

Radiative corrections to deep-inelastic ed - scattering. Case of tensor polarized deuteron

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Abstract

The model-independent radiative corrections to deep-inelastic scattering of unpolarized electron beam off the tensor polarized deuteron target have been considered. The contribution to the radiative corrections due to the hard-photon emission from the elastic electron-deuteron scattering (the so-called elastic radiative tail) is also investigated. The calculation is based on the covariant parametrization of the deuteron quadrupole polarization tensor. The numerical estimates of the radiative corrections to the polarization observables have been done for the kinematical conditions of the current experiment at HERA.

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1 Introduction

The flavor structure of nucleons is described in terms of parton distribution functions. Most of the information on these functions has up to now come from inclusive deep-inelastic scattering process: experiments where only the scattered lepton is detected. The investigation of the nucleon spin structure involves now new types of reactions. For example, the HERMES experiment was specifically designed to perform accurate measurements of semi-inclusive reactions, where besides scattered lepton also some of the produced hadrons are detected [1].

The polarized nuclei of deuterium and helium-3 are used to extract information on the neutron spin-dependent structure function $g_1(x)$ [2]. In analyzing the experimental data on inclusive spin asymmetries for deuterium in order to deduce the spin-dependent structure function g_1^d one should take into account a small effect due to possible tensor polarization in this spin-one target. This is connected with the presence in a deuteron target of an additional tensor polarized structure functions [1]. So far, the spin-structure studies have been focused on the spin-1/2 nucleon. Different spin physics exists for higher-spin hadrons such as the tensor structure in the deuteron. The measurement of these additional spin-dependent structure functions provides important information about non-nucleonic components in spin-one nuclei and tensor structures on the quark-parton level [3]. A general formalism of the deep inelastic

electron–deuteron scattering was discussed in Ref. [4], where new four tensor structure functions $b_i(x)$, $i = 1 - 4$ were introduced. They can be measured using tensor polarized target and unpolarized electron beam. Among these new structure functions, only one, b_1 , is leading twist in QCD [4] and it was found that this function is small for a weakly bound system of nucleons (for example, deuteron). Therefore, the measurement of b_1 for the case of the deuteron can give information about its possible exotic components.

From the theoretical point of view the spin–dependent structure function $b_1(x)$ was investigated in a number of papers. The available fixed targets with $J \geq 1$ are only nuclei (the deuteron is the most commonly used nucleus). If the nucleons in the deuteron are in an S state then $b_1(x) \equiv 0$. For nucleons in a D state, $b_1(x) \neq 0$ in general [4]. It was found [5] that in a quark–parton model the sum rule $\int dx b_1(x) = 0$ is generally true if the sea of quarks and antiquarks is unpolarized (and it was shown how this sum rule is modified in the presence of a polarized sea). Mankiewicz [6] has studied the $b_1(x)$ for the ρ meson and noticed empirically that $\int dx b_1(x) = 0$ in his model. It was shown in Ref. [7] that multiple scattering terms at low x can still lead to $b_1 \neq 0$ even if only the S –wave component is present. Various twist two structure functions of the deuteron (in particular, b_1) have been calculated in a version of the convolution model that incorporates relativistic and binding energy corrections [8]. A simple parametrizations of these structure functions are given in terms of a few deuteron wave function parameters and the free nucleon structure functions. The tensor structure functions were discussed in Ref. [9] for the case of lepton scattering and in hadron reactions such as the polarized proton–deuteron Drell–Yan process.

As it is known, the HERMES experiment has been designed to measure the nucleon spin–dependent structure functions from deep–inelastic scattering of longitudinally polarized positrons and electrons from polarized gaseous targets ($H, D, {}^3He$). In 2000, HERMES collected a data set with a tensor polarized deuterium target for the purpose of making a first measurement of tensor structure function $b_1(x)$. The preliminary results on this structure function are presented in Ref. [10] for the kinematic range $0.002 < x < 0.85$ and $0.1 GeV^2 < Q^2 < 20 GeV^2$. The preliminary result for the tensor asymmetry is sufficiently small to produce an effect of more than 1% on the measurement of g_1^d . The dependence of b_1 on x variable is in qualitative agreement with expectations based on coherent double scattering models [11, 12, 13] and favors a sizeable value of b_1 at low x region. This suggests a significant tensor polarization of the sea–quarks, violating the Close–Kumano sum rule [5].

The radiative corrections to deep–inelastic scattering of unpolarized and longitudinally polarized electron beam on polarized deuteron target were considered in Ref. [14] for the particular case of the deuteron polarization (which can be obtained from the general covariant spin–density matrix [15] when spin functions are the eigenvectors of the spin projection operator). The leading–log model–independent radiative corrections in deep–inelastic scattering of unpolarized electron beam off the tensor polarized deuteron target have been considered in Ref. [16]. The calculation is based on the covariant parametrization of the deuteron quadrupole polarization tensor and use of the Drell–Yan like representation in electrodynamics.

Current experiments at modern accelerators reached a new level of precision and this circumstance requires a new approach to data analysis and inclusion of all possible systematic uncertainties. One of the important source of such uncertainties is the electromagnetic radiative effects caused by physical processes which take place in higher orders of the perturbation theory with respect to the electromagnetic interaction. In present paper we give the covariant description of deep–inelastic scattering of unpolarized electron beam off the tensor polarized deuteron target (the polarization state of the target is described by the spin–density matrix of

the general form) taking into account the radiative corrections

$$e^-(k_1) + d(p) \rightarrow e^-(k_2) + X(p_x). \quad (1)$$

The corresponding approach is based on the covariant parametrization of the deuteron quadrupole polarization tensor in terms of the 4-momenta of the particles in process (1) [16]. We performed also the numerical calculations of the radiative corrections for the kinematical conditions of the experiment [10]. The contribution of the radiative tail from the elastic ed -scattering is considered separately.

2 Born approximation

The standard set of variables which is usually used for the description of deep-inelastic scattering process is

$$x = \frac{-q^2}{2pq}, \quad y = \frac{2pq}{V}, \quad V = 2pk_1, \quad q^2 = -Vxy, \quad q = k_1 - k_2, \quad (2)$$

where q is the 4-momentum of the intermediate heavy photon that probes the deuteron structure. To begin with, we define the deep-inelastic scattering cross section of the process (1) in terms of the leptonic $L_{\mu\nu}$ and hadronic $W_{\mu\nu}$ tensors contraction (in the Born approximation we can neglect the electron mass)

$$\frac{d\sigma}{dx dQ_B^2} = \frac{\pi\alpha^2}{VQ_B^4} \frac{y}{x} L_{\mu\nu} W_{\mu\nu}. \quad (3)$$

Note that only in the Born approximation (without accounting radiative corrections) $q = k_1 - k_2$, $Q_B^2 = -q^2 = 2k_1k_2$.

The Born leptonic tensor (for the unpolarized case) is

$$L_{\mu\nu}^B = q^2 g_{\mu\nu} + 2(k_{1\mu}k_{2\nu} + k_{1\nu}k_{2\mu}). \quad (4)$$

The hadronic tensor is defined by the following way

$$W_{\mu\nu} = (2\pi)^3 \sum_X \delta^{(4)}(k_1 + p - k_2 - p_x) \overline{J_\mu} J_\nu^*,$$

where J_μ is the electromagnetic current for the $\gamma^* + d \rightarrow X$ transition (γ^* is the virtual photon). The sum means summation over the final states and the bar means averaging over the polarizations of the target and summation over the polarizations of the final particles. To write down the hadron tensor in terms of the structure functions we define first the deuteron spin-density matrix (further we do not consider the effect caused by the vector polarization of the deuteron)

$$\rho_{\mu\nu} = -\frac{1}{3}(g_{\mu\nu} - \frac{p_\mu p_\nu}{M^2}) - \frac{i}{2M} \varepsilon_{\mu\nu\lambda\rho} s_\lambda p_\rho + Q_{\mu\nu}, \quad Q_{\mu\nu} = Q_{\nu\mu}, \quad Q_{\mu\mu} = 0, \quad p_\mu Q_{\mu\nu} = 0, \quad (5)$$

here s_μ and $Q_{\mu\nu}$ are the target-deuteron polarization 4-vector and the deuteron quadrupole-polarization tensor. The corresponding hadron tensor has both the polarization-independent and polarization-dependent parts and in general case can be written as

$$W_{\mu\nu} = W_{\mu\nu}(0) + W_{\mu\nu}(V) + W_{\mu\nu}(T), \quad (6)$$

where $W_{\mu\nu}(0)$ corresponds to the unpolarized case and $W_{\mu\nu}(V)(W_{\mu\nu}(T))$ corresponds to the case of the vector (tensor)-polarization of the deuteron target. The $W_{\mu\nu}(0)$ term has the following form

$$W_{\mu\nu}(0) = -W_1\tilde{g}_{\mu\nu} + \frac{W_2}{M^2}\tilde{p}_\mu\tilde{p}_\nu, \quad \tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}, \quad \tilde{p}_\mu = p_\mu - \frac{pq}{q^2}q_\mu, \quad (7)$$

here M is the deuteron mass, and $W_{1,2}$ are the unpolarized structure functions depending on two independent variables x and q^2 . The part of the hadron tensor which is dependent on the quadrupole polarization tensor can be represented as

$$W_{\mu\nu}(T) = \frac{M^2}{(pq)^2} \{ Q_{\alpha\beta}q_\alpha q_\beta (B_1\tilde{g}_{\mu\nu} + \frac{B_2}{pq}\tilde{p}_\mu\tilde{p}_\nu) + B_3q_\alpha(\tilde{p}_\mu Q_{\nu\alpha} + \tilde{p}_\nu Q_{\mu\alpha}) + pqB_4\tilde{Q}_{\mu\nu} \}. \quad (8)$$

Here $B_i (i = 1, 2, 3, 4)$ are the spin-dependent structure functions (caused by the tensor polarization of the target). They are also functions of two variables q^2 and x . Since the hadron tensor $W_{\mu\nu}(T)$ is symmetric under $\mu \leftrightarrow \nu$ the electron beam does not have to be polarized for measuring these new structure functions.

