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HIGH-MOMENTUM-TRANSFER PROCESSES WITH POLARIZED DEUTERONS

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Abstract: On the basis of the nuclear core hypothesis we predict that cross sections for high-energy (e, e′), (e, e′N), (h, N) inclusive reactions and the cross section for the high-momentum-transfer $e+D \to e+N+N$ reaction off polarized deuterons should strongly depend on the deuteron polarization in the kinematics forbidden for the scattering off a free nucleon. A problem of the polarized neuteron structure function extraction from $\vec{e}\vec{D}$ data is analysed. We demonstrate also that the measurement of the $D+N \to \vec{p}+X$ reaction is sufficient to reconstruct the spin-density matrix of a rapid deuteron. On the basis of the perturbative QCD the rule of the minimal change of helicity for leading hadron production in high-energy inclusive hadron processes is suggested.

1. Introduction

In spite of much effort the origin and properties of the short-range nuclear forces ($r \sim 1$ fm) are not well understood. In terms of QCD at these distances a transition occurs from soft hadron physics, where effects of colour confinement dominate, to hard processes, where perturbative QCD is applicable [see the discussion in ref. [1])]. Therefore theoretical and experimental investigations of short-range nuclear structure would give information which is necessary not only for nuclear physics itself but for the construction of a realistic theory of superdense matter (theory of neutron stars, etc.) for QCD itself. At the same time, analysis of experimental data indicates that the short-range nucleus structure has a rather transparent form. All high-energy phenomena, determined by the high-momentum component of the nucleus wave function, investigated until now can be quantitatively described on the basis of the phenomenological nuclear core hypothesis [for a review see ref. [1])].

To study the high-momentum component of the nucleus wave function (WF) one should use processes with large momentum transfer to the nucleons of nuclei, i.e. high-energy processes. Such an investigation is necessarily based on the experimental and the theoretical methods characteristic of elementary particle physics and often called relativistic nuclear physics. Though the field of research is quite new, considerable information on the high-momentum component of the nucleus (deuteron) WF has been accumulated [see ref. [1]) and references therein]. However, the short-range spin structure of the deuteron WF has not been practically

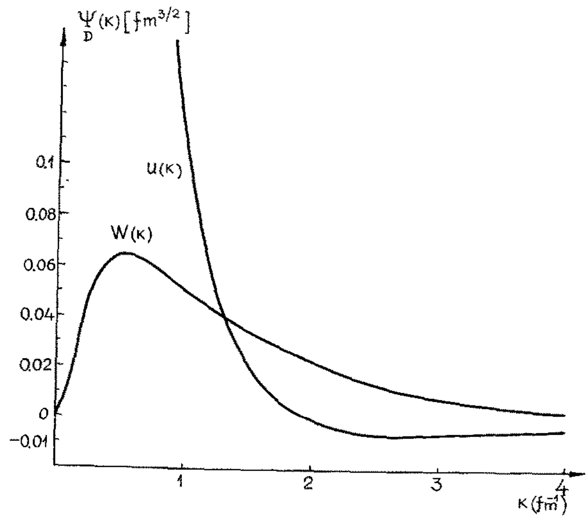

Fig. 1. The momentum dependence of the S- and D-wave deuteron wave functions.

investigated so far† although it is expected to be rather specific (see discussion below and fig. 1).

The aim of this paper is to find feasible ways of measuring the short-range spin structure of the deuteron WF in high-energy processes off the polarized deuteron. We shall concentrate on the inclusive reactions $p(e)+\vec{D}\to N+X$, where a proton (neutron) is registered in the deuteron fragmentation region, and on the reaction $e+\vec{D}\to e+X$ at $x=-q^2/2q_0m_N>1$. An experimental study of such processes is now possible due to the development of polarized deuterium targets [3]) and the acceleration of 9 GeV/c polarized deuterons in Dubna [4]). As a by-product of our investigation we analyse the problem of extracting the polarized neutron structure function from an $\vec{e}\vec{D}$ experiment which is under way in SLAC [3]). We calculate also total and differential cross sections of the $e+\vec{D}\to e+N+N$ reaction at large momentum transfer since a detailed investigation of this reaction could be expected soon using the polarized deuterium gas jet target technique in the electron storage rings developed in Novosibirsk [5]). We discuss also high-energy processes with polarized ^{6}Li which seem necessary for an accurate analysis of experiments with the polarised D^{6}Li targets planned for the FNAL tevatron [6]). In addition, we show in the appendix that study of the reaction $\vec{D}+N\to\vec{p}+X$ gives a possibility of measuring the full spin matrix density of the deuteron. We discuss briefly predictions of perturbative QCD for spin effects in the leading hadron production in fragmentation processes.

† The only exception is the measurement of elastic $p\vec{D}$ scattering [2]). These data are sensitive to the quadrupole form factor and indicate (in agreement with the conventional theory of the deuteron) that the D-wave in the deuteron WF dominates at $k\simeq 0.2$ GeV/c.

For the quantitive description of the deuteron we use in this paper a phenomenological approach based on the nuclear core hypothesis. This hypothesis is in quantitative agreement with the observed behaviour of the S-, D-phases of NN scattering, with the high-momentum-transfer behaviour of elastic and inelastic electromagnetic form factors of the deuteron, and with the form and the value of the observed cross section for cumulative proton and pion production off the unpolarized deuteron [1]). However, the most fascinating consequences of the nuclear core hypothesis – the oscillating behaviour of deuteron WF's in momentum space (fig. 1) and D-wave domination at large nucleon momenta – have not been checked yet. This structure of the deuteron WF is rather untrivial since the total D-wave contribution in the normalization of the deuteron WF is of the order of 6–7% only. Since the phenomena discussed in this paper are determined by the high-momentum component of the deuteron WF D-wave predominance leads to a strong dependence of the cross section on the deuteron spin orientation. We want to emphasize here that alternative models for the deuteron such as bag models and quark counting rules [7]) have not predicted such effects.

It has been suggested in the literature that the nuclear core hypothesis may be checked by measuring electric and quadrupole electromagnetic form factors of the deuteron in elastic eD scattering off the polarized deuteron [see e.g. ref. [8])]. Incoherent phenomena discussed in this paper have a number of obvious advantages for the study of the deuteron WF high-momentum component as compared with elastic eD scattering [this was first explained in ref. [9])]: (i) In incoherent processes at high energy one can measure the deuteron WF directly in momentum space instead of WF convolution as in the case of elastic deuteron form factors. (ii) Absolute values of cross sections are much larger than for elastic eD scattering. (iii) In the kinematical region where the high-momentum-component contribution of the deuteron WF dominates (nucleon momenta larger than 0.2 GeV/c) the cross section of these reactions should strongly depend on the deuteron polarization.

2. (e, e′) reactions on the polarized deuteron

The most reliable method of studying the high-momentum component of the deuteron WF would be the study of the $e+D \to e'+X$ reactions where large momentum q and large energy q_0 is transferred to the hadron system. (All notations correspond to fig. 2.) Repeating formal arguments leading to the parton model [10])

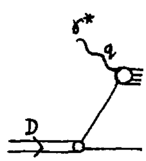

Fig. 2. The impulse approximation diagram for the $e+D \to e+X$ reaction.

it can be strictly proved that at large q^2 and $W^2 = (q + p_D)^2$ the total cross section of these reactions can be calculated by applying the impulse approximation. In this case the final-state interaction is precisely taken into account due to the possibility of applying the closure approximation. As a result the cross section of this process is expressed through the light-cone deuteron WF [11]). For example the structure function $F_{2D}^{\xi}(x, q^2)$ for the scattering of unpolarized electron off the deuteron with polarization ξ is as follows:

$$F_{2D}^{\xi}(x, q^2) = \sum_{N=p,n} \int \frac{d\alpha \, d^2k_t}{\alpha} \rho_{\xi}^{D/N}(\alpha, k_t) F_{2N}(x/\alpha, q^2) . \tag{1}$$

Here $\rho_{\xi}^{D/N}$ is the nucleon density matrix of the deuteron, averaged over nucleon spin (eq. (7)).

For the reader's convenience we reiterate here why it is necessary to use the light-cone deuteron WF instead of the Schrödinger ones for the description of high-energy processes [for a review see ref. [1])]†:

(i) The direct correspondence between description of the deuteron in terms of quantum field theory and the nonrelativistic theory of the deuteron is possible only if vacuum fluctuations characteristic for the scattering process in the relativistic theory are precisely taken into account. This is automatically the case for the Schrödinger (noncovariant) WF of the rapid deuteron, i.e. for the light-cone WF. In this case the space-time development of the scattering process has a form similar to that of nonrelativistic quantum mechanics.

(ii) Theoretically the same argument as above, but probably more transparent, is based on the mathematical criterion that the energy nonconservation in the amplitude of the elementary process in eq. (1) should be finite (not increasing with initial energy); otherwise this amplitude is zero [see p. 230 of ref. [1])].

