VUTH 01-09

Positivity bounds on spin-one distribution and fragmentation functions

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(draft of June 12th, 2001)

We establish a connection between the distribution functions of quarks in spin-one hadrons and the helicity matrix for forward scattering of antiquarks off spin-one hadrons. From positivity of this matrix we obtain inequality relations among the distribution functions. Analogous relations hold also for fragmentation functions. The bounds we obtained can be used to constrain estimates of unknown functions, occurring in particular in semi-inclusive deep inelastic scattering or e^+e^- annihilation with vector mesons in the final state.

I. INTRODUCTION

In recent years some attention has been given to distribution functions characterizing spin-one targets, starting from the work of Hoodbhoy, Jaffe and Manohar [1]. Unfortunately, the only available spin-one target is the deuteron, which is essentially a weakly bound system of two spin-half hadrons. In the approximation of independent scattering off the nucleons, the specific spin-one contribution to the deuteron structure functions is expected to be small, except maybe for the low-x region [2,3]. New precision measurements of the deuteron structure functions coming from HERMES may provide an experimental test of this expectation, as already observed in several papers [4–6].

Instead of pointing the attention to spin-one targets, it is also possible to analyze spin-one final state hadrons in $e^+ e^-$ annihilation or in semi-inclusive deep inelastic scattering. This idea was first considered by Efremov and Teryaev [7]. A systematic study was accomplished by Ji [8], who singled out two new fragmentation functions, the function $\hat{b}_1$, in analogy to the distribution function b_1, and the time-reversal odd (T-odd) function $\hat{h}_{\bar{1}}$. These new fragmentation functions can be observed in the production of vector mesons, *e.g.* ρ, K^*, ϕ. However, these functions require polarimetry on the final-state meson, which can be done by studying the angular distribution of its decay products (*e.g.* $\pi^+\pi^-$ in the case of ρ^0 meson). In this sense, vector meson fragmentation functions represent just a specific contribution to the more general analysis of two-particle production [9,10] near the vector meson mass.

In a recent work [11] we contributed to the study of spin-one distribution and fragmentation functions by defining all possible functions occurring at leading order in $1/Q$ when also partonic transverse momentum is included, extending the work done for spin-half hadrons in [12].

The study of spin-one fragmentation functions has a twofold relevance: it offers new insights in the physics of mesons and it serves as a probe of particular nucleonic distribution functions. With regards to the second issue, we point out that for 1-particle inclusive leptoproduction of a spin-one meson, at leading (zeroth) order in $1/Q$ there are five chiral-odd fragmentation functions that can be observed in combination with the quark transversity distribution function [13], the measurement of which is at present attracting the attention of experiments such as HERMES, COMPASS and RHIC.

In this situation, it seems to be extremely useful to gather as much information as possible about spin-one distribution and fragmentation functions. As a first step in this direction, we present here positivity bounds on them, following what has been done in Ref. [14] for the spin-half case.

II. BOUNDS ON TRANSVERSE MOMENTUM INTEGRATED FUNCTIONS

Distribution functions appear parametrization of the light-cone correlation function [15–17]

$$\Phi_{ij}(x) = \int \frac{\mathrm{d}\xi^-}{2\pi}\, \mathrm{e}^{\mathrm{i}p\cdot\xi}\, \langle P,S,T|\overline{\psi}_j(0)\psi_i(\xi)|P,S,T\rangle\Big|_{\xi^+=\xi_T=0}, \tag{1}$$

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depending on the light-cone fraction $x = p^+/P^+$, where P is the momentum of the target hadron and p the momentum of the outgoing quark. Since we are dealing with spin-one hadrons the polarization state of the target must be specified using a spin vector S and a spin tensor T. In the target rest-frame, we can conveniently parametrize the spin vector and tensor as [1]

$$\boldsymbol{S} = (S_T^x, S_T^y, S_L)\,, \tag{2}$$

$$\boldsymbol{T} = \frac{1}{2}\begin{pmatrix} S_{LL} + S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^{x} \\ S_{TT}^{xy} & S_{LL} - S_{TT}^{xx} & S_{LT}^{y} \\ S_{LT}^{x} & S_{LT}^{y} & -2\,S_{LL} \end{pmatrix}. \tag{3}$$

