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Tensor polarization of vector mesons from quark and gluon fragmentation

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Abstract

We considered the electro- and photoproduction of ρ and other vector mesons with moderately large transverse momentum in the scattering of unpolarized electrons on unpolarized nucleons and nuclei. For these processes we have analyzed how to extract the novel fragmentation functions $\bar{b}_1^q(z), \bar{b}_1^G(z)$, describing tensor polarization, and the photon double spin-flip distribution Δ^γ from the angular distribution of meson decay products.

Deep Inelastic Scattering (DIS) on targets with higher spin [1] offers the possibility to study new spin structure functions. The simplest one is the tensor spin structure function b_1, appearing for target hadrons with spin-1 or higher [1,2], and being the subject of recent model studies [3] and lattice calculations [5].

Another interesting example is the structure function Δ related to the amplitude which describes a spin flip of both the hadron and the photon by two units each [4] (the relevant operators were identified earlier in [6]).

For a photon target, it is proportional to the structure function F_3^γ, whose small-x behaviour was studied recently in [7].

For practical experiments the only available hadronic target with spin 1 is the deuteron. Since both Δ and b_1 are zero, when neutron and proton are scattered independently, the observable effects are expected to be very small. The reason is the weakness of the nuclear binding, as the tensor polarization of any spin-one particle built from two independent spin-$1/2$ constituents is zero.

At the same time, final states including vector mesons are readily produced in hard hadronic and leptonic processes. As quarks in the vector mesons are strongly bound, the latter can have a large tensor polarization (it is a genuine spin-1 object in contrast to the deuteron), which indeed was observed experimentally, see e.g. [8]. This observation suggests, that the proper places to observe effects of b_1 and Δ are their fragmentation counterparts. This is the main subject of the present paper.

We identify the fragmentation analogs of b_1 and Δ which parametrize the tensor polarization of vector mesons, which can be experimentally accessed by the observation of specific angular distributions of their decay products.

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By simultaneously observing high-P_T vector meson production at large and small photon virtualities Q^2 at HERA one could measure new fragmentation functions as well as the (resolved) photon distribution Δ^γ, directly related to the structure function F_3^γ.

We consider the vector meson produced in fragmentation processes of either a leading quark or gluon, and find that they lead to different angular distributions of the decay products. While quarks, at twist-2 level, contribute to b_1 only, the gluons contribute both to b_1 and Δ.

We start with the quark case which is simpler, following [9], where the quark tensor fragmentation function was introduced and investigated, and, in particular, a sum rule for its first moment was derived, which was later obtained for the analogous distribution function by Close and Kumano [10].

We describe the tensor polarization by a Cartesian tensor $S^{\mu\nu}$, which is symmetric, traceless and transverse, and which should be considered at the same footing as the familiar vector polarization S^μ.

$$S^{\mu\nu} = S^{\nu\mu}, \quad S^\mu{}_\mu = 0, \quad S^{\mu\nu} P_\nu = 0. \tag{1}$$

In the rest frame the 3×3 density matrix is reduced, apart of the trace, which describes the spin-averaged cross-section, to the 3 components of the polarization vector S^i and the 5 components of the symmetric traceless tensor S^{ij}.

For pure polarization states the density matrix is just $\rho^{\mu\nu} = \epsilon^\mu \epsilon^{*\nu}$, and its traceless part is

$$S^{\mu\nu} = \frac{1}{2}(\epsilon^\mu \epsilon^{*\nu} + \epsilon^\nu \epsilon^{*\mu}) - \frac{1}{3}\left(g^{\mu\nu} - \frac{P^\mu P^\nu}{M^2}\right)\epsilon^\alpha \epsilon^*_\alpha, \tag{2}$$

where P^μ and M are the meson momentum and mass, respectively.

