

Shadowing of deuteron spin structure functions

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The low x behavior of the twist-two structure functions $F_2^D(x)$, $g_1^D(x)$, $b_1^D(x)$ for the deuteron is examined in the framework of the LPS (Landshoff–Polkinghorne–Short) model for deep inelastic scattering, combined with Glauber theory for quark–nucleon scattering. The quadrupole structure function $b_1^D(x, Q^2)$, which would otherwise vanish for a pure deuteron s-state, turns out to be non-zero once double scattering terms are included. We also observe that, because of nucleon spin correlations in the target, even the unpolarized nuclear structure function requires knowledge of the spin distribution of partons inside individual nucleons. Assuming a simple s-state only in the model of the deuteron, numerical estimates are presented for all twist-two structure functions.

1. Introduction

The light-cone distribution of partons in a nucleus can, at a first level, be obtained by simply convoluting the distribution inside individual nucleons with the distribution of nucleons inside a nucleus. This procedure progressively fails as one approaches the low x , or shadowing region. In the approach of Brodsky and Lu [1], based on the LPS model [2], the virtual photon converts to a $q\bar{q}$ pair upstream of the target at a distance inversely proportional to x , where $x = Q^2/2P \cdot q$. The propagating \bar{q} interacts with the quarks in the nucleus through multiple scattering with the nucleons, treated in the Glauber approximation. Shadowing occurs through the following mechanism: the incident \bar{q} scatters elastically with one nucleon on the front face of the nucleus, continues propagating, and then smashes into a second nucleon. Because the \bar{q} – N amplitude has a dominant imaginary part at high energies, and because the phase of the \bar{q} propagator is also imaginary, the net effect is a reduction of the flux of antiquarks through an interference with the single scattering term.

In this paper, we shall be interested in extending the treatment of Brodsky and Lu [1] to include spin degrees of freedom of the nucleon and nucleus. Specifically, we shall deal with shadowing effects on the structure functions $g_1^D(x)$, $b_1^D(x)$, as well as $F_2^D(x)$,

of the deuteron. These effects could be important for at least two reasons. First, the accurate extraction of F_2 and g_1 for the neutron from e – D experiments requires that nuclear effects be adequately understood. Second, the deuteron, being a $J=1$ system, has a twist-two structure function $b_1^D(x, Q^2)$ whose existence was pointed out in ref. [3], and whose measurement would be of considerable interest. As discussed in ref. [3], the assumptions of the convolution model, together with the absence of a d-state in the deuteron, leads to the vanishing of b_1^D . The contention made there was that a non-zero b_1^D , over and above the small admixture arising from s–d and d–d terms in the wavefunction [4], is a clear indication of the breakdown of the convolution model and is possibly indicative of extra degrees of freedom possessed by partons in a nucleus relative to a nucleon. Here, however, we shall assume a simple conventional model for the deuteron and show that multiple scattering at low x can still lead to $b_1^D \neq 0$ even if only the s-wave component is present.

An interesting fact – that hitherto has not been remarked upon in numerous discussions of shadowing in the deuteron – is that the spin-averaged deuteron structure function, $F_2^D(x, Q^2)$, involves more than spin-averaged information at the nucleon level. This, at first glance, is surprising because one does not expect the symmetric and anti-symmetric structures in

deep inelastic scattering processes to mix with each other in any way. However, as we shall show, in a double scattering process this does occur. Roughly speaking, the product of two antisymmetric terms is a symmetric term. This introduces a small correction to F_2^D which is almost negligible for this very dilute system, but which may be of some importance in denser systems. The same product is responsible for $b_1^D \neq 0$ in our model. Since $b_1^D(x, Q^2)$ is expected to be measured at HERA at some time in the near future, the results obtained here ought to be directly testable [5].

