THE b_1 STRUCTURE FUNCTION AND NUCLEAR PIONS

Gerald A. Miller*
TRIUMF
Vancouver BC, V6T 2A3 Canada
and
Department of Physics
University of Illinois at Urbana-Champaign
1110 West Green Street
Urbana, Illinois 61801

ABSTRACT

The b_1 structure function is measurable in deep inelastic scattering from polarized nuclei with unpolarized beams. The contributions of nuclear pions are evaluated and found to be small, about 2%.

INTRODUCTION

The b_1 structure function is measurable in deep inelastic lepton scattering from polarized nuclei of spin 1. Hoodbhoy et al[1] have pointed out that novel (non-nucleonic) effects might be discernable in such measurements. The purpose here is to evaluate the b_1 structure function for the simplest nuclear target–the deuteron. Much is known about the deuteron wave function, so that one may have reasonable confidence in the computed results for b_1. Moreover, one may use the deuteron and simple counting arguments to guess the results for other targets. The results I present provide another concrete illustration of the formalism presented by Jaffe.[2,3]

Here is an outline of the presentation. I will define the function $b_1(x)$. Then some general criteria to obtain $b_1 \neq 0$ are discussed. The nucleonic contributions for the deuteron target are found to be small, except at very high values of the Bjorken x variable. Then the pionic contributions are evaluated. These amount to about 2% at low x. Finally, the ^{6}Li target is examined, but the pionic effects are also small.

DEFINITIONS

Consider deep inelastic scattering from a polarized J=1 target (T) with J_z=m. (The direction of the virtual photon momentum (z) is used as the spin quantization axis.) The differential cross section can be written as

$$d^2\sigma^{(m)} \propto \ell^{\mu\nu} W^{(m)}_{\mu\nu} \qquad (1)$$

with $\ell^{\mu\nu}$ as the standard lepton tensor and

$$W^{(m)}_{\mu\nu} = \int d^4r\, e^{iq\cdot r} < T,J=1,m| [j_\mu(r),j_\nu(0)]|T,J=1,m>. \qquad (2)$$

The F_1 structure function is obtained for an unpolarized target. Hence

$$F_1(x) = \frac{1}{3} \sum_m W^{(m)}_{11} \qquad (3)$$

where $x = Q^2/(2Mq^0)$. A different structure function b_1 may be defined:

$$b_1(x) = W^{(1)}_{11} - W^{(0)}_{11}. \qquad (4)$$

In the parton model b_1 can be given in terms of quark distribution functions

$$b_1(x) = q^{(0)}(x) - q^{(1)}(x). \qquad (5)$$

Note that $q^{(m)}(x)$ depends on the J_z of the target nucleus. It is independent of the helicity of the quarks. Indeed, $q^{(m)}(x)$ is an average[1]

$$q^{(m)}(x) = \frac{1}{2}(q^{(m)}_{\uparrow}(x) + q^{(m)}_{\downarrow}) \qquad (6)$$

in which the subscript $\uparrow$ indicates the distribution for a quark of positive helicity. (If conservation of parity holds, $q^{(1)}_{\uparrow,\downarrow}(x) = q^{(-1)}_{\downarrow,\uparrow}$.) Given Eqs. (5) and (6), one may investigate the content of the b_1 observable. Note that the beam need not be polarized.

* Permanent address: Physics Dept., FM-15, Univ. of Washington, Seattle, WA 98195.

CRITERIA FOR $b_1(x) \neq 0$

Use (2), (4) and (5) to abstract

$$q^{(m)}(x) = \langle T,J=1,m|O|T,J=1,m\rangle \quad (7)$$

in which O is an integral over a current commutator, $[j_1(r),j_1(o)]$. For $b_1 \neq 0$, O can not be a scalar operator. The Wigner Eckhart (WE) theory and (5) constrain O to be a vector or rank-2 tensor operator. Furthermore the parity condition $q^{(1)} = q^{(-1)}$ along with the WE theorem says that O can't be a vector operator. Thus O is a rank-2 tensor. One may therefore expect that $b_1(x)$ measures effects of tensor forces. This turns out to be the case.

EXAMPLES

It is worthwhile to illustrate the above argument with some specific examples.