At leading twist (leading order in $1/Q$) in inclusive leptoproduction, the relevant part of the correlator is $\Phi\gamma^+$. Its transpose represents the forward amplitude for antiquark-hadron scattering, $M = (\Phi\,\gamma^+)^T$. Being the square of a scattering amplitude, the matrix M in the parton $\otimes$ hadron spin space is positive definite,

$$\begin{aligned} M_{ij,\lambda'\lambda}(x) &= \int \frac{\mathrm{d}\xi^-}{2\pi\sqrt{2}}\, \mathrm{e}^{\mathrm{i}p\cdot\xi}\, \langle P,\lambda'|\psi_{+i}^\dagger(0)\,\psi_{+j}(\xi)|P,\lambda\rangle\Big|_{\xi^+=\xi_T=0} \\ &= \frac{1}{\sqrt{2}}\sum_n \langle P_n|\psi_{+i}|P,\lambda'\rangle^*\,\langle P_n|\psi_{+j}|P,\lambda\rangle\,\delta\left(P_n^+ - (1-x)P^+\right), \end{aligned} \tag{4}$$

where $\psi_+ \equiv \mathcal{P}_+\psi = \frac{1}{2}\gamma^-\gamma^+\psi$ is the good component of the quark field [18] and where $\lambda,\ \lambda' = 1, 0, -1$.

For spin-one hadrons, $\Phi(x)\gamma^+$ is parametrized in terms of five distribution functions

$$\Phi(x)\gamma^+ = \Big\{f_1(x) + g_1(x)\,S_L\,\gamma_5 + h_1(x)\,\gamma_5 \not{S}_T + f_{1LL}(x)\,S_{LL} + \mathrm{i}\,h_{1LT}(x)\,\not{S}_{LT}\Big\}\mathcal{P}_+. \tag{5}$$

The last distribution function is constrained to be zero by time-reversal invariance. Nevertheless, we choose to keep it because in the analysis of fragmentation functions the analogous contribution, $H_{1LT}(z)$, cannot be constrained in the same way. To follow a more systematic naming of the functions when we will include transverse momenta, we felt the need to change the names of the transverse momentum independent functions. We are aware that changing notations often poses some undesired difficulties, but we believe in the convenience of using a notation that harmoniously connects transverse momentum dependent to transverse momentum integrated functions. With the parametrizations in Eq. (3) and Eq. (5), however, our function f_{1LL} is *precisely the same* as the function b_1 of Hoodbhoy, Jaffe and Manohar [1].

To construct the matrix M for the leading order correlation function, only two basis states in Dirac space are relevant, corresponding to good components of the right and the left handed partons. This is particularly transparent when writing Eq. (5) in chiral representation. Therefore, we can effectively reduce the four-dimensional Dirac space to the 2×2 good parton chirality space. In other words, only the scattering of good antiquarks off hadrons gives a non vanishing contribution to leading twist.

The leading-twist scattering matrix takes then the form

$$M_{ij}(x;S,T) = \begin{pmatrix} f_1(x) + g_1(x)\,S_L + f_{1LL}(x)\,S_{LL} & h_1(x)\,(S_T^x + \mathrm{i}\,S_T^y) + \mathrm{i}\,h_{1LT}(x)\,(S_{LT}^x + \mathrm{i}\,S_{LT}^y) \\ h_1(x)\,(S_T^x - \mathrm{i}\,S_T^y) - \mathrm{i}\,h_{1LT}(x)\,(S_{LT}^x - \mathrm{i}\,S_{LT}^y) & f_1(x) - g_1(x)\,S_L + f_{1LL}(x)\,S_{LL}. \end{pmatrix} \tag{6}$$