2. Formalism

The purpose of this section is to derive a convenient expression, in terms of the \bar{q} - N scattering amplitude, for the leading order structure functions of a spin-one hadron measurable in deeply inelastic scattering of leptons. The starting point is the leading order, factorized, forward scattering amplitude of virtual photons from a hadron target with helicity $H=0, \pm 1$ (fig. 1),

$$T_H^{\mu\nu} = \int \frac{d^4k}{(2\pi)^4} \text{Tr}[S_H(P, k)H^{\mu\nu}(k)] . \quad (1)$$

Here $H^{\mu\nu}(k)$ is the readily computable hard part, i.e., the amplitude for forward scattering of a hard virtual photon from an elementary charged constituent. All target information is contained in $S_H(P, k)$,

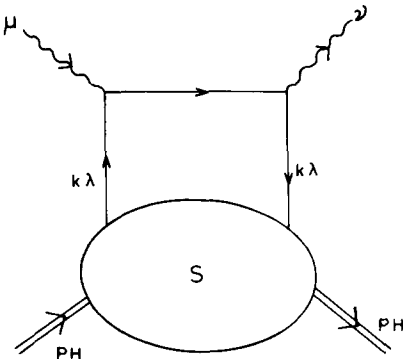


Fig. 1. Amplitude for scattering of a virtual photon from a hadron target with helicity $H=0, \pm 1$ in terms of the \bar{q} -target amplitude $S_H(P, k)$. A crossed diagram must be added to the above.

$$S_H(P, k) = \frac{1}{2\pi} \int d^4\xi \exp(ik \cdot \xi) \times \langle \text{PH} | T[\bar{q}(0)q(\xi)] | \text{PH} \rangle . \quad (2)$$

Quarks cannot, of course, exist as asymptotic states in a scattering process. However, at sufficiently low x in the $\gamma^* \rightarrow q\bar{q}$ process, they exist as almost on-shell states with small k^2 as they traverse the nucleus. It is therefore meaningful to talk of a quark-target amplitude $T_H^\lambda(P, k)$ by appending quark spinors according to the LSZ procedure,

$$T_H^\lambda(P, k) = \bar{u}(k\lambda) \not{k} S_H(P, k) \not{k} u(k\lambda) . \quad (3)$$

This amplitude to scatter quarks of definite helicity λ can be modeled by, for example, the exchange of reggeons. But for now our discussion will not depend on the choice of a particular model. It is useful to expand the 4×4 matrix S in the basis of Dirac matrices,

$$S_H = B_H^\mu \gamma_\mu + C_H^\mu \gamma_\mu \gamma_5 + \dots , \quad (4)$$

where only terms of immediate relevance are displayed. Combining eqs. (3), (4) yields

$$\begin{aligned} B_H^\mu &= i\Delta(k^2) \cdot \frac{1}{2} (T_H^{\lambda=+1} + T_H^{\lambda=-1}) \Delta(k^2) k^\mu \\ &= i \cdot \frac{1}{2} (t_H^{\lambda=+1} + t_H^{\lambda=-1}) k^\mu \\ &\equiv i t_H k^\mu , \end{aligned} \quad (5)$$

and

$$\begin{aligned} C_H^\mu &= i\Delta(k^2) \cdot \frac{1}{2} (T_H^{\lambda=+1} - T_H^{\lambda=-1}) \Delta(k^2) k^\mu \\ &= i \cdot \frac{1}{2} (t_H^{\lambda=+1} - t_H^{\lambda=-1}) k^\mu \\ &\equiv i \delta t_H k^\mu . \end{aligned} \quad (6)$$

$\Delta(k^2)$ is the quark propagator which, for massless quarks, is $1/k^2$ and $t_H^\lambda = \Delta(k^2) T_H^\lambda \Delta(k^2)$ is the full amplitude.

Since S is a certain matrix element of quark operators, it follows from eqs. (2), (4) that we can get an alternative expression for B_H^μ and C_H^μ :

$$\begin{aligned} B_H^\mu &= \frac{1}{8\pi} \int d^4\xi \exp(ik \cdot \xi) \\ &\times \langle \text{PH} | T[\bar{q}(0)\gamma^\mu q(\xi)] | \text{PH} \rangle , \end{aligned} \quad (7)$$

$$\begin{aligned} C_H^\mu &= \frac{1}{8\pi} \int d^4\xi \exp(ik \cdot \xi) \\ &\times \langle \text{PH} | T[\bar{q}(0)\gamma^\mu \gamma_5 q(\xi)] | \text{PH} \rangle . \end{aligned} \quad (8)$$