We used the following notation in formula (8)

$$Q_{\mu\bar{\nu}} = Q_{\mu\nu} - \frac{q_\nu q_\alpha}{q^2}Q_{\mu\alpha}, \quad Q_{\mu\bar{\nu}}q_\nu = 0, \\ \tilde{Q}_{\mu\nu} = Q_{\mu\nu} + \frac{q_\mu q_\nu}{q^4}Q_{\alpha\beta}q_\alpha q_\beta - \frac{q_\nu q_\alpha}{q^2}Q_{\mu\alpha} - \frac{q_\mu q_\alpha}{q^2}Q_{\nu\alpha}, \quad \tilde{Q}_{\mu\nu}q_\nu = 0. \quad (9)$$

Note that the deuteron spin-dependent structure functions B_i are also related to the structure functions b_i , introduced in Ref. [4], in the following way

$$B_1 = -b_1, \quad B_2 = \frac{b_2}{3} + b_3 + b_4, \quad B_3 = \frac{b_2}{6} - \frac{b_4}{2}, \quad B_4 = \frac{b_2}{3} - b_3. \quad (10)$$

When calculating radiative corrections it is convenient to parametrize the polarization state of the deuteron target in terms of the 4-momenta of the particles participating in the reaction under consideration. Therefore, first, we have to find the set of the axes and write them in covariant form in terms of the 4-momenta. If we choose, in the laboratory system of reaction (1), the longitudinal direction \mathbf{l} along the electron beam and the transverse one \mathbf{t} in the plane $(\mathbf{k}_1, \mathbf{k}_2)$ and perpendicular to \mathbf{l} , then

$$S_\mu^{(l)} = \frac{2\tau k_{1\mu} - p_\mu}{M}, \quad S_\mu^{(t)} = \frac{k_{2\mu} - (1 - y - 2xy\tau)k_{1\mu} - xyp_\mu}{d}, \quad S_\mu^{(n)} = \frac{2\varepsilon_{\mu\lambda\rho\sigma}p_\lambda k_{1\rho} k_{2\sigma}}{Vd}, \quad (11)$$

$$d = \sqrt{Vxyb}, \quad b = 1 - y - xy\tau, \quad \tau = M^2/V.$$

We chose one of the axes along the direction \mathbf{l} because in the experiment on measuring the b_1 structure function [10] the direction of the magnetic field, used for the polarization of the deuteron target, is along the positron beam line. The direction of the magnetic field provides the quantization axis for the nuclear spin in the target.

One can verify that the set of the 4-vectors $S_\mu^{(l,t,n)}$ satisfies the following properties

$$S_\mu^{(\alpha)}S_\mu^{(\beta)} = -\delta_{\alpha\beta}, \quad S_\mu^{(\alpha)}p_\mu = 0, \quad \alpha, \beta = l, t, n. \quad (12)$$

One can make sure also that in the rest frame of the deuteron (the laboratory system)

$$S_\mu^{(l)} = (0, \mathbf{l}), \quad S_\mu^{(t)} = (0, \mathbf{t}), \quad S_\mu^{(n)} = (0, \mathbf{n}),$$

$$\mathbf{l} = \mathbf{n}_1, \quad \mathbf{t} = \frac{\mathbf{n}_2 - (\mathbf{n}_1 \mathbf{n}_2) \mathbf{n}_1}{\sqrt{1 - (\mathbf{n}_1 \mathbf{n}_2)^2}}, \quad \mathbf{n} = \frac{\mathbf{n}_1 \times \mathbf{n}_2}{\sqrt{1 - (\mathbf{n}_1 \mathbf{n}_2)^2}}, \quad \mathbf{n}_{1,2} = \frac{\mathbf{k}_{1,2}}{|\mathbf{k}_{1,2}|}. \quad (13)$$

If to add one more 4-vector $S_\mu^{(0)} = p_\mu/M$ to the set (11), we receive the complete set of the orthogonal 4-vectors with the following properties

$$S_\mu^{(m)} S_\nu^{(m)} = g_{\mu\nu}, \quad S_\mu^{(m)} S_\mu^{(n)} = g_{mn}, \quad m, n = 0, l, t, n. \quad (14)$$

This allows to express the deuteron quadrupole polarization tensor, in general case, as follows

$$Q_{\mu\nu} = S_\mu^{(m)} S_\nu^{(n)} R_{mn} \equiv S_\mu^{(\alpha)} S_\nu^{(\beta)} R_{\alpha\beta}, \quad R_{\alpha\beta} = R_{\beta\alpha}, \quad R_{\alpha\alpha} = 0 \quad (15)$$

because the components R_{00} , $R_{0\alpha}$ and $R_{\alpha 0}$ identically equal to zero due to condition $Q_{\mu\nu} p_\nu = 0$.

In the Born approximation the components R_{ln} and R_{tn} do not contribute to the cross section (since the 4-momenta q_μ and $k_{1\mu}$ are orthogonal to the 4-vector $S_\mu^{(n)}$) and the expansion (15) can be rewritten in the following standard form

$$Q_{\mu\nu} = [S_\mu^{(l)} S_\nu^{(l)} - \frac{1}{2} S_\mu^{(t)} S_\nu^{(t)}] R_{ll} + \frac{1}{2} S_\mu^{(t)} S_\nu^{(t)} (R_{tt} - R_{nn}) + (S_\mu^{(l)} S_\nu^{(t)} + S_\mu^{(t)} S_\nu^{(l)}) R_{lt}, \quad (16)$$

here we took into account that $R_{ll} + R_{tt} + R_{nn} = 0$.

In further, the deep-inelastic scattering of the unpolarized electron beam from the tensor polarized deuteron target is considered. Thus, we have to calculate only the convolution of the Born leptonic tensor $L_{\mu\nu}^B$ and hadron tensor $W_{\mu\nu}(T)$ caused by the tensor polarization of the target

$$S^B(T) = L_{\mu\nu}^B W_{\mu\nu}(T) = 8 \frac{\tau}{y} \left\{ -\frac{1}{y^2} [xy^2 B_1 + (a - 1 + y) B_2 + y B_3] Q_0 + \right. \quad (17) \\ \left. + \frac{1}{y} [(2 - y) B_3 - y B_4] Q_1 + B_4 Q_{11} \right\},$$

where $a = xy\tau$, $Q_0 = Q_{\alpha\beta} q_\alpha q_\beta$, $Q_1 = Q_{\alpha\beta} q_\alpha k_{1\beta}$, $Q_{11} = Q_{\alpha\beta} k_{1\alpha} k_{1\beta}$. Using the formulae for the vectors $S_\mu^{(\alpha)}$ we can calculate the convolutions. After simple calculations we have

$$\frac{d\sigma_B(T)}{dx dQ_B^2} = \frac{2\pi\alpha^2}{x Q_B^4} [S_{ll} R_{ll} + S_{tt} (R_{tt} - R_{nn}) + S_{lt} R_{lt}], \quad (18)$$

with

$$S_{ll} = [2xb\tau - y(1 + 2x\tau)^2]G + 2b(1 + 3x\tau)B_3 + (b - a)B_4, \\ S_{lt} = 2\sqrt{\frac{xb\tau}{y}} [2(y + 2a)G + (2 - y - 4b)B_3 + yB_4], \quad (19) \\ S_{tt} = -2xb\tau(G + B_3), \quad G = xyB_1 - \frac{b}{y}B_2.$$

So, in the general case, the cross section of deep-inelastic scattering of unpolarized electron beam from the tensor polarized target is determined, in the Born approximation, by the components of the quadrupole-polarization tensor R_{ll} , R_{lt} and the combination $(R_{tt} - R_{nn})$.

Consider just one more, commonly used, choice of the coordinate axes: components of the deuteron polarization tensor are defined in the coordinate system with the axes along directions \mathbf{L} , \mathbf{T} and \mathbf{N} in the rest frame of the deuteron, where

$$\mathbf{L} = \frac{\mathbf{k}_1 - \mathbf{k}_2}{|\mathbf{k}_1 - \mathbf{k}_2|}, \quad \mathbf{T} = \frac{\mathbf{n}_1 - (\mathbf{n}_1 \mathbf{L}) \mathbf{L}}{\sqrt{1 - (\mathbf{n}_1 \mathbf{L})^2}}, \quad \mathbf{N} = \mathbf{n}. \quad (20)$$

The corresponding covariant form of set (20) reads

$$S_\mu^{(L)} = \frac{2\tau(k_1 - k_2)_\mu - yp_\mu}{M\sqrt{yh}} , \quad S_\mu^{(T)} = \frac{(1 + 2x\tau)k_{2\mu} - (1 - y - 2x\tau)k_{1\mu} - x(2 - y)p_\mu}{\sqrt{Vxbh}} \quad (21)$$

$$S_\mu^{(N)} = S_\mu^{(n)} , \quad h = y + 4x\tau ,$$

and the expansion of the deuteron polarization tensor is defined in full analogy with (16)

$$Q_{\mu\nu} = [S_\mu^{(L)}S_\nu^{(L)} - \frac{1}{2}S_\mu^{(T)}S_\nu^{(T)}]R_{LL} + \frac{1}{2}S_\mu^{(T)}S_\nu^{(T)}(R_{TT} - R_{NN}) + (S_\mu^{(L)}S_\nu^{(T)} + S_\mu^{(T)}S_\nu^{(L)})R_{LT} . \quad (22)$$

These two sets of the orthogonal 4-vectors are connected by means of orthogonal matrix which describes the rotation in the plane perpendicular to direction $\mathbf{n} = \mathbf{N}$

$$S_\mu^{(L)} = \cos\theta S_\mu^{(l)} + \sin\theta S_\mu^{(t)} , \quad S_\mu^{(T)} = -\sin\theta S_\mu^{(l)} + \cos\theta S_\mu^{(t)} , \quad (23)$$

$$\cos\theta = \frac{y(1 + 2x\tau)}{\sqrt{yh}} , \quad \sin\theta = -2\sqrt{\frac{xb\tau}{h}} .$$

The part of the differential cross section that depends on the tensor polarization can be written as follows in this set of axes

$$\frac{d\sigma_B(T)}{dx dQ_B^2} = \frac{2\pi\alpha^2}{xQ_B^4} [S_{LL}R_{LL} + S_{TT}(R_{TT} - R_{NN}) + S_{LT}R_{LT}] , \quad (24)$$

$$S_{LL} = -hG + 2bB_3 + \frac{B_4}{h} [(1 - y)(y - 2x\tau) - 2a(y + x\tau)] ,$$

$$S_{TT} = \frac{2xb\tau}{h}B_4 , \quad S_{LT} = 2\sqrt{\frac{xb\tau}{y}}(2 - y)(B_3 + \frac{y}{h}B_4) . \quad (25)$$

3 Radiative corrections

In this work we consider only QED radiative corrections to the deep-inelastic scattering process (1). We confine ourselves to the calculation of the so-called model-independent radiative corrections when the photons are radiated from the lepton line and the vacuum polarization is also taken into account. The reason is that it gives the main contribution to radiative corrections due to the smallness of the electron mass, and can be calculated without any additional assumptions. Nevertheless, these radiative corrections depend on the shape of the deuteron structure functions (both spin-independent and spin-dependent) by their dependence on the x and Q^2 variables.