The result contains the dependence on the hadron spin vector and tensor. We aim at obtaining a 6×6 matrix encompassing also the hadron spin space, with no dependence on the spin vector and tensor. To this goal, we need to use the hadron spin density matrix, $\rho(S,T)$, and invert the relation

[1]Note that the the parametrization of T is different from the one of Ref. [11]. There is a factor $-2/3$ difference between the old definition of S_{LL} and the new, so that now $-1/3 \le S_{LL} \le 2/3$.

$$M_{ij}(x;S,T) = \rho_{\lambda'\lambda}(S,T)\; M_{i\lambda,\,j\lambda'}(x). \tag{7}$$

The spin density matrix for spin-one hadrons can be decomposed on a Cartesian basis of 3×3 matrices, using the (rank-one) spin vector, S^i, and the rank-two spin tensor, T^{ij},

$$\boldsymbol{\rho} = \frac{1}{3}\left(\mathbf{1} + \frac{3}{2}S^i\boldsymbol{\Sigma}^i + 3\,T^{ij}\boldsymbol{\Sigma}^{ij}\right), \tag{8}$$

where the matrices $\boldsymbol{\Sigma}^i$ are the generalization of the Pauli matrices to the three-dimensional case and where we chose

$$\boldsymbol{\Sigma}^{ij} = \frac{1}{2}\left(\boldsymbol{\Sigma}^i\boldsymbol{\Sigma}^j + \boldsymbol{\Sigma}^j\boldsymbol{\Sigma}^i\right) - \frac{2}{3}\mathbf{1}\,\delta^{ij}. \tag{9}$$

Inverting Eq. (7), we can eventually reconstruct the 6×6 leading-twist scattering matrix

$$M_{i\lambda,\,j\lambda'}(x) = \\ \begin{pmatrix} f_1+g_1-\frac{f_{1LL}}{3} & 0 & 0 & 0 & \sqrt{2}\,(h_1+\mathrm{i}\,h_{1LT}) & 0 \\ 0 & f_1+\frac{2\,f_{1LL}}{3} & 0 & 0 & 0 & \sqrt{2}\,(h_1-\mathrm{i}\,h_{1LT}) \\ 0 & 0 & f_1-g_1-\frac{f_{1LL}}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & f_1-g_1-\frac{f_{1LL}}{3} & 0 & 0 \\ \sqrt{2}\,(h_1-\mathrm{i}\,h_{1LT}) & 0 & 0 & 0 & f_1+\frac{2\,f_{1LL}}{3} & 0 \\ 0 & \sqrt{2}\,(h_1+\mathrm{i}\,h_{1LT}) & 0 & 0 & 0 & f_1+g_1-\frac{f_{1LL}}{3} \end{pmatrix}, \tag{10}$$

where all the functions on the right-hand side depend only on x.

From the positivity of the diagonal elements we obtain the bounds:

$$f_1(x) \geq 0 \tag{11}$$

$$-\frac{3}{2}\,f_1(x) \leq f_{1LL}(x) \;\leq\; 3\,f_1(x) \tag{12}$$

$$|g_1(x)| \leq f_1(x) - \frac{1}{3}\,f_{1LL}(x) \;\leq\; \frac{3}{2}\,f_1(x), \tag{13}$$

while positivity of 2-dimensional minors gives the bound [19]

$$[h_1(x)]^2 + [h_{1LT}(x)]^2 \leq \frac{1}{2}\Big[f_1(x) + \frac{2}{3}\,f_{1LL}(x)\Big]\,\Big[f_1(x) + g_1(x) - \frac{f_{1LL}(x)}{3}\Big], \tag{14}$$

This bound is a generalization of the Soffer bound and must be fulfilled by any spin-1 target. When $h_{1LT}=0$ due to time-reversal invariance, the bound can be reduced to

$$|h_1(x)| \leq \sqrt{\frac{1}{2}\Big[f_1(x) + \frac{2}{3}\,f_{1LL}(x)\Big]\,\Big[f_1(x) + g_1(x) - \frac{f_{1LL}(x)}{3}\Big]}. \tag{15}$$

