Quantum-chromodynamic evolution of six-quark states

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The evolution of six-quark color-singlet state distribution amplitudes is formulated as an application of perturbative quantum chromodynamics to nuclear wave functions. We derive and solve a set of coupled evolution equations for the deuteron S-wave amplitude. The solution of the evolution equations leads to a general matrix representation of anomalous dimensions which can be used to analyze the deuteron wave function at short distances.

I. INTRODUCTION

In the past few years a number of new applications of perturbative quantum chromodynamics to nuclear physics have been explored[1] including a qualitative description of the nuclear force in terms of quark exchange, and detailed predictions for the electromagnetic interactions of nuclei at large momentum transfer. Since the basic scale of QCD, $\Lambda_{\overline{\rm MS}}$ (where $\overline{\rm MS}$ is the modified minimal-subtraction scheme), is phenomenologically of the order of a few hundred MeV or less, QCD predicts a transition from the traditional meson and nucleon degrees of freedom of nuclear physics to quark and gluon degrees of freedom at internucleon separations of a fermi or less. In addition, because of asymptotic freedom, perturbative QCD calculations should become relevant at momentum-transfer scales of the order of 1 GeV or even less.[2]

Recently, we have presented detailed QCD predictions for the asymptotic high-Q^2 behavior of the deuteron form factor which are, in principle, exact dynamical predictions of nuclear physics.[3] One of the most convenient and physical formalisms for analyzing exclusive processes with large transverse momenta is the QCD evolution formalism, based on a reformulation of the Bethe-Salpeter equation at equal light-cone time. In this paper, we present a detailed derivation of a set of six-quark evolution equations for the deuteron S-wave amplitude and a convenient way to solve the derived equations. We then construct a general matrix representation of the anomalous dimensions from explicit solutions of the evolution equations in order to analyze the deuteron wave function at short distances.

The evolution of the amplitude for simpler hadrons such as quark-antiquark meson[4,5] and three-quark baryon[4] systems have already been formulated and solved. While these conventional hadrons have only one color-singlet representation, the six-quark systems considered here have five independent color-singlet representations. The formulation of the evolution equation for totally antisymmetric six-quark states is not trivial even though it is a natural extension of the three-quark case.[4] We have presented a general method for solving the QCD evolution equations which govern relativistic multiquark wave functions.[6] We have also applied it to a four-quark toy system in $SU(2)_C$ and derived some constraints on the effective force between two baryons.[7] However, since the antisymmetric representation of a multiquark wave function must be constructed explicitly, it is hard, in practice, to solve the multiquark evolution equation. In this paper we avoid this problem by exploiting the permutation symmetry of the evolution kernel.

In Sec. II, a completely antisymmetric six-quark wave function is constructed and an example of an explicit representation is presented. In Sec. III we derive a set of evolution equations for the deuteron S-wave amplitude through a generalized kernel equation for a completely antisymmetric six-quark wave function. A convenient way to solve these coupled evolution equations is presented in Sec. IV. In Sec. V the general matrix representation for the anomalous dimension is obtained. Results for the leading anomalous dimension are given in detail. Discussions and conclusions are followed in Sec. VI. In Appendix A we describe a general method determining color-singlet representations and explain the methods leading to the explicit representations given in Sec. II. In Appendix B we present the color-factor calculations and the detailed expressions for the orthogonal kernels which have specific permutation symmetries.

II. SIX-QUARK STATES

Six-quark states can be classified by their symmetries under $SU(3)_C$ (color), $SU(2)_T$ (isospin), $SU(2)_S$ (spin), and spatial symmetry. Since the physical states are color singlets, the Young symmetry of the color-singlet states of the six-quark system is $f_C=(222)$ or ⊞ . In the six-quark system, there are five independent color-singlet states corresponding to five different Yamanouchi labels[8] of (222) symmetry. The explicit representations of the five independent color-singlet states and their correspondence to Yamanouchi labels are given in Appendix A.

