Convenient parametrization for deep inelastic structure functions of the deuteron

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The spin, as well as spin-averaged, twist-two structure functions of the deuteron are calculated in a version of the convolution model that incorporates relativistic and binding energy corrections. A simple parametrization of these structure functions is given in terms of a few deuteron wave-function parameters and the free nucleon structure functions. This allows for an easy comparison of structure functions calculated using different deuteron models.

Deep inelastic scattering (DIS) from nuclear targets with spin $J \geq 1$ is interesting for a variety of reasons. Among possible targets, the deuteron has a position of privilege because its wave function is known far better than that of any other nucleus. The fact that it is a rather dilute neutron-proton bound state allows for the extraction of neutron structure functions F_1^n, F_2^n, g_1^n, and g_2^n, which, because of the absence of neutron targets, cannot be directly measured. The extraction of g_1^n is particularly important in view of the current controversies regarding the spin of the proton as deduced from the measurements of g_1^p. But DIS from a polarized deuteron target is important for other reasons as well: It was noted rather recently [1] that a $J=1$ target has a total of eight different structure functions which are potentially measurable. One of these, denoted by $b_1(x,Q^2)$ in Ref. [1], is of considerable interest since it provides a clear measure of possible exotic effects in nuclei, i.e., the extent to which the nuclear ground state deviates from being a composite of nucleons only. A DIS experiment with polarized deuterons planned at HERA will allow for the measurement of all deuteron structure functions [2].

In view of the experimental importance, it would perhaps be timely to collect together a set of *simple* expressions by means of which one could calculate the effect of nuclear Fermi motion and binding upon various deuteron structure functions. To this end, in this Brief Report, we use Jaffe's formulation [3] of the convolution model, which correctly embodies relativistic Fermi motion corrections as well as binding energy effects. The limitations of the convolution model have been dwelt upon earlier [3,4] and will not concern us here. Other calculations of deuteron structure functions have been performed by a number of authors using various other formulations of the convolution model [5]. However, the results to be presented below are considerably simpler, and this simplicity affords for an easy comparison between structure functions obtained from different deuteron wave functions. Moreover, spin-dependent and spin-independent structure functions are obtained simultaneously. We also take this opportunity to correct for errors given in the expression for b_1^D in Ref. [1].

The results for the deuteron structure functions F_1^D, b_1^D, and g_1^D are summarized below:

$$F_1^D(x)=(\alpha_1+\alpha_2+2\alpha_3)F_1(x)+(\alpha_2+4\alpha_3)xF_1'(x) +\alpha_3x^2F_1''(x)\ , \tag{1}$$

$$b_1^D(x)=(\beta_1+\beta_2+2\beta_3)F_1(x)+(\beta_2+4\beta_3)xF_1'(x) +\beta_3x^2F_1''(x)\ , \tag{2}$$

$$g_1^D(x)=(\gamma_1+\gamma_2+2\gamma_3)g_1(x)+(\gamma_2+4\gamma_3)xg_1'(X) +\gamma_3x^2g_1''(x)\ , \tag{3}$$

where the constants α_1 are defined by

$$\alpha_1=1\ , \tag{4a}$$

$$\alpha_2=\left\langle\frac{\varepsilon}{M}\right\rangle-\frac{1}{3}\left\langle\frac{p^2}{M^2}\right\rangle_{ss}-\frac{1}{3}\left\langle\frac{p^2}{M^2}\right\rangle_{dd}\ , \tag{4b}$$

$$\alpha_3=\frac{1}{6}\left\langle\frac{p^2}{M^2}\right\rangle_{ss}+\frac{1}{6}\left\langle\frac{p^2}{M^2}\right\rangle_{dd}\ , \tag{4c}$$

the β_1 by

$$\beta_1=0\ , \tag{5a}$$

$$\beta_2=\frac{4}{5\sqrt{2}}\left\langle\frac{p^2}{M^2}\right\rangle_{sd}-\frac{1}{5}\left\langle\frac{p^2}{M^2}\right\rangle_{dd}\ , \tag{5b}$$