An analogous calculation can be performed for fragmentation functions, which describe the hadronization process of a parton into the final detected hadron. In this case the transverse momentum independent correlator is [20]

$$\Delta_{ij}(z) = \sum_X \int \frac{\mathrm{d}\xi^-}{2\pi\sqrt{2}}\, e^{ik\cdot\xi} \langle 0|\psi_i(\xi)|P_h, S_h, T_h; X\rangle\langle P_h, S_h, T_h; X|\overline{\psi}_j(0)|0\rangle\Big|_{\xi^+=\xi_T=0}, \tag{16}$$

depending on the light-cone momentum fraction $z = P_h^-/k^-$, where P_h is the momentum of the outgoing hadron and k the momentum of the fragmenting quark.

The analysis of the leading-order scattering matrix can be developed in complete analogy to the distribution correlation function. The matrix $M = \Delta\gamma^-$ now represents a hadron-quark decay matrix, which is again positive definite. Instead of using the hadron spin density matrix, we should employ the hadron decay matrix, expressing it in terms of vector and tensor analyzing powers A_L, A_T, A_{LL}, etc. The final result, however, would still have the form of Eq. (10), with the z-dependent fragmentation functions D_1, G_1, D_{1LL}, H_1 and H_{1LT} replacing the x-dependent distribution functions f_1, g_1, f_{1LL}, h_1 and h_{1LT}. Despite the differences in notation, our fragmentation functions correspond to the ones of Ji [8] (i.e. $D_{1LL} = \hat{b}_1$ and $H_{1LT} = \hat{h}_{\bar{1}}$). Note that in the context of a fragmentation process it is not justified to discard T-odd functions. We point out that the description in terms of the matrix in spin space allows a connection to the analysis performed in [21].

It is particularly useful to set bounds on the function H_{1LT}, which is a possible candidate to probe the transversity distribution of the nucleon in inclusive deep-inelastic vector meson production. It appears in connection with the transversity distribution in a $\langle \sin(\phi^\ell_{\pi\pi} + \phi^\ell_S)\rangle$ transverse spin asymmetry, where ϕ^ℓ_S and $\phi^\ell_{\pi\pi}$ are the azimuthal angles with respect to the lepton scattering plane of, respectively, the spin of the target and the difference of the momenta of the two decay particles [13]. The equivalent of Eq. (14) for fragmentation functions implies the bounds

$$\begin{aligned}
[H_{1LT}(z)]^2 + [H_1(z)]^2 &\leq \frac{1}{2}\Big[D_1(z) + \frac{2}{3}\,D_{1LL}(z)\Big]\left[D_1(z) + G_1(z) - \frac{D_{1LL}(z)}{3}\right] \\
&\leq \left[D_1(z) + \frac{2}{3}\,D_{1LL}(z)\right]\left[D_1(z) - \frac{1}{3}\,D_{1LL}(z)\right] \;\leq\; \frac{9}{8}\,[D_1(z)]^2 . \qquad (17)
\end{aligned}$$

The less restrictive bounds are particularly relevant because in the specific case vector meson decay the vector analyzing powers are zero, making it impossible to measure the fragmentation functions H_1 and G_1. Note that some information on the function D_{1LL} can be already extracted from experimental measurements [22,23].