We shall now use the technique of collinear expansion [6] to reduce these expressions. To this end, define the light-cone vectors

$$p^\mu = \frac{1}{\sqrt{2}} (P, 0, 0, P), \quad n^\mu = \frac{1}{\sqrt{2}} \left(\frac{1}{P}, 0, 0, -\frac{1}{P} \right),$$

$$k_T^\mu = (0, \mathbf{k}_T, 0), \quad (9)$$

which obey $p \cdot n = 1$ and $p^2 = n^2 = p \cdot k_T = n \cdot k_T = 0$. In terms of these vectors the quark momentum k^μ is expressible as

$$k^\mu = x p^\mu + \frac{k^2 + k_T^2}{2x} n^\mu + k_T^\mu, \quad (10)$$

and the target momentum P^μ is

$$P^\mu = p^\mu + \frac{1}{2} M^2 n^\mu. \quad (11)$$

The collinear expansion method, in which the hadron is viewed as an almost collinear beam of partons moving rapidly in a given direction with limited transverse momentum, corresponds to expanding $q(\xi)$ about $\xi^\mu = \xi^- n^\mu$. To leading order – which corresponds to total neglect of transverse momentum – we have that $q(\xi) = q(\lambda n) + \dots$. Inserting this into eq. (8) yields,

$$B_H^\mu = \frac{1}{4} (2\pi)^3 \delta^2(\mathbf{k}_T) \delta(k^-) \frac{1}{P} \int \frac{d\lambda}{2\pi} \exp(i\lambda x) \langle \text{PH} | \bar{q}(0) \gamma^\mu q(\lambda n) | \text{PH} \rangle, \quad (12)$$

and

$$C_H^\mu = \frac{1}{4} (2\pi)^3 \delta^2(\mathbf{k}_T) \delta(k^-) \frac{1}{P} \int \frac{d\lambda}{2\pi} \exp(i\lambda x) \langle \text{PH} | \bar{q}(0) \gamma^\mu \gamma_5 q(\lambda n) | \text{PH} \rangle. \quad (13)$$

The power of the collinear method is that one may systematically incorporate non-zero transverse momentum, albeit at the cost of including multi-parton distributions [7].

We now turn to the issue of defining quark distributions for a spin-one target. Rather than follow the somewhat cumbersome route as in ref. [3], we shall simply extend the procedure of ref. [7] for spin-half targets to spin-one. Lorentz covariance and simple power counting of mass dimensions leads to the expansion,

$$\int \frac{d\lambda}{2\pi} \exp(i\lambda x) \langle \text{PH} | \bar{q}(0) \gamma^\mu q(\lambda n) | \text{PH} \rangle = 2f_1(x) p^\mu + 2b_1(x) \Theta^{\mu\nu} n_\nu + \dots, \quad (14)$$

and

$$\int \frac{d\lambda}{2\pi} \exp(i\lambda x) \langle \text{PH} | \bar{q}(0) \gamma^\mu \gamma_5 q(\lambda n) | \text{PH} \rangle = 2g_1(x) (s \cdot n) p^\mu + \dots, \quad (15)$$

where $\Theta^{\mu\nu}$ is a traceless, symmetric, tensor constructed from the polarization vector of the target,

$$\Theta^{\mu\nu}(H) = \frac{1}{2} [E^{\mu*}(H) E^\nu(H) + E^{\nu*}(H) E^\mu(H)] - \frac{1}{3} (P^\mu P^\nu - M^2 g^{\mu\nu}), \quad (16)$$

and s^μ is the target spin vector,

$$s^\mu(H) = \frac{-i}{M^2} \epsilon^{\mu\alpha\beta\gamma} E_\alpha^*(H) E_\beta(H) P_\gamma. \quad (17)$$

The terms omitted in eqs. (14), (15) are of twist-three and higher. Equating B_H^μ and C_H^μ obtained in eqs. (5), (6) with that obtained in eqs. (7)–(16), and setting $\mu = +$, yields

$$\frac{i}{(2\pi)^3} t_H k^+ = \frac{1}{2} \delta^2(\mathbf{k}_T) \delta(k^-) \left(f_1(x) + b_1(x) \frac{1}{P^2} \Theta^{++} \right), \quad (18)$$

and

$$\frac{i}{(2\pi)^3} \delta t_H k^+ = \frac{1}{2} \delta^2(\mathbf{k}_T) \delta(k^-) (s \cdot n) g_1(x). \quad (19)$$