(iii) At high energies, due to the approximate Feynman and Bjorken scaling energy, nonconservation in the fragmentation region of the rapid deuteron can be neglected.

First, we restate some definitions [1]). The D→NN light-cone vertex function for the deuteron with momentum $P \to \infty$ in the direction of the 3-axis can be written in the form

$$\varepsilon_{\mu} G_{\mu}(\alpha, p_t) = \bar{u}(p_1)(G_1(M_{NN}^2)\varepsilon_{\mu}\gamma_{\mu} + G_2(M_{NN}^2)(p_1 - p_2)_{\mu}\varepsilon_{\mu})u(-p_2) , \tag{2}$$

where p_1, p_2 are the nucleon momenta. ε_{μ} is the polarization vector of the two-nucleon system. For transverse deuteron polarizations, ε_{μ} coincides with the deuteron polarization vector, though ε_L has the form

$$\varepsilon_L = (P, P + M_{NN}^2/2P, 0, 0)/M_{NN} ,$$

where $M_{NN}^2 = 4(m^2 + p_t^2)(2 - \alpha)/\alpha$ is the square mass of the two-nucleon system.

† Actually in many cases the final answer is rather close to the nonrelativistic one.

$\frac{1}{2}\alpha$, p_t are the light-cone variables for the nucleon with momentum p_1:

$$p_1 = (\tfrac{1}{2}\alpha P + (m^2 + p_t^2)/\alpha P, \alpha P/2, p_t) .$$

In the kinematical region where the deuteron can be described as the two-nucleon system the light-cone deuteron WF practically coincides with WF of the conventional theory of the deuteron, extracted on the basis of phase analyses. The proof is based on the observation that the form of the Weinberg–Schrödinger equations follows from the neglect of other degrees of freedom and the symmetry between nucleons [1]). (The main difference from the nonrelativistic quantum mechanics is in the relationship between the scattering amplitude and the deuteron WF.) As a result G_1, G_2 are expressed through the nonrelativistic S- and D-wave deuteron WF's $u(k)$, $w(k)(\int (u^2(k)+w^2(k))\, d^3k = 1)$ as follows:

$$G_1(M_{NN}^2) = (u(k) - w(k)/\sqrt{2})\tfrac{1}{4}\sqrt{\varepsilon} ,$$
$$G_2(M_{NN}^2) = -\sqrt{\varepsilon}/8k^2(u(k)(1-m/\varepsilon) + w(k)\sqrt{\tfrac{1}{2}}(2+m/\varepsilon)) , \tag{3}$$

where $\varepsilon = \sqrt{m^2+k^2}$. We also introduced here the three-vector k [refs. [11,12])] which can be interpreted as the internal momentum of the nucleon in the deuteron rest frame:

$$M_{NN}^2 = 4(m^2+k^2) , \qquad k_3 = (\alpha-1)\sqrt{m^2+k^2} , \qquad k_t = p_t . \tag{4}$$

It is convenient also to express the D→NN vertex using two-component spinors:

$$\varepsilon_\mu G_\mu(\alpha, p_t) = \varphi^* A_\varepsilon \varphi \equiv \varphi^*(A_\varepsilon^0 + A_\varepsilon \sigma)\varphi , \tag{5}$$

where

$$A_L^0 = 0 , \qquad A_L^t = -k_t G_2(k)(1-\alpha) 2M_{NN}/\sqrt{\alpha(2-\alpha)} ,$$
$$A_L^3 = -2M_{NN}(G_1(k)\tfrac{1}{2}\alpha(2-\alpha) - mG_2(k)(1-\alpha)^2/\sqrt{\alpha(2-\alpha)} .$$

For

$$\varepsilon \equiv \varepsilon_T ,$$
$$A_T^0 = i\varepsilon_{3\beta\gamma}\varepsilon_\gamma k_\beta G_1(k) 2/\sqrt{\alpha(2-\alpha)} ,$$
$$A_T^{i=1,2} = 2(G_1(k)m(\varepsilon e_i) - 2(k\varepsilon)k_i G_2(k))/\sqrt{\alpha(2-\alpha)} ,$$
$$A_T^3 = 2(k\varepsilon)(1-\alpha)(G_1(k) + 2mG_2(k))/\sqrt{\alpha(2-\alpha)} . \tag{6}$$

For most of the reactions considered below the cross section can be expressed through the nucleon density matrix of the deuteron with polarization ξ, averaged over nucleon polarizations: $\rho_\xi^{D/N}(\alpha, k_t)$. (For a brief discussion of the general case see the appendix.) Using eqs. (5) and (6) we obtain

$$\rho_\xi^{D/N}(\alpha, k_t) = \frac{2}{2-\alpha}\,\mathrm{Sp}\,A_\xi (A_{\xi^*})^*$$
$$= \frac{\sqrt{m^2+k^2}}{2-\alpha}(u^2(k) + \tfrac{1}{2}w^2(k) - \sqrt{2}\,u(k)w(k) + 3(k\xi)(k\xi^*)/k^2$$
$$\times(\sqrt{2}u(k)w(k) + \tfrac{1}{2}w^2(k)))\,. \qquad (7)$$

$\rho_\xi^{D/N}$ as given by eq. (7) is invariant under Lorentz boosts in the 3-axis direction, i.e. at fixed α and k_t; $\rho_\xi^{D/N}$ is independent of $p_+ = (p_D)_0 + (p_D)_3$. This is not the case for the full nucleon density matrix due to rotation of the spin of the nucleon; due to the lack of this effect $\rho_\xi^{D/N}$ is rather similar to the corresponding nonrelativistic expression.

In the case of the unpolarized electron the cross section of eN scattering is independent of the nucleon polarization. As a result one can use the density matrix $\rho_\xi^{D/N}$. Substituting variables α, k_t by $\boldsymbol{k}$ in eq. (1) and using eq. (7) we obtain the basic formula of this section[†]:

$$F_{2D}(x, q^2) = \int (u^2(k) + w^2(k))P_\xi(k)(F_{2p}(x/\alpha, q^2) + F_{2n}(x/\alpha, q^2))\,d^3k\,, \qquad (8)$$

where

$$P_\xi(k) = 1 + (3(k\xi)(k\xi^*)/k^2 - 1)(\sqrt{\tfrac{1}{2}}u(k)w(k) + \tfrac{1}{2}w^2(k))/(u^2(k) + w^2(k))\,. \qquad (9)$$

In the case of the unpolarized deuteron the factor $P_\xi(k)$ should be substituted by 1.

To obtain the nonrelativistic limit of eq. (9) one has only to substitute $\sqrt{m^2+k^2}$ in eq. (4) for α by m.

It is convenient to represent the magnitude of spin effects in the form of the ratio

$$R(x, q^2) = (\tfrac{1}{2}(\sigma_+ + \sigma_-) - \sigma_0)/\langle\sigma\rangle\,, \qquad (10)$$

where $\langle\sigma\rangle = \frac{1}{3}(\sigma_+ + \sigma_- + \sigma_0)$. Indices $(+, -, 0)$ denote deuteron helicities. In the deuteron rest frame the deuteron spin is quantized in the direction of the γ^* momentum. Note that in the unpolarized electron case $\sigma_+ = \sigma_-$ due to space parity conservation. Evidently in the physical region R can vary from -3 to 1.5.

In fig. 3 we give results of calculation of R on the basis of eq. (8) for the Reid (soft-core) and Hamada–Johnston (hard-core) deuteron WF's. Oscillation of R is due to dominance of the D-wave in a wide range of nucleon momenta (see fig. 1). Since the essential region of integration in k in eq. (9) is rather large, the deuteron WF is considerably averaged. As a result the absolute value of R is comparatively small even at large x.

[†] Here we restrict ourselves by consideration of $F_{2D}^{\xi}(x, q^2)$ since in this case the contact terms which could not be calculated through the contribution of large longitudinal distances are not enhanced.

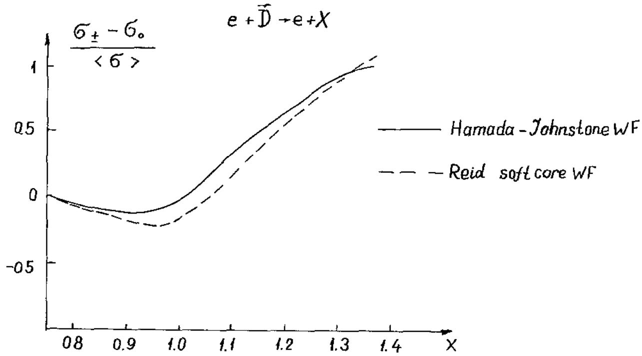

Fig. 3. The dependence of $F_{2D}^{\xi}(x, q^2)$ on the deuteron polarization.

In reactions which will be considered below the deuteron WF is not averaged or averaged over smaller interval in k. As a result variation of R will be considerably larger.