The completely antisymmetric six-quark representation

$|(1^6)[654321]\rangle_{f_T,f_{CSO}}$ (Ref. 9), which has Young symmetries for isospin [$SU(2)_T$] labeled by f_T, is given by[10]

$$|(1^6)[654321]\rangle_{f_T,f_{CSO}} = \frac{1}{(N_T)^{1/2}} \sum_{Y_T} \eta_T |f_T Y_T\rangle_{f_T} |\tilde{f}_T \tilde{Y}_T\rangle_{f_{CSO}} , \quad (2.1)$$

where Y_T is the allowed Yamanouchi label of f_T (f_T symmetry has N_T different Yamanouchi labels) and the phase $\eta_T = \pm 1 = (-1)^{Y_T}$ depends on whether Y_T is obtained from the Yamanouchi label with the indices in natural order by an even or odd number of transpositions. The dual symmetry states in the *CSO* space (represented by wavy lines) are used to construct the overall antisymmetric representations. It is convenient to construct a basis of completely antisymmetric six-quark representations from the combined color-spin symmetry. We thus introduce color spin as an intermediate representation. For example, the specific projection of the completely antisymmetric six-quark representation $|(1^6)[654321]\rangle_{f_T,f_{CS},f_O}$,[9] which has Young symmetries for color-spin [$SU(6)_{CS}$], and orbital symmetry labeled by f_{CS}, and f_O, respectively, is represented by[10]

$$|(1^6)[654321]\rangle_{f_T,f_{CS},f_O} = \frac{1}{(N_T)^{1/2}} \sum_{Y_T} \sum_{Y_{CS}} \sum_{Y_O} \eta_T \langle f_{CS} Y_{CS}, f_O Y_O | \tilde{f}_T \tilde{Y}_T \rangle |f_T Y_T\rangle |f_{CS} Y_{CS}\rangle |f_O Y_O\rangle , \quad (2.2)$$

where Y_{CS} and Y_O are the allowed Yamanouchi labels of f_{CS} and f_O, and $\langle f_{CS} Y_{CS}, f_O Y_O | \tilde{f}_T \tilde{Y}_T\rangle$ are Clebsch-Gordan coefficients of the permutation group S_6. Now we further decompose f_{CS} into f_C and f_S using Clebsch-Gordan coefficients $\langle f_C Y_C, f_S Y_S | f_{CS} Y_{CS}\rangle$ so that Eq. (2.1) becomes

$$|(1^6)[654321]\rangle_{f_T,f_C,f_S,f_O} = \frac{1}{(N_T)^{1/2}} \sum_{Y_T} \sum_{Y_{CS}} \sum_{Y_O} \sum_{Y_C} \sum_{Y_S} (-1)^{Y_T} \langle f_{CS} Y_{CS}, f_O Y_O | \tilde{f}_T \tilde{Y}_T\rangle \langle f_C Y_C, f_S Y_S | f_{CS} Y_{CS}\rangle$$
$$\times |f_T Y_T\rangle |f_C Y_C\rangle |f_S Y_S\rangle |f_O Y_O\rangle . \quad (2.3)$$

Since our purpose is to formulate and solve a generalized evolution equation, it is useful to project Eq. (2.1) onto the light-cone momentum space of six quarks, each carrying light-cone longitudinal-momentum fraction $x_i = (q_0^i + q_3^i)/(P_0 + P_3)$ of the deuteron's momentum P_μ ($\sum_{i=1}^6 x_i = 1$) and transverse momentum $q_\perp^i$ ($\sum_{i=1}^6 q_\perp^i = 0$). The corresponding light-cone wave function of the six-quark system $\Psi_d(x_i, q_\perp^i)$ is defined by

$$\Psi_d(x_i, q_\perp^{(i)}) = \langle x_i, q_\perp^{(i)} | (1^6)[654321]\rangle_{f_T,f_{CS},f_O}$$
$$= \frac{1}{(N_T)^{1/2}} \sum_{Y_T=1}^{N_T} \sum_{Y_{CS}=1}^{N_{CS}} \sum_{Y_O=1}^{N_O} (-1)^{Y_T} \langle f_{CS} Y_{CS}, f_O Y_O | \tilde{f}_T \tilde{Y}_T\rangle \psi_o(x_i, q_\perp^{(i)}) |f_T Y_T\rangle |f_{CS} Y_{CS}\rangle , \quad (2.4)$$

where the orbital wave function is given by

$$\psi_o(x_i, q_\perp^{(i)}) = \langle x_i, q_\perp^{(i)} | f_O Y_O\rangle . \quad (2.5)$$

The remaining $SU(2)_T$ and $SU(6)_{CS}$ symmetries ($|f_T Y_T\rangle$ and $|f_{CS} Y_{CS}\rangle$) are given by specific tensor representations.