The $S^{\mu\nu}$-dependent twist-2 terms in the quark fragmentation function [11] are described by the following expression:

$$\begin{aligned} &zP^+ \int \frac{dx^-}{4\pi} \exp\left(-iP^+ \frac{x^-}{z}\right) \tfrac{1}{3}\mathrm{Tr}_{\mathrm{Color}} \tfrac{1}{2}\mathrm{Tr}_{\mathrm{Dirac}} \sum_{P,X} \langle 0|\gamma^\mu \psi(0)|P,X\rangle\langle P,X|\bar{\psi}(x)|0\rangle \\ &\quad = \bar{f}(z,\mu^2) P^\mu + M^2\left(\bar{f}_{LT}(z,\mu^2) S^{\mu\nu} n_\nu + \bar{f}_{LL}(z,\mu^2) P^\mu S^{\rho\nu} n_\rho n_\nu\right), \end{aligned} \tag{3}$$

where $z = P^+/k^+$, μ is a factorization scale, $\bar{f}(z,\mu^2)$ is the standard spin-averaged term, n_ν is a light-cone vector normalized by the condition $P \cdot n = 1$, and we introduced the two tensor fragmentation functions $\bar{f}_{LT}(z,\mu^2)$ and $\bar{f}_{LL}(z,\mu^2)$.

We are not considering any power corrections here, and in this limit only the purely longitudinal component [9] of the tensor $S^{\mu\nu}$ contributes.

$$S^{\mu\nu} \Rightarrow \frac{P^\mu P^\nu}{M^2} S_{zz}, \tag{4}$$

where the meson is assumed to move along the z axis in the infinite momentum frame. The coefficient S_{zz} is the component of the tensor polarization in the meson rest frame, when the direction of the z axis is chosen along the meson momentum in the infinite momentum frame (before the boost to the rest frame), Consequently only the sum of the functions $\bar{f}_{LT}$ and $\bar{f}_{LL}$ is entering, which is just f_{Al} in (20) of [9] and the analog of b_1 for the distribution case:

$$\bar{b}_1^q(z,\mu^2) = \bar{f}_{LT}(z,\mu^2) + \bar{f}_{LL}(z,\mu^2). \tag{5}$$

The tensor polarization dependence of a given hard cross-section is obtained by the convolution of (3) with the hard scattering kernel. The factor P^μ in (3), (4) is playing the role of the unpolarized quark density matrix.

Consequently, the hadron cross-section is just a convolution of the *unpolarized* partonic cross-section with this new function

$$\frac{d\sigma}{dt} = \int dz \bar{b}_1^q(z,\mu^2) \frac{d\hat{\sigma}}{d\hat{t}}(z,\mu^2), \tag{6}$$

where $d\hat{\sigma}/d\hat{t}$ is the corresponding spin-averaged partonic cross-section.

Let us consider the analogous contribution due to gluons. Restricting ourselves to the terms symmetric in μ and ν (the antisymmetric terms are related to the vector polarization which is not discussed in this paper) and averaging over gluons colours and spins, we get [11]:

$$\begin{aligned} &-\frac{z}{16\pi P^+}\int dx^- \exp\left(-iP^+\frac{x^-}{z}\right)\sum_{P,X}\langle 0|G^b_{+\mu}(0)|P,X\rangle\langle P,X|G^b_{+\nu}(x)|0\rangle \\ &\quad = \bar{G}(z,\mu^2)g^\perp_{\mu\nu} + \bar{\Delta}(z,\mu^2)S^\perp_{\mu\nu} + M^2\bar{b}_1^G(z,\mu^2)g^\perp_{\mu\nu}S_{\rho\sigma}n^\rho n^\sigma, \end{aligned} \tag{7}$$

where $g^\perp_{\mu\nu} = g_{\mu\nu} - g^{\|}_{\mu\nu}$ with $g^{\|}_{\mu\nu} = P_\mu n_\nu + P_\nu n_\mu - M^2 n_\mu n_\nu$ and G is the familiar spin-averaged term. The transverse component of the polarization tensor is

$$S^\perp_{\mu\nu} = \tfrac{1}{2}\left(g^\perp_{\mu\alpha}g^\perp_{\nu\beta} + g^\perp_{\nu\alpha}g^\perp_{\mu\beta} - g^\perp_{\mu\nu}g^\perp_{\alpha\beta}\right)S^{\alpha\beta} \approx g^\perp_{\mu\alpha}g^\perp_{\nu\beta}S^{\alpha\beta}, \tag{8}$$

where symmetry and tracelessness were taken into account to get the second expression, valid, when the mass corrections to $g^\perp_{\mu\nu}$ are neglected.