Integrating over k_T and k^2 at fixed x , and projecting out the structure functions by appropriately weighted sums over spins yields

$$f_1(x) = \frac{i}{(2\pi)^3} \int dk^2 d^2 k_T \sum_H f(H) t_H(s, k^2), \quad (20)$$

$$b_1(x) = \frac{i}{(2\pi)^3} \int dk^2 d^2 k_T \sum_H b(H) t_H(s, k^2), \quad (21)$$

$$g_1(x) = \frac{i}{(2\pi)^3} \int dk^2 d^2 k_T \sum_H g(H) \delta t_H(s, k^2). \quad (22)$$

In the above, the amplitudes are weighted as follows: $f(1) = f(-1) = f(0) = \frac{1}{3}$ for the spin averaged distri-

bution; $b(1)=b(-1)=-\frac{1}{2}$ and $b(0)=1$ for the quadrupole distribution; and $g(1)=-g(-1)=\frac{1}{2}$ and $g(0)=0$ for the axial distribution.

We shall only briefly consider the analytic properties of the amplitude $t_H^\lambda(s, k^2)$ because this discussion overlaps considerably with that in ref. [2]. Suffice it to say that the k^2 plane contains two cuts – the s cut and the u cut, the position of each cut depending on the value of x because,

$$s = (P+k)^2, \\ = M^2(1+x) + k^2 \left(1 + \frac{1}{x}\right) + \frac{k_T^2}{x}, \quad (23)$$

and u is obtained by $x \rightarrow -x$ in the above. For $|x| > 1$, the integral over k^2 vanishes since the contour in the complex k^2 plane encloses no singularities. For $0 < x < 1$, the dominant contribution comes from incident antiquarks annihilating with quarks in the target,

$$f_1(x) = \frac{2xC}{(2\pi)^3(1-x)} \int_{s_0} ds d^2k_T \operatorname{Im} \sum_H f(H) t_H(s, k^2), \quad (24)$$

$$b_1(x) = \frac{2xC}{(2\pi)^3(1-x)} \int_{s_0} ds d^2k_T \operatorname{Im} \sum_H b(H) t_H(s, k^2), \quad (25)$$

$$g_1(x) = \frac{2xC}{(2\pi)^3(1-x)} \int_{s_0} ds d^2k_T \operatorname{Im} \sum_H g(H) \delta t_H(s, k^2) \quad (26)$$

where C is the parton wavefunction renormalisation constant [8]. Eqs. (24)–(26) represent the generalizations of eq. (1) of the paper by Brodsky and Lu [1]. We shall now consider application to the simplest $J=1$ system, the deuteron.

3. Application

Applying the formalism derived above to the deuteron requires specifying the forward \bar{q} - D invariant amplitude, $T_H^\lambda(s, k^2)$, in terms of the \bar{q} - N amplitude, $T_h^\lambda(s, k^2)$. This is easily done if one uses Glauber theory [9] and neglects Fermi motion, which should be an adequate approximation at low x ,

$$T_H^\lambda = T_{h_p}^\lambda + T_{h_n}^\lambda + i\xi T_{h_p}^\lambda T_{h_n}^\lambda. \quad (27)$$

Here ξ is an overlap factor which determines the strength of the double scattering term,

$$\xi(s, k^2) = \frac{1}{16\pi r_0^2 P_{\text{CM}} \sqrt{s}} \\ \times \int d^3r |\psi(r)|^2 \exp[-(\mathbf{r} - \hat{\mathbf{k}} \mathbf{r} \cdot \hat{\mathbf{k}})^2 / 4r_0^2]. \quad (28)$$

In arriving at eqs. (27), (28) it has been assumed that the quark helicity λ remains unchanged in the scattering process, which is true for massless quarks. The parameter r_0 arises from postulating that the non-forward amplitude has the conventional form

$$T_h^\lambda(s, k^2, q^2) = T_h^\lambda(s, k^2) \exp(-\frac{1}{2}r_0^2 q^2). \quad (29)$$

In using Glauber theory one actually starts from the lab frame amplitude; the factor $P_{\text{CM}} \sqrt{s}$ in eq. (28) arises from making the transformation of the lab frame amplitude to the invariant amplitude.