In the analysis of data from the polarized deuteron target, consisting of a polarized deuteron and unpolarized nuclei, it is necessary to take into account that the background from nuclei increases as x increases, since the high-momentum component in the nucleus WF is considerably larger than that for the deuteron. For example, it was predicted in ref. [13]), and it is now confirmed experimentally [14]), that the ratio of structure functions for ^{12}C and D at $x = 1$ is of order 40 and increases rather rapidly with x.

The calculation given above is valid in the kinematical region where the description of the deuteron as a two-nucleon system is well founded, i.e. at $x \leqslant 1.4$–1.5.

3. On the problem of the polarized neutron structure function extraction from eD scattering

At present considerable attention is given to the experimental check of QCD predictions for the structure function of the polarized neutron. The perturbative QCD predicts that helicities of the nucleon and leading quark should coincide at $x \to 1$ [ref. [15])]. This prediction is in reasonable agreement with SLAC data in the case of $\vec{e}\vec{p}$ scattering [3]). For investigation of parton spin structure of the neutron it is planned to measure $\vec{e}\vec{D}$ scattering [3]). However, the presence of a D-wave in the deuteron violates the direct relationship between the helicities of the deuteron and nucleons (e.g. in the deuteron with helicity 1 the nucleon can have helicity $-\frac{1}{2}$). For certainty we restrict ourselves to the case of high electron energies and quantize the deuteron spin along the γ^* momentum direction (3-axis).

Let us first calculate $h_+^D(x, q^2) - h_-^D(x, q^2)$, i.e. the difference of the quark distributions with helicities $+\frac{1}{2}$ and $-\frac{1}{2}$ in the deuteron with helicity 1. It is expressed through the difference of the nucleon structure functions h_+^{N}, h_-^{N} [we use here the conventions of ref. [10])]:

$$h_+^{\mathrm{D}}(x,q^2)-h_-^{\mathrm{D}}(x,q^2)=\sum_{\mathrm{N=p,n}}\int\frac{\mathrm{d}\alpha\,\mathrm{d}^2k_{\mathrm{t}}}{\alpha(1-\frac{1}{2}\alpha)}\,\mathrm{Sp}\,(A_\xi\sigma_3A_{\xi^*}^*)\frac{2}{\alpha}(h_+^{\mathrm{N}}(x/\alpha,q^2)$$
$$-h_-^{\mathrm{N}}(x/\alpha,q^2)\,,$$

where ξ is the polarization vector of the deuteron of helicity 1. Using eqs. (5), (6) we finally obtain

$$\begin{aligned}h_+^{\mathrm{D}}&(x,q^2)-h_-^{\mathrm{D}}(x,q^2)\\&=\sum_{\mathrm{N=p,n}}\int 2/\alpha\;\mathrm{d}^3k\Big(u^2(k)\Big(1-\frac{k_{\mathrm{t}}^2(\varepsilon-m)}{(2-\alpha)\varepsilon k^2}\Big)\\&\quad-\sqrt{2}\,u(k)w(k)\Big(1-\frac{k_{\mathrm{t}}^2(2\varepsilon+m)}{2\varepsilon(2-\alpha)k^2}\Big)+w^2(k)\Big(0.5-\frac{k_{\mathrm{t}}^2(2m+\varepsilon)}{2\varepsilon k^2(2-\alpha)}\Big)\Big)\\&\quad\times(h_{+(x/\alpha,\,q^2)}^{\mathrm{N}}-h_-^{\mathrm{N}}(x/\alpha,q^2))\,.\end{aligned}\tag{11}$$

In the region of small x we can in the first approximation neglect the x/α variation of $h_\pm^{\mathrm{N}}$ as compared to a rather sharp variation of $u(k)$, $w(k)$. As a result in the lowest order in k^2/m^2 we obtain for the asymmetry

$$A^{\mathrm{D(N)}}(x,q^2)=(h_+^{\mathrm{D(N)}}(x,q^2)-h_-^{\mathrm{D(N)}}(x,q^2))/(h_+^{\mathrm{D(N)}}(x,q^2)+h_-^{\mathrm{D(N)}}(x,q^2))$$

a rather simple expression:

$$A^{\mathrm{D}}(x,q^2)=(1\text{–}1.5\,P_{\mathrm{D}})A^{\mathrm{N}}(x,q^2)\,.\tag{11'}$$

Here $P_{\mathrm{D}}=(6\text{–}7)\times10^{-2}$ is the D-wave probability in the deuteron. Thus, an effective nucleon polarization in the deuteron is about 90%. As an illustration of eq. (11) we calculate the ratio of asymmetries A^{D} and A^{P} neglecting the neuteron structure function (fig. 4). In the calculation we use the Reid soft-core WF and the simple

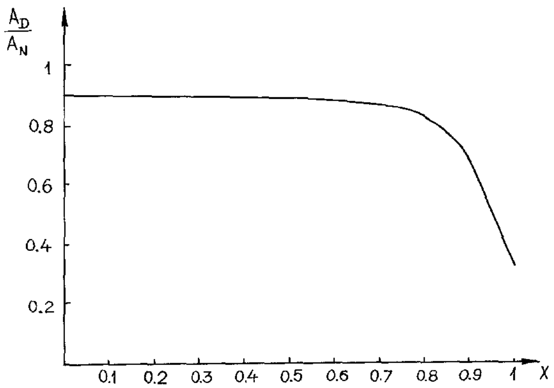

Fig. 4. The ratios of deuteron and nucleon asymmetries for the model described in the text.

parametrization of $A^{p}(x)=2x-x^2$, which reasonably fits SLAC data [3]) and has correct QCD behaviour at $x\to 1$. It can be seen from the figure that eq. (11′) provides an excellent approximation to eq. (11) up to $x\leq 0.5$. Thus the extraction of $A^{n}(x)$ from the deuterium data appears to be quite simple in a wide x-range.

Similarly it is possible to obtain expressions for the structure functions $k^{D}_{\pm}(x,q^2)$ for the deuteron polarized along 1-axis through the similar nucleon structure functions $k^{N}_{\pm}(x,q^2)$, defined in ref. [10]):

$$k^{D}_{+}(x,q^2)-k^{D}_{-}(x,q^2)=\sum_{N=p,n}\int d^3k(2/\alpha)16(G_1^2(k)m+G_1(k)G_2(k)(2m^2(1-\alpha)^2 - (\alpha^2-\alpha-1)k_t^2)/\alpha/(2-\alpha))(k^{N}_{+}(x/\alpha,q^2)-k^{N}_{-}(x/\alpha,q^2)\,. \qquad (12)$$

For small x eq. (12) can be simplified, leading to an expression similar to eq. (11′).

4. Electrodesintegration of polarized deuteron at large Q^2

4.1. TOTAL CROSS SECTION

In refs [16,1]) we have calculated the cross section of the reaction

$$e+D\to e+p+n \qquad (13)$$

and found it to be in a reasonable agreement with the measurements of ref. [17]) for $q^2=0.8$–$6\ \mathrm{GeV}^2$ not far from the threshold, i.e. in the kinematics where the high-momentum component of the deuteron WF gives a dominant contribution. Evidently, the reaction (13) with the polarized deuteron [which can now be studied both inclusively, using e.g. SLAC polarized deuteron target [3]), or even semiinclusively in electron storage rings [5])] can provide valuable information about the spin structure of the high-momentum component of the deuteron WF. Therefore, we shall derive here expressions for both inclusive and differential cross sections for reaction (13).

As above, to take into account the space-time structure of the scattering process characteristic for relativistic theory we shall describe the deuteron by the light-cone WF in the infinite momentum frame with $P_D\to\infty$ along the 3-axis. We choose $q_+=q_0+q_z=0$ in order to suppress vacuum $N\bar{N}$ pair production by γ^* [cf. discussion of the calculation of elastic deuteron form factor in ref. [1])]. As a result the cross section of the reaction (13) is described by the sum of two typical diagrams of noncovariant light-cone perturbation theory, figs. 5a,b. (In the diagrams $E-p_z$ is not conserved which substitutes the energy nonconservation characteristic for the time-ordered diagrams of nonrelativistic theory.) Fig. 5a corresponds to the impulse approximation; fig. 5b takes into account the interference term. Of course, one should take into account scattering from both the proton and neutron. As in

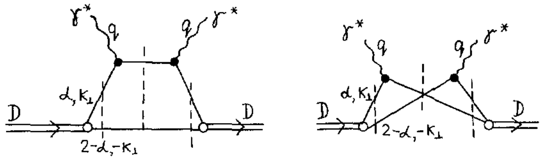

Fig. 5. Impulse approximation and interference diagrams for the $e+D \to e'+p+n$ reaction.

refs. [17,1]) we restrict ourselves to the consideration of the kinematical region where M_{NN}, i.e. the invariant mass of the two-nucleon system produced, is sufficiently large: $M_{NN} - 2M_N \geq 50$ MeV, where the final-state interaction between two nucleons is small. In the chosen rest frame

$$q_+ = 0\,, \qquad q_- = 2\nu/M_D\,, \qquad \nu = (p_D q)\,, \qquad q^2 = -q_t^2\,. \tag{14}$$

Note that due to the choice $q_+ = 0$ the invariant energy is conserved in the elementary γ^*NN vertex.