$$\beta_3=-\frac{2}{5\sqrt{2}}\left\langle\frac{p^2}{M^2}\right\rangle_{sd}+\frac{1}{10}\left\langle\frac{p^2}{M^2}\right\rangle_{dd}\ , \tag{5c}$$

and the γ_i by

$$\gamma_1=\cos^2\alpha-\frac{1}{2}\sin^2\alpha-\frac{1}{3}\left\langle\frac{p^2}{M^2}\right\rangle_{ss} +\frac{2}{15\sqrt{2}}\left\langle\frac{p^2}{M^2}\right\rangle_{sd}+\frac{7}{30}\left\langle\frac{p^2}{M^2}\right\rangle_{dd}\ , \tag{6a}$$

$$\gamma_2=\left\langle\frac{\varepsilon}{M}\right\rangle\left[\cos^2\alpha-\frac{1}{2}\sin^2\alpha\right]-\frac{1}{3}\left\langle\frac{p^2}{M^2}\right\rangle_{ss} -\frac{4}{15\sqrt{2}}\left\langle\frac{p^2}{M^2}\right\rangle_{sd}+\frac{1}{30}\left\langle\frac{p^2}{M^2}\right\rangle_{dd}\ , \tag{6b}$$

$$\gamma_3=\frac{1}{6}\left\langle\frac{p^2}{M^2}\right\rangle_{ss}+\frac{2}{15\sqrt{2}}\left\langle\frac{p^2}{m^2}\right\rangle_{sd} -\frac{1}{60}\left\langle\frac{p^2}{M^2}\right\rangle_{dd}\ . \tag{6c}$$

In the above, $\langle \varepsilon/M \rangle$ is the ratio of the binding energy of the deuteron (2.2 MeV) to the nucleon mass M and the various averaged quantities are

$$\left\langle \frac{p^2}{M^2} \right\rangle_{ss} = \cos^2\alpha \int_0^\infty dp\, p^2 u_s^2(p) \frac{p^2}{M^2} , \tag{7a}$$

$$\left\langle \frac{p^2}{M^2} \right\rangle_{dd} = \sin^2\alpha \int_0^\infty dp\, p^2 u_d^2(p) \frac{p^2}{M^2} , \tag{7b}$$

$$\left\langle \frac{p^2}{M^2} \right\rangle_{sd} = \cos\alpha \sin\alpha \int_0^\infty dp\, p^2 u_s(p) u_d(p) \frac{p^2}{M^2} . \tag{7c}$$

The normalization of the deuteron's s- and d-state wave functions is

$$\int dp\, p^2 u_s^2(p) \left[1 + \frac{p^2}{4M^2} \right] = \int dp\, p^2 u_d^2(p) \left[1 + \frac{p^2}{4M^2} \right] = 1 ,$$

and $\cos\alpha$ and $\sin\alpha$ are defined by

$$|D\rangle = \cos\alpha |L=0\rangle + \sin\alpha |L=2\rangle . \tag{8}$$

In Eqs. (1)–(3), $F_1(x) = F_1^P(x) + F_1^N(x)$ is the isoscalar unpolarized structure function and $g_1(x) = g_1^P(x) + g_1^N(x)$ is the isoscalar polarized structure function. Observe that for zero Fermi motion and binding, we have from Eqs. (1)–(3) that $F_1^D = F_1$, $b_1^D = 0$, and $g_1^D = (\cos^2\alpha - \frac{1}{2}\sin^2\alpha) g_1$. The primed quantities in Eqs. (1)–(3) (F_1', g_1', etc.) are the appropriate derivatives evaluated at x.