1. Two non-interacting nucleons (total spin=1) at rest. In this case, the use of (6) and (7) leads to

$$q^{(m)}(x) = \sum_{M_1,M_2} \langle 1m|\tfrac{1}{2}m_1\tfrac{1}{2}m_2\rangle^2$$

$$\frac{1}{2}[q_\uparrow^{N,m_1} + q_\downarrow^{N,m_1} + q_\uparrow^{N,m_2} + q_\downarrow^{N,m_2}] \quad (8)$$

in which $q_\uparrow^{N,m}$ is the distribution of a positive helicity quark in a nucleon of J_z=m. Eq. (8) leads to

$$q^{(1)}(x) = q_\uparrow^{N,\frac{1}{2}} + q_\downarrow^{N,\frac{1}{2}} \quad (9)$$

and

$$q^{(0)}(x)=\frac{1}{2}[q_\uparrow^{N,\frac{1}{2}}+q_\downarrow^{N,\frac{1}{2}}+q_\uparrow^{N,-\frac{1}{2}}+q_\downarrow^{N,-\frac{1}{2}}]. \quad (10)$$

The parity condition says that $q_\downarrow^{N,-\frac{1}{2}} = q_\uparrow^{N,\frac{1}{2}}$ and $q_\uparrow^{N,-\frac{1}{2}} = q_\downarrow^{N,\frac{1}{2}}$. Thus $q^{(1)}(x)=q^{(0)}(x)$ and $b_1(x)$=0.

2. Two nucleons in a relative s-state. Again J=1, but now the nucleons have relative isotropic motion. It takes but a moment's thought to realize that again $b_1(x)$ must vanish. These two examples are not surprising, since there are no tensor operators. Let's turn to a more realistic example.

NUCLEONIC CONTRIBUTIONS

Now consider deep inelastic scattering from polarized deuterons (D). (This topic was also considered in Ref. 4). We expect a non-zero result since the tensor force provided by the one pion exchange potential is manifest in the deuteron quadrupole moment.

I use a well-known formalism[5,6] to compute the effect of nucleons moving in the nuclear (D) target. One has a convolution of two probabilities

$$q^{(m)}(x)= \int_x^\infty dy\, q^N(x/y)\, f^{(m)}(y) \quad (11)$$

in which $f^{(m)}(y)$ is a nucleon momentum distribution

$$f^{(m)}(y) = \int d^4p\, [1+\frac{p^3}{\sqrt{p^2 + M^2}}]$$

$$S_D^{(m)}(p)\, \delta(y - \frac{p^o+p^3}{M_D}) \quad (12)$$

where M is the nucleon mass, M_D is the deuteron mass, and

$$S_D^{(m)}(p) = \sum_{m,m'} \langle D,m|b^+_{\vec{p},m}$$

$$\delta(-p_0 + M_D - H)\; b_{\vec{p},m'}|D,m\rangle. \quad (13)$$

For deuterons at rest, one has $p_0 = M - 2.2\text{MeV} - \frac{\vec{p}^2}{2M}$. No closure approximation is necessary.

Standard techniques may be used to show

$$b_1(x) \int dy\, (F_1^p(x/y)+F_1^n(x/y))\Delta f_{sd}(y) \quad (14)$$

where

$$\Delta f_{sd}(y) = \frac{-4\sqrt{2}}{8\pi}\int d^3p\, u(p)w(p)\,(3\cos^2\theta-1)$$

$$\delta\left(\frac{p\cos\theta+p^0}{M} - y\right)\left[1 + \frac{p\cos\theta}{M}\right]. \quad (15)$$

Here only the s-d interference terms are kept. The functions u(p) and w(p) are the usual s and d-state radial wavefunctions. The tensor nature of the operator θ leads to the $3\cos^2\theta$-1 term. In doing the calculations, I found that $\Delta f_{sd}(y)$ could not be approximated as a sum of a delta function and the second derivative of a delta function.

