Size and scaling of the double-helicity-flip hadronic structure function

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Recently, Jaffe and Manohar identified a new leading-twist structure function $\Delta(x,Q^2)$, which can be measured in deep-inelastic scattering from polarized targets with spin ≥ 1 . We calculate the scaling behavior of $\Delta(x,Q^2)$ in QCD, both by computing the anomalous dimensions of the associated operators and by finding the splitting function for the appropriate parton density. We also estimate the size of the lowest moment of $\Delta(x,Q^2)$ in the bag model. In an appendix we give the complete cross section for electroproduction from a spin-1 target of arbitrary polarization.

I. INTRODUCTION

To a good approximation, nuclei are simply bound states of neutrons and protons. Thus, to lowest order, deep-inelastic scattering off a nucleus can be regarded as deep-inelastic scattering off the bound nucleons. However, the number of nuclear structure functions increases steadily with the spin of the nucleus, 1 providing additional information about the nuclear substructure. Hoodbhoy, Jaffe, and Manohar showed that two of the additional structure functions for spin-1 hadrons, $b_1(x,Q^2)$ and $b_2(x,Q^2)$, occur at leading twist and satisfy a Callan-Gross relation $b_2 = 2xb_1$. These structure functions arise from the traditional quark and gluon operators found in spin- $\frac{1}{2}$ scattering.

Later it was realized that there is another tower of gluonic operators that contributes for targets of spin $\geq 1.^3$ These operators occur at leading twist and give rise to a third structure function $\Delta(x,Q^2)$ for spin-1 targets. In terms of the helicity amplitudes of Ref. 2, $\Delta = A_{+-,-+}$; it flips the photon helicity by two units. The gluonic operators that contribute to $\Delta(x,Q^2)$ are unique in that their expectation values in states of spin < 1 are identically zero. Therefore, $\Delta(x,Q^2)$ probes the gluonic content of a nucleus which cannot be identified with any of the constituent neutrons, protons, or virtual pions. For this reason $\Delta(x,Q^2)$ would be very interesting to measure.

In this paper we explore the structure function $\Delta(x,Q^2)$ further. The logarithmic scaling behavior of $\Delta(x,Q^2)$ can be calculated easily in QCD. In Sec. II we do this by computing the anomalous dimensions of the associated twist-two operators. We also present a complementary parton-model analysis, using the Altarelli-Parisi splitting-function technique. In Sec. III we estimate, using the MIT bag model, the lowest moment of the structure function for a Δ^{++} . This provides a guide to the magnitude of $\Delta(x,Q^2)$ that might be found in an experiment. In Sec. IV we present our conclusions. In an appendix we reanalyze the kinematics for deepinelastic scattering from a spin-1 target and present the

most general cross section for scattering unpolarized electrons off a polarized spin-1 target.

II. SCALING BEHAVIOR

We begin by summarizing some of the results of Ref. 4. In the operator-product expansion of the time-ordered product of two hadronic currents,

$$\mathcal{T}_{\mu\nu}(q) \equiv i \int d^4x \ e^{iq \cdot x} T(j_{\mu}(x)j_{\nu}(0)) \ , \tag{2.1}$$

there are additional terms,

$$\frac{1}{2}\mathcal{T}_{\mu\nu}(q) = \cdots + \sum_{n=2,4,6,\dots} \frac{2^n q^{\mu_1} \cdots q^{\mu_n}}{(Q^2)^n} C_n(Q^2) \times O_{\mu\nu\mu_1\mu_2\cdots\mu_n}, \qquad (2.2)$$

where the coefficient functions are

$$C_n(Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \text{Tr} Q^2 \frac{2}{n+2}$$
 (2.3)

and we have the new tower of operators

$$O_{\mu\nu\mu_1\mu_2\cdots\mu_n} \equiv \left[\frac{i}{2}\right]^{n-2} \operatorname{Tr}_{\text{colors}} \mathscr{S} \{G_{\mu\mu_1} \overrightarrow{\mathbf{D}}_{\mu_3} \cdots \overrightarrow{\mathbf{D}}_{\mu_n} G_{\nu\mu_2}\} . \tag{2.4}$$

