University of Pennsylvania

FNCE 1000 F24

FNCE 1000 Notes

by

Jefferson Ding

Contents

1 Time Value of Money

"The value of a dollar today is worth more than the value of a dollar in the future"

1.1 Foundational Concepts

I **Future Value (FV)** — The value of an assest in the future, follows:

$$\textbf{FV} = \text{principal} + \text{interest}$$

II **Present Value (PV)** — The vale of an asset today with a respect to cash flow in the future, follows:

$$\textbf{PV} = \frac{C_t}{(1+r)}$$

where C_t is cash flow at time t, $\frac{1}{1+r}$ is the discount factor, and r is the discount rate. For a general case, over n periods, we have:

$$\textbf{PV} = \sum_{t=1}^{n} \frac{C_t}{(1+r)^t}$$

III **Net Present Value (NPV)** — The difference between the present value of cash inflows and outflows.

$$\text{NPV} = \text{PV of future cash inflows} - \text{required investment}$$

we accept a project if NPV > 0.

Example

Required investment: \$3,500,000
Horizon: One year holding period
Projected value in one year: \$4,000,000
Required return: 5%

$$\textbf{PV} = \frac{4,000,000}{1.05} = 3,809,523.81$$
$$\textbf{NPV} = 3,809,523.81 - 3,500,000 = 309,523.81$$

Thus, we will accept the project.

Over a time horizon of n years, we have:

$$\textbf{NPV} = N_0 + \sum_{t=1}^{n} \frac{C_t}{(1+r)^t}$$

IV **Discount Factor** — The factor by which future cash flows are discounted to get the present value.

$$\mathbf{DF}_t = \frac{1}{(1+r)^t}$$

where r is the discount rate and t is the time period.

Discount factors have a decreasing property, i.e. $\mathbf{DF}_{t+1} < \mathbf{DF}_t$ due to the time value of money.

Therefore, we can equivilently write the **NPV** as:

$$\mathbf{NPV} = \sum_{t=0}^{n} C_t \cdot \mathbf{DF}_t$$

V **Seperation Theorem** — he value of an investment to an individual is not dependent on consumption preferences. This theorem holds with the following assumptions:

i Trading is costless and access to the financial markets is free.

ii Borrowing and lending opportunities are available.

iii Distortions that have a significant impact on market prices can not exist (for example impact of liquidity or taxes)

2 Simple Cash Flow Streams

In a general case, a simple cash flow stream can be described as:

$$\mathbf{PV} = \sum_{t=1}^{n} \frac{C_t}{(1+r_t)^t}$$

To simplify, with a flat term structure in interest rates, i.e. $r_t = r$ for all t, we can keep the discount rate constant.

Recall, the summation of a geometric series:

$$\sum_{t=1}^{n} x^t = \frac{x(1-x^n)}{1-x} \tag{1}$$

and when $|x| < 1$, we have:

$$\sum_{t=1}^{\infty} x^t = \frac{x}{1-x} \tag{2}$$

2.1 Constant Perpetuity

A perpetuity is a stream of cash flows that continues indefinitely. The present value of a perpetuity is given by:

$$\textbf{PV} = \sum_{t=1}^{\infty} \frac{C}{(1+r)^t}$$

where C is the cash at each period and r is the discount rate. Thus, applying (2), setting $x = \frac{1}{1+r}$, we have:

$$\textbf{PV} = \frac{C}{r} \tag{3}$$

Example

Cash flow stream is an annual payment of $100 per year forever. The annual interest rate is 8%. Using the definition of present value we have:

$$\textbf{PV} = \sum_{t=1}^{\infty} \frac{100}{(1+0.08)^t}$$

using (3):

$$\textbf{PV} = \frac{100}{0.08} = 1,250$$

2.2 Growing Perpetuity

A growing perpetuity is a stream of cash flows that continues indefinitely with a constant growth rate. The present value of a growing perpetuity by first principals is given by:

$$\textbf{PV} = \sum_{t=1}^{\infty} \frac{C \cdot (1+g)^t}{(1+r)^t}$$

where C is the cash at each period, r is the discount rate, and g is the growth rate. Thus, applying (2), setting $x = \frac{1+g}{1+r}$, we have:

$$\textbf{PV} = \frac{C}{r-g} \tag{4}$$

Example

Cash flow stream is an annual payment of $100 first year (at time 1). The annual flow continues forever growing at an annual rate of 5%. The annual interest rate is 8%.

$$\mathbf{PV} = \sum_{t=1}^{\infty} \frac{100 \cdot (1+0.05)^t}{(1+0.08)^t}$$

using (4):

$$\mathbf{PV} = \frac{100}{0.08 - 0.05} = 3,333.33$$

2.3 Delayed Perpetuity

A perpetuity that pays C dollars per year starting at time n that goes on forever. The present value of a delayed perpetuity is given by:

$$\mathbf{PV} = \sum_{t=n}^{\infty} \frac{C}{(1+r)^t} = \frac{C}{r} \cdot \frac{1}{(1+r)^{n-1}}$$

note that when the first payment, which happens at time n, the formula above is for one year before the first payment.

