

Machine Vision

October 9, 2018

1 Lecture 1

Bernoulli Distribution

$$Pr(x) = \lambda^x (1 - \lambda)^{1-x}, \lambda \in [0, 1], x \in \{0, 1\}$$

$$Pr(x) = \text{Bern}_x[\lambda]$$

Beta Distribution

$$Pr(\lambda) = \frac{\Gamma[\alpha + \beta]}{\Gamma[\alpha]\Gamma[\beta]} \lambda^{\alpha-1} (1 - \lambda)^{\beta-1}, \alpha, \beta > 0$$

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt = (z-1)!$$

$$E[\lambda] = \frac{\alpha}{\alpha + \beta}$$

$$B(p, q) = \frac{q-1}{p+q+1} B(p, q-1)$$

α, β decide the coin fact λ

Categorical Distribution

$$Pr(x = k) = \lambda_k$$

$$Pr(x) = \text{Cat}_x[\lambda]$$

Dirichlet Distribution

$$Pr(\lambda_1 \dots \lambda_K) = \frac{\Gamma[\sum_{k=1}^K \alpha_k]}{\prod_{k=1}^K \Gamma[\alpha_k]} \prod_{k=1}^K \lambda_k^{\alpha_k-1}$$

$$Pr(\lambda_1 \dots \lambda_K) = \text{Dir}_{\lambda_1 \dots \lambda_K}[\alpha_1, \alpha_2, \dots, \alpha_K]$$

Univariate Normal Distribution

$$Pr(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp[-0.5(x - \mu)^2/\sigma^2]$$
$$Pr(x) = \text{Norm}_x[\mu, \sigma^2]$$

Normal Inverse Gamma Distribution

$$Pr(\mu, \sigma^2) = \frac{\sqrt{\gamma}\beta^\alpha}{\sigma\sqrt{2\pi}\Gamma[\alpha]} \left(\frac{1}{\sigma^2}\right)^{\alpha+1} \exp\left[-\frac{2\beta + \gamma(\delta - \mu)^2}{2\sigma^2}\right]$$
$$Pr(\mu, \sigma^2) = \text{NormInvGam}_{\mu, \sigma^2}[\alpha, \beta, \gamma, \delta]$$

Multivariate Normal Distribution

$$Pr(\mathbf{x}) = \frac{1}{(2\pi)^{D/2} |\Sigma|^{1/2}} \exp[-0.5(\mathbf{x} - \mu)^T \Sigma^{-1} (\mathbf{x} - \mu)]$$

Normal Inverse Wishart

$$Pr(\mu, \Sigma) = \frac{\gamma^{D/2} |\Psi|^{\alpha/2} |\Sigma|^{-\frac{\alpha+D+2}{2}}}{(2\pi)^{D/2} 2^{\frac{\alpha D}{2}} \Gamma_D(\frac{\alpha}{2})} \exp\left\{-\frac{1}{2}(Tr(\Psi \Sigma^{-1})) + \gamma(\mu - \delta)^T \Sigma^{-1} (\mu - \delta)\right\}$$

Conjugate Distribution and Conjugate prior

Conjugate Distribution is between prior and posterior

Prior is the conjugate prior of the likelihood function.