Here \mathcal{Q} is the quark charge matrix, $G_{\alpha\beta}$ is the gluon field-strength tensor, and \mathcal{S} symmetrizes the indices $\mu_1, \mu_2, \ldots, \mu_n$ and removes traces from all Lorentz indices. If the quantities $A_n(\mathcal{Q}^2)$ are defined in terms of the matrix element of $O_{\mu\nu\mu_1\mu_2\cdots\mu_n}$ in a spin-1 target with polarizations E, E' by

$$\langle pE'|O_{\mu\nu\mu_{1}\mu_{2}\cdots\mu_{n}}|pE\rangle$$

$$=\frac{1}{2}\mathcal{S}\{[(p_{\mu}E'_{\mu_{1}}^{*}-p_{\mu_{1}}E'_{\mu}^{*})(p_{\nu}E_{\mu_{2}}-p_{\mu_{2}}E_{\nu})$$

$$+(\mu\leftrightarrow\nu)]p_{\mu_{1}}\cdots p_{\mu_{n}}\}A_{n}(Q^{2}), \qquad (2.5)$$

then the moments of $\Delta(x, Q^2)$,

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$$M_n^{\Delta}(Q^2) = \int_0^1 dx \ x^{n-1} \Delta(x, Q^2) \ ,$$
 (2.6)

are related to the $A_n(Q^2)$ (for n = 2, 4, 6, ...) by

$$M_n^{\Delta}(Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \text{Tr} Q^2 \frac{A_n(Q^2)}{n+2}$$
 (2.7)

Hence the scaling behavior of $\Delta(x, Q^2)$ is completely specified by the scaling of these operators.

A. Anomalous dimensions

The new operators have Lorentz structures unique at twist two in QCD and, therefore, do not mix with other operators under renormalization. We can greatly simplify the perturbative calculations by working with the operators O_n obtained by contracting the $O_{\mu\nu\mu_1\mu_2\cdots\mu_n}$ with $\phi^\mu\phi^\nu\eta^{\mu_1}\eta^{\mu_2}\cdots\eta^{\mu_n}$, where ϕ and η are nonparallel null vectors with a vanishing inner product $\phi\cdot\eta=0$. The symmetrization and the removal of traces is then automatic. We can compute the anomalous dimensions to $O(\alpha)$ by renormalizing to one loop the gluon two-point function with operator insertion $\Gamma_{O_n}^{(2)}(0;k)$ (zero external momentum flowing into the operator vertex). The diagrams that occur to this order in the perturbative expansion of $\Gamma_{O_n}^{(2)}(0;k)$ are shown in Fig. 1. At the tree level,

$$\Gamma_{O_n}^{(2)}(0;k) = \delta^{ab}(\eta \cdot k)^{n-2} (\phi \cdot k \eta^{\alpha} - \eta \cdot k \phi^{\alpha})$$

$$\times (\phi \cdot k \eta^{\beta} - \eta \cdot k \phi^{\beta}) . \tag{2.8}$$

The upper right graph of Fig. 1(b) vanishes in dimensional regularization and from the remaining graphs we find strict multiplicative renormalization. With the operator renormalization defined by

$$\Gamma_{O_n \text{ren}}^{(2)}(0;k) = Z_A Z_{O_n} \Gamma_{O_n \text{bare}}^{(2)}(0;k)$$
, (2.9)

the anomalous dimensions of the new operators are

$$\gamma_{n} = \mu \frac{\partial}{\partial \mu} \ln Z_{O_{n}}$$

$$= -\frac{\alpha_{s}(Q^{2})}{2\pi} \left[\left[4 \sum_{j=2}^{n} \frac{1}{j} + \frac{1}{3} \right] C_{2}(G) + \frac{4}{3} T(R) \right],$$
(2.10)

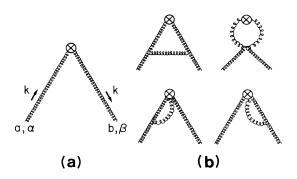


FIG. 1. Perturbative expansion of $\Gamma_{O_n}^{(2)}(0;k)$. (a) Tree level; (b) one-loop graphs.

where $C_2(G)$ and T(R) are the Casimir elements of the gauge group and the fermion representation, respectively; for QCD, $C_2(G)=3$ and $T(R)=n_f/2$, where n_f is the number of quarks. Note that in the large-n limit the anomalous dimensions are proportional to $\ln n$ as is usual for operators in gauge theories.