2.4 Constant Annuity

You will recieve a fixed payment of C dollars every year for a specified number of years. That is, a perpetuity with a finite horizon. The present value of a constant annuity is given by:

$$\mathbf{PV} = \sum_{t=1}^{n} \frac{C}{(1+r)^t}$$

thus, substituting $x = \frac{1}{1+r}$ into (1):

$$\mathbf{PV} = \frac{C}{r}\left(\frac{1}{r} - \frac{1}{(1+r)^n}\right)$$

isolating C, we find the annuity factor:

$$\mathbf{AF} = \frac{1}{r} - \frac{1}{(1+r)^n}$$

2.5 Growing Annuity

Similar to a growing perpetuity, you recieve a fixed payment of C dollars at the end of each year with C growing at a constant annual rate of g. By first principals:

$$\mathbf{PV} = \sum_{t=1}^{n} \frac{C \cdot (1+g)^{t-1}}{(1+r)^t}$$

factoring constant C, we find:

$$\mathbf{PV} = C \cdot \sum_{t=1}^{n} \frac{(1+g)^{t-1}}{(1+r)^t}$$

thus, substituting $x = \frac{1+g}{1+r}$ into (1):

$$\mathbf{PV} = C \cdot \frac{1 - \left(\frac{1+g}{1+r}\right)^n}{r-g}$$

and the annuity factor is for a growing annuity is in general form is:

$$\mathbf{AF} = \frac{1 - \left(\frac{1+g}{1+r}\right)^n}{r-g}$$

3 Interest Rates and Compounding

A **simple interest** rate is when interest is paid only on the principal investment. That is, in simple terms:

$$\mathbf{FV} = \text{principal} + \text{principal} \cdot r \cdot t$$

where r is the interest rate and t is the time period.

3.1 Compound Interest

A **compound interest** is interest paid on the initial investment and on interest accumulated in previous periods. That is:

$$\mathbf{FV} = \text{principal} \cdot (1+r)^t$$

The compounding interveal is the time between interest payments, or in other words, the frequency of which interest is added to the principal. The effective annual rate (EAR) is the annual rate of interest when compounding occurs more than once a year. The formula for the EAR is:

$$\mathbf{EAR} = \left(1 + \frac{r}{m}\right)^m - 1$$

where r is the nominal rate and m is the number of compounding periods per year. Thus, to find the rate for each compounding period, we have:

$$R = \frac{r}{m}$$

The concept of continuous compounding is when the compounding period is infinitely small. The formula for continuous compounding is:

$$\mathbf{FV} = \text{principal} \cdot e^{r \cdot t}$$

We can also do the reverse, that is, given the EAR, we can find the nominal rate of continuous compouding by:

$$r = \ln(1 + r)$$

An **amortization schedule** is a table that details each loan payment's allocation between interest and principal. An example of an amortization schedule is shown below:

Example

Consider a 3-year \$100,000 loan at a annual rate of 8%. We first use the annuity formula to find the annual payment:

$$\$100,000 = C \cdot \mathbf{AF} = C \cdot \left(\frac{1}{0.08} - \frac{1}{(1+0.08)^3}\right)$$

Now, we can solve for C to find the annual payment.

$$C = \$38803.35$$

Now, we can construct the amortization schedule:

Time	**Prin. Outstanding**	**Payment**	**Int. Paid**	**Prin. Paid**
0	100,000			
1	69,196.65	38,803.35	8000.00	30803.35
2	35,929.03	38,803.35	5535.73	33267.32
3	0	38,803.35	2874.32	35,929.03

Table 1: Amortization Schedule for 100,000 Loan

4 Bonds

A bond is an obligation to pay a fixed amount of money to the lender plus a specified coupon rate. A *Pure Discount Bond* is a bont where the rate is 0, that is, the bond face value only is payed at maturity.

Bond Pricing is the process of determining the fair price of a bond. We can calculate the price of a bond by using the ask and bid rates. The **ask rate** is the price at which the bond is sold, and the **bid rate** is the price at which the bond is bought. The **bid-ask spread** is the difference between the ask and bid rates.

The **yield to maturity** (YTM) is the rate of return anticipated on a bond if it is held until the maturity date. We can say that if:

- The bond is selling at par, then the YTM = coupon rate.
- The bond is selling at a discount, then the YTM > coupon rate.
- The bond is selling at a premium, then the YTM < coupon rate.