There exist two contributions for radiative corrections when we take into account the corrections of the order of α . The first one is caused by virtual and soft photon emission that cannot affect the kinematics of process (1). The second one arises due to the radiation of a hard photon

$$e^-(k_1) + d(p) \rightarrow e^-(k_2) + \gamma(k) + X(p_x) . \quad (26)$$

The leptonic tensor, corresponding to the hard-photon radiation, is well known [17, 18]. For the case of unpolarized electron beam it can be written as

$$L_{\mu\nu}^\gamma = A_0\tilde{g}_{\mu\nu} + A_1\tilde{k}_{1\mu}\tilde{k}_{1\nu} + A_2\tilde{k}_{2\mu}\tilde{k}_{2\nu} , \quad (27)$$

where

$$A_0 = -\frac{(q^2 + \chi_1)^2 + (q^2 - \chi_2)^2}{\chi_1 \chi_2} - 2m^2 q^2 \left(\frac{1}{\chi_1^2} + \frac{1}{\chi_2^2} \right), \quad A_1 = -4 \left(\frac{q^2}{\chi_1 \chi_2} + \frac{2m^2}{\chi_2^2} \right),$$

$$A_2 = -4 \left(\frac{q^2}{\chi_1 \chi_2} + \frac{2m^2}{\chi_1^2} \right), \quad \tilde{k}_{i\mu} = k_{i\mu} - \frac{q k_i}{q^2} q_\mu, \quad (i = 1, 2),$$

with $\chi_{1,2} = 2k k_{1,2}$, m is the electron mass, $q^2 = \chi_2 - \chi_1 - Q_B^2$, and in this chapter $q = k_1 - k_2 - k$. The hadronic tensor in this case has the same form as the hadronic tensor in the Born approximation, but momentum transfer q differs from the Born one and the structure functions B_i depend on the new momentum q . Here and further we neglect the terms which are zero at $m \rightarrow 0$.

We consider the hard-photon (with the energy $\omega > \Delta\varepsilon$, where $\Delta \ll 1$) emission process using the approach of paper [19] where it was applied to the process of deep-inelastic scattering on unpolarized target. Let us introduce the variables suitable for this process

$$z = \frac{M_x^2 - M^2}{V} = \frac{q^2 + 2pq}{V}, \quad r = -\frac{q^2}{Q_B^2}, \quad x' = \frac{-q^2}{2pq} = \frac{xyr}{xyr + z}, \quad \chi_{1,2} = 2k k_{1,2},$$

where M_x is the invariant mass of the hadron system produced in the scattering of the photon (with the virtuality q^2) by the target.

Note that the physical meaning of z variable is following: the quantity z shows the degree of deviation from the elastic process ($ed \rightarrow ed$). So, the value $z = 0$ corresponds to the elastic ed -scattering threshold and the value $z = \varepsilon_d/\varepsilon_1$ (ε_d is the deuteron bound energy, ε_1 is the electron beam energy in the laboratory system) corresponds to the $ed \rightarrow enp$ reaction threshold (quasi-elastic ed - scattering).

The convolution of the leptonic and hadronic tensors may be represented as

$$S^\gamma(T) = AA_0 + BA_1 + CA_2, \quad (28)$$

$$A = NQ_0[3B_1 + \frac{2\tau}{c}B_2 + \frac{c}{2xyr}(B_2 + 2B_3 + B_4)],$$

$$B = N\{Q_0[\frac{V}{2c}B_2 - V\frac{Q_B^2 + \chi_1}{2rQ_B^2}(B_2 + B_3) + \frac{(Q_B^2 + \chi_1)^2}{4rQ_B^2}(B_1 +$$

$$+ \frac{Vc}{2rQ_B^2}(B_2 + 2B_3 + B_4))] + VQ_1[B_3 - \frac{Q_B^2 + \chi_1}{2rQ_B^2}c(B_3 + B_4)] + \frac{V}{2}cQ_{11}B_4\},$$

$$C = N\{Q_0[\frac{V}{2}\frac{(1-y)^2}{c}B_2 + V\frac{Q_B^2 - \chi_2}{2rQ_B^2}(1-y)(B_2 + B_3) + \frac{(Q_B^2 - \chi_2)^2}{4rQ_B^2}(B_1 +$$

$$+ \frac{Vc}{2rQ_B^2}(B_2 + 2B_3 + B_4))] + VQ_2[B_3(1-y) + \frac{Q_B^2 - \chi_2}{2rQ_B^2}c(B_3 + B_4)] + \frac{V}{2}cQ_{22}B_4\},$$

with $N = 4\tau/Vc^2$, $c = z + xyr$.

The quantities Q_0 , Q_1 , Q_2 , Q_{11} and Q_{22} are the convolutions of the deuteron quadrupole-polarization tensor and 4-momenta and they can be expressed in terms of the scalar products of the 4-momenta of the particles participating in the reaction and the set of 4-vectors $S_\mu^{(l,t,n)}$. So, these convolutions are

$$Q_0 = Q_{\alpha\beta}q_\alpha q_\beta = [(l \cdot q)^2 - \frac{1}{2}(t \cdot q)^2 - \frac{1}{2}(n \cdot q)^2]R_{ll} + 2l \cdot qt \cdot qR_{lt} + 2n \cdot ql \cdot qR_{ln} +$$

$$\begin{aligned}
& +2n \cdot qt \cdot q R_{tn} + \frac{1}{2}[(t \cdot q)^2 - (n \cdot q)^2](R_{tt} - R_{nn}), \\
Q_1 = Q_{\alpha\beta} q_\alpha k_{1\beta} &= (l \cdot k_1 l \cdot q - \frac{1}{2}t \cdot k_1 t \cdot q) R_{ll} + (l \cdot k_1 t \cdot q + t \cdot k_1 l \cdot q) R_{lt} + l \cdot k_1 n \cdot q R_{ln} + \\
& + t \cdot k_1 n \cdot q R_{tn} + \frac{1}{2}t \cdot k_1 t \cdot q (R_{tt} - R_{nn}), \tag{29}
\end{aligned}$$

$$Q_{11} = Q_{\alpha\beta} k_{1\alpha} k_{1\beta} = [(l \cdot k_1)^2 - \frac{1}{2}(t \cdot k_1)^2] R_{ll} + 2l \cdot k_1 t \cdot k_1 R_{lt} + \frac{1}{2}(t \cdot k_1)^2 (R_{tt} - R_{nn}),$$

$$Q_2 = Q_1(k_1 \rightarrow k_2), \quad Q_{22} = Q_{11}(k_1 \rightarrow k_2), \quad i \cdot a = S_\mu^{(i)} a_\mu, \quad i = l, t, n.$$

Here we used the following conditions: $R_{ll} + R_{tt} + R_{nn} = 0$, $n \cdot k_1 = n \cdot k_2 = 0$. For the set of the 4-vectors $S_\mu^{(l,t,n)}$ we have also $t \cdot k_1 = 0$.

It is convenient to separate the poles in the term $(\chi_1 \chi_2)^{-1}$ using the relation

$$\frac{1}{\chi_1 \chi_2} = \frac{1}{Q_B^2} \frac{1}{1-r} \left(\frac{1}{\chi_1} - \frac{1}{\chi_2} \right).$$

Then the radiative correction (caused by the hard-photon emission) to the differential cross section of deep-inelastic scattering of unpolarized electron beam by the tensor polarized target has the following form

$$\frac{d\sigma^\gamma}{dx dQ_B^2} = \frac{\alpha y}{Vx} \int \frac{d^3 \vec{k}}{2\pi\omega} \Sigma(z, r), \tag{30}$$

where ω is the energy of the hard photon and

$$\Sigma(z, r) = \frac{\alpha^2(q^2)}{Q_B^4} \left\{ R_0(z, r) + \left(\frac{1}{\chi_1} - \frac{1}{\chi_2} \right) R_1(z, r) + \frac{m^2}{\chi_1^2} R_{1m}(z, r) + \frac{m^2}{\chi_2^2} R_{2m}(z, r) \right\}, \tag{31}$$

$$R_0 = -\frac{2}{r^2} A, \quad R_1 = \frac{1}{r-1} \left[\left(1 + \frac{1}{r^2} \right) Q_B^2 A - \frac{4}{r} (B+C) \right], \quad R_{1m} = 2 \left(\frac{Q_B^2}{r} A - \frac{4}{r^2} C \right), \quad R_{2m} = 2 \left(\frac{Q_B^2}{r} A - \frac{4}{r^2} B \right).$$

It is convenient to write down the integral in Eq. (30) as

$$I = \int \frac{d^3 \vec{k}}{2\pi\omega} \Sigma(z, r) = I_{1m} + I_{2m} + I_R, \tag{32}$$

where we separate the contributions proportional to m^2

$$I_{1m} = \int \frac{d^3 \vec{k}}{2\pi\omega} \frac{\alpha^2(q^2)}{Q_B^4} \frac{m^2}{\chi_1^2} R_{1m}(z, r), \quad I_{2m} = \int \frac{d^3 \vec{k}}{2\pi\omega} \frac{\alpha^2(q^2)}{Q_B^4} \frac{m^2}{\chi_2^2} R_{2m}(z, r). \tag{33}$$

Let us consider first the integrals I_{im} , $i = 1, 2$. In this case the numerator of the integrands in I_{1m} (I_{2m}) is calculated in the approximation $\chi_1 = 0$ ($\chi_2 = 0$) [19]. The hard-photon phase-space is written as

$$\frac{d^3 \vec{k}}{2\pi\omega} = \frac{dz}{z_+ - z} \frac{\omega^2 d\Omega_k}{2\pi}, \quad z_+ = y(1-x). \tag{34}$$

Using the invariance of $\omega^2 d\Omega_k$, we can do the integration over the angular variables $d\Omega_k$ in the most suitable coordinate system, namely: in the coordinate frame $\vec{k}_1 - \vec{k}_2 + \vec{p} = 0$ (c.m.s. of the scattered electron and produced hadronic system). We obtain

$$\int \frac{\omega^2 d\Omega_k}{2\pi} \frac{m^2}{\chi_{1,2}^2} = \frac{1}{2}.$$

Integral I_{1m} . We calculate the integrand in the approximation $\chi_1 = 0$ (besides the denominator). This approximation corresponds to the emission of the collinear photon along the initial-electron momentum. In this case the variables acquire the following values

$$r_1 = \frac{1-y+z}{1-xy}, \quad q_1^2 = -r_1 Q_B^2, \quad x'_1 = \frac{xyr_1}{z+xyr_1}.$$