III. BOUNDS ON TRANSVERSE MOMENTUM DEPENDENT FUNCTIONS

To include the dependence on the parton transverse momentum, $\boldsymbol{p}_T$, the correlation function can be defined as [12]

$$\Phi_{ij}(x, \boldsymbol{p}_T) = \int \frac{\mathrm{d}\xi^-\,\mathrm{d}\boldsymbol{\xi}_T}{(2\pi)^3}\; \mathrm{e}^{\mathrm{i}p\cdot\xi}\,\langle P,S,T|\overline{\psi}_j(0)\psi_i(\xi)|P,S,T\rangle\Big|_{\xi^+=0}, \qquad (18)$$

For convenience, the correlation function can be decomposed in several terms in relation to the polarization state of the target, *i.e.* $\Phi = \Phi_U + \Phi_L + \Phi_T + \Phi_{LL} + \Phi_{LT} + \Phi_{TT}$. To leading order in $1/Q$, these terms can be decomposed as (we identify T-odd terms by enclosing them in round parentheses)

$$\Phi_U(x, \boldsymbol{p}_T) = \frac{1}{4}\left\{ f_1(x, p_T^2)\,\not{n}_+ + \left(h_1^\perp(x, p_T^2)\,\sigma_{\mu\nu}\frac{p_T^\mu}{M} n_+^\nu \right) \right\}, \qquad (19)$$

$$\Phi_L(x, \boldsymbol{p}_T) = \frac{1}{4}\left\{ g_{1L}(x, p_T^2)\,S_L\,\gamma_5\,\not{n}_+ + h_{1L}^\perp(x, p_T^2)\,S_L\,\mathrm{i}\sigma_{\mu\nu}\gamma_5 n_+^\mu \frac{p_T^\nu}{M} \right\}, \qquad (20)$$

$$\begin{aligned}
\Phi_T(x, \boldsymbol{p}_T) = \frac{1}{4}\Bigg\{ & g_{1T}(x, p_T^2)\,\frac{\boldsymbol{S}_T\cdot\boldsymbol{p}_T}{M}\,\gamma_5\,\not{n}_+ + h_{1T}(x, p_T^2)\,\mathrm{i}\sigma_{\mu\nu}\gamma_5 n_+^\mu S_T^\nu \\
& + h_{1T}^\perp(x, p_T^2)\,\frac{\boldsymbol{S}_T\cdot\boldsymbol{p}_T}{M}\,\mathrm{i}\sigma_{\mu\nu}\gamma_5 n_+^\mu \frac{p_T^\nu}{M} \\
& + \left(f_{1T}^\perp(x, p_T^2)\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu \frac{p_T^\rho}{M} S_T^\sigma \right) \Bigg\}, \qquad (21)
\end{aligned}$$

$$\Phi_{LL}(x, \boldsymbol{p}_T) = \frac{1}{4}\left\{ f_{1LL}(x, p_T^2)\,S_{LL}\,\not{n}_+ + \left(h_{1LL}^\perp(x, p_T^2)\,S_{LL}\,\sigma_{\mu\nu}\frac{p_T^\mu}{M} n_+^\nu \right) \right\}, \qquad (22)$$

$$\begin{aligned}
\Phi_{LT}(x, \boldsymbol{p}_T) = \frac{1}{4}\Bigg\{ & f_{1LT}(x, p_T^2)\,\frac{\boldsymbol{S}_{LT}\cdot\boldsymbol{p}_T}{M}\,\not{n}_+ + \left(g_{1LT}(x, p_T^2)\,\epsilon_T^{\mu\nu} S_{LT\,\mu}\frac{p_{T\,\nu}}{M}\,\gamma_5\,\not{n}_+ \right) \\
& + \left(h_{1LT}'(x, p_T^2)\,\mathrm{i}\sigma_{\mu\nu}\gamma_5 n_+^\mu \epsilon_T^{\nu\rho} S_{LT\,\rho} \right) \\
& + \left(h_{1LT}^\perp(x, p_T^2)\,\frac{\boldsymbol{S}_{LT}\cdot\boldsymbol{p}_T}{M}\,\sigma_{\mu\nu}\frac{p_T^\mu}{M} n_+^\nu \right) \Bigg\}, \qquad (23)
\end{aligned}$$

$$\Phi_{TT}(x, \boldsymbol{p}_T) = \frac{1}{4}\Bigg\{ f_{1TT}(x, p_T^2)\,\frac{\boldsymbol{p}_T\cdot\boldsymbol{S}_{TT}\cdot\boldsymbol{p}_T}{M^2}\,\not{n}_+$$