B. Parton-model analysis

We can also study the scaling of $\Delta(x,Q^2)$ by using the method of Altarelli and Parisi⁵ to determine the evolution of the corresponding parton density. In addition to obtaining the splitting function, from which the anomalous dimensions of the moments of $\Delta(x,Q^2)$ can be recovered, this provides us with a more physical picture of the scaling.

Consider a spin-1 hadronic target aligned (i.e., circularly polarized) in the x direction in the infinite-momentum frame and let $g_{\hat{x},\hat{y}}(x,Q^2)$ be the probability of finding a gluon of momentum fraction x linearly polarized along the x,y direction. It was found in Ref. 4 that $\Delta(x,Q^2)$ can be expressed in terms of the parton density

$$a(x,Q^2) \equiv -2[g_{\hat{x}}(x,Q^2) - g_{\hat{y}}(x,Q^2)]$$

a

$$\Delta(x,Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \text{Tr} Q^2 x^2 \int_x^1 \frac{dy}{y^3} a(y,Q^2) . \qquad (2.11)$$

Therefore, the moments of $\Delta(x,Q^2)$ are related to those of $a(x,Q^2)$, $M_n^a(Q^2)$, by

$$M_n^{\Delta}(Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \text{Tr} Q^2 \frac{M_n^a(Q^2)}{n+2}$$
 (2.12)

and if we can find how $a(x,Q^2)$ evolves with scale, we will have determined the scaling of $\Delta(x,Q^2)$.

The evolution of $a(x,Q^2)$ is determined by the master equation

$$\frac{\partial}{\partial \ln Q^2} a(x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} a(y, Q^2) P_a \left[\frac{x}{y} \right],$$
(2.13)

where $P_a(z) = P_{xx}(z) - P_{xy}(z)$ and $P_{xx,xy}(z)$ is the probability of finding a gluon linearly polarized in the x,y direction with momentum fraction z inside a gluon linearly polarized in the x direction. We can compute the splitting function $P_a(z)$ to $O(\alpha)$ from the three-gluon vertex for z < 1. The singularities in the splitting function at z = 1 occur only in $P_{xx}(z)$ and hence are the same as in $P_{gg}(z) = P_{xx}(z) + P_{xy}(z)$, the well-known spin-averaged splitting function. We find

$$P_{a}(z) = 2C_{2}(G) \left[\frac{z}{(1-z)_{+}} + \left[\frac{11}{12} - \frac{1}{3} \frac{T(R)}{C_{2}(G)} \right] \delta(z-1) \right]. \quad (2.14)$$

Taking moments of both sides of the master equation (2.13) we see that the $M_n^a(Q^2)$ evolve according to

$$\frac{d}{d \ln \sqrt{Q^2}} M_n^a(Q^2) = \gamma_n M_n^a(Q^2) , \qquad (2.15)$$

where the anomalous dimensions are proportional to the moments of the splitting function, $\gamma_n = [\alpha_s(Q^2)/\pi] M_n^{P_a}$; explicit evaluation of $M_n^{P_a}$ using (2.14) reproduces the perturbative results of (2.10).

III. BAG-MODEL ESTIMATE OF THE LOWEST MOMENT OF $\Delta(x, Q^2)$

We cannot, of course, compute $\Delta(x,Q^2)$. But as a measure of the new structure function we can estimate its lowest moment for the spin- $\frac{3}{2}$ Δ^{++} using the MIT bag model. In Ref. 4 an expression was derived from (2.5) for this moment in terms of a matrix element of colorelectric $(E_k^a \equiv G_{0k}^a)$ and -magnetic $(B_k^a \equiv -\frac{1}{2}\epsilon_{ijk}G_{ij}^a)$ fields in a state with spin (≥ 1) aligned in the x direction:

$$\int_{0}^{1} dx \, x \, \Delta(x, Q^{2})$$

$$= \frac{\alpha_{s}(Q^{2})}{16\pi} \operatorname{Tr} Q^{2} \frac{1}{(p^{+})^{2}}$$

$$\times \langle p, s_{x} | (E_{2} - B_{1})^{2} - (E_{1} + B_{2})^{2} | p, s_{x} \rangle , \qquad (3.1)$$

where $p^+ = (p^0 + p^3)/\sqrt{2}$ and the sum over color indices is suppressed. We can estimate this moment for the Δ^{++} by using the bag model to relate the color field matrix element of (3.1) to the Δ^{++} -proton mass splitting.