After integrating over the hard-photon angular variables, the integral I_{1m} can be represented as follows

$$\begin{aligned} I_{1m} &= \frac{1}{Q_B^4} \int_0^{z_m} \frac{dz}{z_+ - z} \alpha_1^2 N_1 \Sigma_1(z), \\ \Sigma_1(z) &= \Sigma_{1ll} R_{ll} + \Sigma_{1lt} R_{lt} + \Sigma_{1tt} (R_{tt} - R_{nn}), \\ \Sigma_{1tt} &= b \frac{Q_B^2}{r_1} (G_t + B_{3t}), \quad \Sigma_{1lt} = -\frac{V}{r_1} \sqrt{\frac{xyb}{\tau}} [(y-1+r_1)B_{4t} + (a-3b+r_1)B_{3t} + 2(a-b+r_1)G_t], \\ \Sigma_{1ll} &= \frac{V}{2\tau r_1} \{(a-b)(y-1+r_1)B_{4t} + 2b(b-2a-r_1)B_{3t} - [2ab - (a-b+r_1)^2]G_t\}, \\ G_t &= xyB_{1t} - \frac{b}{y-1+r_1} B_{2t}, \quad \alpha_1 = \alpha(q_1^2), \quad N_1 = \frac{4\tau}{(z+xyr_1)^2}, \\ z_m &= z_+ - \rho, \quad \rho = 2\Delta\varepsilon\sqrt{(\tau+z_+)/V}, \quad B_{it} = B_i(q_1^2, x'_1), \quad i = 1-4. \end{aligned} \tag{35}$$

It is convenient to extract explicitly the contribution containing the infrared divergence. To do this we should add and subtract in the numerator of the integrand its value at $z = z_+$. At this value we have: $r_1 = 1$, $\alpha_1 = \alpha$, $N_1 = 4\tau/y^2$, $x'_1 = x$. So, the integral I_{1m} can be written as

$$I_{1m} = \frac{1}{Q_B^4} \int_0^{z_+} \frac{dz}{z_+ - z} \left[\alpha_1^2 N_1 \Sigma_1(z) - \alpha^2 \frac{4\tau}{y^2} \Sigma_1(z_+) \right] + \frac{Vx}{\pi y} \ln\left(\frac{\rho}{z_+}\right) \frac{d\sigma_B}{dx dQ_B^2}. \tag{36}$$

Integral I_{2m} . Calculation of the integrand is performed in the approximation $\chi_2 = 0$ that corresponds to the emission of the collinear photon along the final-electron momentum. In this case the variables acquire the following values

$$r_2 = \frac{1-z}{1-z_+}, \quad q_2^2 = -r_2 Q_B^2, \quad x'_2 = \frac{xyr_2}{1-r_2(1-y)}.$$

After integrating over the hard-photon angular variables, the integral I_{2m} is represented as follows

$$\begin{aligned} I_{2m} &= \frac{1}{Q_B^4} \int_0^{z_m} \frac{dz}{z_+ - z} \alpha_2^2 N_2 \Sigma_2(z), \\ \Sigma_2(z) &= \Sigma_{2ll} R_{ll} + \Sigma_{2lt} R_{lt} + \Sigma_{2tt} (R_{tt} - R_{nn}), \\ \Sigma_{2tt} &= b Q_B^2 (r_2 G_s + B_{3s}), \quad \Sigma_{2lt} = -V \sqrt{\frac{xyb}{\tau}} \left\{ (y-1+\frac{1}{r_2})B_{4s} + (a-3b+\frac{1}{r_2})B_{3s} + 2[1+(a-b)r_2]G_s \right\}, \\ \Sigma_{2ll} &= \frac{V}{2\tau r_2} \{(a-b)[1-r_2(1-y)]B_{4s} - 2b[1+(2a-b)r_2]B_{3s} - [2abr_2^2 - (1+ar_2-br_2)^2]G_s\}, \\ G_s &= xyB_{1s} - \frac{b}{1-r_2(1-y)} B_{2s}, \quad \alpha_2 = \alpha(q_2^2), \quad N_2 = \frac{4\tau}{(z+xyr_2)^2}, \quad B_{is} = B_i(q_2^2, x'_2), \quad i = 1-4. \end{aligned} \tag{37}$$

The contribution containing the infrared divergence is extracted explicitly in a similar manner as it was done for the I_{1m} integral. At the value $z = z_+$ we have: $r_2 = 1$, $\alpha_2 = \alpha$, $N_2 = 4\tau/y^2$, $x'_2 = x$. So, the integral I_{2m} is rewritten as

$$I_{2m} = \frac{1}{Q_B^4} \int_0^{z_+} \frac{dz}{z_+ - z} \left[\alpha_2^2 N_2 \Sigma_2(z) - \alpha^2 \frac{4\tau}{y^2} \Sigma_2(z_+) \right] + \frac{Vx}{\pi y} \ln\left(\frac{\rho}{z_+}\right) \frac{d\sigma_B}{dx dQ_B^2}. \quad (38)$$

The radiative corrections due to the virtual photon exchange and real soft-photon emission (with energy $\omega < \Delta\varepsilon$) can be related to the Born cross section as (note that the vacuum polarization effects are included in the Born cross-section through the dependence of the coupling constant α on the virtual-photon momentum)

$$\frac{d\sigma^{(S+V)}}{dx dQ_B^2} = \delta^{SV} \frac{d\sigma_B}{dx dQ_B^2}, \quad (39)$$

where the factor δ^{SV} is [19]

$$\delta^{SV} = \frac{\alpha}{\pi} \left[(L-1) \ln \frac{(\Delta\varepsilon)^2}{\varepsilon_1 \varepsilon_2} + \frac{3}{2} L - \frac{1}{2} \ln^2 \left(\frac{\varepsilon_1}{\varepsilon_2} \right) - \frac{\pi^2}{6} - 2 - f(\cos^2 \frac{\theta}{2}) \right], \quad L = \ln \frac{Q_B^2}{m^2}, \quad (40)$$

here $\varepsilon_1(\varepsilon_2)$ is the initial (final) electron energy, and θ is the electron scattering angle in the coordinate frame $\vec{k}_1 - \vec{k}_2 + \vec{p} = 0$. The function f is defined as

$$f(x) = \int_0^x \frac{dt}{t} \ln(1-t).$$

The quantities $\varepsilon_1(\varepsilon_2)$ and θ can be expressed in terms of the invariant variables

$$\varepsilon_1 = \frac{V(1-xy)}{2\sqrt{V(\tau+z_+)}} , \quad \varepsilon_2 = \frac{V(1-z_+)}{2\sqrt{V(\tau+z_+)}} , \quad \cos^2 \frac{\theta}{2} = \frac{1-y-xy\tau}{(1-xy)(1-z_+)}. \quad (41)$$

Then the radiative correction δ^{SV} is finally rewritten as

$$\delta^{SV} = \frac{\alpha}{2\pi} \left\{ -1 - \frac{\pi^2}{3} - 2f\left[\frac{1-y-xy\tau}{(1-xy)(1-z_+)}\right] - \ln^2\left(\frac{1-xy}{1-z_+}\right) + (L-1)(3 + 2\ln\left[\frac{\rho^2}{(1-xy)(1-z_+)}\right]) \right\}. \quad (42)$$

Integral I_R . When calculating this integral we use the results of Ref. [19]. Besides the integrals calculated in that paper we need the following integrals

$$\int \frac{d^3\vec{k}}{2\pi\omega} F(z, r) \chi_1, \quad \int \frac{d^3\vec{k}}{2\pi\omega} F(z, r) \chi_1^2. \quad (43)$$

To calculate these integrals we write the hard-photon phase space in the form

$$\frac{d^3\vec{k}}{2\pi\omega} = \frac{Q_B^2}{2\sqrt{y^2+4a}} \frac{d\varphi}{2\pi} dz dr. \quad (44)$$

Since, in our case, the function F does not depend on the φ variable, we can integrate over this variable. We do it in the above mentioned coordinate frame. The results are

$$i_1 = \int \frac{d\varphi}{2\pi} \chi_1 = \frac{Q_B^2}{y^2+4a} \left[(2-y)(y-c) - (1-r)(y+2a) \right], \quad (45)$$

$$i_2 = \int \frac{d\varphi}{2\pi} \chi_1^2 = \frac{1}{2} \left[3i_1^2 - \frac{Q_B^4(1-xy)^2}{y^2 + 4a}(r - r_1)^2 \right].$$

After simple calculations the integral I_R is (here we omit the contributions proportional to the R_{ln} and R_{tn} components)

$$\begin{aligned} I_R = & \frac{1}{2Q_B^4} \sum_{i=1}^4 \sum_{m,n} R_{mn} \left\{ \frac{L_1}{1-xy} \int_0^{z_m} \frac{dz}{1-r_1} G_i^{mn}(z, r_1) + \frac{L_2}{1-z_+} \int_0^{z_m} \frac{dz}{1-r_2} \tilde{G}_i^{mn}(z, r_2) + \right. \\ & + \frac{1}{1-xy} \int_0^{z_m} dz \int_{r_-}^{r_+} \frac{dr}{|r-r_1|} \left[\frac{G_i^{mn}(z, r)}{1-r} - \frac{G_i^{mn}(z, r_1)}{1-r_1} \right] + \\ & + \frac{1}{1-z_+} \int_0^{z_m} dz \int_{r_-}^{r_+} \frac{dr}{|r-r_2|} \left[\frac{\tilde{G}_i^{mn}(z, r)}{1-r} - \frac{\tilde{G}_i^{mn}(z, r_2)}{1-r_2} \right] + \\ & \left. + \frac{Q_B^2}{\sqrt{y^2 + 4a}} \int_0^{z_m} dz \int_{r_-}^{r_+} dr \frac{\alpha^2}{r^2} B_i \left[C_{0i}^{mn}(z, r) + i_1 C_{1i}^{mn}(z, r) + i_2 C_{2i}^{mn}(z, r) \right] \right\}, \end{aligned} \quad (46)$$

where

$$\begin{aligned} L_1 &= \ln \left[\frac{Q_B^2(1-xy)^2}{m^2 xy(\tau + z_+)} \right], \quad L_2 = \ln \left[\frac{Q_B^2(1-z_+)^2}{m^2 xy(\tau + z_+)} \right], \\ r_{\pm}(z) &= \frac{1}{2xy(\tau + z_+)} \left[2xy(\tau + z) + (z_+ - z)(y \pm \sqrt{y^2 + 4a}) \right], \\ G_i^{mn}(z, r) &= \frac{\alpha^2}{r^2} (1-r) B_i A_i^{mn}(z, r), \quad \tilde{G}_i^{mn}(z, r) = \frac{\alpha^2}{r^2} (1-r) B_i \tilde{B}_i^{mn}(z, r), \end{aligned} \quad (47)$$

with $m, n = l, t, n$. Note that the structure functions B_i are the functions of two independent variables $q^2 = -rQ_B^2$ and $x' = xyr/(z + xyr)$. The expressions for the coefficients A_i^{mn} , B_i^{mn} , C_{ki}^{mn} , $k = 0, 1, 2$ are given in the Appendix A. The contributions proportional to the R_{ln} and R_{tn} components are considered in a more detail in the Appendix B.