We now briefly review the physics going into the derivation of the above results. DIS of leptons from a target is determined by the imaginary part of the forward Compton scattering amplitude $A_{hH,h'H'} = \varepsilon^{\mu *}(h') W_{\mu\nu}(H',H) \varepsilon^\nu(h)$, where h and h' are the incoming and outgoing photon helicities and H and H' are the corresponding target helicities. In terms of these amplitudes,

$$F_1^D(x) = \tfrac{1}{3}(A_{++,++} + A_{+-,+-} + A_{+0,+0}) , \tag{9a}$$

$$g_1^D(x) = -\tfrac{1}{2}(A_{++,++} - A_{+-,+-}) , \tag{9b}$$

$$b_1^D(x) = -\tfrac{1}{2}(A_{++,++} + A_{+-,+-} - 2A_{+0,+0}) , \tag{9c}$$

where the symmetric part of the helicity amplitude (corresponding to the contraction of the symmetric part of $W_{\mu\nu}$ with the photon polarization vector) is given by

$$A_{hH,hH}^{\rm sym}(x) = \int_0^2 dy \int_0^1 dz\, \delta(x - yz) \sum_{s=\uparrow\downarrow} f_s^H(y) A_{hs,hs}^{\rm sym}(z) , \tag{10}$$

where

$$f_s^H(y) \equiv \int d^3p\, \phi_s^{H\dagger}(\mathbf{p})(1+\alpha_3)\phi_s^H(\mathbf{p}) \delta \left[y - \frac{p\cos\theta + E(p)}{M} \right] . \tag{11}$$

$f_s^H(y)$ can be interpreted as the probability of finding a nucleon with spin s in a deuteron with helicity H with light-cone momentum fraction y of the deuteron momentum. $A_{hs,hs}(z)$ is the helicity amplitude for forward Compton scattering of a virtual photon off a quark having momentum fraction z of the nucleon. The light-cone character of the DIS process leads to the occurrence of α_3 in Eq. (11). The importance of this, even for nonrelativistic nucleons (such as in the deuteron), was noted by Jaffe [3]. $\phi_s^H(\mathbf{p})$ is obtained by decomposing the state $|H\rangle$ according to whether the nucleon spin is $\uparrow$ or $\downarrow$ along the axis defined by the virtual photon (from which θ is measured), $\phi_s^H(\mathbf{p}) = \langle \mathbf{p}s|H\rangle$. The wave function of a nucleon in the deuteron in terms of the relative momentum $\mathbf{p} = \frac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2)$, assuming that the nucleon is adequately described by a Dirac equation with a static potential, is

$$\phi_{m_{s_1}}^H(\mathbf{p}) = \sum_{L=0,2} \begin{bmatrix} u_L(p) \\ \dfrac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E+M} v_L(p) \end{bmatrix} C_{m_{s_1} m_{s_2} m_s}^{1/2\, 1/2\, s} C_{m_L\, m_s\, H}^{L\ S\ J} Y_{L,m_L}(\hat{\mathbf{p}}) |\tfrac{1}{2} m_{s_2}\rangle . \tag{12}$$

Because the nucleon is lightly bound, it is adequate to take $E \approx M$ and $u_L(p) \approx v_L(p)$. Since the lower component is itself small, and of order p/M relative to the upper component, we shall ignore corrections to this approximation. The second nucleon state $|\frac{1}{2} m_{s_2}\rangle$ remains unchanged in the scattering process. Using this wave function, it follows from Eq. (9a) that

$$F_1^D(x) = \tfrac{2}{3} \int_x^2 \frac{dy}{y} [f_{1/2}^1(y) + f_{1/2}^{-1}(y) + f_{1/2}^0(y)] F_1\left[\frac{x}{y}\right] = \int_x^2 \frac{dy}{y} \Delta f(y) F_1 \left[\frac{x}{y} \right] , \tag{13}$$

where

$$\Delta f(y) = \int d^3p \frac{1}{4\pi} [\cos^2\alpha\, u_s^2(p) + \sin^2\alpha\, u_d^2(p)] \left[1 + \frac{p\cos\theta}{M} + \frac{p^2}{4M^2} \right] \delta \left[y - \frac{p\cos\theta + E(p)}{M} \right] . \tag{14}$$