A Δ^{++} aligned in the x direction is represented in the bag model by three up quarks (assumed massless) all in the lowest mode of the bag cavity and spin up in the x direction. Since the three quarks have the same mass, the color-electric field $\mathbf{E}^a = \sum_{i=1}^3 \mathbf{E}^a_i$ (the sum of the contributions from each quark) is proportional to $\sum_{i=1}^3 \lambda_i^a$ which annihilates the color singlet. Hence the matrix element reduces to

$$\langle (B_1)^2 - (B_2)^2 \rangle = 2M_\Delta \int_{\text{bag}} [(B_1)^2 - (B_2)^2].$$
 (3.2)

In the bag model, the Δ^{++} -proton mass difference arises solely from the color-magnetostatic energy, which is also

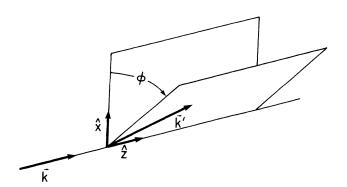


FIG. 2. Definition of the azimuthal scattering angle ϕ .

given by an integral quadratic in the color-magnetic fields:

$$E_M = -\sum_{a} \sum_{i < j} \int_{\text{bag}} \mathbf{B}_i^a \cdot \mathbf{B}_j^a . \tag{3.3}$$

Evaluating these integrals we find

$$\langle (B_1)^2 - (B_2)^2 \rangle = -0.91(2M_{\Delta})(M_{\Delta} - M_{\text{proton}})$$
 (3.4)

so that for three flavors of quarks $Q = \text{diag}(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$, the lowest moment of $\Delta(x, Q^2)$ is given by

$$\int_0^1 dx \ x \Delta(x, Q^2) = -0.012 \alpha_s(Q^2) \ . \tag{3.5}$$

It should be noted that this calculation, while dependent on bag-model assumptions, is independent of bag-model parameters.

IV. CONCLUSIONS

The small estimate of the lowest moment of $\Delta(x,Q^2)$ in Δ^{++} , expected to be smaller still in a nuclear target, suggests that $\Delta(x,Q^2)$ may prove difficult to measure. However, because of its unique identification with gluons outside individual nucleons, it would be very interesting to investigate. Luckily, a measurement of $\Delta(x,Q^2)$ can be obtained for free in any experiment designed to measure g_2 for a target with spin ≥ 1 . From Eq. (A3) in the Appendix, we see that $\Delta(x,Q^2)$ is most readily extracted when scattering unpolarized electrons off a target which is polarized perpendicular to the beam, the same target configuration as is needed to measure g_2 . Recently, the HERMES Collaboration at the DESY ep collider HERA has proposed to use this fact to measure or set limits on $\Delta(x,Q^2)$ for a deuteron target.

Note added. After the completion of this work we learned that the anomalous dimensions of the parton density $a(x,Q^2)$ have been calculated by Artru and Mekki [Z. Phys. C 45, 669 (1990)].

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APPENDIX: KINEMATICS FOR A SPIN-1 TARGET

In Ref. 2 the structure functions for a spin-1 target were studied under the assumption that the double-helicity-flip amplitude $A_{+-,-+}$ fell as $1/Q^2$. Because of this, the structure functions b_3 and b_4 do not scale properly, while b_1 and b_2 contain terms proportional to $A_{+-,-+}$. Thus, we find it useful to redefine the structure functions by