Let us discuss briefly the singularities in the I_R integral. The value $r = 1$ corresponds to the real soft-photon emission (we have infrared divergence at this point), and the value $r = r_1(r_2)$ corresponds to the emission of the collinear photon along the initial-(final-) electron momentum (the so-called collinear divergence). The singularity at the point $z = z_+$ is the infrared one. The divergence at $r = 1$ point is unphysical one. It arises during the integration procedure due to the separation of the poles in the expression $(\chi_1 \chi_2)^{-1}$. It is necessary to extract explicitly the collinear and infrared divergences in the above formula.

The integrand in the above expression can be written in the form which does not contain explicitly the infrared divergences if we add term (39) to it. To do this we use the following transformations

$$\begin{aligned} \ln \left[\frac{\varphi_i(x, y)}{xy(\tau + z_+)} \right] \frac{G(z, r_i)}{1-r_i} + \int_{r_-}^{r_+} \frac{dr}{|r-r_i|} \left[\frac{G(z, r)}{1-r} - \frac{G(z, r_i)}{1-r_i} \right] = \\ = P \int_{r_-}^{r_+} \frac{dr}{(1-r)|r-r_i|} \left[G(z, r) - G(z, r_i) \right], \quad i = 1, 2, \end{aligned} \quad (48)$$

where $\varphi_1(x, y) = (1-xy)^2$, $\varphi_2(x, y) = (1-z_+)^2$, and the symbol P denotes the principal value of integral. The total radiative correction (which is the sum of the contribution due to the hard-photon emission and the contribution due to the real soft-photon emission and

virtual-photon contribution) to the part of the differential cross section caused by the tensor polarization of the target is written as

$$\frac{d\sigma}{dx dQ_B^2} = \frac{d\sigma_B}{dx dQ_B^2} + \delta^{tot}, \quad (49)$$

where

$$\begin{aligned} \delta^{tot} = & \frac{\alpha}{2\pi} \left\{ 3L + 2(L-1) \ln \left[\frac{z_+^2}{(1-xy)(1-z_+)} \right] - \ln^2 \left(\frac{1-xy}{1-z_+} \right) - 4 - \frac{\pi^2}{3} - \right. \\ & - 2f \left[\frac{b}{(1-xy)(1-z_+)} \right] \left. \right\} \frac{d\sigma_B}{dx dQ_B^2} + \frac{\alpha y}{x Q_B^4} \int_0^{z_+} \frac{dz}{z_+ - z} \left[\alpha_1^2 N_1 \Sigma_1(z) + \alpha_2^2 N_2 \Sigma_2(z) - \alpha^2 \frac{8\tau}{V y^2} \Sigma_1(z_+) \right] + \\ & + \frac{\alpha y}{2x V Q_B^4} \sum_{i=1}^4 \sum_{mn} \left\{ L \int_0^{z_+} \frac{dz}{z_+ - z} \left[G_i^{mn}(z, r_1) - G_i^{mn}(z_+, 1) - \tilde{G}_i^{mn}(z, r_2) + \tilde{G}_i^{mn}(z_+, 1) \right] + \right. \\ & \left. + \frac{Q_B^2}{\sqrt{y^2 + 4a}} \int_0^{z_+} dz \int_{r_-}^{r_+} dr \frac{\alpha^2}{r^2} B_i \left[C_{0i}^{mn}(z, r) + i_1 C_{1i}^{mn}(z, r) + i_2 C_{2i}^{mn}(z, r) \right] + R_i^{mn} \right\}. \end{aligned} \quad (50)$$

The term R_i^{mn} has different form depending on the integration region of the variable r . For the regions $r_- \leq r \leq r_1$ and $r_2 \leq r \leq r_+$ the function R_i^{mn} has the following form (in these regions $r \neq 1$ and therefore the divergence at the point $r = 1$ is absent)

$$\begin{aligned} R_i^{mn} = & \frac{1}{1-xy} \int_0^{z_+} dz \int_{r_-}^{r_+} \frac{dr}{(1-r)|r-r_1|} \left[G_i^{mn}(z, r) - G_i^{mn}(z, r_1) \right] + \\ & + \frac{1}{1-z_+} \int_0^{z_+} dz \int_{r_-}^{r_+} \frac{dr}{(1-r)|r-r_2|} \left[\tilde{G}_i^{mn}(z, r) - \tilde{G}_i^{mn}(z, r_2) \right]. \end{aligned} \quad (51)$$

For the region $r_1 < r < r_2$ we have

$$\begin{aligned} R_i^{mn} = & \int_0^{z_+} dz \ln \left(\frac{1-r_-}{r_+ - 1} \right) \left\{ g_{i1}^{mn}(z, 1) - f_{i1}^{mn}(z, 1) + \frac{1}{z_+ - z} \left[g_{i0}^{mn}(z, 1) - g_{i0}^{mn}(z, r_1) + \right. \right. \\ & \left. \left. + f_{i0}^{mn}(z, 1) - f_{i0}^{mn}(z, r_2) \right] \right\} + \int_0^{z_+} dz \int_{r_-}^{r_+} \frac{dr}{1-r} \left\{ g_{i1}^{mn}(z, r) - g_{i1}^{mn}(z, 1) - \right. \\ & \left. - f_{i1}^{mn}(z, r) + f_{i1}^{mn}(z, 1) + \frac{1}{1-xy} \left[F^{mn}(z, r) - F^{mn}(z, 1) \right] - \frac{1}{1-z_+} \left[\tilde{F}^{mn}(z, r) - \tilde{F}^{mn}(z, 1) \right] \right\}, \end{aligned} \quad (52)$$

where we introduce the following notation

$$\begin{aligned} G_i^{mn}(z, r) &= g_{i0}^{mn}(z, r) + \Delta_1 g_{i1}^{mn}(z, r), \quad \tilde{G}_i^{mn}(z, r) = f_{i0}^{mn}(z, r) + \Delta_2 f_{i1}^{mn}(z, r), \\ F^{mn}(z, r) &= \frac{1}{r - r_1} \left[g_{i0}^{mn}(z, r) - g_{i0}^{mn}(z, r_1) \right], \quad \tilde{F}^{mn}(z, r) = \frac{1}{r - r_2} \left[f_{i0}^{mn}(z, r) - f_{i0}^{mn}(z, r_2) \right], \\ \Delta_1 &= (1-xy)r - a - b - z, \quad \Delta_2 = (1-y+xy)r + z - 1. \end{aligned} \quad (53)$$

In obtaining the above formula we use the relation

$$P \int_{r_-}^{r_+} \frac{dr}{1-r} \Psi(r) = \int_{r_-}^{r_+} \frac{dr}{1-r} \left[\Psi(r) - \Psi(1) \right] + \Psi(1) \ln \left(\frac{1-r_-}{r_+ - 1} \right). \quad (54)$$

At last, let us consider the part of the integral I which is caused by the R_{ln} and R_{tn} components of the deuteron quadrupole-polarization tensor. As stated above, these components do not contribute to the cross section treated in the Born approximation. If these terms are

integrated over the whole region of the φ variable, then these integrals are equal to zero as well (this result is due to the fact that only one plane is remained after such integration). We discuss this problem in more detail in Appendix B.

Note that the integration limits in formula (50) over the variable z are given somewhat schematically. This integral contains two contributions (we neglect here the contribution of the radiative tail from the quasi-elastic scattering). One of them is the so-called inelastic contribution and the integration region for it over the variables r and z is presented in Fig. 1 by the dashed triangle. The integration over z variable for this contribution must be carried out from $z_{min} = (M_{th}^2 - M)/V$ to z_+ , where M_{th} is the inelastic threshold ($M_{th} = M + m_\pi$). The second contribution, related to the radiative tail of the elastic peak, is given by the interval $z = 0, r_-(0) \leq r \leq r_+(0)$.

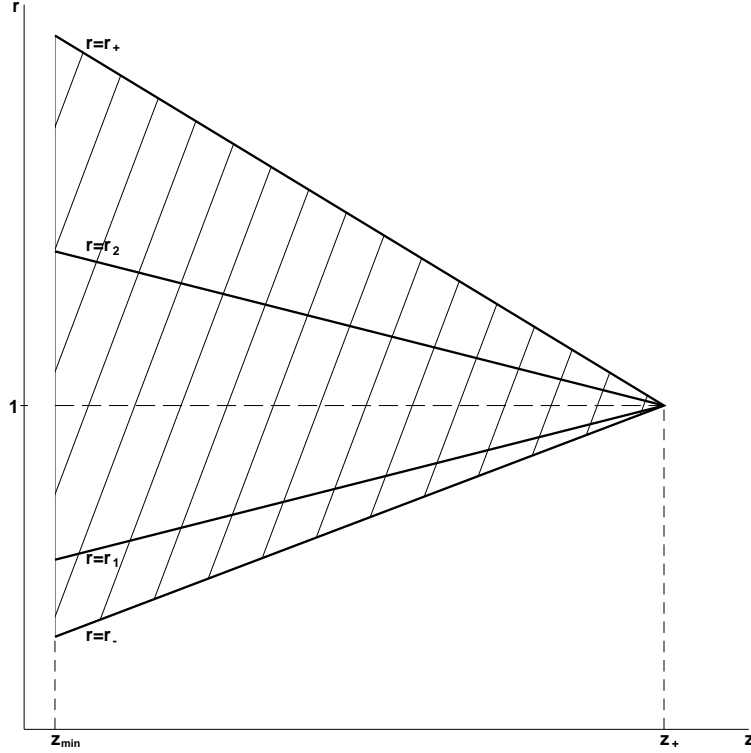


Figure 1: The integration domain with respect to the r and z variables.