A **Forward Rate** is the rate of interest that can be locked in today for a future investment. For example, you are able to lock in a rate for a $1000 investment one year in the future. To calculate the forward rate, we use the formula:

$$(1 + r_2)^2 = (1 + r_1)(1 + f)$$

where r_1 is the one year rate, r_2 is the two year rate, and f is the forward rate.

5 Stocks

A share of common stock simply represents a claim on the assets of a corporation. The value of a share of stock can come from:

- Dividends
- Capital gains (or losses)

thus, the expected return for holding a share for one period can be generalized to:

$$\text{Expected Return Rate} = r = \frac{\text{DIV}_1 + (P_1 - P_0)}{P_0}$$

thus, viewing r as a discount rate, we can find the present value:

$$P_0 = \frac{\text{DIV}_1}{r} + \frac{P_1}{(1 + r)}$$

and, now to find P_1, we can use the same formula:

$$P_1 = \frac{\text{DIV}_2}{r} + \frac{P_2}{(1 + r)}$$

by this generalization, we can find the present value of a stock over a time horizon as:

$$P_0 = \sum_{t=1}^{T} [\frac{\text{DIV}_t}{(1+r)^t}] + \frac{P_T}{(1+r)^T}$$

under the assumption that as $T \to \infty$, $\frac{P_T}{(1+r)^T} \to 0$, that is intuitively, the discount will be so large that the value of the stock will be negligible, we have:

$$P_0 = \sum_{t=1}^{\infty} \frac{\text{DIV}_t}{(1+r)^t}$$

which outlines a key concept in stock valuation where the value of a share of common stock is the present value of all future dividends that are expected to be paid.

Thus, the valuation of a stock can be seen as a calculation of simple cash flow streams, where the cash flow is the dividend and depending on the dividend policy, we can have a constant, growing or some combination of cash flows.

5.1 Interpreting Discount Rates

The discount rate r is the expected return on the stock. The discount rate can be broken down into **dividend yield**, that is, the expected dividend divided by the current price, and the **capital gains yield**, that is, the expected price appreciation divided by the current price. Thus, we can write the discount rate as:

$$r = \frac{\text{DIV}_1}{P_0} + g = \frac{\text{DIV}_1}{P_0} + \frac{P_1 - P_0}{P_0}$$

The earnings of a company can be broken down into dividend payouts and retained earnings, normalizing by the number of shares, we have the **earnings per share** (EPS). The **dividend payout ratio** is the percentage of earnings paid out as dividends, and the **plowback ratio** is the percentage of earnings retained by the company. Always, $1 =$ dividend payout ratio $+$ plowback ratio.

EPS is a key metric for investors as it provides a measure of a company's profitability, that is, the amount of profit generated per share of stock. Using EPS, we can calculate the **price-to-earnings ratio** (P/E).

Another concept is **Return on Equity** (ROE), which is a measure of a company's profitability that takes a company's net income and divides it by the company's equity. ROE is a measure of how well a company is using its assets to generate profit. It must be part of the consideration when evaluating growth in a stock. We have the follwoing relationship:

$$\text{EPS}_2 = \text{EPS}_1 + \text{Retained Earnings} \cdot \text{ROE}$$

using the plowback ratio, we can find the growth rate:

$$g = \text{ROE} \cdot k$$

Example

Consider IBM at \$201 per share, with a dividend of \$6.74 per share, and a EPS of \$10.87. We can find k, the plowback ratio by:

$$k = 1 - \frac{6.74}{10.87} = 0.38$$

assuing a 10% ROE, we can find the growth rate:

$$g = 0.10 \cdot 0.38 = 0.038$$

thus, the discount rate is:

$$r = \frac{6.74}{201} + 0.038 = 0.072$$

using FV, we can find the price of the stock in one year:

$$201 \cdot (1 + 0.072) = 215.04$$

or solving for current value using cash flow streams:

$$P_0 = \frac{6.74}{0.072 - 0.038} = 198$$

5.2 Price-to-Earnings Ratio

The price-to-earnings ratio (P/E) is a measure of the price investors are willing to pay for a dollar of earnings. It's defined as:

$$\text{P/E} = \frac{P_0}{E_1}$$

where P_0 is the current price of the stock and E_1 is the expected earnings per share.

A higher P/E ratio indicates that investors are willing to pay more for a dollar of earnings and are expecting higher growth in the future. A lower P/E ratio indicates that investors are willing to pay less for a dollar of earnings and are expecting lower growth in the future.