Considering only the s state of the deuteron, for which we have $H = h_p + h_n$, inserting eqs. (27), (28) into eqs. (24)–(26) yields

$$f_1^D(x) = f_1^p(x) + f_1^n(x) \\ + \frac{2xC}{1-x} \int \frac{ds d^2k_T}{(2\pi)^3} \xi \operatorname{Im} i(T^p T^n + \frac{1}{3} \delta T^p \delta T^n) A_F^2, \quad (30)$$

$$b_1^D(x) = \frac{-4xC}{1-x} \int \frac{ds d^2k_T}{(2\pi)^3} \xi \operatorname{Im} i(\delta T^p \delta T^n) A_F^2, \quad (31)$$

$$g_1^D(x) = g_1^p(x) + g_1^n(x) \\ + \frac{2xC}{1-x} \int \frac{ds d^2k_T}{(2\pi)^3} \xi \operatorname{Im} i(T^p \delta T^n + T^n \delta T^p) A_F^2. \quad (32)$$

The contribution from q - N scattering is trivially obtained from the above by crossing. The \bar{q} - N helicity averaged and difference amplitudes may be parameterized using Regge theory. For $T^{p,n}$, the pomeron is the dominant contribution, supplemented by reggeon contributions at lower values of s . We have used the parameterization used in ref. [1]. For $\delta T^{p,n}$, the pomeron does not contribute. Thus it will have little effect at very small x . The leading order contribution from reggeons is parameterized as,

$$\begin{aligned} \delta T^{p,n}(s, \nu^2) \\ = \sigma [i\beta_0(\nu^2) + (1-i)s^{-1/2}\beta_{-1/2}(\nu^2) \\ + is^{-1}\beta_{-1}(\nu^2)] , \end{aligned} \quad (33)$$

where $\beta_\alpha(\nu^2)$ with $\nu^2 \equiv -k^2$ has the same functional form as in ref. [1].

4. Results and discussion

The basic input into the calculation of the deuteron quantities is the quark–nucleon amplitude. For the isospin averaged $F_2 = xf_1$, we used the same parameters as Brodsky and Lu [1]. For g_1^p a reasonably good fit (see fig. 3) to the experimental data can be made using the form in eq. (33), and with parameters of reasonable sizes. These values are displayed in table 1. There is no data available as yet for g_1^n but different predictions of these are available. We have fitted the parameters in Δt^n to one such prediction. Also needed is the s-state wavefunction [10], for which it is convenient to use a sum of gaussians because the integral in eq. (28) can be done trivially. A d-wave component can also be included, but the shadowing corrections due to it are rather small because, in addition to the d state having only (3–4)% probability, nucleons tend to stay further apart because of the centrifugal barrier.

Our prediction for the shadowing contribution to F_2 for the deuteron is shown in fig. 2. In common with other recent works [11,12] on this, we find that even for a system as dilute as the deuteron, this is not negligible. Quite interestingly, there is even a small amount of anti-shadowing at large x -values, a manifestation of the constructive interference of \bar{q} – N amplitudes. The amount of shadowing we find, how-

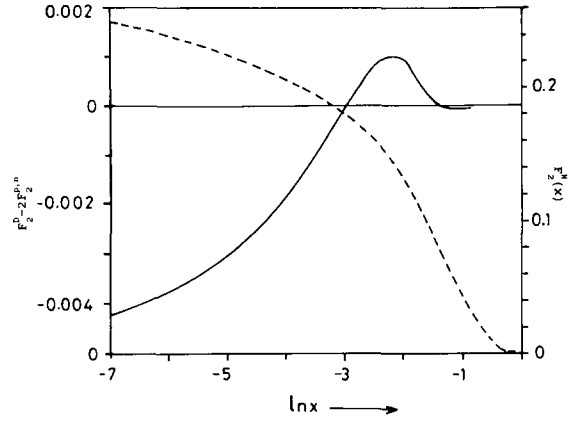


Fig. 2. The contribution of the double scattering term in $F_2^D(x)$ (solid line) and the spin averaged nucleon structure function (dashed line) given in ref. [1], normalized to 0.25 at $x=0.001$.