The contribution of the elastic radiative tail (i.e. the inclusion of radiative corrections to the elastic ed -scattering) to the total radiative correction δ^{tot} can be obtained from formula (50) by simple substitution in the hadronic tensor

$$B_i(q^2, x') \rightarrow -\frac{1}{q^2} \delta(1 - x') B_i^{(el)}, \quad i = 1 - 4, \quad (55)$$

where $B_i^{(el)}$ are expressed in terms of the deuteron electromagnetic form factors as

$$B_1^{(el)} = \eta q^2 G_M^2, \quad B_2^{(el)} = -2\eta^2 q^2 [G_M^2 + \frac{4G_Q}{1+\eta} (G_C + \frac{\eta}{3} G_Q + \eta G_M)], \quad (56)$$

$$B_3^{(el)} = 2\eta^2 q^2 G_M (G_M + 2G_Q), \quad B_4^{(el)} = -2\eta q^2 (1 + \eta) G_M^2, \quad \eta = -q^2/4M^2.$$

Here G_C , G_M and G_Q are the deuteron charge monopole, magnetic dipole and quadrupole form factors, respectively. These form factors have the following normalizations:

$$G_C(0) = 1, \quad G_M(0) = (M/m_n)\mu_d, \quad G_Q(0) = M^2Q_d,$$

where m_n is the nucleon mass, $\mu_d(Q_d)$ is the deuteron magnetic (quadrupole) moment and their values are: $\mu_d = 0.857$, $Q_d = 0.2859 fm^2$. After substitution of $B_i^{(el)}$ into formula (50) we have to do a trivial integration over z variable using a $\delta(1-x') = xy\delta(z)$.

4 Numerical estimations

We calculate the radiative corrections for the kinematical conditions of the HERMES experiment [10]. The energy of the positron beam is 27.6 GeV. The HERMES has provided the first direct measurement of the structure function b_1 in the kinematic range $0.002 < x < 0.85$ and $0.1 GeV^2 < Q^2 < 20 GeV^2$. A cylindrical target cell confines the polarized gas along the positron beam line, where a longitudinal magnetic field provides the quantization axis for the nuclear spin. The corresponding tensor atomic polarization is $T=0.83$ (for the definition of this quantity see the Appendix C).

The analysis of the experimental data was performed in the approximation $b_3 = b_4 = 0$. In further numerical estimation we also neglect these functions.

The deuteron spin-dependent structure function b_1 is extracted from the measured tensor asymmetry A_{zz} via the relation [10]

$$b_1 = -\frac{3}{2}A_{zz}\frac{(1+\gamma^2)F_2^d}{2x(1+R)}, \quad (57)$$

where the deuteron spin-independent structure function F_1^d has been expressed in terms of the ratio $R = \sigma_L/\sigma_T = F_2^d(1+4M^2x^2/Q^2)/2xF_1^d - 1$ [20], γ is a kinematic factor ($\gamma^2 = 4M^2x^2/Q^2$). Here $\sigma_T(\sigma_L)$ is the cross section for the absorption of the transversely (longitudinally) polarized virtual photons by the unpolarized target. The Born cross section of the deep-inelastic scattering of unpolarized electron beam by unpolarized target has the form

$$\frac{d\sigma_B^{un}}{dx dQ_B^2} = \frac{4\pi\alpha^2}{xQ_B^4}[(1-y-xy\tau)F_2^d(x, Q^2) + xy^2F_1^d(x, Q^2)]. \quad (58)$$

The structure functions $F_{1,2}^d$ are related to the structure functions $W_{1,2}$ (introduced in the formula (7)) by the following way: $W_1 = 2F_1^d$, $W_2 = 4(\tau/y)F_2^d$. The deuteron spin-independent structure function $F_2^d = F_2^p(1 + F_2^n/F_2^p)/2$ is calculated using parameterizations for the proton structure functions F_2^p [21] and the ratio F_2^n/F_2^p [22]. The deuteron spin-dependent structure function b_2 has also been extracted from the experiment using the Callan-Gross relation

$$b_2 = 2x\frac{1+R}{1+\gamma^2}b_1. \quad (59)$$

According to the preliminary results of the HERMES experiment the tensor asymmetry can be parametrized as [23]

$$A_{zz} = -1.56 \cdot 10^{-2}(1 - 1.74x - 1.45\sqrt{x}). \quad (60)$$

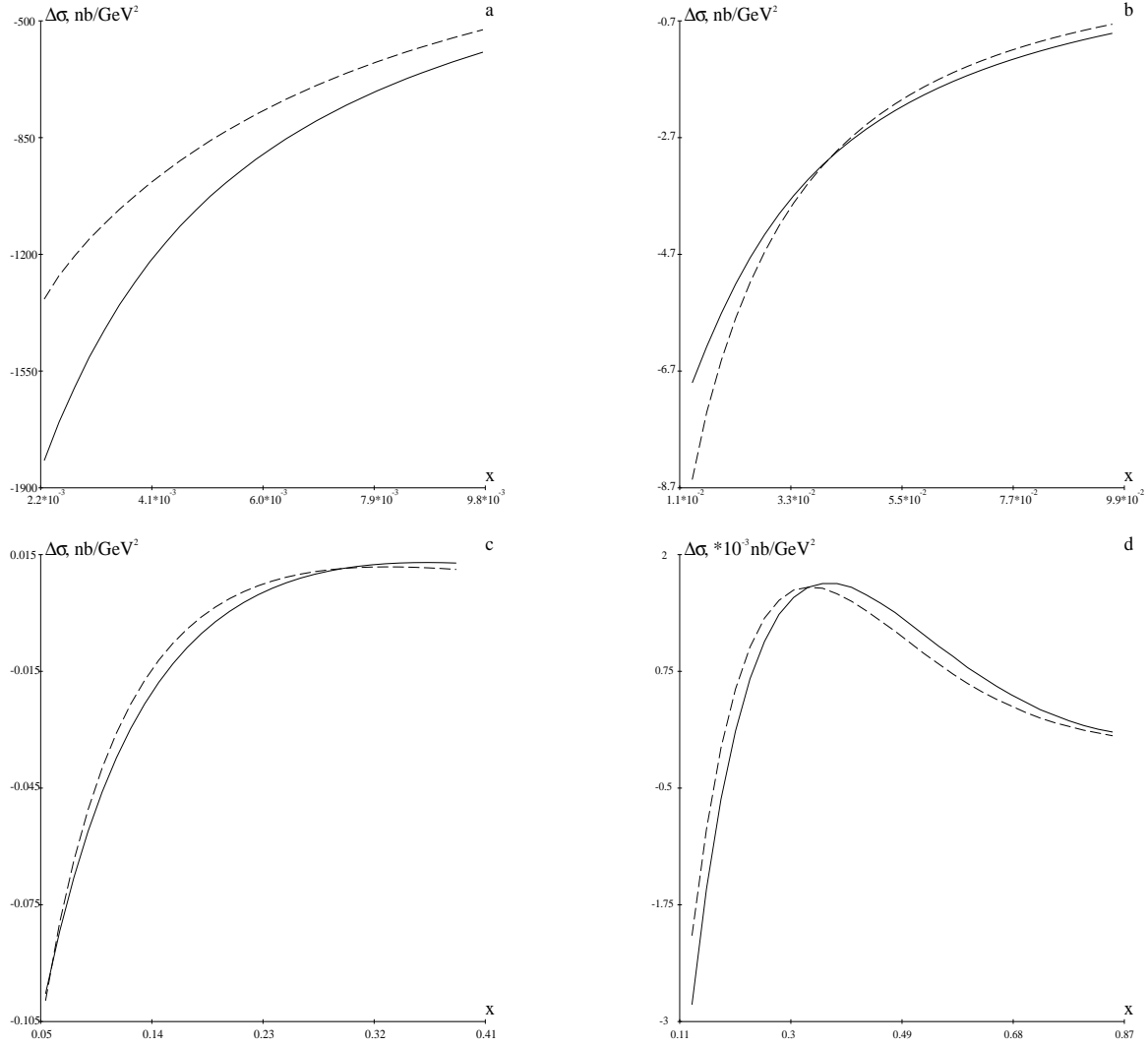


Figure 2: The spin-dependent part of the cross section calculated for the kinematical conditions of the HERMES experiment [10]. The solid line is the Born approximation, and the dotted line corresponds to the inclusion of the radiative corrections. The Q^2 values are the following: $a - 0.1 \text{ GeV}^2$, $b - 1 \text{ GeV}^2$, $c - 4 \text{ GeV}^2$, $d - 10 \text{ GeV}^2$.

The influence of the radiative correction on the spin-dependent part of the Born cross-section is shown in Fig.2 as a function of the variable x for various Q^2 values. The inclusion of the radiative correction shifts the zero value of b_1 and b_2 to the smaller x -value region (see Fig. 2 c and d). In the range of low x ($x \sim 10^{-3} - 10^{-2}$) the value of radiative correction changes from 10% to 30% as compared with the Born contribution. It is the region that of the utmost importance for b_1 -measurements. According to the theoretical predictions [11, 12, 13] the structure function b_1 increases very rapidly in this region and this fact was confirmed in the HERMES experiment [10].

From our estimation we conclude that the radiative corrections to process (1) are not small, especially for low x region, and they have to be taken into account in the data analysis.

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5 Appendix A

In this Appendix we present the formulae for the coefficients A_i^{mn} , B_i^{mn} , and C_{ji}^{mn} , ($m, n = l, t$, $i = 1 - 4$, $j = 0, 1, 2$) that determining the cross-section of the hard-photon emission process (see formula (50)).

The coefficients determining the contribution proportional to the B_1 structure function are:

$$\begin{aligned}
A_1^{ll} &= -\frac{n_1}{\tau}[(\bar{r} - \Delta_1)^2 - 2a(b + \Delta_1)], \quad B_1^{ll} = \frac{n_1}{\tau}\{[(2a - b)r + 1 + \Delta_2]^2 - ar(2 + 3ar)\}, \\
C_{01}^{ll} &= -\frac{VN}{\tau}\{(\bar{r} - \Delta_1)^2 + a[3a(1 + r^2) - 2(b + \Delta_1)]\}, \quad C_{11}^{ll} = -6N(c + 2a), \quad C_{21}^{ll} = -6N\frac{\tau}{V}, \\
A_1^{lt} &= \frac{2n_1}{Md}Q_B^2(2b + \Delta_1)(\bar{r} - \Delta_1), \quad B_1^{lt} = \frac{2n_1}{Md}Q_B^2(\Delta_2 - 2br)[(a - b)r + 1 + \Delta_2], \quad (A.1) \\
C_{01}^{lt} &= -4n_2Q_B^2[a(1 + r^2)(y + 2a) - 2b\bar{r} - \Delta_1(c + 2a - 2b)], \quad C_{11}^{lt} = -4n_2[(y + 4a)(c + 2a) - 2a(\bar{r} + 2b)], \\
C_{21}^{lt} &= -8n_2\frac{\tau}{V}(y + 2a), \quad A_1^{tt} = -\frac{n_1}{b}\frac{Q_B^2}{V}[b^2 + (b + \Delta_1)^2], \quad B_1^{tt} = \frac{n_1}{b}\frac{Q_B^2}{V}(2b^2r^2 - 2br\Delta_2 + \Delta_2^2), \\
C_{01}^{tt} &= -\frac{NQ_B^2}{2b}[(1 + r^2)(y^2 + 4a - 2ab) + (2b + \Delta_1)^2 + \Delta_1^2], \quad C_{11}^{tt} = \frac{2N}{b}[b(1 + y + 2a - r) + (1 + a)\Delta_1], \\
C_{21}^{tt} &= -\frac{N}{d^2}[y^2 + 2a(2 - b)].
\end{aligned}$$