Note the important difference between quarks and gluons. While both are contributing to the function $\bar{b}_1$ [9,12], it is only gluons, which provide the leading twist contribution to $\bar{\Delta}$. Physically, the reason is that a (on-shell) gluon is described by a 2×2 matrix of Stokes parameters (in contrast to a massless quark whose polarization state is completely characterized by just one number, namely its helicity) and thus carries additional information on the parent hadron.

The functions $\bar{b}_1$ and $\bar{\Delta}$ describe different components of the meson tensor polarization. While $\bar{b}_1$ in the meson rest frame is related to the component $S_{zz} = -S_{xx} - S_{yy}$, $\bar{\Delta}$ is describing another degree of freedom, namely $S_{xx} - S_{yy}$.

To come to the observable quantities one should first recall [13,9] that S^{ij} is actually the polarization, selected by the detector. By isolating specific angular correlations in the final state one defines S^{det}_{ij}. More precisely, the measured cross-section allows to extract the tensor polarization S^{scat}_{ij} according to:

$$d\sigma = d\sigma_0\left(1 + S^{\text{scat}}_{ij}S^{\text{det}}_{ij}\right), \tag{9}$$

where $d\sigma_0$ is the spin-averaged cross-section. The angular distribution for e.g. the decay into a pair of scalar particles (pions, kaons) in the rest frame is of the form:

$$\frac{d^2\sigma}{d\theta d\phi} \sim 1 + S^{\text{scat}}_{zz}(3\cos^2\theta - 1) + \left(S^{\text{scat}}_{xx} - S^{\text{scat}}_{yy}\right)\sin^2\theta\cos 2\phi, \tag{10}$$

where θ is the polar angle with respect to the z axis and ϕ is the azimuthal angle with respect to the reaction plane. The standard choice of the z axis is along the meson momentum $\boldsymbol{P}$ in the target rest frame. Both angles are measured in the vector meson rest frame.

To our knowledge, the unifying treatment of b_1 and Δ as describing different components of the polarization tensor is absent in the literature, although the appearance of Δ in the description of the difference of deutron cross-sections with orthogonal linear polarization was already presented in the first paper of [4].

Recall [9], that for $\bar{b}_1$, the short-distance subprocesses are the same as in the unpolarized case, as they are projected onto the same structure. At the same time, the structure $S^\perp_{\mu\nu}$ may be considered as a density matrix of the linearly polarized gluons. As a result, $\overline{\Delta}$ is picking out that part of the final gluon density matrix which describes its linear polarization.

Such partonic subprocesses were quite extensively studied in the past. Let us briefly discuss, which hadronic reactions are of potential importance for us.

The linear polarization of produced gluons may emerge in scattering processes with unpolarized initial particles. In particular, the gluon linear polarization in the $q\bar{q}$ and qg subprocesses of pp scattering [14] is zero in the Born approximation for massless quarks and requires to take into account one-loop contributions. The resulting polarization is sensitive to the QCD colour structure and is reasonably large (of the order 10%). The observation of a $\cos 2\phi$ dependence in the vector mesons decays, correlated with the predicted angular dependence of the polarization of the scattered gluon, would give access to $\overline{\Delta}$. However, as the resulting coefficients in the angular distribution are not expected to be large, high statistics data (say, from RHIC) would be required.

Another promising candidate is electron-positron annihilation [15]. The relevance of vector mesons (especially isosinglet ones, which are supposed to be strongly coupled to gluons) was already mentioned there. However, the related comprehensive analysis, explicitly specifying the relevant fragmentation functions in a way presented here, is, to our knowledge, absent in the literature. The last remark applies also to recent studies [16] of the linear polarization of gluons accompanying $t\bar{t}$ quarks pairs.

The gluon linear polarization in DIS is appearing due to the linear polarization of the virtual photon. The situation is different for large and small Q^2. For large Q^2 the gluon polarization is calculable perturbatively. The gluon linear polarization in the QCD Compton scattering subprocess was studied in detail some years ago [18]. The obtained numerical results are fairly large, in particular, for HERA kinematics. As it is clear from our analysis, the studies of the azimuthal dependences of decay products are providing access to the tensor fragmentation functions.