Similar to the case of deep inelastic scattering considered in sect. 2, the contribution of the impulse approximation differs from the expressions for the nonpolarized case [eqs. (3.3) and (3.7) of ref. [1])[†]] by the factor $P_\xi(k)$ defined above (eq. (9)).

It can be written in the form which has close resemblance to the nonrelativistic expressions[††],[†††]:

$$\begin{aligned} W_2^{\xi(\mathrm{imp})}(\nu, q^2) &= \tfrac{1}{2} M_D \int d^3 k_f P_\xi(k_i)[u^2(k_i) + w^2(k_i)] \\ &\quad \times \left(\frac{m^2 + k_i^2}{m^2 + k_f^2}\right)^{1/2} \delta\left(\tfrac{1}{4}W^2 - m^2 - k_f^2\right) \sum_{N=p,n} [F_{1N}^2(q^2) - q^2/4m^2 F_{2N}^2(q^2)] \\ &= \tfrac{1}{4} M_D \int d\Omega_f k_f P_\xi(k_i)(u^2(k_i) + w^2(k_i))((m^2 + k_i^2)/(m^2 + k_f^2))^{1/2} \\ &\quad \times \sum_{N=p,n} (F_{1N}^2(q^2) - q^2/4m^2 F_{2N}^2(q^2))\,. \end{aligned} \tag{15}$$

Here F_1, F_2 are Dirac nucleon form factors, k_i is the "light-cone" momentum of the nucleon in the deuteron, and k_f is the momentum of the spectator nucleon in the center of mass of the produced two-nucleon system. The relationship between k_i and α, k_t is given by eq. (3). For k_f

$$\alpha(k_f) = \alpha\,, \qquad k_{f_t} = k_{i_t} - \tfrac{1}{2}\alpha q_t\,, \qquad W^2 = 4(m^2 + k_{f_t})/(2-\alpha)/\alpha\,. \tag{16}$$

Similar expressions can be derived for $W_1(q^2)$, cf. eq. (3.7) of ref. [1]).

[†] In the right-hand side of eq. (3.7) [1]) the factor α^{-1} is missing. There are also misprints in eq. (3.9) which are corrected below.

[††] We use here the natural definition of W_{2D}^{ξ} as the leading term in the cross section at small scattering angles.

[†††] Hereafter we give results for W_{2D}^{ξ} only, because it is more accessible for experimental studies and because the contact terms can not be suppressed for W_{1D} as effectively as for W_{2D} [see discussion in ref. [1])].

The contribution of the interference diagrams fig. 5b is as follows (for certainty we choose here $q_t = q_1$):

$$W^{\xi}_{2D}(\nu, q^2)_{int} = \tfrac{1}{2}M_D(\mathrm{Sp}\,(A^{\xi}(k)(F^{p}_{1}(q^2) + iq_t\sigma_2F^{p}_{2}(q^2)/2m)$$
$$\times (A^{\xi^*}(\tilde{k}))^*(F^{n}_{1}(q^2) - iq_t\sigma_2F^{n}_{2}(q^2)/2m)$$
$$+ (q \to -q))\delta(\tfrac{1}{4}W^2 - m^2 - k_f^2)\frac{d\alpha\, d^2k_t}{\alpha(2-\alpha)}, \tag{17}$$

where $k = k(\alpha, k_t)$, $\tilde{k} = k(\alpha, k_t + q_t)$ and A was defined in eq. (5). Changing the integration variable to k_f we finally obtain

$$W^{\xi}_{2D}(\nu, q^2)_{int} = \tfrac{1}{2}M_D \int \frac{k_f\, d\Omega_f}{\sqrt{m^2 + k_f^2}}\{(F_{1p}(q^2)F_{1n}(q^2) + q_t^2F_{2p}(q^2)F_{2n}(q^2))$$
$$\times (A^{\varepsilon}_2\widetilde{A}^{\varepsilon}_2 + A^{\varepsilon}_0\widetilde{A}^{\varepsilon}_0) + (F_{1p}(q^2)F_{1n}(q^2) - q_t^2F_{2p}(q^2)F_{2n}(q^2))$$
$$\times (A^{\varepsilon}_1\widetilde{A}^{\varepsilon}_1 + A^{\varepsilon}_3\widetilde{A}^{\varepsilon}_3) + q_t(F_{2p}(q^2)F_{1n}(q^2)$$
$$+ F_{1p}(q^2)F_{2n}(q^2))(A^{\varepsilon}_3\widetilde{A}^{\varepsilon}_1 - A^{\varepsilon}_1\widetilde{A}^{\varepsilon}_3)$$
$$+ iq_t(F_{1p}(q^2)F_{2n}(q^2) - F_{2p}(q^2)F_{1n}(q^2))(A^{\varepsilon}_2\widetilde{A}^{\varepsilon}_0 + A^{\varepsilon}_0\widetilde{A}^{\varepsilon}_2) + (q_t \to -q_t)\,. \tag{18}$$

$A^{\varepsilon}_i \equiv A^{\varepsilon}_i(k)$ was defined in eq. (6) and $\widetilde{A}^{\varepsilon}_i \equiv (A^{\varepsilon^*}_i(\tilde{k}))^*$. The fourth term is cancelled after integration over Ω_f, since it is odd under the substitution $(k_f)_1 \to -(k_f)_1$. However, it contributes to the differential cross section. The simplest way to express k, $\tilde{k}$ through k_f is to make the Lorentz boost of the momentum of the final-state two-nucleon system by $-q_t$,

$$k_t = k_{f_t} - \tfrac{1}{2}\alpha q_t\,, \qquad \tilde{k}_t = k_{f_t} + \tfrac{1}{2}(2-\alpha)q_t\,, \qquad \alpha_i = \alpha_f = \alpha\,, \tag{19}$$

because for the Lorentz boosts in the transverse direction the transverse momentum is partitioned in the proportion of the light-cone fractions $\frac{1}{2}\alpha$ and $\frac{1}{2}(2-\alpha)$ [see e.g. ref. [18])]. k_f is determined from eq. (16).

To obtain $W_{2D}(\nu, q_2)_{int}$ for the unpolarized case one should average over the deuteron polarizations:

$$W_{2D}(\nu, q^2)_{int} = \tfrac{1}{3}\sum_{\xi} W_{2D}(\nu, q^2)_{int}\,.$$

To calculate W_{2D} for fixed helicity states of the deuteron it is convenient to use the polarization vector e_2 which is not changed under Lorentz boosts in the 1–3 plane, necessary for the transformation to the γ^*D c.m.s. Evidently, e_2 is a superposition of the deuteron states with helicities ± 1. Due to helicity and space parity conservation $W^{+}_{2D} = W^{-}_{2D} = W^{2}_{2D}$. W^{0}_{2D} can be expressed through W^{+}_{2D} and W_{2D}, using $W^{0}_{2D} = 3W_{2D}(\nu, q^2) - 2W^{+}_{2D}(\nu, q^2)$.

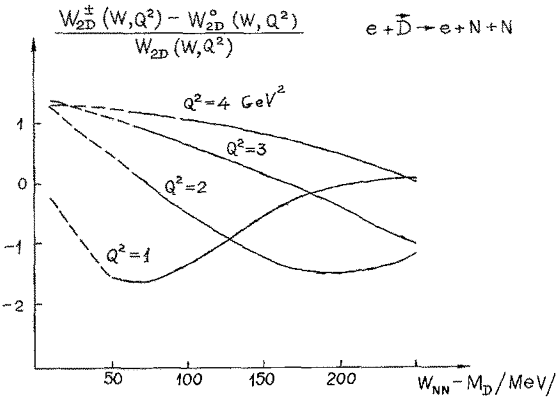

Fig. 6. The dependence of $W_{2D}^{\xi}(\nu, q^2)$ for the $e+\vec{D}\to e'+p+n$ reaction on the deuteron polarization.

The results of the calculations of $R=(W_{2D}^{+}(\nu,q^2)-W_{2D}^{0}(\nu,q^2))/W_{2D}(\nu,q^2)$ with Reid soft-core and Hamada-Johnston hard-core WF's (which lead to practically coinciding curves) are presented in fig. 6. In the region $W-M_D<50$ MeV the curves are broken to emphasize that the final-state interaction, not accounted for in our analysis, could be of importance here. Note that the predicted dependence of R is stronger than in the deep inelastic case (fig. 3) because in the near-threshold reaction a much narrower region of nucleon momenta k contributes in the integrand.

4.2. DIFFERENTIAL CROSS SECTION

Now let us derive expressions for $W_{2D}^{\xi}(\nu, q^2, \Omega_f)$, where Ω_f are the angular variables, which characterize the angular distribution of nucleons in the γ^*D c.m.s. To do this, note that the derived expressions are invariant under boosts in the 3-axis direction. In particular, they are valid in the frame where $p_D=(m_D,0,0,0)$ and $q_+=0$. Thus, it is sufficient to omit integration over Ω_f in eqs. (15) and (18) and to take into account the rotation of the deuteron spin from the deuteron rest frame to the c.m.s.

