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FNCE 1000 F24

FNCE 1000 Notes

by

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Contents

1 Time Value of Money

"The value of a dollar today is worth more than the value of a dollar in the future"

1.1 Foundational Concepts

I **Future Value (FV)** — The value of an assest in the future, follows:

$$\mathbf{FV} = \text{principal} + \text{interest}$$

II **Present Value (PV)** — The vale of an asset today with a respect to cash flow in the future, follows:

$$\mathbf{PV} = \frac{C_t}{(1+r)}$$

where C_t is cash flow at time t, $\frac{1}{1+r}$ is the discount factor, and r is the discount rate. For a general case, over n periods, we have:

$$\mathbf{PV} = \sum_{t=1}^{n} \frac{C_t}{(1+r)^t}$$

III **Net Present Value (NPV)** — The difference between the present value of cash inflows and outflows.

$$\text{NPV} = \text{PV of future cash inflows} - \text{required investment}$$

we accept a project if NPV > 0.

Example

Required investment: \$3,500,000
Horizon: One year holding period
Projected value in one year: \$4,000,000
Required return: 5%

$$\mathbf{PV} = \frac{4,000,000}{1.05} = 3,809,523.81$$
$$\mathbf{NPV} = 3,809,523.81 - 3,500,000 = 309,523.81$$

Thus, we will accept the project.

Over a time horizon of n years, we have:

$$\mathbf{NPV} = N_0 + \sum_{t=1}^{n} \frac{C_t}{(1+r)^t}$$

IV **Discount Factor** — The factor by which future cash flows are discounted to get the present value.

$$\mathbf{DF}_t = \frac{1}{(1+r)^t}$$

where r is the discount rate and t is the time period.

Discount factors have a decreasing property, i.e. $\mathbf{DF}_{t+1} < \mathbf{DF}_t$ due to the time value of money.

Therefore, we can equivilently write the **NPV** as:

$$\mathbf{NPV} = \sum_{t=0}^{n} C_t \cdot \mathbf{DF}_t$$

V **Seperation Theorem** — he value of an investment to an individual is not dependent on consumption preferences. This theorem holds with the following assumptions:

i Trading is costless and access to the financial markets is free.

ii Borrowing and lending opportunities are available.

iii Distortions that have a significant impact on market prices can not exist (for example impact of liquidity or taxes)

2 Simple Cash Flow Streams

In a general case, a simple cash flow stream can be described as:

$$\mathbf{PV} = \sum_{t=1}^{n} \frac{C_t}{(1+r_t)^t}$$

To simplify, with a flat term structure in interest rates, i.e. $r_t = r$ for all t, we can keep the discount rate constant.

Recall, the summation of a geometric series:

$$\sum_{t=1}^{n} x^t = \frac{x(1-x^n)}{1-x} \tag{1}$$

and when $|x| < 1$, we have:

$$\sum_{t=1}^{\infty} x^t = \frac{x}{1-x} \tag{2}$$

2.1 Constant Perpetuity

A perpetuity is a stream of cash flows that continues indefinitely. The present value of a perpetuity is given by:

$$\mathbf{PV} = \sum_{t=1}^{\infty} \frac{C}{(1+r)^t}$$

where C is the cash at each period and r is the discount rate. Thus, applying (2), setting $x = \frac{1}{1+r}$, we have:

$$\mathbf{PV} = \frac{C}{r} \tag{3}$$

Example

Cash flow stream is an annual payment of \$100 per year forever. The annual interest rate is 8%. Using the definition of present value we have:

$$\mathbf{PV} = \sum_{t=1}^{\infty} \frac{100}{(1+0.08)^t}$$

using (3):

$$\mathbf{PV} = \frac{100}{0.08} = 1,250$$

2.2 Growing Perpetuity

A growing perpetuity is a stream of cash flows that continues indefinitely with a constant growth rate. The present value of a growing perpetuity by first principals is given by:

$$\mathbf{PV} = \sum_{t=1}^{\infty} \frac{C \cdot (1+g)^t}{(1+r)^t}$$

where C is the cash at each period, r is the discount rate, and g is the growth rate. Thus, applying (2), setting $x = \frac{1+g}{1+r}$, we have:

$$\mathbf{PV} = \frac{C}{r-g} \tag{4}$$

Example

Cash flow stream is an annual payment of \$100 first year (at time 1). The annual flow continues forever growing at an annual rate of 5%. The annual interest rate is 8%.

$$\mathbf{PV} = \sum_{t=1}^{\infty} \frac{100 \cdot (1+0.05)^t}{(1+0.08)^t}$$

using (4):

$$\mathbf{PV} = \frac{100}{0.08 - 0.05} = 3,333.33$$

2.3 Delayed Perpetuity

2.4 Constant Annuity

2.5 Growing Annuity

3 Interest Rates and Compounding