The coefficients determining the contribution proportional to the B_2 structure function are:

$$\begin{aligned}
A_2^{ll} &= \frac{n_3}{\tau}[b(1 + r^2) + (1 - r + ry)\Delta_1][(\bar{r} - \Delta_1)^2 - 2a(b + \Delta_1)], \\
B_2^{ll} &= -\frac{n_3}{\tau}[b(1 + r^2) - \Delta_2(\bar{r} - 2a)]\{[(2a - b)r + 1 + \Delta_2]^2 - ar(2 + 3ar)\}, \\
C_{02}^{ll} &= -\frac{NV}{c}\{(7 - 3y)c^2 + 3a(5 - y + r)c + 3a^2(3 + r^2) - ar[5 + 3(a + b)^2]\}, \\
C_{12}^{ll} &= -3\tau\frac{N}{c}[4(a + c) - cy], \quad C_{22}^{ll} = -6N\frac{\tau^2}{cV}, \\
A_2^{lt} &= -2\frac{n_3Q_B^2}{Md}(2b + \Delta_1)(\bar{r} - \Delta_1)[b(1 + r^2) + (1 - r + ry)\Delta_1], \\
B_2^{lt} &= -2\frac{n_3Q_B^2}{Md}(\Delta_2 - 2br)[1 + \Delta_2 + (a - b)r][b(1 + r^2) + (a + b - r)\Delta_2], \quad (A.2)
\end{aligned}$$

$$\begin{aligned}
C_{02}^{lt} &= \frac{NV^2}{Mdc} \left\{ 2ac \left[y\bar{r} + (3b+a)(1+r) - y - 8a \right] - c^2 \left[2a + (2-y)(y+4a) \right] + 2a \left[2a(b-a+r) + (y+2a)(r - \right. \\
&\quad \left. - a(1+r^2) + r(a+b)^2 \right] \right\}, \quad C_{12}^{lt} = -4n_2 \frac{\tau}{c} \left[2a(1-3r+2\bar{r}+4c) + cy(2+b-a) \right], \quad C_{22}^{lt} = -8n_2 \frac{\tau^2}{cV} (y+2a), \\
A_2^{tt} &= \frac{xy}{b} n_3 Z_1 [2b^2 + \Delta_1(2b + \Delta_1)], \quad B_2^{tt} = -\frac{xy}{b} n_3 Z_2 [2b^2 r^2 + \Delta_2(\Delta_2 - 2br)], \\
C_{02}^{tt} &= \frac{VN}{2bc} \left\{ -2c^2 \left[a + (1+a)(2-y) \right] + c \left[(3-2y+a^2+b^2)(\bar{r}-2a) + 4(ab+b-a^2) + 4r(a-b^2) \right] - \right. \\
&\quad \left. - 2a \left[(r-a)^2 + b^2 \right] + (1+2a-2b+a^2+b^2) \left[r - a(1+r^2) + (a+b)^2 r \right] \right\}, \\
C_{12}^{tt} &= -\frac{VN}{2cd^2} \left\{ c \left[1 + 7a(1+a) - b(1+b) + (a+b)(a^2+b^2) \right] + 4a \left[(a-b)(1-r) + a^2 + b^2 - r \right] \right\}, \\
C_{22}^{tt} &= -\frac{N\tau}{cd^2} \left[y^2 + 2a(2-b) \right].
\end{aligned}$$

The coefficients determining the contribution proportional to the B_3 structure function are:

$$\begin{aligned}
A_3^l &= n_3 \frac{c}{\tau} \left\{ (a+\bar{r}) \left[2Z_1 + r\Delta_1(2a+r-\Delta_1) \right] - \Delta_1 \left[r^2(r-\Delta_1) + 2(b+\Delta_1) + r(a+b)(a+r-\Delta_1) \right] \right\}, \\
B_3^l &= -n_3 \frac{c}{\tau} \left\{ 2Z_2 \left[1 + (2a-b)r + \Delta_2 \right] + 3a\Delta_2 \left[(b-a)r - 1 - \Delta_2 \right] \right\}, \\
C_{03}^l &= \frac{VN}{2} \left[c(6a-16+9y) + 6a(y-3-r) \right], \quad C_{13}^l = -3\tau N(2-y), \quad C_{23}^l = 0, \quad (A.3) \\
A_3^{lt} &= -n_3 \frac{cQ_B^2}{Md} \left\{ 2Z_1(3b-a-r) + \Delta_1 \left[4r(1+b^2+3ab) - 2a(1+r^2) + c(ar-3+5br) \right] \right\}, \\
B_3^{lt} &= n_3 \frac{cQ_B^2}{Md} \left\{ 2Z_2 \left[(3b-a)r - 1 \right] + \Delta_2 \left[(1+ar)(a-6b) - (a+3b)(r^2+\Delta_2) + r(b^2-1) + b + \right. \right. \\
&\quad \left. \left. + \Delta_2(3r-2b) \right] \right\}, \quad C_{03}^{lt} = n_2 V \left\{ 4a \left[2b\bar{r} - y^2 + 4(b^2-a) \right] - c \left[3y(2-y) + 8a(1+a+2b) \right] \right\}, \\
C_{13}^{lt} &= -2N \frac{M}{d} (2-y)(y+2a), \quad C_{23}^{lt} = 0, \\
A_3^{tt} &= n_3 xy \frac{c}{b} \left\{ (\Delta_1 - 2b) \left[b(1+r^2) + (1-r+ry)\Delta_1 \right] + b\Delta_1 \left[1 + r(b-a+\Delta_1) \right] \right\}, \\
B_3^{tt} &= n_3 xy \frac{c}{b} \left\{ (2br - \Delta_2) \left[b(1+r^2) + (1-r-y)\Delta_2 \right] + b\Delta_2 \left[r(b-a+r) - \Delta_2 \right] \right\}, \\
C_{03}^{tt} &= \frac{VN}{2b} \left\{ 3b-a-(a^2+b^2)(2+a+b) + r \left[y^2 + 2y(2b-a) + 2a(3-a) \right] + c \left[y(1+y+3a) - 4(1+a) - 2ab \right] \right\}, \\
C_{13}^{tt} &= -\frac{VN}{2d^2} (2-y) \left[y^2 + 2a(2-b) \right], \quad C_{23}^{tt} = 0.
\end{aligned}$$

The coefficients determining the contribution proportional to the B_4 structure function are:

$$\begin{aligned}
A_4^l &= n_3 \frac{c^2}{\tau} \left\{ (b-a)(1+r^2) + \Delta_1 \left[1 + r(2a-b) \right] \right\}, \quad B_4^l = n_3 \frac{c^2}{\tau} \left[(a-b)(1+r^2) + \Delta_2(a+\bar{r}) \right], \\
C_{04}^l &= -\frac{cNV}{2} \left[1 + 3(b-a) \right], \quad C_{14}^l = C_{24}^l = 0, \\
A_4^{lt} &= n_3 \frac{c^2 Q_B^2}{Md} \left\{ 2b(1+r^2) + \Delta_1 \left[1 - r(3b-a) \right] \right\}, \quad B_4^{lt} = n_3 \frac{c^2 Q_B^2}{Md} \left[-2b(1+r^2) + \Delta_2(\bar{r}-2b) \right],
\end{aligned}$$

$$C_{04}^{lt} = -c \frac{NV^2}{2Md} [1 + 4ab - (a - b)^2], \quad C_{14}^{lt} = C_{24}^{lt} = 0, \quad (\text{A.4})$$

$$A_4^{tt} = -xyrc^2 \Delta_1 n_3, \quad B_4^{tt} = -xyc^2 \Delta_2 n_3, \quad C_{04}^{tt} = -\frac{cNV}{2}(y + 2a), \quad C_{14}^{tt} = C_{24}^{tt} = 0,$$

here we use the following notation

$$c = z + xyr, \quad \bar{r} = a - b + r, \quad n_1 = \frac{N}{2} \frac{1 + r^2}{1 - r} V Q_B^2, \quad n_2 = \frac{NV}{2Md}, \quad n_3 = \frac{N}{2c} \frac{V^2}{1 - r},$$

$$d^2 = bQ_B^2, \quad \Delta_1 = (1 - xy)r - a - b - z, \quad \Delta_2 = (1 - y + xy)r + z - 1, \quad N = \frac{4\tau}{Vc^2},$$

$$Z_1 = b(1 + r^2) + \Delta_1(1 - r + yr), \quad Z_2 = b(1 + r^2) + \Delta_2(1 - y - r).$$

6 Appendix B

In this Appendix we consider the part of the integral I which is caused by the R_{ln} and R_{tn} components of the deuteron quadrupole-polarization tensor (these components do not contribute to the differential cross section treated in the Born approximation). Let us define the integral which is caused by the R_{ln} component

$$I_{ln} = \int \frac{d^3 \vec{k}}{2\pi\omega} \Sigma_{ln}(z, r, \varphi) R_{ln}, \quad (\text{B.1})$$

with

$$\Sigma_{ln}(z, r, \varphi) = \frac{\alpha^2(q^2)}{Q_B^4} \frac{2VN}{Mr^2} n \cdot q \left(\frac{P_{1ln}}{\chi_1} - \frac{P_{2ln}}{\chi_2} + U_{0ln} + U_{1ln} \chi_1 \right),$$

$$P_{1ln} = \frac{V}{1 - r} \left\{ cgyx(1 + r^2)B_1 + g[c(1 - r(1 - y)) + a(1 + r^2) - 4fr]B_2 + [2a - fr + \right.$$

$$\left. + \frac{1}{2}(3(1 - r + yr) + 2ar)]B_3 + \frac{1}{2}c[1 + (a - b)r]B_4 \right\}, \quad (\text{B.2})$$

$$P_{2ln} = -\frac{V}{1 - r} \left\{ -xy(1 + r^2)(c + 2ar)B_1 + \frac{2}{c}[-ar(a(1 + r^2) - 4fr) + \frac{c^2}{2}(1 - y - r) + \right.$$

$$\left. + \frac{c}{2}(4fr - a(1 + 3r^2 + 2yr - 2r))]B_2 + [r(f - 2ar) - \frac{c}{2}(2a + 3(r + y - 1))]B_3 - \frac{c}{2}(a - b + r)B_4 \right\},$$

$$U_{0ln} = 2g(cB_1 + \tau B_2) + 2\tau(2 - y)(B_2 + B_3), \quad U_{1ln} = \frac{4\tau}{V}(B_1 + \frac{\tau}{c}B_2),$$

and $c = z + xyr$, $f = 1 + (1 - y)^2$, $g = 1 + 2a/c$, $n \cdot q = S_\mu^{(n)} q_\mu$.