$$\begin{split} W^{\mu\nu} &= -g^{\mu\nu}F_1 + \frac{p^{\mu}p^{\nu}}{\nu}F_2 + i\frac{g_1}{\nu}\epsilon^{\mu\nu\lambda\sigma}q_{\lambda}s_{\sigma} + i\frac{g_2}{\nu}\epsilon^{\mu\nu\lambda\sigma}q_{\lambda}\left[s_{\sigma} - \frac{s\cdot q}{\nu}p_{\sigma}\right] + \left[-g^{\mu\nu}\Delta_1 + \frac{p^{\mu}p^{\nu}}{\nu}\Delta_2\right]\left[\frac{M^2}{\kappa\nu^2}q\cdot E\ q\cdot E^* - \frac{1}{3}\right] \\ &+ \frac{\kappa-1}{\sqrt{\kappa\nu}}\Delta_3\left[p^{\mu}q\cdot E^*(E^{\nu} - \frac{q\cdot E}{\kappa\nu}p^{\nu}) + p^{\mu}q\cdot E\left[E^{*\nu} - \frac{q\cdot E^*}{\kappa\nu}p^{\nu}\right] + (\mu\leftrightarrow\nu)\right] \\ &+ \frac{\kappa}{2}\Delta_4\left\{\left[-g^{\mu\nu} + \frac{2x}{\kappa\nu}p^{\mu}p^{\nu}\right]\left[\frac{M^2}{\kappa\nu^2}q\cdot Eq\cdot E^* - 1\right] + \left[\left[E^{\mu} - \frac{q\cdot E}{\kappa\nu}p^{\mu}\right]\left[E^{*\nu} - \frac{q\cdot E^*}{\kappa\nu}p^{\nu}\right] + (\mu\leftrightarrow\nu)\right]\right\}, \end{split} \tag{A1}$$

where $v=p\cdot q$, $Q^2=-q^2\geq 0$, $x=Q^2/2v$, and $s_\sigma=-i\epsilon_{\sigma\alpha\beta\tau}E^{*\alpha}E^{\beta}p^{\tau}$, and we have dropped terms proportional to q^μ or q^ν . The variable $(\kappa-1)=M^2Q^2/v^2=4x^2M^2/Q^2$ is a measure of the approach to the Bjorken limit.

Written in terms of helicity amplitudes, we have

$$F_{1} = \frac{1}{3} (A_{++,++} + A_{+-,+-} + A_{+0,+0}), F_{2} = \frac{2x}{3\kappa} (A_{++,++} + A_{+-,+-} + A_{+0,+0} + 2A_{0+,0+} + A_{00,00}),$$

$$g_{1} = \frac{1}{2\kappa} [A_{+-,+-} - A_{++,++} + \sqrt{\kappa - 1} (A_{+0,0+} + A_{+-,00})],$$

$$g_{2} = \frac{1}{2\kappa} [A_{++,++} - A_{+-,+-} + \frac{1}{\sqrt{\kappa - 1}} (A_{+0,0+} + A_{+-,00})],$$

$$\Delta_{1} = \frac{-1}{2} (A_{++,++} + A_{+-,+-} - 2A_{+0,+0}), \Delta_{2} = \frac{-x}{\kappa} (A_{++,++} + A_{+-,+-} - 2A_{+0,+0} + 2A_{0+,0+} - 2A_{00,00}),$$

$$\Delta_{3} = \frac{1}{2\sqrt{\kappa(\kappa - 1)}} (A_{+0,0+} - A_{+-,00}), \Delta_{4} = \frac{1}{\kappa} \Delta = \frac{1}{\kappa} A_{+-,-+}.$$
(A2)

These structure functions all scale. They are defined so that all of the analysis of b_1 and b_2 in Ref. 2 can be applied directly to Δ_1 and Δ_2 . 8 Also $\Delta = \Delta_4$ in the Bjorken limit.

With these definitions we can now give the cross section in the laboratory frame for scattering unpolarized electrons off polarized spin-1 hadrons. Let the incoming electron momentum define the z axis and let ϕ be the azimuthal angle between the x-z plane and the plane formed by the beam and the scattered electron. (See Fig. 2.) We can write the cross section in terms of a density matrix ρ for the target polarization in a Cartesian basis. It is