The integrand of eq. (16) averaged over $d\Omega_f$ gives the proton and neutron distributions. Since the interference term does not change under the substitution $k_f\to -k_f$ it contributes equally to the proton and neutron spectra. All asymmetry originates from the impulse approximation term. Indeed since the square of the proton form factor is larger than that of the neutron, it is more probable for the proton to fly in the γ^* direction.

If γ^* is absorbed by the neutron, the momentum of the proton in the frame with $q_+=0$ and $\boldsymbol{p}_D=0$ is determined from the condition that +, t-components of momenta flowing in fig. 5a are conserved. Thus, in this reference frame

$$k_{f_t}=k_t\,, \qquad k_{p+}=\sqrt{m^2+k^2}+k_{p3}=\tfrac{1}{2}\alpha m_D\,, \tag{20}$$

while for the neutron momentum after absorption of γ^* we have

$$k_{n_t} = -k_t + q_t\,, \qquad k_{n+} = \sqrt{m^2 + k_n^2} + k_{n3} = (2-\alpha)/2m_D\,. \tag{21}$$

The mass squared of the pn system produced is

$$4\frac{m^2 + (k_t - \frac{1}{2}\alpha q_t)^2}{\alpha(2-\alpha)} = W^2\,. \tag{22}$$

The transformation to the c.m.s. with the z-axis along the $\boldsymbol{q}$ direction can be made by applying the standard Lorentz boost $L(W, \boldsymbol{q})$, which we shall not write down explicitly.

The impulse approximation contribution to $W^{\xi}_{2,p}(\nu, q^2, \Omega_p)$ due to the γ^* interaction with the neutron is as follows:

$$W^{\xi}_{2,p(1)}(\nu, q^2, \Omega_p) = \frac{\sqrt{m^2+k^2}}{4\sqrt{m^2+k_f^2}}[u^2(k) + w^2(k)]k_f P^{\xi}(k)$$
$$\times [F^2_{1n}(q^2) - q^2/4m^2 F_{2n}(q^2)]\,, \tag{23}$$

where $k = k(L^{-1}(W, q), \boldsymbol{k}_f)$ is given by eq. (20). The contribution of the interaction with proton is given by

$$W^{\xi}_{2p(2)}(\nu, q^2, \Omega_p) = \frac{1}{4}\sqrt{\frac{m^2+k^2}{m^2+k^2}}[u^2(k)+w^2(k)][F^2_{1p}(q^2) - (q^2/4m)F^2_{2p}(q^2)]\,, \tag{24}$$

where $k = k(L^{-1}(W, q), -\boldsymbol{k}_f)$ is given by eq. (21).

We shall not give here a detailed discussion of the ξ-dependence of the differential cross section. Let us briefly discuss only the most interesting case when W is sufficiently large and $k(\boldsymbol{k}_f)$ and $k(-\boldsymbol{k}_f)$ differ considerably. In this case the interference contribution is negligible and it can be easily derived from the kinematics which of the nucleons absorbed γ^*. Evidently in the region $k \sim 0.2$ GeV/c, where $w(k)$ and $u(k)$ are comparable, the cross section strongly depends on ξ and this dependence is even stronger than for the total cross section. The crucial difference between the equations derived above and the equations which could be derived in the fixed nucleon approximation, the Bethe–Salpeter approach of ref. [19]), or in nonrelativistic quantum mechanics, is that the momentum of the nucleon spectator p_s in the deuteron rest frame does not coincide with momentum k in the argument of the deuteron WF [cf. refs. [11,1]) and sect. 2]. Moreover, the difference of p_s and k depends on q^2 and ν. This effect arises due to the accurate treatment in eqs. (16) of the space-time development of strong interaction and account of the vacuum fluctuations. As a result the ratio $R = W^{\xi}_2(\nu, q^2, p_s)/W_2(\nu, q^2, p_s)$ at fixed p_s should depend, though rather weakly, on ν, q^2. In most of reactions which were analysed in ref. [1]) the difference between p_s and k leads to small effects only for $p_s \lesssim 0.3$ GeV/c. However due to the fast variation of $u(k)$ for $k \sim 0.2$ GeV/c in the reaction considered the relativistic effect of the q^2, ν variation of $R(q^2, \nu, p_s)$ could be observed even at momenta $p_s \sim 0.2$ GeV/c. Observation of this effect would provide an interesting qualitative test of the light-cone description of the deuteron, since in nonrelativistic quantum mechanics or in the approach of ref. [19]) this effect is absent for the arbitrary deuteron WF. To summarize, we have demonstrated in this section that study of the $e+\vec{D} \to e+N+N$ reaction at high q^2 provides an effective method of studying the short-range deuteron structure.

5. Inclusive reactions $e, h+\vec{D} \to p+X$

In this section we shall consider the inclusive production of protons in the lepton (hadron) scattering off the polarized deuteron in the deuterium fragmentation region forbidden for the scattering off the free nucleon. One of these processes $p+D \to p+X$ will be studied in the near future in Dubna with polarized deuteron beams [4]).

First, let us give a qualitative picture of the expected effect in the deuteron rest frame at not-too-large nucleon momenta within the deuteron ($k/m \ll 1$). It is well known that in the coordinate space, due to the presence of the D-wave deuteron with zero helicity, it looks like a ball flattened in the direction of the 3-axis (the spin of deuteron and nucleons are quantized in the direction of initial particles momentum which is chosen as the 3-direction). The form of the deuteron with helicity ± 1 differs from the case of zero helicity by a space rotation of this figure. Evidently due to the properties of Furie transformation in momentum space the zero helicity deuteron has the form of a cucumber. As a result the yield of the backward spectators is maximal for zero deuteron helicity. The effects discussed above should reveal themselves most clearly in the processes dominated by the high-momentum component of the deuteron WF since in this case the D-wave contribution is enhanced. In hadronic processes it is necessary to take into account Glauber-type screening of the spectrum of cumulative protons. It is evident from the geometry of the scattering process that Glauber-type screening is smallest in the case of zero helicity since the deuteron with zero helicity is more flat in the 3-direction than that for helicity ± 1[†].

Evidently, processes of deep inelastic lepton scattering off the deuteron with cumulative proton production are the simplest for theoretical interpretation. In this case, the impulse approximation is valid for lepton interaction with the nucleons of the deuteron. Moreover, corrections to the spectator mechanism are small in a wide kinematical region [16,1]). In the deuteron rest frame we have a rather simple description of the fast backward (cumulative) proton production [9,1]):

$$\sigma^{\xi}(p_s) = \sigma^{\text{in}}((2-\alpha)\nu, q^2)\rho_{\xi}^{D/N}(k)\,. \tag{25}$$

† A similar effect for $\sigma_{\text{tot}}(\text{pD})$ was found in ref. [20]).

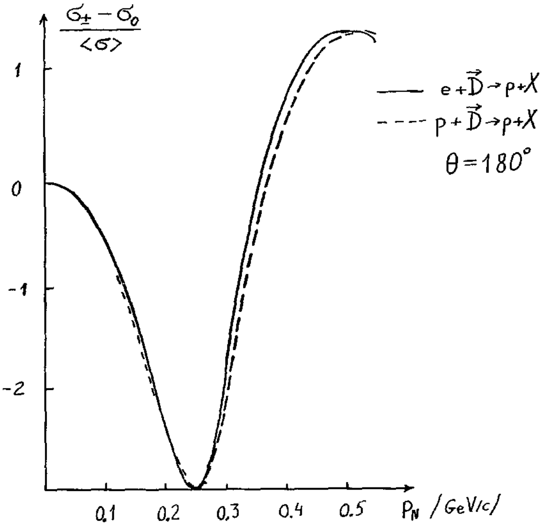

Fig. 7. $(\sigma_\pm - \sigma_0)/\langle\sigma\rangle$ for the backward nucleon production in high-energy $e\vec{D}$ and $p\vec{D}$ scattering.