The probability amplitude for the constituents with light-cone momentum fraction x_i to combine into the hadron with relative transverse momentum up to the scale Q^2 is given by the distribution amplitude $\Phi_d(x_i, Q)$ defined by

$$\Phi_d(x_i, Q) \equiv \int^{Q^2} \prod_{i=1}^{6} \left[\frac{d^2 q_\perp^{(i)}}{16\pi^3} \right] 16\pi^3 \delta^2 \left[\sum_i q_\perp^{(i)} \right] \times \Psi_d^{(Q)}(x_i, q_\perp^{(i)}) , \quad (2.6)$$

where the Q dependence of $\Psi_d^{(Q)}$ comes from the renormalization of the quark fields.[11]

As an explicit example of a six-quark representation, we give a specific representation for $f_T = (33)$, $f_{CS} = (222)_C \times (6)_S$, and $f_O = (6)$ ($T=0$, $S = S_Z = 3$, and S wave):

$$\Phi_d(x_i, Q) = \frac{a_0}{48\sqrt{5}} [-\epsilon_{ijk}\epsilon_{lmn}(\epsilon_{ad}\epsilon_{be}\epsilon_{cf} + \epsilon_{ae}\epsilon_{bd}\epsilon_{cf} + \epsilon_{ad}\epsilon_{bf}\epsilon_{ce} + \epsilon_{af}\epsilon_{bd}\epsilon_{ce})$$
$$+ \epsilon_{ijl}\epsilon_{kmn}(\epsilon_{ac}\epsilon_{be}\epsilon_{df} + \epsilon_{ac}\epsilon_{bf}\epsilon_{de} + \epsilon_{ae}\epsilon_{bc}\epsilon_{df} + \epsilon_{af}\epsilon_{bc}\epsilon_{de}) - (\epsilon_{ijm}\epsilon_{kln} + \epsilon_{ijn}\epsilon_{klm})(\epsilon_{ac}\epsilon_{bd}\epsilon_{ef} + \epsilon_{ad}\epsilon_{bc}\epsilon_{ef})$$
$$+ (\epsilon_{ikm}\epsilon_{jln} + \epsilon_{ikn}\epsilon_{jlm} + \epsilon_{jkm}\epsilon_{iln} + \epsilon_{jkn}\epsilon_{ilm})\epsilon_{ab}\epsilon_{cd}\epsilon_{ef} - (\epsilon_{ikl}\epsilon_{jmn} + \epsilon_{jkl}\epsilon_{imn})(\epsilon_{ab}\epsilon_{ce}\epsilon_{df} + \epsilon_{ab}\epsilon_{cf}\epsilon_{de})]$$
$$\times a_i^\dagger(1) b_j^\dagger(2) c_k^\dagger(3) d_l^\dagger(4) e_m^\dagger(5) f_n^\dagger(6) x_1 x_2 x_3 x_4 x_5 x_6 \left[\ln \left[\frac{Q^2}{\Lambda^2} \right] \right]^{-\gamma_0} , \quad (2.7)$$

where the indices $i,j,\ldots,n$ and $a,b,\ldots,f$ are the color (r,y,b) and isospin (u,d) indices, respectively. The ϵ_{ijk}'s and ϵ_{ab}'s are the completely antisymmetric Cartesian tensors of $SU(3)_C$ and $SU(2)_T$. The coefficient a_0 is the normalization of the orbital distribution amplitude (see Sec. V). The leading anomalous dimension is γ_0. A detailed calculation of γ_0 for the various six-quark states and the tensor representations in Eq. (2.7) will be given in Sec. V and the Appendix A, respectively.