For nonrelativistic nucleons, $E(p)=M-\varepsilon+p^2/2M$, where ε is the binding energy of the deuteron. Similarly, it can be shown that

$$b_1^D(x)=\int_x^2 \frac{dy}{y}[2f_{1/2}^0(y)-f_{1/2}^1(y)-f_{1/2}^{-1}(y)]F_1\left[\frac{x}{y}\right]$$
$$=\int_x^2 \frac{dy}{y}\Delta b(y)F_1\left[\frac{x}{y}\right], \tag{15}$$

where

$$\Delta b(y)=\int d^3p\left[-\frac{3}{4\pi\sqrt{2}}\sin\alpha\cos\alpha\, u_s(p)u_d(p)+\frac{3}{16\pi}\sin^2\alpha\, u_d^2(p)\right]$$
$$\times(3\cos^2\theta-1)\left[1+\frac{p\cos\theta}{M}+\frac{p^2}{4M^2}\right]\delta\left[y-\frac{p\cos\theta+E(p)}{M}\right]. \tag{16}$$

(Equation (15) replaces the corresponding equation for b_1 in Ref. [1] where some Clebsch-Gordan coefficients were evaluated incorrectly, and the lower component of the nucleon wave function is in error by a factor of $\frac{1}{2}$.) Similarly, the antisymmetric part of the helicity amplitude is

$$A_{hH,hH}^{\text{asym}}(x)=\int_0^2 dy\int_0^1 dz\,\delta(x-yz)\sum_{s=\uparrow\downarrow} g_s^H(y)A_{hs,hs}^{\text{asym}}(z), \tag{17}$$

where

$$g_s^H(y)=\int d^3p\,\phi_s^{H\dagger}(\mathbf{p})(1+\alpha_3)\gamma_5\phi_s^H(\mathbf{p})\delta\left[y-\frac{p\cos\theta+E(p)}{M}\right]. \tag{18}$$

The spin-averaged distribution function is essentially the difference in the momentum distribution of the nucleon with momentum fraction y with spin $\uparrow$ and $\downarrow$ in the deuteron of helicity H modified by relativistic effects,

$$g_1^D(x)=\int_0^2 dy\int_0^1 dz\,\delta(x-yz)[g_{1/2}^{-1}(y)-g_{1/2}^1(y)]g_1(z)$$
$$=\int_x^2\frac{dy}{y}\Delta g(y)g_1\left[\frac{x}{y}\right],$$

where

$$\Delta g(y)=\frac{1}{4\pi}\int d^3p\left[\cos^2\alpha\, u_s^2(p)+\frac{1}{\sqrt{2}}\cos\alpha\sin\alpha\, u_s(p)u_d(p)(3\cos^2\theta-1)\right.$$
$$\left.+\frac{1}{2}\sin^2\alpha\, u_d^2(p)(3\cos^2\theta-2)\right]\left[1+\frac{p\cos\theta}{M}-\frac{p^2(1-\cos^2\theta)}{4M^2}\right]\delta\left[y-\frac{p\cos\theta+E(p)}{M}\right]. \tag{19}$$

For a nonrelativistic nucleon, $p^2/M^2\ll 1$. Therefore, $\Delta f(y)$, $\Delta b(y)$, and $\Delta g(y)$ are concentrated near $y=1$ and so can be approximated by a distribution at $y=1$. The

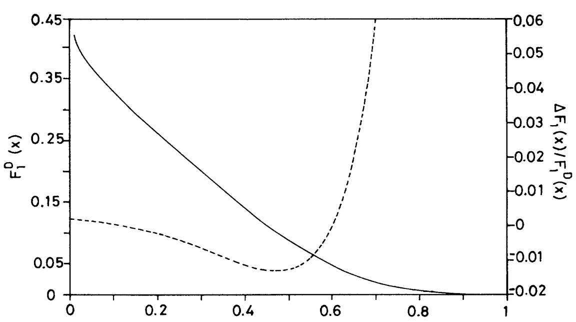

FIG. 1. $F_1(x)$ (solid curve) in the convolution model for a deuteron target. The spin-averaged structure functions of the nucleons are taken from Ref. [7]. Also shown is $\Delta F_1^D/F_1^D$ (dashed curve).