$$
\begin{aligned}
&- \left(g_{1TT}(x, p_T^2)\, \epsilon_T^{\mu\nu} S_{TT\,\nu\rho} \frac{p_T^\rho p_{T\,\mu}}{M^2}\, \gamma_5\, \not{n}_+ \right) \\
&- \left(h'_{1TT}(x, p_T^2)\, \mathrm{i}\sigma_{\mu\nu}\gamma_5 n_+^\mu \epsilon_T^{\nu\rho} S_{TT\,\rho\sigma} \frac{p_T^\sigma}{M} \right) \\
&+ \left(h_{1TT}^\perp(x, p_T^2)\, \frac{\boldsymbol{p}_T \cdot \boldsymbol{S}_{TT} \cdot \boldsymbol{p}_T}{M^2}\, \sigma_{\mu\nu} \frac{p_T^\mu}{M} n_+^\nu \right) \Bigg\} .
\end{aligned} \tag{24}
$$

Following steps analogous to the previous section, we can reconstruct the complete 6×6 scattering matrix. The inclusion of $\boldsymbol{p}_T$ dependence makes all the entries of the matrix to be non-zero. It is also convenient to define the functions

$$
h_{1LT}(x, p_T^2) = h'_{1LT}(x, p_T^2) + \frac{\boldsymbol{p}_T^2}{2M^2}\, h_{1LT}^\perp(x, p_T^2), \tag{25}
$$

$$
h_{1TT}(x, p_T^2) = h'_{1TT}(x, p_T^2) + \frac{\boldsymbol{p}_T^2}{2M^2}\, h_{1TT}^\perp(x, p_T^2). \tag{26}
$$

In the rest of this Section, unless otherwise specified, all the functions are understood to depend on the variables x and p_T^2.

In principle, if we fully exploit the condition of the scattering matrix to be positive definite, we can write several relations involving an increasing number of different functions. We feel this to be an excessive task if compared to the exiguity of information we have on the involved functions. Therefore, here we choose to focus on the relations stemming from positivity of the two-dimensional minors of the matrix.

Because of the symmetry properties of the matrix, only nine independent inequality relations between the different functions are produced. To simplify the discussion, it is useful to identify some of the T-odd functions as imaginary parts of the T-even functions, which become then complex scalar functions. The following replacements are required

$$
g_{1T} - \mathrm{i}\, f_{1T}^\perp \to g_{1T}, \tag{27}
$$

$$
f_{1LT} - \mathrm{i}\, g_{1LT} \to f_{1LT}, \tag{28}
$$

$$
f_{1TT} - \mathrm{i}\, g_{1TT}^\perp \to f_{1TT}, \tag{29}
$$

$$
h_1 + \mathrm{i}\, h_{1LT} \to h_1, \tag{30}
$$

$$
h_{1T}^\perp - \mathrm{i}\, h_{1LT}^\perp \to h_{1T}^\perp. \tag{31}
$$

Assuming the above relations, we can write the following inequalities:

$$
|h_1|^2 \le \frac{1}{2} \left(f_1 + \frac{2\, f_{1LL}}{3} \right) \left(f_1 + g_1 - \frac{f_{1LL}}{3} \right), \tag{32}
$$

$$
\frac{|p_T|^2}{2M^2}\, |g_{1T} + f_{1LT}|^2 \le \left(f_1 + \frac{2\, f_{1LL}}{3} \right) \left(f_1 + g_1 - \frac{f_{1LL}}{3} \right), \tag{33}
$$

$$
\frac{|p_T|^2}{2M^2}\, |g_{1T} - f_{1LT}|^2 \le \left(f_1 + \frac{2\, f_{1LL}}{3} \right) \left(f_1 - g_1 - \frac{f_{1LL}}{3} \right), \tag{34}
$$