$\rho_\xi^{D/N}(k)$ is given by eq. (7). The relationship between the cumulative proton momentum p_s and inner momentum k† is given by

$$k_\perp = p_{s_t}, \qquad \tfrac{1}{2}\alpha = (\sqrt{m^2 + p_s^2} - p_{s3})/m_D = 1 - k_3/\sqrt{m^2 + k^2}. \tag{26}$$

As a result we obtain for $R = (\frac{1}{2}(\sigma_+ + \sigma_-) - \sigma_0)/\sigma$ a rather simple expression:

$$R(p_s) = \frac{3(k_\perp^2 - k_z^2)}{k^2}\left[\frac{u(k)w(k)\sqrt{2} + \frac{1}{2}w^2(k)}{u^2(k) + w^2(k)}\right]. \tag{27}$$

In nonrelativistic quantum mechanics $(p_s/m \ll 1)$ p_s and k_s coincide. In this case R has the form

$$R^{\text{nonrel.q.m.}}(p_s) = \frac{3(p_\perp^2 - p_z^2)}{p^2}\left(\frac{u(p)w(p)\sqrt{2} + \frac{1}{2}w^2(p)}{u^2(p) + w^2(p)}\right). \tag{28}$$

It is worthwhile to emphasize that eqs. (27) and (28) predict a different momentum dependence at fixed angle and a different angular dependence at fixed nucleon momentum (fig. 8). It can be seen from fig. 8 that the calculation based on eq. (28) leads to $R \sim \cos 2\theta$, though a rather complicated angular dependence follows from eq. (27) (θ is the angle between p_s and the 3-axis). In the forward direction the contribution of the direct mechanism becomes important. However, it could be suppressed in a wide kinematical range by choosing a sufficiently large x [refs. [16,1])].

† The difference between k and p_s is due to the account in eq. (25) of the space-time picture characteristic for high-energy processes in relativistic theory [11,1]).

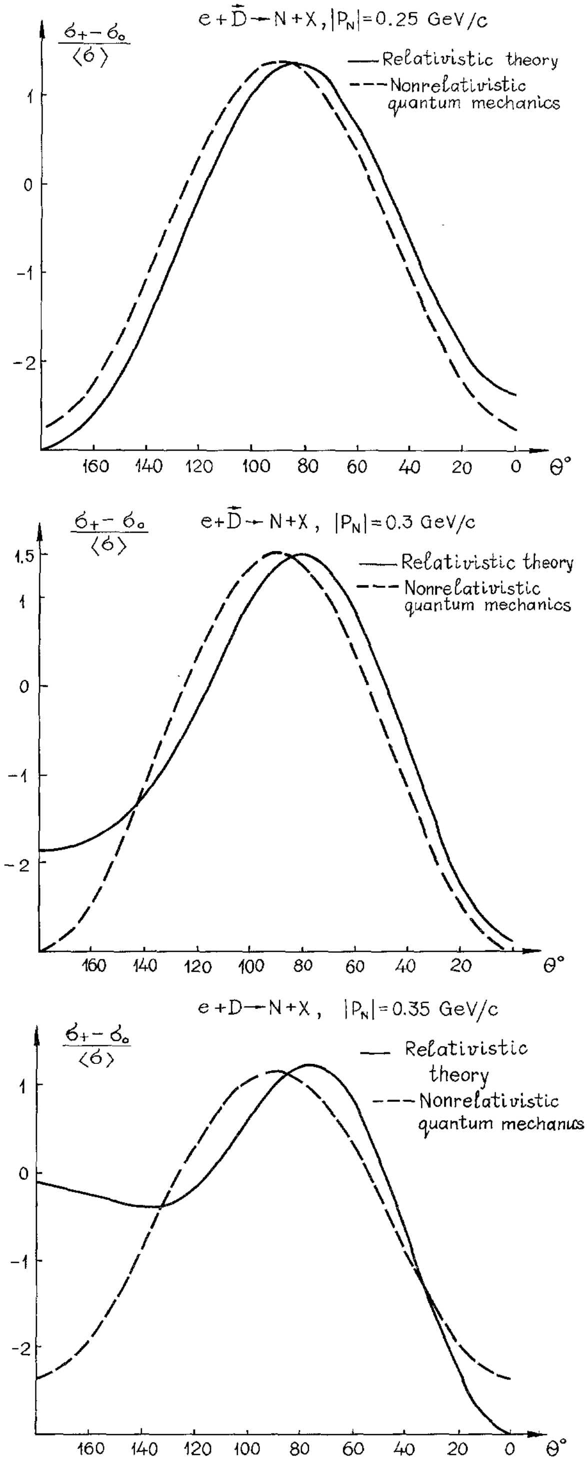

Fig. 8. Angular dependence of $(\sigma_{\pm}-\sigma_0)/\langle\sigma\rangle$ for the spectator distribution in the $\mathrm{e}+\vec{\mathrm{D}}\to\mathrm{N}+X$ reaction at different nucleon momenta. Solid and dashed lines are predictions of relativistic theory and nonrelativistic quantum mechanics, respectively.

In the case of hadron processes it is necessary to take into account Glauber-type screening [1]). The calculation of eikonal diagrams of the Glauber–Gribov model for fast hadron scattering off nuclei leads to the formula

$$\alpha \frac{\mathrm{d}\sigma^{\mathrm{D/p}}}{\mathrm{d}\alpha\, \mathrm{d}^2 p_\mathrm{t}} = \frac{\sigma_{\mathrm{in}}^{\mathrm{hN}}}{2-\alpha}\left[P^\xi(k)(u^2(k)+w^2(k))\sqrt{m^2+k^2} - \int G^\xi(k,k')f^2(q_\mathrm{t})\frac{\mathrm{d}^2 q_\mathrm{t}}{(2\pi)^2}\right.$$
$$\left. + \tfrac{1}{4}\int G^\xi(k_1,k_2)f(q_{1\mathrm{t}})f(q_{1\mathrm{t}}+q_{2\mathrm{t}})f(q_{2\mathrm{t}})\frac{\mathrm{d}^2 q_{1\mathrm{t}}}{(2\pi)^2}\frac{\mathrm{d}^2 q_{2\mathrm{t}}}{(2\pi)^2}\right]. \tag{29}$$

Here $f(q)$ is the elastic hN amplitude. $\mathrm{Im}\, f(0) = \sigma_{\mathrm{tot}}^{\mathrm{hN}}$.

$$G^\xi(a,b) = 4\sum_{i=0}^{3} A_i^\xi(a)(A_i^{\xi*}(b))^* \tag{30}$$

and $k' = k(\alpha, p_\mathrm{t}+q_\mathrm{t})$, $k_1 = k(\alpha, p_\mathrm{t}+q_{1\mathrm{t}})$, $k_2 = k(\alpha, p_\mathrm{t}+q_{2\mathrm{t}})^\dagger$. The $A_i(a)$ were defined in eq. (6). The calculation based on eq. (29) (dashed line in fig. 7) shows that shadowing effects do not significantly change the ratio $R = (\sigma_+ + \sigma_- - \sigma_0)/\langle\sigma\rangle$. This is because at fixed α the integration in eq. (29) extends over a small interval in q_t. As a result the differences between k', k_1, k_2 and k are not large. Similarly, corrections from the direct mechanism (i.e. the contribution of nucleons produced in the scattering off the fast backward nucleon of the deuteron) should lead to a comparatively small change of R. With $p_\perp$ increased, corrections to R from the direct mechanism and Glauber screening increase, since in this case the integration extends over a rather wide interval of momentum transfer.

Let us now discuss background processes due to the final-state interaction. It seems now that the basic background process at intermediate energies ≤ 10 GeV is the two-step process (fig. 9) $\mathrm{p}+\mathrm{D}\to\mathrm{N}+\Delta+\mathrm{N}\to 3\mathrm{N}$ discussed e.g. in refs. [21,22]). Its contribution was probably observed experimentally in ref. [23]) and it is quite significant at large angles and nucleon momenta $0.3<p<0.4$ GeV/c. In the approximation of π-exchange for the elementary $\mathrm{N}+\mathrm{N}\to\mathrm{N}+\Delta$ amplitude the ratio of mechanism contributions of fig. 9 to the cross sections of reactions $\mathrm{p}+\vec{\mathrm{D}}\to\mathrm{p}+X$ and $\mathrm{n}+\vec{\mathrm{D}}\to\mathrm{p}+X$ is equal to 5 [ref. [22])]. Therefore, one can try to subtract contribution of this mechanism by comparing the cross sections for the $\vec{\mathrm{D}}+\mathrm{p}\to\mathrm{p}+X$, $\vec{\mathrm{D}}+\mathrm{D}\to\mathrm{p}+X$ or $\vec{\mathrm{D}}+\mathrm{p}\to\mathrm{p(n)}+X$ reactions. Another way to suppress the mechanism of fig. 9 is to study the inelastic reaction $\vec{\mathrm{D}}+\mathrm{p}\to\mathrm{p}+\mathrm{p}+X$, where one proton has

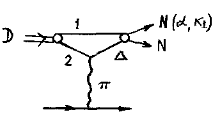

Fig. 9. The triangle diagram contribution to the fast backward nucleon production.

† In the difference from ref. [1]) we give here an expression for $G(a, b)$ accounting for the rotation of the nucleon spin, which numerically is quite a small effect.

$\alpha > 1$ and the invariant mass of the two-proton system is different from the sum of masses $m_\Delta + m_N$. At sufficiently large energies there exists a more simple method for suppressing mechanisms like fig. 7† and similar rescatterings of N*: it is enough to select events with sufficiently large multiplicity [1]).