higher derivatives of the δ function are ignored as their coefficients are negligibly small. The terms retained here are of the order of $p^2/M^2\approx 10^{-2}$ and $\varepsilon/M\approx 10^{-2}$. The factors of order ε^2/M^2 and p^4/M^4 are also neglected. After doing some algebra, $\Delta f(y)$, $\Delta b(y)$, and $\Delta g(y)$ are expressible to $O(p^2/M^2)$ in terms of the parameters defined in Eqs. (4)–(6) as

$$\begin{bmatrix}\Delta f(y)\\ \Delta b(y)\\ \Delta g(y)\end{bmatrix}=\begin{bmatrix}\alpha_1\\ \beta_1\\ \gamma_1\end{bmatrix}\delta(y-1)+\begin{bmatrix}\alpha_2\\ \beta_2\\ \gamma_2\end{bmatrix}\delta'(y-1)$$
$$+\begin{bmatrix}\alpha_3\\ \beta_3\\ \gamma_3\end{bmatrix}\delta''(y-1).$$

To illustrate the use of the results derived in this Brief Report, we take the deuteron wave function correspond-

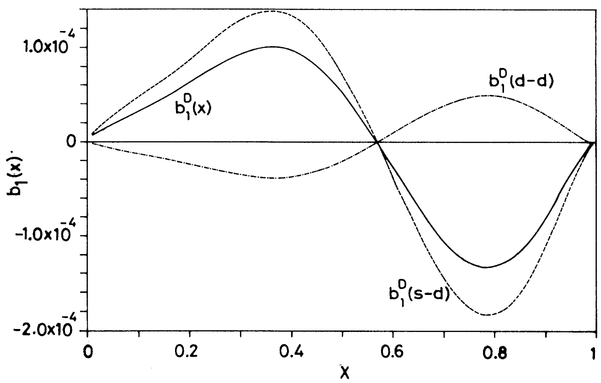

FIG. 2. $b_1^D(x)$ (solid curve), the *s-d* contribution to $b_1^D(x)$ (dashed curve), and the *d-d* contribution to $b_1^D(x)$ (dot-dashed curve).

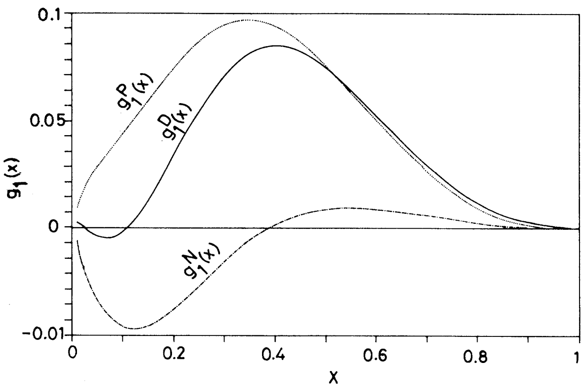

FIG. 3. $g_1^D(x)$ (solid curve), $g_1^P(x)$ (dotted curve), and $g_1^N(x)$ (dot-dashed curve). The nucleon structure functions are taken from Ref. [8].

ing to the Reid soft core potential [6] and typical nucleon structure functions. These are graphed in Figs. 1–3 for F_1^D, b_1^D, and g_1^D. $\Delta F_1^D/F_1^D$, where $\Delta F_1^D=F_1^D-F_1^P-F_1^N$, is plotted in Fig. 1 along with F_1^D. The Fermi motion of the nucleon in the deuteron plays an important role at intermediate values of x in this ratio. The binding energy term dominates its behavior at large x. The behavior of $g_1^D(x)$ is more involved. At small x, $g_1^D(x)$ is quite small because the g_1's of the neutron and proton tend to cancel each other. For $x>0.45$, it starts decreasing and approaches zero for $x\rightarrow 1$. It peaks at $x\approx 0.4$, the value of which is less than the sum of the g_1's of the nucleons for pure s state, while for large x values, g_1^D is greater than $g_1^P+g_1^N$.

There is no *s-s* term in $b_1^D(x)$, while *s-d* and *d-d* contributions tend to cancel each other. The dominant contribution to $b_1^D(x)$ comes from the interference term, that is, the *s-d* term. The zeroth moment of $b_1^D(x)$ is zero, which merely expresses the conservation of nucleon number and follows immediately from Eq. (15). The first moment is calculated to be -6.65×10^{-4}. This is a small number, and hence deviations from this will be a good signature of exotic effects in the deuteron wave function.

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