

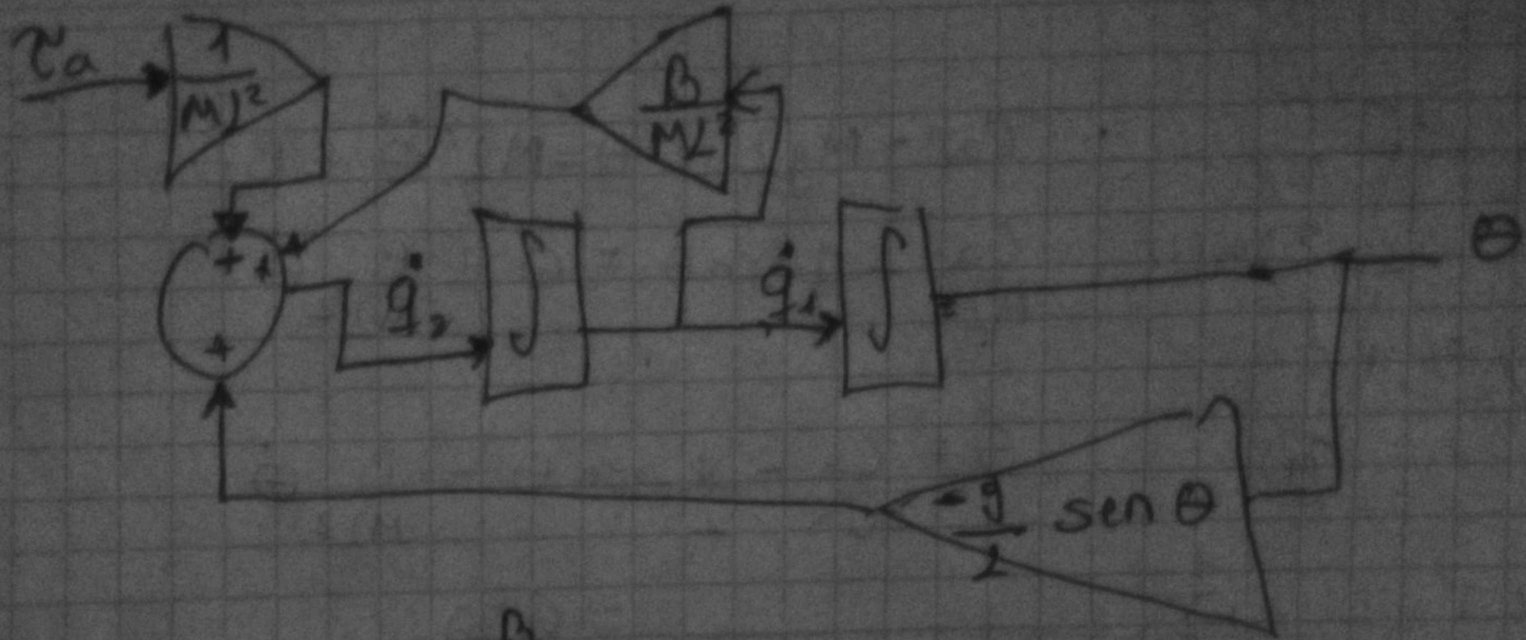
$$\frac{\Theta(s)}{T_a(s)} = \frac{1}{\frac{gML\,2\pi}{s^2+(2\pi)^2} + Bs}$$

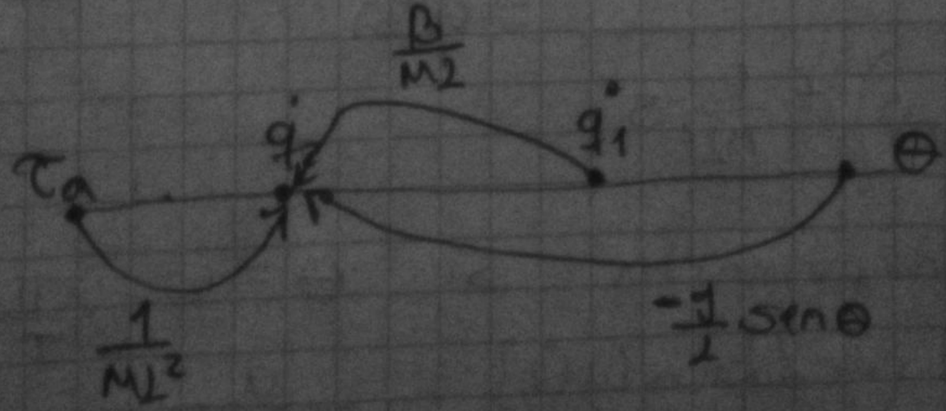
Funcion de transferencia //

$q_1 = \theta$; $q_2 = \dot{q}_1 = \dot{\theta}$; $\dot{q}_2 = \ddot{\theta}$

$$\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\frac{g}{L}\,\text{sen}\,\theta & \frac{B}{ML^2}\end{bmatrix}\begin{bmatrix}q_1\\ q_2\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{ML^2}\end{bmatrix}\tau_a$$

$$\theta = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}q_1\\ q_2\end{bmatrix}$$


τ_a
$\frac{1}{Ml^2}$
$\frac{B}{Ml^2}$
+
+
+
$\dot{q}_2$
$\dot{q}_1$
$\int$
$\int$
Θ
$-\frac{g}{l}$ sen Θ


$\frac{B}{Ml}$
$\dot{q}_2$
$\dot{q}_1$
τ_a
Θ
$\frac{1}{Ml^2}$
$-\frac{1}{l}$ sen Θ