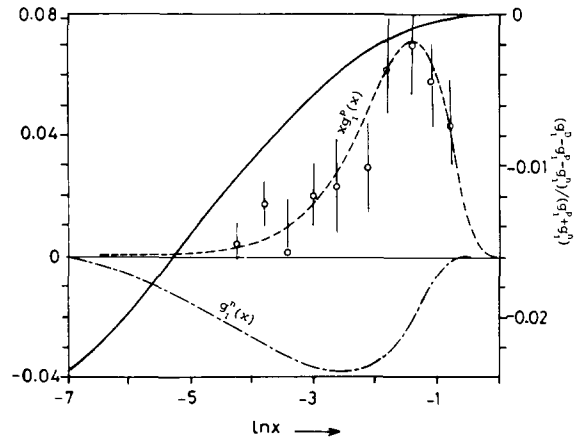


Fig. 3. The shadowing contribution to $g_1^D(x)$ (solid curve) computed from eq. (32). The dashed line represents the curve for xg_1^p corresponding to the parameterization given in eq. (33). The data points are taken from ref. [13]. The curve for g_1^n (dash-dotted) is approximated by g_1^n given in Schäfer [14].

Table 1
Parameters characterizing δT for the proton and neutron.

	Parameter									
	f_0 (GeV ²)	$f_{-1/2}$ (GeV ³)	f_{-1} (GeV ⁴)	n_0	$n_{-1/2}$	n_{-1}	$\bar{\nu}_0^2$ (GeV ²)	$\bar{\nu}_{-1/2}^2$ (GeV ²)	$\bar{\nu}_{-1}^2$ (GeV ²)	r_0^2 (GeV/c) ²)
proton	0.09	0.044	0.31	2	4	2	0.1	1	1	10
neutron	0.015	0.018	0.005	2	4	2	0.1	1	1	10

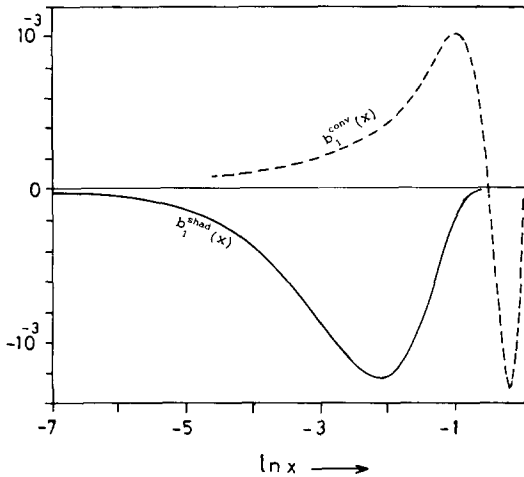


Fig. 4. $b_1^D(x)$ evaluated from eq. (31) (solid line). The dashed line is $b_1^D(x)$ in the convolution model [4] and includes s-d and d-d terms.

ever, is significantly less than in refs. [11,12] which, also differs significantly among themselves. For the moment needed in tests of the Gottfried sum rule, we find

$$\int_{0.001}^{0.99} dx \frac{\Delta F_2}{x} = -0.013,$$

instead of the value -0.060 in ref. [12]. It is not clear why the predictions of three different models of shadowing differ so much. It is not, however, because of the $\delta t \delta t$ term neglected in refs. [11,12]. This term is unimportant relative to the $t-t$ term except at larger values of x , where the whole effect is small anyway.

In fig. 3, we display our prediction for the shadowing of the deuteron's g_1 . Again, it is quite non-negligible. There do not appear to be other calculations of this quantity with which our results could be compared. However, it seems rather clear that extraction of the neutron's g_1 for small values of x will require subtraction of the deuteron's nuclear effects and that some model dependence is inevitable.

The $\delta t \delta t$ term, while negligible for F_2^D , is totally

responsible for $b_1^D(x)$ and leads to the curve shown in fig. 4 if Fermi motion is neglected. The contribution from Fermi motion alone, taken from ref. [4], is also shown. Both are rather small, but shadowing is dominant at small x . This is rather interesting; for other structure functions, shadowing is a small, but unfortunately necessary correction. But for $b_1^D(x)$, shadowing accounts for most of the contribution at small x . Measurement of b_1^D in this region would, therefore, shed considerable light on the nature of the shadowing process.

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