$$\begin{split} \frac{d\sigma}{dx\,dy\,d\phi} &= \frac{4\alpha^2 ME}{Q^4} \left\{ xy^2 F_1 + \lambda F_2 - \frac{2}{3} (xy^2 \Delta_1 + \lambda \Delta_2) \mathrm{Tr}(I_1\rho) + 4x \left(1 - y/2\right) \mathrm{sin}\beta \Delta_3 \mathrm{Tr}(I_2\rho) + 2x \lambda \Delta_4 \mathrm{Tr}(I_3\rho) \right. \\ & + \left. \left(xy^2 \Delta_1 + \lambda \Delta_2 - x \lambda \Delta_4 \right) \mathrm{sin}\beta \left[\mathrm{sin}\beta \, \mathrm{Tr}(I_1\rho + I_3\rho) - \mathrm{cos}\beta \, \mathrm{Tr}(I_2\rho) \right] \right. \\ & \left. \left. \left(1 - y/2 \right) \Delta_3 \mathrm{sin}^2 \beta \left[\mathrm{cos}\beta \, \mathrm{Tr}(I_1\rho + I_3\rho) + \mathrm{sin}\beta \, \mathrm{Tr}(I_2\rho) \right] \right\} \;, \end{split} \tag{A3}$$

where $y = v/p \cdot k$, and we have introduced the variable $\lambda = 1 - y - (\kappa - 1)y^2/4$, as well as the matrices

$$I_1 = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & -1 \end{bmatrix}, \tag{A4}$$

$$I_2 = \begin{bmatrix} 0 & 0 & \cos\phi \\ 0 & 0 & \sin\phi \\ \cos\phi & \sin\phi & 0 \end{bmatrix}, \tag{A5}$$

$$I_3 = \frac{1}{2} \begin{bmatrix} \cos 2\phi & \sin 2\phi & 0\\ \sin 2\phi & -\cos 2\phi & 0\\ 0 & 0 & 0 \end{bmatrix} . \tag{A6}$$

The angle β is the angle between the photon momentum $\mathbf{q} = \mathbf{k} - \mathbf{k}'$ and the beam momentum \mathbf{k} and is given by $\sin \beta = \sqrt{\lambda(\kappa - 1)/\kappa}$. Note that $\sin \beta$ is O(1/Q) and that we have not gone to the Bjorken limit. From this equation we see that Δ_3 , like g_2 , is kinematically suppressed by 1/Q.

For a target with spin component $m_J = 1$ along the x axis, the density matrix can be written $\rho = E_x^i E_x^{*j}$, where $E_x^i = (1/\sqrt{2})(0,1,i)$. Then the cross section becomes

$$\frac{d\sigma}{dx\,dy\,d\phi} = \frac{4\alpha^2 ME}{Q^4} \left[xy^2 F_1 + \lambda F_2 + \frac{1}{6} (xy^2 \Delta_1 + \lambda \Delta_2) - \frac{1}{2} x \lambda \Delta_4 \cos 2\phi - \frac{1}{4} (xy^2 \Delta_1 + \lambda \Delta_2 - x \lambda \Delta_4) \sin^2 \beta (1 + \cos 2\phi) \right] + 2x \left(1 - y/2 \right) \Delta_3 \sin^2 \beta \cos \beta (1 + \cos 2\phi) \right]. \tag{A7}$$

We see that Δ_4 gives the leading $\cos 2\phi$ dependence, while Δ_1 , Δ_2 , and Δ_3 give corrections suppressed by $1/Q^2$.

For completeness we also give the cross section for a longitudinally polarized target. The density matrix is $\rho = E_z^i E_z^{*j}$, where $E_z^i = (1/\sqrt{2})(1,i,0)$, and the cross section is

$$\frac{d\sigma}{dx\,dy\,d\phi} = \frac{4\alpha^2 ME}{Q^4} \left[xy^2 F_1 + \lambda F_2 - \frac{1}{3} (xy^2 \Delta_1 + \lambda \Delta_2) + \frac{1}{2} (xy^2 \Delta_1 + \lambda \Delta_2 - x\lambda \Delta_4) \sin^2 \beta - 4x (1 - y/2) \Delta_3 \sin^2 \beta \cos \beta \right]. \tag{A8}$$

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⁷The HERMES Collaboration (unpublished).

⁸In the Bjorken limit they are related by $b_1 = \Delta_1 + \frac{1}{2}\Delta_4$ and $b_2 = \Delta_2 + x\Delta_4$.

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