The angular distribution of the decay pions is given by (10). The fragmentation functions $\bar{b}_1$ of all quarks and gluons contribute to the polar angle dependence:

$$S_{zz}^{\mathrm{scat}} = \frac{\sum_q \int dxdz\, q(x) \left[\bar{b}_1^q(z) \frac{d\hat{\sigma}^q}{d\hat{t}} + \bar{b}_1^G(z) \frac{d\hat{\sigma}^G}{d\hat{t}} \right]}{\sum_q \int dxdz\, q(x) \left[\bar{f}^q(z) \frac{d\hat{\sigma}^q}{d\hat{t}} + \overline{G}(z) \frac{d\hat{\sigma}^G}{d\hat{t}} \right]} . \tag{11}$$

Here $q(x)$ is the standard spin-averaged quark distribution. We drop for brevity here and in the following formulas the dependence on the factorization scale which should be, as usual, of the order of the transverse meson momentum.

The azimuthal angle dependence at the leading twist level is directly related to the gluon fragmentation function $\overline{\Delta}$ (Fig. 1):

$$S_{xx}^{\mathrm{scat}} - S_{yy}^{\mathrm{scat}} = \frac{\sum_q \int dxdz\, q(x)\, \overline{\Delta}(z) \left[\frac{d\hat{\sigma}_{xx}^G}{d\hat{t}} - \frac{d\hat{\sigma}_{yy}^G}{d\hat{t}} \right]}{\sum_q \int dxdz\, q(x) \left[\bar{f}^q(z) \frac{d\hat{\sigma}^q}{d\hat{t}} + \overline{G}(z) \frac{d\hat{\sigma}^G}{d\hat{t}} \right]} , \tag{12}$$

where $\frac{d\hat{\sigma}_{ii}}{d\hat{t}}$(i = x,y) is the partonic cross-section with the produced gluon being linearly polarized along the i axis, so that:

$$\frac{d\hat{\sigma}_{xx}^G}{d\hat{t}} + \frac{d\hat{\sigma}_{yy}^G}{d\hat{t}} = \frac{d\hat{\sigma}^G}{d\hat{t}} . \tag{13}$$

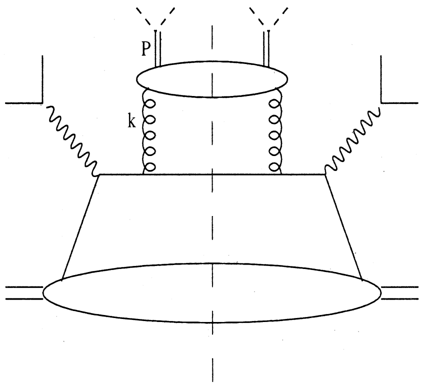

Fig. 1. Typical diagram for the high P_T production of a ρ meson in semi-inclusive DIS

One of the two integrations in the parton momentum fractions may be eliminated by making use of the delta-function in the parton-cross section. Taking also into account that the distribution function decreases rapidly with growing x, which is required to produce the fast gluon with small z, we conclude that small z gives only negligible contribution and restrict the integration to the leading hadron region $z \sim 1$. This corresponds to the case of the free gluon polarization, considered in [18], and which is most important for the experimental studies.

The experimental measurement of angular distributions allows to determine the l.h.s. of the above equations. Developing estimates [9] and models [3,5] for tensor fragmentations functions, one may evaluate the size of the expected experimental signal.

For small Q^2, there is a significant resolved contribution parametrized by the photon structure function (Fig. 2). The term, proportional to $\overline{\Delta}$ is picking out, due to helicity conservation. The double helicity flip term is

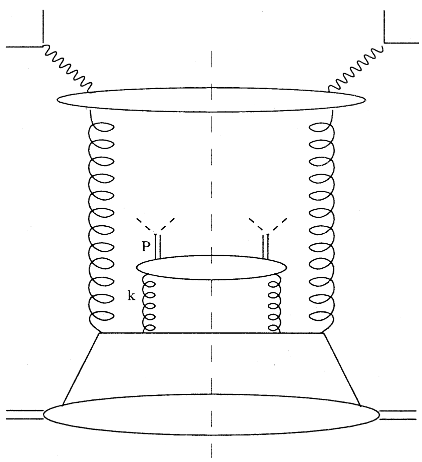

Fig. 2. Typical digram for photoproduction of a ρ meson with high P_T

given by the distribution of gluons in the photon Δ^γ, whose convolution with the quark box diagram results in the structure function F_3^γ.