The second integral, caused by the R_{tn} component, is defined as

$$I_{tn} = \int \frac{d^3 \vec{k}}{2\pi\omega} \Sigma_{tn}(z, r, \varphi) R_{tn}, \quad (\text{B.3})$$

where the integrand is

$$\Sigma_{tn}(z, r, \varphi) = \frac{\alpha^2(q^2)}{Q_B^4} \frac{2VN}{dr^2(r - 1)} n \cdot q \left(\frac{P_{1tn}}{\chi_1} - \frac{P_{2tn}}{\chi_2} + U_{0tn} + U_{1tn} \chi_1 \right),$$

$$\begin{aligned}
P_{1tn} &= Q_B^2 \left\{ \bar{f}xy(1+r^2)B_1 - \frac{\bar{f}}{c} [a(1+r^2) - 4r(f+4y)]B_2 + \right. \\
&\quad \left. + \bar{f}[1+r(y-1)](B_2+B_3) + brc(B_3+B_4) + 2br(1-y)B_3 \right\}, \\
P_{2tn} &= -Q_B^2 \left\{ xy\bar{g}(1+r^2)B_1 + \frac{\bar{g}}{c} [a(1+r^2) - 4fr]B_2 + \right. \\
&\quad \left. + \bar{g}(r-1+y)(B_2+B_3) - bc(B_3+B_4) - 2br(1-y)B_3 \right\}, \\
U_{0tn} &= (r-1) \left[-2xy\bar{f}(B_1 + \frac{\tau}{c}B_2) + (2-y)(2a+y)(B_2+B_3) \right], \\
U_{1tn} &= \frac{1}{V} \left\{ (r-1)(2a+y)G_1 - \bar{f}G_2 - \frac{y-2}{Mxy} [M(2a+y) - 2\tau d](B_2+B_3) - \frac{d}{Mxy} [c-2a(r-2)]G_2 \right\}, \\
G_1 &= 3B_1 + \frac{2\tau}{c}B_2 + \frac{c}{2xyr}(B_2+2B_3+B_4), \quad G_2 = -B_1 - \frac{c}{2xyr}(B_2+2B_3+B_4),
\end{aligned} \tag{B.4}$$

and $d^2 = bQ_B^2$, $\bar{f} = b - a - z + r(1 - xy)$, $\bar{g} = z - 1 + r(a - b + xy)$.

As before, we calculate the above integrals in c.m.s. of the hard photon and undetected hadron system: $\vec{k}_1 - \vec{k}_2 + \vec{p} = 0$. The electron momenta \vec{k}_1 and \vec{k}_2 define the xz plane, z axis is directed along the deuteron momentum \vec{p} . Then the hard-photon momentum \vec{k} is determined by the azimuthal (φ) and polar (θ) angles, and the phase space of the hard photon can be written as

$$\frac{d^3\vec{k}}{2\pi\omega} = \frac{Q_B^2}{2\sqrt{y^2+4a}} \frac{d\varphi}{2\pi} dz dr, \tag{B.5}$$

where ω is the hard-photon energy.

The quantity $n \cdot q$ can be written in this coordinate system as $n \cdot q = \bar{n} \sin\varphi$, where \bar{n} is a factor independent on φ . Then the integration over the φ variable in region $(0, 2\pi)$ leads to the following result: $I_{ln} = I_{tn} = 0$. So, the R_{ln} and R_{tn} components of the deuteron quadrupole-polarization tensor do not contribute to the differential cross section of deep-inelastic scattering of unpolarized electron beam by the tensor polarized target. This result is due to the fact that only the scattered-electron variables are measured (it corresponds to the HERA experimental conditions, for example).

If the hard-photon is detected then the I_{ln} and I_{tn} survive and the expressions for Σ_{ln} and Σ_{tn} have to be taken into account

7 Appendix C

In this Appendix we give some formulae describing the polarization state of the deuteron target for different cases. For the case of arbitrary polarization of the target it is described by the general spin-density matrix (in general case it is defined by 8 parameters) which in the coordinate representation has the form

$$\rho_{\mu\nu} = -\frac{1}{3}(g_{\mu\nu} - \frac{p_\mu p_\nu}{M^2}) - \frac{i}{2M} \varepsilon_{\mu\nu\lambda\rho} s_\lambda p_\rho + Q_{\mu\nu}, \quad Q_{\mu\nu} = Q_{\nu\mu}, \quad Q_{\mu\mu} = 0, \quad p_\mu Q_{\mu\nu} = 0, \tag{C.1}$$

where p_μ is the deuteron 4-momentum, s_μ and $Q_{\mu\nu}$ are the deuteron polarization 4-vector and the deuteron quadrupole-polarization tensor.

In the deuteron rest frame the above formula is written as

$$\rho_{ij} = \frac{1}{3}\delta_{ij} + \frac{i}{2}\varepsilon_{ijk}s_k + Q_{ij}, \quad ij = x, y, z. \quad (\text{C.2})$$

This spin-density matrix can be written in the helicity representation using the following relation

$$\rho_{\lambda\lambda'} = \rho_{ij}e_i^{(\lambda)*}e_j^{(\lambda')}, \quad \lambda, \lambda' = +, -, 0, \quad (\text{C.3})$$

where $e_i^{(\lambda)}$ are the deuteron spin functions which have the deuteron spin projection λ on to the quantization axis (z axis). They are

$$e^{(\pm)} = \mp \frac{1}{\sqrt{2}}(1, \pm i, 0), \quad e^{(0)} = (0, 0, 1). \quad (\text{C.4})$$

The elements of the spin-density matrix in the helicity representation are related to the ones in the coordinate representation by such a way

$$\rho_{\pm\pm} = \frac{1}{3} \mp \frac{1}{2}s_z - \frac{1}{2}Q_{zz}, \quad \rho_{00} = \frac{1}{3} + Q_{zz}, \quad \rho_{+-} = -\frac{1}{2}(Q_{xx} - Q_{yy}) + iQ_{xy}, \quad (\text{C.5})$$

$$\rho_{+0} = -\frac{1}{2\sqrt{2}}(s_x - is_y) - \frac{1}{\sqrt{2}}(Q_{xz} - iQ_{yz}), \quad \rho_{-0} = -\frac{1}{2\sqrt{2}}(s_x + is_y) + \frac{1}{\sqrt{2}}(Q_{xz} + iQ_{yz}), \quad \rho_{\lambda\lambda'} = (\rho_{\lambda'\lambda})^*.$$

To obtain these relations we use $Q_{xx} + Q_{yy} + Q_{zz} = 0$.

The polarized deuteron target which is described by the population numbers n_+ , n_- and n_0 is often used in the spin experiments. Here n_+ , n_- and n_0 are the fractions of the atoms with the nuclear spin projection on to the quantization axis $m = +1$, $m = -1$ and $m = 0$, respectively. If the spin-density matrix is normalized to 1, i.e. $Spp = 1$, then we have $n_+ + n_- + n_0 = 1$. Thus, the polarization state of the deuteron target is defined in this case by two parameters: the so-called V (vector) and T (tensor) polarizations

$$V = n_+ - n_-, \quad T = 1 - 3n_0. \quad (\text{C.6})$$

Using the definitions for the quantities $n_{\pm,0}$

$$n_{\pm} = \rho_{ij}e_i^{(\pm)*}e_j^{(\pm)}, \quad n_0 = \rho_{ij}e_i^{(0)*}e_j^{(0)}, \quad (\text{C.7})$$

we have the following relation between V and T parameters and parameters of the spin-density matrix in the coordinate representation (in the case when the quantization axis is directed along the z axis)

$$n_0 = \frac{1}{3} + Q_{zz}, \quad n_{\pm} = \frac{1}{3} \mp \frac{1}{2}s_z - \frac{1}{2}Q_{zz}, \quad (\text{C.8})$$

or

$$T = -3Q_{zz}, \quad V = -s_z. \quad (\text{C.9})$$

References

- [1] D. Ryckbosch, arXiv:hep-ex/0311021.
- [2] Spin Muon Collaboration. D. Adams, B. Adeva, E. Arik et al., Phys. Lett. **B 357**, 248 (1995); P. L. Anthony, R. G. Arnold, H. R. Band et al., Phys. Rev. **D 54**, 6620 (1996).
- [3] S. Kumano, Preprint MKPH-T-93-03, Univ. Mainz, Germany, 1993.
- [4] P. Hoodbhoy, R.L. Jaffe and A. Manohar, Nucl. Phys. **B312**, 571 (1989).
- [5] F. E. Close, S. Kumano, Phys. Rev. **D 42**, 2377 (1990).
- [6] L. Mankiewicz, Phys. Rev. **D 40**, 255 (1989).
- [7] H. Khan and P. Hoodbhoy, Phys. Lett. **B298**, 181 (1993).
- [8] H. Khan and P. Hoodbhoy, Phys. Rev. **C44**, 1219 (1991).
- [9] S. Kumano, arXiv:hep-ph/0012341.
- [10] M. Contalbrigo, arXiv:hep-ex/0211014.
- [11] N. N. Nikolaev et al., Phys. Lett. **B 398**, 245 (1997).
- [12] J. Edelmann et al., Phys. Rev. **C 57**, 254 (1998).
- [13] K. Bora et al., Phys. Rev. **D 57**, 6906 (1998).
- [14] I. V. Akushevich and N. M. Shumeiko, J. Phys. G: Nucl. Part. Phys. **20**, 513 (1994).
- [15] S. Y. Choi, T. Lee and H. S. Song, Phys. Rev. **D 40**, 2477 (1989).
- [16] G. I. Gakh, N. P. Merenkov, Pis'ma v ZhETF **73**, 659 (2001).
- [17] E. A. Kuraev, N. P. Merenkov, V. S. Fadin, Yad. Fiz. **45**, 782 (1987); Sov. J. Nucl. Phys. **45**, 486 (1987).
- [18] T. V. Kukhto, and N. P. Shumeiko, Nucl. Phys. **B219**, 412 (1983).
- [19] E. A. Kuraev, N. P. Merenkov, V. S. Fadin, Yad. Fiz. **47**, 1593 (1988).
- [20] L. W. Whitlow et al., Phys. Lett. **B250**, 193 (1990).
- [21] ALLM97 parameterization:H. Abramowicz et al., hep-ph/9712415.
- [22] NMC Coll., P. Amaudruz et al., Nucl. Phys. **B371**, 3 (1992).
- [23] M. Contalbrigo, private communication.