It is well known that the measurement of fast deuteron polarization is a difficult problem. The reaction discussed here provides an effective method to measure deuteron polarization. The region of small nucleon momenta $\leqslant 0.15$ GeV/c is best for this task, since at these momenta nucleons are at large relative distances and therefore uncertainties in the calculation of $R(p_s)$ are small††. Another method is to measure the helicity of the nucleon spectator as at $p_t = 0$, $\lambda_p = \lambda_{D/2}$ (for $\lambda_D = \pm 1$). Here λ_N (λ_D) is the helicity of the nucleon (deuteron). Actually complete reconstruction of the nucleon spin density matrix on the basis of this reaction is possible (see appendix).

6. QCD and reactions with polarized deuteron: spin effects in soft hadronic reactions at $x_F \sim 1$

It was explained in ref. [1]) that the light-cone two-nucleon description of the dueteron is justified theoretically up to nucleon momenta $k \leqslant 0.8$–1.0 GeV/c only. QCD analysis [16,1]) indicates that at larger nucleon momenta ($\alpha \gtrsim 1.7$) the contribution of different quark–gluon configurations should dominate in different reactions. For example, the 6q configuration determines $F_{2D}(x, Q^2)$ behaviour at $x \gtrsim 1.5$ and $Q^2 \to \infty$. On the contrary, in the $h + D \to p(\pi) + X$ reaction the dominant contribution is given by the so-called gluon mechanism [16]) (fig. 10). According to this mechanism a nucleon is formed in the final state after soft interaction (denoted in fig. 10 by the blob) of 3 quarks and 3 hard gluons emitted by the other 3 quarks. Evidently, to have a non-negligible overlapping integral between the 3q+3g state and the

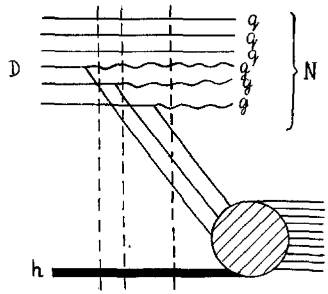

Fig. 10. The gluon mechanism for production of leading nucleons in the $h + D \to p + X$ reaction.

† Actually the cross section of Δ production fastly decreases with E_{inc}. However, there exists a smaller scaling contribution due to inclusive Δ production [1]).

†† The large difference R from zero makes this reaction a convenient polarization monitor for deuterium targets.

nucleon, the momenta of nucleons in the soft WF (i.e. before emission of hard gluons) should be rather large. However, the soft WF at $k \geqslant 0.15$ GeV/c rather strongly depends on the deuteron polarization. Thus, even in the $\alpha \sim 2$ region R should significantly differ from 0.

Since there is no specific cut on the nucleon transverse momenta in the integral over the soft WF, the D-wave with the "wrong" nucleon polarization should give a large contribution. This would lead to a considerable depolarization of produced nucleons even at $p_t \sim 0$, which at present cannot be quantitatively calculated. On the contrary, in the two nucleon approximation $\lambda_p \simeq \lambda_{D/2}$ for $|\lambda_D| = 1$ up to small corrections due to Glauber screening terms (see sect. 4 and appendix). The perturbative QCD is known to describe reasonably the x-dependence of the fragmentation processes b → c [refs. [24,16])]. Thus, one can try to apply it to estimate the polarization transfer in the processes of limiting fragmentation: $\vec{b} \to \vec{c}$. If the s-wave dominates in quark WFs of N, Δ, ρ, . . . , and weak parton effects are inessential, it is easy to demonstrate within the perturbative QCD that the QCD selection rules for helicities in $q\bar{q}g$ vertices lead to the rule of minimal helicity change:

(a) in the transitions like N → ρ, A_1 and $\pi \to$ N, Δ . . . , where the dominant contribution is given at large x by the gluon mechanism:

$$\lambda_M = \lambda_B \pm \tfrac{1}{2}\,; \tag{31}$$

(b) for transitions where the leading hadron is produced in the configuration without leading gluons: N → Λ, Δ, $\bar{p}$, Ω; $\pi^+ \to \rho^-$, K^{*-} helicity is conserved:

$$\lambda_M = \lambda_{M'}\,; \qquad \lambda_B = \lambda_{B'}\,. \tag{32}$$

Note that the s-wave approximation for the hadron WF is generally quite reasonable. However in the case of the gluon mechanism this assumption is more questionable since it is used for the $q\bar{q}g$ configuration in the meson WF. Consequently eq. (32) appears to be more well founded than eq. (31).

Evidently eqs. (31), (32) lead also to certain spin alignment effects in the production of the leading hadron. For example, in the fragmentation π, K, p → h(Δ, Σ^*, Ω · · ·).

7. Oscillation effects in the cumulative pion production off the polarized deuteron

In ref. [1]) we have analysed the fast pion production in the deuteron fragmentation region (the so-called cumulative effect): the $h + D \to \pi + X$ reaction for $x_\pi \geqslant 1$. It was demonstrated that Glauber screening for the inclusive cross section is cancelled with pion emission in the h-interactions with both nucleons of the deuteron. Since the cancellation is just a consequence of the AGK combinatorics [25]) it is valid in the polarized deuteron case as well. Therefore, the impulse approximation can be used to describe the cross section, provided the final-state interactions of hadrons in the elementary interactions are neglected. Similar to the case of the deuteron structure function (eq. (1)), the inclusive cross section $G_h^{\vec{D}/\pi}(x_\pi, p_t)$ is given by the convolution of the elementary $h+N\to\pi+X$ inclusive cross section and the polarized deuteron density matrix $\rho_\xi^{D/N}(\alpha, k_t)$ defined in eq. (4):

$$G_h^{\vec{D}/\pi}(x_\pi, p_t) \equiv E_\pi \frac{d^3\sigma^{h+\vec{D}\to\pi+X}}{d^3p_\pi}$$

$$= \sum_{N=p,n} \int \frac{d\alpha\, d^2k_t}{\alpha} \rho_\xi^{D/N}(\alpha, k_t)\beta \frac{d\sigma^{h+N\to\pi+X}}{d\beta\, d^2\tau_t}(x/\alpha, p_{\pi t}-(x/\alpha)k_t)\,. \quad (33)$$

Here $\frac{1}{2}x_\pi$ is the light-cone fraction of the deuteron momentum carried by the cumulative pion. x can be expressed through the pion momentum p_π in the deuteron lab frame as follows:

$$x_\pi = \frac{\sqrt{m_\pi^2+p_\pi^2}-(\boldsymbol{p}_\pi \boldsymbol{p}_h)/|\boldsymbol{p}_h|}{\frac{1}{2}m_D}\,. \quad (34)$$

In two important cases which we restricted in this paper it is justified to use the cross section of the elementary $h+N\to\pi+N$ reaction off the unpolarized nucleon:

(a) for the real polarization vector ξ, for $\lambda_D=0$ or for the sum of the cross sections for $\lambda_D=\pm1$;

(b) for $p_{\pi t}=0$. In this case the contribution of spin-dependent terms in $G_h^{N/\pi}$ is cancelled due to integration over k_t.

In other cases it is necessary to know the cross section for the $h+N\to\pi+X$ scattering of the polarized nucleon which is poorly known now. Numerical estimates of the ratio $R(x_\pi, p_{\pi t}) = ((\sigma^+ + \sigma^-)/2-\sigma^0)/\langle\sigma\rangle$ reveal an oscillation type behaviour of R at large x_π similar to one predicted for the deep inelastic scattering (fig. 3). Observation of such oscillation for the cumulative pion production off the polarized deuteron would provide an independent test of its origin since other models as far as we know do not predict oscillations.

8. Inclusive reactions with polarized ^{6}Li

Recently, a program of experiments with the polarized D^{6}Li target has been proposed for the FNAL tevatron [6]. Here we shall estimate the effective polarization of nucleons in such a target. We shall suggest also several experiments with $^6\overrightarrow{\text{Li}}$ which could help to improve this estimate.

We explained in sect. 2 that in the deuteron nucleons are depolarized due to the D-wave. A straightforward calculation of the Clebsch–Gordan coefficients shows that for the transversely polarized D with $|\lambda_D|=1$ the effective nucleon polarization λ_N is (cf. eq. (11))

$$\lambda_N = \tfrac{1}{2}(1-1.5P_D) \simeq 0.45\,. \quad (35)$$

Here $P_D \simeq 6$–7% is the D-wave probability in the deuteron.