$$
\frac{|p_T|^4}{2M^4}\, |h_{1T}^\perp|^2 \le \left(f_1 + \frac{2\, f_{1LL}}{3} \right) \left(f_1 - g_1 - \frac{f_{1LL}}{3} \right), \tag{35}
$$

$$
\frac{|p_T|^6}{M^6}\, {h_{1TT}^\perp}^2 \le \left(f_1 - g_1 - \frac{f_{1LL}}{3} \right)^2, \tag{36}
$$

$$
\frac{|p_T|^2}{4M^2} \left(h_1^\perp + \frac{2\, h_{1LL}^\perp}{3} \right)^2 \le \left(f_1 + \frac{2\, f_{1LL}}{3} \right)^2, \tag{37}
$$

$$
\frac{|p_T|^2}{M^2}\, h_{1TT}^2 \le \left(f_1 + g_1 - \frac{f_{1LL}}{3} \right)^2, \tag{38}
$$

$$
\frac{|p_T|^2}{M^2} \left[{h_{1L}^\perp}^2 + \left(h_1^\perp - \frac{h_{1LL}^\perp}{3} \right)^2 \right] \le \left(f_1 + g_1 - \frac{f_{1LL}}{3} \right) \left(f_1 - g_1 - \frac{f_{1LL}}{3} \right), \tag{39}
$$

$$
\frac{|p_T|^4}{4M^4}\, |f_{1TT}|^2 \le \left(f_1 + g_1 - \frac{f_{1LL}}{3} \right) \left(f_1 - g_1 - \frac{f_{1LL}}{3} \right). \tag{40}
$$

A further simplification can be performed by introducing the positive real functions

$$A(x, p_T^2) \equiv f_1(x, p_T^2) + g_1(x, p_T^2) - \frac{1}{3} f_{1LL}(x, p_T^2), \tag{41}$$

$$B(x, p_T^2) \equiv f_1(x, p_T^2) + \frac{2}{3}\, f_{1LL}(x, p_T^2), \tag{42}$$

$$C(x, p_T^2) \equiv f_1(x, p_T^2) - g_1(x, p_T^2) - \frac{1}{3} f_{1LL}(x, p_T^2) \tag{43}$$

and express the combinations of distribution functions occurring in the various entries of the matrix as

$$\begin{aligned} h_1 &= a\sqrt{\tfrac{1}{2}\,A\,B}, & h_1^{\perp} + \tfrac{2}{3}\,h_{1LL}^{\perp} &= f\,B, \\ g_{1T} + f_{1LT} &= b\sqrt{A\,B}, & h_{1TT} &= g\,A, \\ g_{1T} - f_{1LT} &= c\sqrt{C\,B}, & h_{1L}^{\perp} - \mathrm{i}\left(h_1^{\perp} - \tfrac{1}{3}\,h_{1LL}^{\perp}\right) &= h\sqrt{A\,C}, \\ h_{1T}^{\perp} &= d\sqrt{C\,B}, & f_{1TT} &= j\sqrt{A\,C}, \\ h_{1TT}^{\perp} &= e\,C, \end{aligned} \tag{44}$$

where the coefficients a, b, c, d, h, j are complex functions of the variables x and p_T^2, and the coefficients e, f, g are real functions of the same variables. Time-reversal invariance in the distribution sector constrains the coefficients a, b, c, d, h, j to be real functions of the variables x and p_T^2 and the coefficients e, f, g to vanish.