In the following section, we will derive a set of evolution equations for the deuteron S-wave distribution amplitude. Since the deuteron is an isospin singlet and the S wave is a symmetric orbital, the Young symmetry in each quantum space is given by $f_T=(33)_T$, $f_{CS}=(222)_{CS}$, and $f_O=(6)_O$. Thus, an example of the ground-state S wave of the deuteron distribution amplitude is represented by

$$|(1^6)\,\ydiagram{1,1,1,1,1,1}>_D=\frac{1}{\sqrt{5}}\sum_{Y_T}\sum_{Y_{CS}}\eta_T|\,\ydiagram{3,3}\,Y_T>|\,\ydiagram{2,2,2}\,Y_{CS}>|\,\ydiagram{6}\,[111111]_O> , \tag{2.8}$$

where $(6)_O$ has only one Yamanouchi label, $[111111]_O$ [the Clebsch-Gordan coefficient in Eq. (2.2) is trivially given by 1]. We will concentrate on the leading term in the high-Q^2 limit. It is an eigensolution of the evolution equation which will be derived in the next section. The leading term is the lowest power term of x_i-dependent polynomials, i.e., $x_1x_2x_3x_4x_5x_6$. For example, we can see that the basis element given by Eq. (2.7) is an eigensolution because only the symmetric tableau is allowed for the spin in $S_Z=3$ case. However, in general, and specifically for the deuteron, the eigensolutions will be given by mixing of basis elements in spin space.

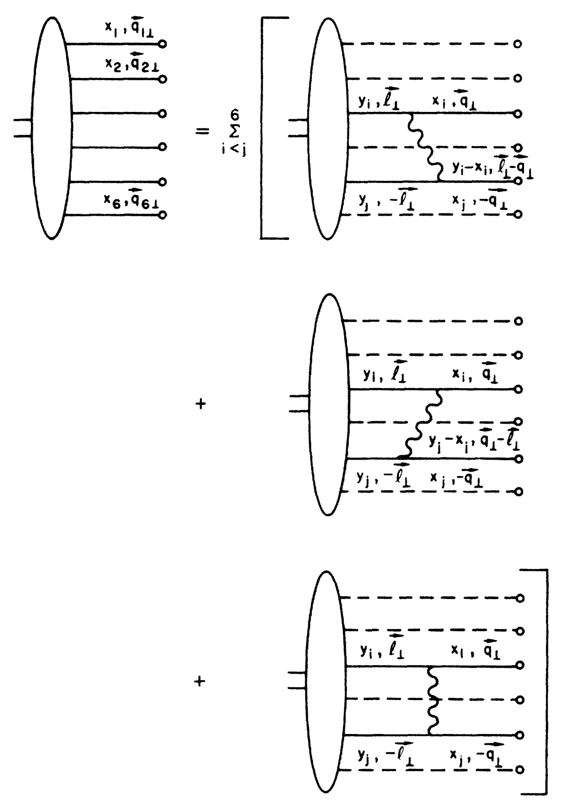

FIG. 1. The leading-order contributions to the kernel of the six-quark wave function in light-cone perturbation theory. The longitudinal momentum of particle i ($i=1,2,\ldots,6$) before and after the interactions is y_i and x_i, respectively; the i and j particles interact with transverse-momentum transfer of order Q. Fifteen diagrams are included by summation over i and j with $i\neq j$. The Feynman rules of light-cone perturbation theory in the light-cone gauge are summarized in Ref. 4.

III. EVOLUTION EQUATIONS FOR THE DEUTERON

Each eigensolution of a six-quark state satisfies a kernel equation of the form

$$K\,|(1^6)[654321]\rangle_e=e\,|(1^6)[654321]\rangle_e\ , \tag{3.1}$$

where $|(1^6)[654321]\rangle_e$ is an eigensolution with the eigenvalue e given by a linear combination of completely antisymmetric representations (basis elements). The kernel K is calculated to leading order in $\alpha_s(Q^2)$ from one-gluon exchange by using light-cone perturbation theory. It is given explicitly in Sec. V. The pairwise one-gluon-exchange diagrams are shown in Fig. 1. Since the isospin representation does not change through gluon exchange, the $SU(2)_T$ symmetry has no mixing and is fixed. For example, $f_T=(33)$ in the deuteron case. Also, the six-quark states are always color-singlet states, i.e., $f_C=(222)$. Thus, in general, the evolution equation has the following form in the basis of Eq. (2.3):