The measurable observables depend on the ratio of $b_1(x)/F_1(x)$. My numerical results, using the Bonn and Paris potentials, are that $b_1(x)/F_1(x)$ essentially vanishes except for $x \gtrsim 0.7$. For such x values $F_1(x)$ is very small, so that the unpolarized cross section is very small. The even smaller differences between $W_{11}^{(1)}$ and $W_{11}^{(0)}$ are therefore not detectable for $x \gtrsim 0.7$.

PIONIC CONTRIBUTIONS

The pionic contribution to lepton-nucleus deep-inelastic scattering (DIS) has been studied in Refs. 7 and 8. If nucleons exchange pions, the DIS can take place between the incident lepton and the virtual pion. A pion on its way from one nucleon to another is bashed to bits by the incident virtual photon. Definitive experimental evidence for the presence of this effect does not yet exist. This pionic effect contributes mainly at low x, where shadowing also occurs. The presence of (at least) two different effects obscures the interpretations of the current experimental results.

Pions are associated with tensor forces, so that one expects a contribution to b_1. A sufficiently large effect would provide definitive evidence for the participation of nuclear pions in DIS.

Proceed by applying the formulation of Ref. 6. The pionic contribution to the nuclear quark distribution function for a target of J_z=m is given by

$$\Delta_\pi q^{(m)}(x)=\int_x^\infty \frac{dy}{y} q^\pi(x/y)\, f_\pi^{(m)}(y) \qquad (16)$$

$$f_\pi^{(m)}(y)=2y\int \frac{d^4k}{(2\pi)^4}\,\delta\!\left(y-\frac{k^+}{M}\right)\, \chi_\pi^{D,m}(k) \qquad (17)$$

and

$$\chi_\pi^{D,m}(k)=2\pi \int d^4\xi\, e^{-ik\cdot\xi} \langle D,m|\phi(\xi)\, \delta(-k^0 + H-M_D)\, \phi(0)|D,m\rangle \qquad (18)$$

The function $f_\pi^{(m)}(y)$ is the pionic version of the nucleonic function of Eq. (12). The pion field operator is $\phi(\xi)$. The energy of the pion (k^0) is determined by the spectrum of $H-M_D$ in the intermediate state $\phi(0)|D,m\rangle$. Numerical calculations[6,7] show that $H-M_D$ tends to be fairly small. Thus approximate the delta function of Eq. (18) by $\delta(k^0)$. This sets the pion energy to zero. (Recall that the energy of the virtual pion of the one pion exchange potential is zero). The procedure of using $H-M_D$ to determine k^0, differs from that of Ref. 8. In that work k^0 is $\sqrt{\vec{k}^2+m_\pi^2}$. We believe the approach of eq. (18) is the appropriate way to make use of the available information regarding nuclear pion dynamics. This is because the dynamics employ the Feynman diagram approach.

Now evaluate $f_\pi^{(m)}(y)$. Use the expansion

$$\phi(\vec{\xi}) = \sum_i \int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2\omega_p}} \left(a_i(\vec{p})\, e^{i\vec{p}\cdot\vec{\xi}} + a_i^\dagger(\vec{p})\, e^{-i\vec{p}\cdot\vec{\xi}}\right) \qquad (19)$$

where $a_i(\vec{p})$ destroys a pion of momentum $\vec{p}$ and isospin index i. The use of (19) in (17) and (18) (along with the $\delta(k^0)$) leads to

$$f_\pi^{(m)}=y\int d^4k \frac{4M}{\omega_k}\delta(k^0)\,\delta\!\left(y-\frac{(k^0+k^3)}{M}\right) \rho_\pi^{(m)}(\vec{k}) \qquad (20)$$

with

$$\rho_\pi^{(m)}(\vec{k}) = \sum_i \langle D,m|a_i^\dagger(\vec{k})\, a_i(\vec{k})|D,m\rangle . \qquad (21)$$