The scattering matrix can now be expressed in a concise way as

$$M_{i\lambda,\, j\lambda'}(x, \boldsymbol{p}_T) = \begin{pmatrix} A & \mathrm{e}^{-\mathrm{i}\phi}\, b\sqrt{A\,B} & \mathrm{e}^{-2\mathrm{i}\phi}\, j\sqrt{A\,C} & \mathrm{e}^{\mathrm{i}\phi}\, h\sqrt{A\,C} & a\sqrt{A\,B} & -\mathrm{i}\mathrm{e}^{-\mathrm{i}\phi}\, g\,A \\ \mathrm{e}^{\mathrm{i}\phi}\, b^*\sqrt{A\,B} & B & \mathrm{e}^{-\mathrm{i}\phi}\, c\sqrt{C\,B} & \mathrm{e}^{2\mathrm{i}\phi}\, d\sqrt{C\,B} & -\mathrm{i}\mathrm{e}^{\mathrm{i}\phi}\, f\,B & a^*\sqrt{A\,B} \\ \mathrm{e}^{2\mathrm{i}\phi}\, j^*\sqrt{A\,C} & \mathrm{e}^{\mathrm{i}\phi}\, c^*\sqrt{C\,B} & C & -\mathrm{i}\mathrm{e}^{3\mathrm{i}\phi}\, e\,C & \mathrm{e}^{2\mathrm{i}\phi}\, d^*\sqrt{C\,B} & -\mathrm{e}^{\mathrm{i}\phi}\, h^*\sqrt{A\,C} \\ \mathrm{e}^{-\mathrm{i}\phi}\, h^*\sqrt{A\,C} & \mathrm{e}^{-2\mathrm{i}\phi}\, d^*\sqrt{C\,B} & \mathrm{i}\mathrm{e}^{-3\mathrm{i}\phi}\, e\,C & C & -\mathrm{e}^{-\mathrm{i}\phi}\, c^*\sqrt{C\,B} & \mathrm{e}^{-2\mathrm{i}\phi}\, j^*\sqrt{A\,C} \\ a^*\sqrt{A\,B} & \mathrm{i}\mathrm{e}^{-\mathrm{i}\phi}\, f\,B & \mathrm{e}^{-2\mathrm{i}\phi}\, d\sqrt{C\,B} & -\mathrm{e}^{\mathrm{i}\phi}\, c\sqrt{C\,B} & B & -\mathrm{e}^{\mathrm{i}\phi}\, b^*\sqrt{A\,B} \\ \mathrm{i}\mathrm{e}^{\mathrm{i}\phi}\, g\,A & a\sqrt{A\,B} & -\mathrm{e}^{-\mathrm{i}\phi}\, h\sqrt{A\,C} & \mathrm{e}^{2\mathrm{i}\phi}\, j\sqrt{A\,C} & -\mathrm{e}^{-\mathrm{i}\phi}\, b\sqrt{A\,B} & A \end{pmatrix}. \tag{45}$$

Positivity of the two-dimensional minors of this matrix requires the absolute value of the coefficients a to j to be smaller or equal to unity. It is possible to consider higher dimensional minors to extract other useful bounds. The choice of the appropriate minors will depend on the availability of measurements of specific functions.

IV. CONCLUSIONS

In this letter, we studied the leading-order part of the correlation functions for parton distribution inside spin-one targets or parton fragmentation into spin-one hadrons. We cast the correlation functions in the form of scattering matrices in the parton chirality space $\otimes$ hadron spin space. Positive definiteness of these matrices allowed us to set bounds to the involved distribution and fragmentation functions. These elementary bounds can act as important guidances to estimate the magnitude of otherwise unknown functions and, consequently, the magnitude of spin and azimuthal asymmetries appearing in several processes containing spin-one hadrons. In particular, these bounds are significant for one-particle-inclusive deep inelastic scattering and e^+e^- annihilation with vector mesons in the final state. Among our results, we presented a bound that generalizes the Soffer inequality to the case of spin-one targets. Furthermore, we proposed a bound on the fragmentation function H_{1LT}. Since the latter can appear in connection with the transversity distribution in a $\langle \sin(\phi_{\pi\pi}^{\ell} + \phi_S^{\ell})\rangle$ asymmetry, our bound can be useful in the extraction of the transversity distribution, where, for instance, it could be important to constrain the absolute normalization of the function.

ACKNOWLEDGMENTS

This work is supported by the Foundation for Fundamental Research on Matter (FOM) and the Dutch Organization for Scientific Research (NWO).

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