Introduction to Statistical Data Science

October 4, 2018

1 Lecture 1

Basic Rules of Probability

Frequently we will say $p(x) \propto f(x)$ for some non-negative function $f(x)$

Then we can conclude that:

$$p(x) = \frac{f(x)}{\sum_y f(y)}$$

For joint distribution,

$$\sum_x p(x) = \sum_x \sum_y p(x, y) = 1$$

Independence

If $p(x|y) = p(x)$ for all states of x and y, then the variables x and y are said to be independent as $x \perp\!\!\!\perp y$.

If x and y are independent, then x and y are uncorrelated. However, in general, x and y are uncorrelated, then cannot conclude that x and y are independent.

Conditional Independence

$$\mathcal{X} \perp\!\!\!\perp \mathcal{Y} | \mathcal{Z}$$

$$p(\mathcal{X}, \mathcal{Y} | \mathcal{Z}) = p(\mathcal{X} | \mathcal{Z}) p(\mathcal{Y} | \mathcal{Z})$$

and

$$p(\mathcal{X} | \mathcal{Y}, \mathcal{Z}) = p(\mathcal{X} | \mathcal{Z})$$

Conditional independence does not imply marginal independence:

$$p(x, y) = \sum_z p(x|z)p(y|z)p(z) \neq \sum_z p(x|z)p(z) \sum_z p(y|z)p(z)$$