The interesting dynamical quantity is the pionic density $\rho_\pi^{(m)}(\vec{k})$. In lowest order in the pion nucleon coupling constant f ($f^2 = 0.08\,(4\pi)$), I find

$$\rho_\pi^{(m)}(\vec{k}) = \frac{-3}{\omega_k^2}\frac{f^2}{m_\pi^2}\frac{1}{(2\pi)^3} \langle D,m|\vec{\sigma}_1\cdot\vec{k}\;\vec{\sigma}_2\cdot\vec{k}\,e^{i\vec{k}\cdot\vec{r}}|D,m\rangle \qquad (22)$$

with $\vec{\sigma}_1$ as nucleon Pauli spin operator, and $\vec{r}$ is the nucleon-nucleon separation. The point of all of these equations is to obtain the operator $\vec{\sigma}_1\cdot\vec{k}\;\vec{\sigma}_2\cdot\vec{k}\,e^{i\vec{k}\cdot\vec{r}}$. This can be decomposed into a scalar term contributing to $F_1(x)$, and a tensor term. This tensor operator gives the biggest contribution (b_1^π) to b_1 that I have found.

Now I discuss the numerical results. One promising feature is that $b_1^\pi(x)$ turns out to be larger than the pionic contribution to $F_1(x)$. However, both effects are controlled by the same operator so that the difference is not great. The computed results can be parametrized by

$$b_1^\pi(x)/F_1(x) \simeq 0.02\ (x-0.3) \qquad (23)$$

for $x \lesssim 0.6$. $F_1(x)$ is dominated by nucleon terms and is relatively large. Seeing the b_1 effect therefore requires great experimental accuracy.

I have not computed all possible terms. One such occurs from the pressure of $\Delta\Delta$ components in the deuteron wavefunction. However, any such effects contribute to F_1 and b_1. No significant "EMC effect" has been observed for deuterium targets, so that one can not expect to find $b_1^\pi(x)F/_1(x)$ much larger than in Eq. (23). Therefore I ask about heavier targets.

HEAVIER TARGETS

The deuteron is a large spread out system. Only small non-nucleonic effects are anticipated. Heavier targets are more dense and the interesting effects could be larger. For example, the lowest order approximation used in obtaining Eq. (22) is not expected to be valid. This is because the pionic propagator is modified by attractive pion-nucleon interactions.

Can heavier targets be used to amplify the value of b_1? Consider ^{6}Li as a specific example. This J=1 nucleus can be thought of as a proton-neutron J=1 pair plus ^{4}He. The ^{4}He core does not contribute directly to b_1 although it does modify the pion propagator. Hence there is only one active J=1 pair in ^{6}Li. I expect $b_1^\pi(^6\mathrm{Li}) > b_1^\pi(\mathrm{D})$.

But the quantity relevant for observation is the ration b_1/F_1. The function F_1 is approximately proportional to the nucleon number, A. Thus for b_1/F_1 to be larger for ^{6}Li than D, one needs $b_1^\pi(^6\mathrm{Li}) > 3\ b_1^\pi(\mathrm{D})$. Numerical estimates based on using a denser nucleon wave function to evaluate Eq. (22) show that such a triple fold increase is unlikely. Indeed, to obtain a 10% effect for b_1/F_1 one needs $b_1^\pi(^6\mathrm{Li}) \approx 15\ b_1^\pi(\mathrm{D})$! Thus one expects b_1 effects for ^{6}Li or even ^{7}Li to be very small. Using still heavier targets will not increase b_1/F_1 since $F_1 \sim A$.

SUMMARY

The function b_1 can be measured with a polarized target but unpolarized beam. Should such experiments become feasible, the search for $b_1 \neq 0$ could shed light on nuclear pions. However, the predicted effects are very small: $b_1/F_1 \lesssim 0.02$.

REFERENCES

1. P. Hoodbhoy, R. L. Jaffe and A. Manohar, Nucl. Phys. B312, 571 (1989).
2. R. L. Jaffe, this workshop.
3. R. L. Jaffe and A. Manohar, MIT CTP 1640, Nov. 1988.
4. L. L. Frankfurt and M. I. Strickman, Nucl. Phys. A405, 557 (1983).
5. R. L. Jaffe, in Relativistic Dynamics and Quark Nuclear Physics. M. B. Johnson and A. Picklesimer eds.,(Wiley New York, 1986).
6. H. Jung and G. A. Miller, Phys. Lett. B200, 351 (1988).
7. M. Ericson and A. W. Thomas, Phys. Lett. 132B, 112 (1983).
8. E. L. Berger, et al., Phys. Rev. D29, 398 (1984).