

KEY



endpoint



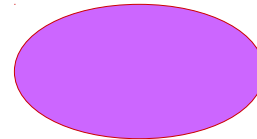
**Mathematica
notebook**



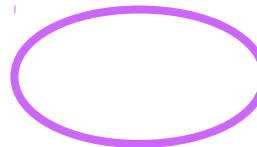
**Python
script**



Bash/terminal



**INPUT/OUTPUT
User modifiable**



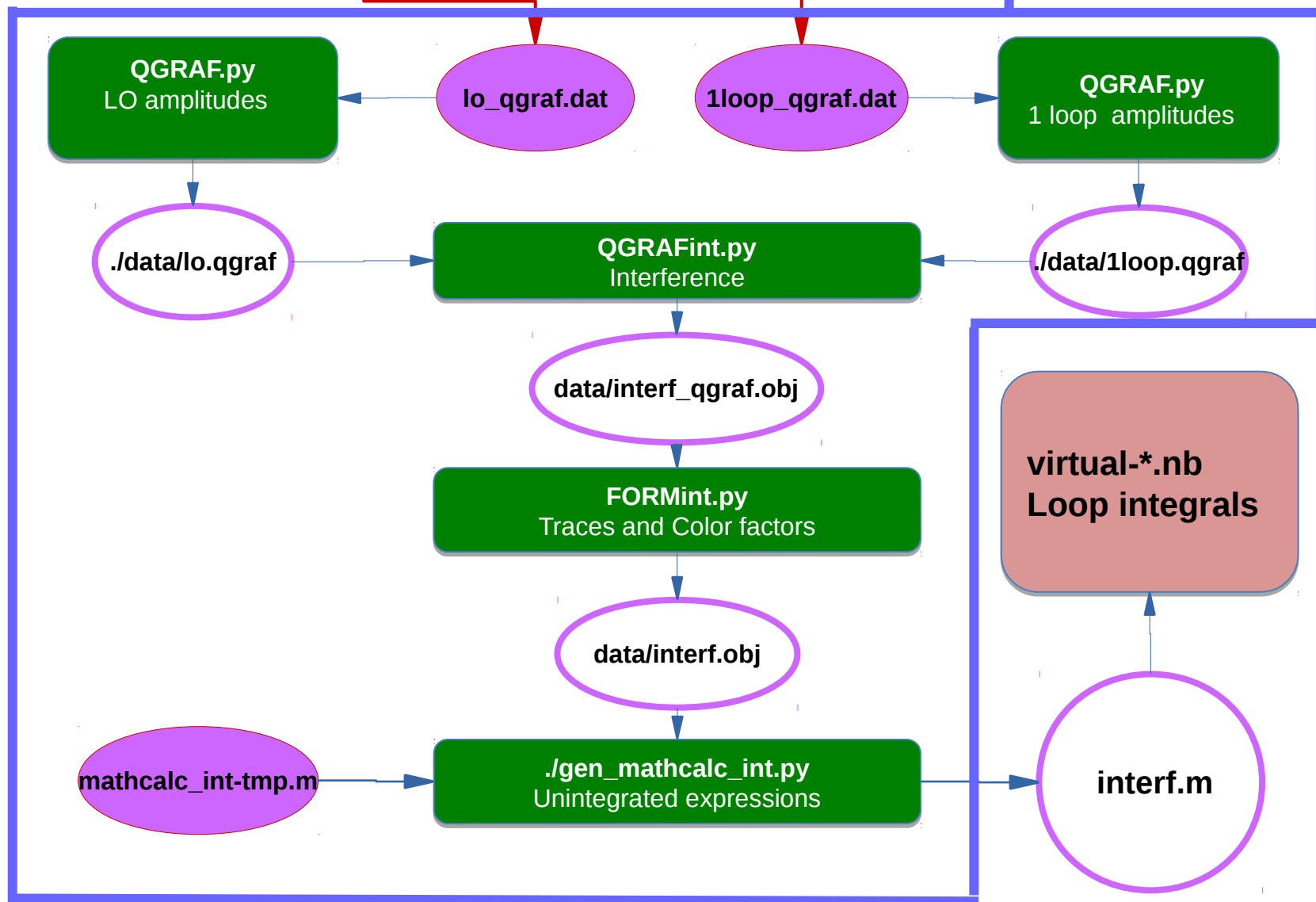
**INPUT/OUTPUT
automatic**

(1)