In the framework of the cluster model ^{6}Li can be described as the sum of D and ^{4}He clusters. The variational calculations of ref. [26]) indicate that the D-cluster is somewhat compressed as compared to the free deuteron, leading to an increase of the high-momentum component of the D-cluster by a factor $\gamma \sim 1.5$. Since the D-wave gives a dominant contribution to the deuteron high-momentum component a similar increase of the D-wave probability in the D-cluster is expected. Consequently, an effective polarization of nucleons in a totally polarized D^{6}Li target with processes described by the impulse approximation, can be estimated as

$$\begin{aligned}\lambda_{\mathrm{N}}^{\mathrm{eff}} &= \tfrac{1}{2}\frac{(1-\frac{3}{2}P_{\mathrm{D}}^{(D)})\sigma_{\mathrm{D}}+(1-\frac{3}{2}P_{\mathrm{D}}(^6\mathrm{Li})\sigma_{^6\mathrm{Li}}}{\sigma_{\mathrm{D}}+\sigma_{^6\mathrm{Li}}} \\ &= \tfrac{1}{2}\frac{(1-\frac{3}{2}P_{\mathrm{D}}^{(\mathrm{D})}+(1-\frac{3}{2}P_{\mathrm{D}}(D)\gamma))\sigma_{\mathrm{D}}}{\sigma_{\mathrm{D}}+\sigma_{^6\mathrm{Li}}} \\ &= \tfrac{1}{4}(1-\tfrac{3}{4}P_{\mathrm{D}}(1+\gamma)),\end{aligned} \tag{36}$$

where $P_{\mathrm{D}} \equiv P_{\mathrm{D}}(D)$, $P_{\mathrm{D}}(^6\mathrm{Li})$ are the probabilities of D-waves in D, ^{6}Li. Here we substituted σ_A by $\frac{1}{2}A(\sigma_{\mathrm{p}}+\sigma_{\mathrm{n}})$. The additional multiplier $\frac{1}{2}$ in eq. (36) as compared to eq. (35) is due to the spin-0 ^{4}He cluster in ^{6}Li. Thus, the D-wave admixture in the D, ^{6}Li WF's leads to a decrease of the effective polarization of the D^{6}Li target by about 12%.

To determine γ more accurately and to check incidentally the cluster structure of the ^{6}Li WF at high nucleon momenta, one should perform for $^6\overrightarrow{\mathrm{Li}}$ the experimental program analogous to one discussed in the previous sections for $\vec{\mathrm{D}}$. In the cluster model it is easy to derive several useful relations between cross sections of scattering off ^{6}Li and D with polarizations $\xi_1 = \xi_2 = \xi$. Similar to ref. [1]) we have

$$G_{\xi,\mathrm{a}}^{6\mathrm{Li/p},\pi}(\alpha, p_{\mathrm{t}}) = \gamma G_{\xi,\mathrm{a}}^{\mathrm{D/p},\pi}(\alpha, p_{\mathrm{t}}) + G_{\mathrm{a}}^{4\mathrm{He/p},\pi}(\alpha, p_{\mathrm{t}}) \tag{37}$$

for $\alpha > 1$. Thus, in the region $\alpha > 1$

$$G_{\xi,\mathrm{a}}^{6\mathrm{Li/p},\pi}(\alpha, p_{\mathrm{t}}) - G_{\mathrm{a}}^{6\mathrm{Li/p},\pi}(\alpha, p_{\mathrm{t}}) = \gamma[G_{\xi,\mathrm{a}}^{\mathrm{D/p},\pi}(\alpha, p_{\mathrm{t}}) - G_{\mathrm{a}}^{\mathrm{D/p},\pi}(\alpha, p_{\mathrm{t}})]\,. \tag{38}$$

Besides,

$$W_2^{\xi}(\nu, q^2) - W_{2^6\mathrm{Li}}(\nu, q^2) = \gamma(W_{2\mathrm{D}}^{\xi}(\nu, q^2) - W_{2\mathrm{D}}(\nu, q^2)) \tag{39}$$

for $x = -q^2/2m_{\mathrm{N}}\nu > 1$ and sufficiently large q^2. Similar relations are valid for the nucleon polarization in the p, e + $^6\overrightarrow{\mathrm{Li}} \to$ p + X and p, e + $\vec{\mathrm{D}} \to$ p + X reactions at $\alpha > 1$.

9. Conclusions

In this paper we have considered in length a new and potentially quite rich field of research: hard nuclear reactions with polarized deuterons. We believe that inclusion of a new measured variable – deuteron spin – will lead to disappearance of the somewhat dull smooth curves known from measurements with the unpolarized deuteron. Such measurements would determine more definitely up to what nucleon momenta the two-nucleon description of D remains a good approximation and where the 6q (quark–gluon) component of the deuteron WF comes into play.

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Note added in proof: Recently a certain difference between the bound nucleon wave function and the free one has been discovered by EMC for iron [Phys. Lett. **123B** (1983) 275] and confirmed by the SLAC data. Our analysis [LNPI preprint no. 838 (1983)] indicates that similar (though smaller) effects should be present for the deuteron. It would be instructive to study such an effect in the case of polarized D since the deformation of a bound nucleon could be different for S- and D-waves. The deep inelastic $e+\vec{D}\to e'+N+X$ reaction could provide an effective tool for this purpose, since the selection of a fast backward nucleon enhances the contribution of small internucleon distances.

Appendix

The reaction $\vec{D}+p\to p+X$ is a method of complete determination of the spin density matrix of a fast deuteron.

It is well known that it is rather difficult to determine the spin density matrix of a fast deuteron, $\Omega_{\alpha\beta}$. Here we shall present equations necessary for the measurement of Ω through the study of the angular distribution and polarization of nucleons produced in the $\vec{D}+p\to p+X$ reaction. To avoid any significant theoretical uncertainties in the determination of Ω it is sufficient to restrict the measurements to the region of nucleon momenta $p_N \leqslant 0.1$–0.15 GeV/c in the deuteron rest frame. In this case nucleons in the deuteron are far apart and therefore Glauber screening effects and the final-state interaction are small. The validity of the impulse approximation in this kinematics for the unpolarized deuteron with $p_D \geqslant 4$ GeV/c is confirmed experimentally [23a,b]).

Similarly to eq. (7) it is easy to demonstrate, by a quite straightforward calculation, that the nucleon momentum density matrix $\rho_{\Omega}^{D/N}(\boldsymbol{k})$ for deuteron with spin density matrix Ω has the form

$$\begin{aligned}\rho_{\Omega}^{D/N}(\alpha, p_t) = \frac{2}{2-\alpha}\sum_{a,b}\Omega_{ab}\,\mathrm{Sp}\,(G_1(M_{NN}^2)\varepsilon_a\gamma_a \\ +(p_1-p_2, \varepsilon_a)G_2(M_{NN}^2))(m+\hat{p}_1)(1-\gamma_5\hat{s})(G_1(M_{NN}^2)\tilde{\varepsilon}_b^*\gamma_b \\ +(p_1-p_2, \varepsilon_b^*)G_2(M_{NN}^2))(m-\hat{p}_2)\,,\end{aligned} \tag{A.1}$$

where for $a=1,2$ $\varepsilon_a=\varepsilon_{\mathrm{T}}$ and for $a=3$ $\varepsilon_a=\varepsilon_{\mathrm{L}}$, which was defined after eq. (2). s is the four-vector of nucleon polarization. In the case when nucleon polarization is not measured eq. (A.1) is simplified and the cross section has the form

$$E\frac{\mathrm{d}^3\sigma^{\vec{\mathrm{D}}/\mathrm{p}}}{\mathrm{d}^3p}=\sigma_{\mathrm{tot}}^{\mathrm{NN}}\frac{\sqrt{m^2+k^2}}{2-\alpha}\Big\{u^2(k)+\tfrac{1}{2}w^2(k)-\sqrt{2}u(k)w(k)$$
$$+\frac{3k_ak_b}{k^2}\Omega_{ab}[\tfrac{1}{2}w^2(k)+\sqrt{2}u(k)w(k)]\Big\}$$
$$\approx\sigma_{\mathrm{tot}}^{\mathrm{NN}}(m-p_3)\Big(u^2(p)+\tfrac{1}{2}w^2(p)-\sqrt{2}u(p)w(p)$$
$$+\frac{3p_ap_b}{p^2}\Omega_{ab}(\tfrac{1}{2}w^2(p)+\sqrt{2}w(p)u(p))\Big)\,, \tag{A.2}$$

where α, p, k are related according to eq. (26). Remember that p is the momentum of the spectator nucleon in the deuteron rest frame. In the derivation of eq. (A.2) we used the normalization condition for the deuteron density matrix: $\mathrm{Sp}\,\Omega=1$.

It is evident from eqs. (A.1), (A.2) that a complete reconstruction of Ω is possible. To measure the symmetric part of $\Omega(\Omega_{ab}+\Omega_{ba})$ it is sufficient to study the angular distribution of produced nucleons; to measure the imaginary part of $\Omega(\Omega_{ab}-\Omega_{ba})$ it is necessary to study the spin density matrix of the emitted nucleons at one point. For example, for the stripping nucleon ($k=0$) the nucleon polarization matrix $p(\eta)=\frac{1}{2}(I+(\sigma\eta))$ has the form

$$p(\eta)=\tfrac{1}{2}(I+i\varepsilon_{abc}\Omega_{ab}\sigma_c)\,. \tag{A.3}$$

Thus,

$$\eta_c=i\varepsilon_{abc}\Omega_{ab}\,. \tag{A.4}$$

We want to stress once more that in the region discussed all realistic WF's predict a practically identical D-wave in the deuteron.

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