5.3 Earning Streams

In some cases, all the earnings of a company are paid out as dividends, that is, the plowback ration is 0 and DIV=EPS. In this case, we can easily value the stock as:

$$P_0 = \frac{\text{EPS}_1}{r}$$

where r is the discount rate. and the P/E ration is simply $\frac{1}{r}$. This is usually the case for more mature companies, or value stocks.

In other cases, where a firm has positive NPV projects, the plowback ratio is positive, and the firm is expected to grow. We can now value the stock as:

$$P_0 = \frac{\text{EPS}_1}{r} + \text{NPVGO}$$

where NPVGO is the net present value of growth opportunities. The P/E ratio is now:

$$\text{P/E} = \frac{1}{r} + \frac{\text{NPVGO}}{\text{EPS}_1}$$

this is usually the case for growth stocks.

A TI-Nspire™ Finance Functions

This appendix provides an overview of the finance functions available in the TI-Nspire calculator, their uses, and examples.

A.1 Finance Solver

The Finance Solver is used to compute various financial values, such as present value, future value, interest rate, number of periods, and payments. This function allows users to input known values and solve for the unknown.

Finance Solver

Suppose you want to calculate the monthly payment on a loan of $10,000 at an annual interest rate of 5% over 5 years. You would set the following values:

- N = 60 (months)
- I = 5 (annual interest rate)
- PV = -10000 (loan amount)
- FV = 0 (since the loan is fully paid off)
- PmtAt = 0 (end of the period)

Solve for the payment (`Pmt`).

A.2 Time Value of Money (TMV) Functions

These functions are used to calculate various aspects of time value of money, such as the number of periods, interest rate, present value, payment amount, and future value.

- `tvmN(I,PV,Pmt,FV,[PpY],[CpY],[PmtAt])`: Calculates the number of payment periods.
- `tvmI(N,PV,Pmt,FV,[PpY],[CpY],[PmtAt])`: Calculates the interest rate per year.
- `tvmPV(N,I,Pmt,FV,[PpY],[CpY],[PmtAt])`: Calculates the present value.
- `tvmPmt(N,I,PV,FV,[PpY],[CpY],[PmtAt])`: Calculates the amount of each payment.
- `tvmFV(N,I,PV,Pmt,[PpY],[CpY],[PmtAt])`: Calculates the future value.

TMV Functions

Calculate the future value of an investment where $1,000 is invested at 6% annual interest for 10 years. Set the following:

- N = 10
- I = 6
- PV = -1000
- Pmt = 0 (no additional payments)
- FV = Solve for this

The future value is $1,790.85.

A.3 Amortization Functions

These functions calculate the principal and interest components of payments over a range of periods.

- `amortTbl(NPmt,N,I,PV,[Pmt],[FV],[PpY],[CpY],[PmtAt])`: Returns an amortization table as a matrix.
- `bal(NPmt,N,I,PV,[Pmt],[FV],[PpY],[CpY],[PmtAt])`: Calculates the balance after a specified payment.
- `ΣInt(NPmt1,NPmt2,N,I,PV,[Pmt],[FV],[PpY],[CpY],[PmtAt])`: Calculates the sum of interest during a specified range of payments.
- `ΣPrn(NPmt1,NPmt2,N,I,PV,[Pmt],[FV],[PpY],[CpY],[PmtAt])`: Calculates the sum of principal during a specified range of payments.

Amortization Functions

Given a loan of $5,000 at an interest rate of 5% for 3 years, compute the amortization table for the first 12 months. Use:

- NPmt = 12 (first 12 months)
- N = 36 (total payments)
- I = 5 (annual interest rate)
- PV = -5000

The amortization table will display the amount paid towards principal and interest for each month.

A.4 Cash Flow Functions

These functions calculate metrics such as the net present value (NPV) and the internal rate of return (IRR) for cash flows.

- `npv(rate,CF0,CFList,[CFFreq])`: Calculates the net present value of cash flows.
- `irr(CF0,CFList,[CFFreq])`: Calculates the internal rate of return for cash flows.

Cash Flow Functions

Suppose you have an investment with the following cash flows: an initial investment of $1,000 and cash inflows of $300, $400, $500 over 3 years. The discount rate is 8%. Use the `npv` function to find the NPV of the investment.

A.5 Interest Conversion Functions

These functions help convert between nominal and effective interest rates.

- `eff(nomRate,CpY)`: Converts nominal interest rate to effective rate.
- `nom(effectiveRate,CpY)`: Converts effective interest rate to nominal rate.

Interest Conversion Functions

Convert a nominal interest rate of 5% compounded quarterly to its effective rate using:

- Nominal Rate = 5%
- CpY = 4 (quarterly)

The effective rate is approximately 5.095%.