VIRTUAL CHAIN FROM SCRATCH
Run “./setup new ; cd workspace_virtual”
Open and modify: lo_qgraf.dat
1loop_qgraf.dat

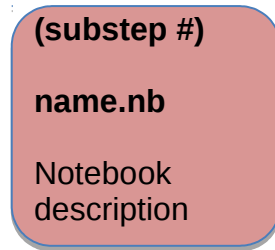
Run: “bash runchain_virtual”
Generates “interf.m”

(2)



(3)

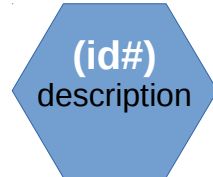
KEY



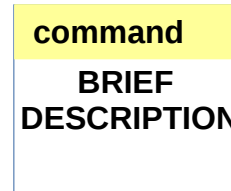
**Mathematica
notebook**



**Mathematica
Function
(user level)**



**Mathematica
expression**



**Mathematica
Function
(internal)**



**Ongoing
checks**

(3.1)

virtual-*.nb

Loop integrals

(0)

interfij
Raw expression
from "interf.m"

//PASSARINOVELTMAN

Implements PV.
Defined as the
chain below

(1)

Reduced term:
Function of
Scalar integrals

- $B0[\{\}]$
- $CO[\{\}]$
- $DO[\{\}]$

//TENSORINTEGRALS

Classified integrals:

bubble $[\{\},\{\}]$
triangle $[\{\},\{\}]$
box $[\{\},\{\}]$

//FORMFACTORS

**Decomposed into
Form Factors:**

Following Eqs in ref1:
Bubble: A.1,A.2
Triangle: A.6-A.8
Box: A.23-A.25

//REDUCEBOX

PV for box:
Solved Eqs shown in
Table A.2 of ref1.
Only C and B
form factors present

//REDUCETRIANGLE

PV for triangle:
Solved Eqs shown in
Table A.1 of ref1.
Only B
form factors present

//REDUCEBUBBLE

PV for bubble:
Solved Eqs A.3 of ref1.
No form factors left

(3.2)

virtual-*.nb

Loop integrals

(1)

Reduced term:
Function of
Scalar integrals
B0,C0,D0

//EVALSCALARINT

**Replaces scalar integrals
by solutions in ref2**

(2)

Loop Integrated:
Function of
epsilon, contains
Log,Li2

//EVALB0

Replaces B0.

Implemented as replacements:

term /.
B0[a_] → B0[a, \[Mu]]/.
B0 → B0eval

//EVALC0

Replaces C0.

Implemented as replacements:

term /.
C0[a_] → C0[a, \[Mu]]/.
C0 → C0eval

//EVALD0

Replaces D0.

Implemented as replacements:

term /.
D0[a_] → D0[a, \[Mu]]/.
D0 → D0eval

NOTE: Integrals in ref2 are defined in Eqs. (2.1)

This implies that for our calculation each result in ref2

Should be multiplied by an extra factor of :

$$\frac{i2^{2\epsilon-4}\pi^{\epsilon-2}\Gamma(1-\epsilon)^2\Gamma(\epsilon+1)}{\Gamma(1-2\epsilon)}$$

Finally, expand up to order $\mathcal{O}(\epsilon^0)$, so that

expressions in notebook look different than in ref2

(3.2)

virtual-*.nb

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B0eval[momshifts,\[Mu]]

arguments:

momshifts={ Δ_1, Δ_2 }

$\sqrt{[Mu]}$ =dim reg scale

Steps:

1) calc indep momentum
 $s=(\Delta_2-\Delta_1)^2$

2) test if $s < 0$, $s > 0$ or $s = 0$

3) if $s < 0$, use Eq.(4.4) in
ref2, else if

4) if $s > 0$, use Eq.(4.4) in
ref2, with the replacements:

4.1) $\text{Log}[a_] \rightarrow \text{Log}[a] + i \sqrt{[Pi]}$

4.2) $s \rightarrow -s$

else

5) if $s = 0$ set integral to 0
(scaleless case)

(3.2)

virtual-*.nb

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Replaces D0.