$$S_{xx}^{\mathrm{scat}} - S_{yy}^{\mathrm{scat}} = \frac{\sum_q \int dx_g dx_q dz \frac{2(1-y_\gamma)}{y_\gamma} \Delta^\gamma(x_g) q(x_q) \bar{\Delta}(z) \left[\frac{d\hat{\sigma}_{xx}^G}{dt} - \frac{d\hat{\sigma}_{yy}^G}{dt} \right]}{\sum_q \int dx_g dx_q dz \frac{1+(1-y_\gamma)^2}{y_\gamma} G^\gamma(x_g) q(x_q) \left[\bar{f}^q(z) \frac{d\hat{\sigma}^q}{dt} + \bar{G}(z) \frac{d\hat{\sigma}^G}{dt} \right]}, \tag{14}$$

where G^γ is the spin-averaged distribution of gluons in the photon, while $y_\gamma = 2\nu/s$, $y_\gamma x_g$ are the fractions of the electron momentum, carried by the photon and gluon, respectively. This equation is just Eq. (12) in which nominator and denominator were convoluted with the appropriate gluon distribution. Depending on the experimental situation, one might also have to integrate over y_γ.

Several additional comments might help to better understand this formula.

Firstly, note that the Weizsäcker-Williams factors differ for the linearly polarized and unpolarized case (cf. also [4]) and their ratio is just the linear polarization of the emitted (almost on-shell) photon [19]:

$$\xi = \frac{2(1-y_\gamma)}{1+(1-y_\gamma)^2}. \tag{15}$$

This formula, which may be easily recovered from, say, [19], has a simple physical interpretation. Recall that $(1 \pm (1-y_\gamma)^2)/y_\gamma$ are the DGLAP spin-averaged and spin-dependent kernels, respectively. The factors $1/y_\gamma$ and $(1-y_\gamma)^2)/y_\gamma$ are then the kernels for the productions of the photon of positive and negative helicity by the electron of positive helicity, so that $1/\sqrt{y_\gamma}$ and $(1-y_\gamma)/\sqrt{y_\gamma}$ may be understood as the respective helicity amplitudes (up to inessential phases). The interference term, corresponding to the double helicity flip, is then $2(1-y_\gamma)/y_\gamma$, and the ratio 'double helicity flip over spin-averaged case' becomes $2(1-y_\gamma)/(1+(1-y_\gamma)^2)$ in agreement with (15). The polarization is maximal for $y_\gamma \to 0$, when flip and non-flip amplitudes are equal. If the integration over y_γ is performed in (14), the contribution of this region is enhanced by the factor $1/y_\gamma$.

Secondly, other partonic subprocesses besides QCD Compton scattering (Fig. 2) contribute both in the numerator and denominator. The numerator can receive an extra contribution from gluon-gluon scattering. It is dominant at small x and requires a separate investigation following the lines of [7]. In the present paper we assume P_T to be large enough to neglect such a term. The denominator describing the spin averaged cross-section receives an additional contribution from all possible combinations of quarks (antiquarks distributions are assumed to be small) and gluons. At the same time, the quark-quark subprocess dominates at large x, corresponding to very large P_T and small cross-sections, while the gluon-gluon scattering is dominant at small x. So, one should add only the convolution of the quark distribution in the photon with the gluon distribution in the nucleon neglected just for brevity.

Thirdly, the analyzing power of the short distance production of linearly polarized gluons by linearly polarized photons by the QCD Compton subprocess was found [17] to be fairly large. The analysing power for the QCD Compton subprocess of elastic scattering of linearly polarized gluons is actually the same, as it is controlled only by chirality and angular momentum conservation.

Let us conclude: we have analyzed how to extract from the angular distribution of ρ and other vector mesons produced in DIS of unpolarized electrons on unpolarized nucleons and nuclei the novel fragmentation functions $\bar{b}_1^q(z)$, $\bar{b}_1^G(z)$ (from Eq. (11)), $\bar{\Delta}(z)$ (from Eq. (12)) and the photon distribution $\Delta^\gamma(x)$, directly related to the structure function $F_3^\gamma(x)$, from photoproduction (Eq. (14)).

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