Implemented as replacements:

term /.
D0[a_] → D0[a, \[Mu]]/.
D0 → D0eval

C0eval[momshifts,\[Mu]]

arguments:

momshifts={ $\Delta_1, \Delta_2, \Delta_3$ }

$\sqrt{[Mu]}$ =dim reg scale

Steps:

1) calc indep momenta²
 $mom1=(\Delta_2-\Delta_1)^2$,
 $mom2=(\Delta_3-\Delta_2)^2$,
 $mom3=(\Delta_1-\Delta_3)^2$

C0type[mom1, mom2, mom3]

2) check if type 1 or type 2, (in ref2 Eq. 4.5 & 4.6 respectively)

C0type1eval[mom1, mom2, mom3]

2a)

- 1) find $s \equiv$ non-vanishing indep. mom²
- 2) test if $s < 0$, $s > 0$ or $s = 0$
- 3) Use Eq. 4.5 ref2
- 4) if $s > 0$ replace:
 $\log[a_] \rightarrow \text{Log}[-a] + i \sqrt{[Pi]}$,
else
- 5) leave unevaluated, i.e. return
 $C0[momshifts, \sqrt{[Mu]}]$

C0type2eval[mom1, mom2, mom3]

2b)

- 1) find two non-vanishing indep. Mom²
(defined as array {s1,s2})
- 2) test if $s_i < 0$ or $s_i > 0$
- 3) use Eq. 4.6 ref2, use “dummy” functs:
 $\log1[-\sqrt{[Mu]}/s1]$
 $\log2[-\sqrt{[Mu]}/s2]$
- 4) if $s1$ and/or $s2 < 0$, replace:
 $\log1[a_] \rightarrow \text{Log}[-a] + i \sqrt{[Pi]}$ and/or
 $\log2[a_] \rightarrow \text{Log}[-a] + i \sqrt{[Pi]}$

(3.2)

virtual-*.nb

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Replaces D0.

Implemented as replacements:

term /.
D0[a_] → D0[a, \[Mu]]/.
D0 → **D0eval**

D0eval[momshifts,\[Mu]]

arguments:

momshifts={ $\Delta_1, \Delta_2, \Delta_3, \Delta_4$ }

$\sqrt{[Mu]}$ =dim reg scale

Steps:

1) *define

$M_1 = (\Delta_2 - \Delta_1)$, $M_2 = (\Delta_3 - \Delta_2)$
 $M_3 = (\Delta_4 - \Delta_3)$
 $M_4 = -(M_1 + M_2 + M_3)$
 $M_5 = M_1 + M_2$, $M_6 = M_2 + M_3$
 $s_{12} = M_5^2$, $s_{23} = M_6^2$

► 2) find cases $M_i^2 \neq 0$, $i=1,2,3,4$. (none for prompt ph and only 1 for SIDIS)

D0type[M_1, M_2, M_3, M_4]

3) check if type 1 or type 2, (in ref2 Eq. 4.18 & 4.19 respectively)

D0type1eval[mo
mshifts,\[Mu]]

NOT IN SIDIS
(check,
update draft)

D0type2eval[momshifts,\[Mu]]

3b)(with same definitions*)

1) three invariants: M_i^2 , s_{12} , s_{12}

2) use Eq. (4.19), two Dilogs
and four logs (dummy functions):
 $Li_{21}[1 - M_i^2/s_{12}]$, $Li_{22}[1 - M_i^2/s_{23}]$
 $Log1[-(\mu^2/M_i^2)]$, $log2[-(\mu^2/s_{12})]$
 $Log3[-(\mu^2/s_{23})]$, $log4[(s_{12}/s_{12})]$

3) if $s_{12} > 0$

$log2(a) \rightarrow log(-a) + i\pi$

$Li_{21}(a) \rightarrow$

$-Li_2\left(\frac{1}{a}\right) - \frac{1}{2}log^2(a) - i\pi log(a) + \frac{\pi^2}{3}$

4) repeat 3) with s_{23} , $log3$, Li_{22}

5) if only one of s_{12} or $s_{23} < 0$

$log4(a) \rightarrow log(-a) + i\pi$