$$\sum_{f_{CS}}\sum_{f_S}\sum_{f_O}\sum_{Y_{CS}}\sum_{Y_C}\sum_{Y_S}\sum_{Y_O}C_{f_Of_S}^{f_{CS}}\langle f_CY_C,f_SY_S\,|\,f_{CS}Y_{CS}\rangle\langle f_{CS}Y_{CS},f_OY_O\,|\,\tilde f_T\tilde Y_T\rangle K\,|f_CY_C\rangle\,|f_SY_S\rangle\,|f_OY_O\rangle$$
$$=e\sum_{f_{CS}}\sum_{f_S}\sum_{f_O}\sum_{Y_{CS}}\sum_{Y_C}\sum_{Y_S}\sum_{Y_O}C_{f_Of_S}^{f_{CS}}\langle f_CY_C,f_SY_S\,|\,f_{CS}Y_{CS}\rangle\langle f_{CS}Y_{CS},f_OY_O\,|\,\tilde f_T\tilde Y_T\rangle\,|f_CY_C\rangle\,|f_SY_S\rangle\,|f_OY_O\rangle\ . \tag{3.2}$$

The unknown coefficients $C_{f_Of_S}^{f_{CS}}$ and the eigenvalue e are obtained by solving Eq. (3.2). In Eq. (3.2) the possible Young tableaux f_{CS} and f_O in the sums are determined by the Clebsch-Gordan series of S_6 to produce the CSO Young-tableau $\tilde f_T$. Likewise, the possible Young tableaus f_S are determined to produce f_{CS} after combining with f_C. Each possible combination of f_S, f_{CS}, and f_O gives an equation (3.2). The combinations of f_S, f_{CS}, and f_O

are given by coefficients of $C^{f_{CS}}_{f_O f_S}$ and has a corresponding eigenvalue e.

In general, there are many possible combinations of f_S, f_{CS}, and f_O contributing to Eq. (3.2), since only f_T and f_C are fixed and mixed Young symmetries are allowed. However, if we constrain ourselves to some special cases, then only a few equations actually need to be solved. For example, the leading term in the high-Q^2 limit of the $S_Z=3$ and $T=0$ amplitude has only one possible combination of f_S, f_{CS}, and f_O [i.e., $f_T=(33)$, $f_C=(222)$, $f_S=(6)$, $f_{CS}=(222)$, $f_O=(6)$], and only one equation needs be constructed for this special case. Therefore, we can easily see that the explicit representation given by Eq. (2.7) in an eigensolution itself, as we already mentioned in Sec. II.

In any case, the unknown coefficients $C^{f_{CS}}_{f_O f_S}$ and eigenvalue e in Eq. (3.2) can be determined after a given set of equations (corresponding to the number of possible combinations of f_S, f_{CS}, and f_O) are solved.

In this paper we will concentrate on analyzing the asymptotic amplitude which dominates exclusive processes at large transverse momenta such as the asymptotic high-Q^2 behavior of the deuteron form factor. However, the general equation is given by Eq. (3.2), and the method which we present in the rest of this section can be applied to arbitrary cases. Since a deuteron is isospin singlet and the dominant degree of freedom in the high-Q^2 limit is S wave, the eigensolutions which we are considering have $f_T=(33)$ and $f_O=(6)$. In the $S_Z=1$ case, one has mixing between $f_S=(6)_S$ and $f_S=(42)_S$.

For the $f_T=(33)_T, f_O=(6)_O$ case, Eq. (3.2) becomes

$$\sum_{f_S}\sum_{Y_C}\sum_{Y_S} C^{(222)}_{(6)f_S}\langle (222)_C Y_C, f_S Y_S \mid (222)_{CS} Y_{CS}\rangle K \mid (222)_C Y_C\rangle \mid f_S Y_S\rangle \mid (6)_O[111111]_O\rangle$$

$$= e\sum_{f_S}\sum_{Y_C}\sum_{Y_S} C^{(222)}_{(6)f_S}\langle (222)_C Y_C, f_S Y_S \mid (222)_{CS} Y_{CS}\rangle \mid (222)_C Y_C\rangle \mid f_S Y_S\rangle \mid (6)_O[111111]_O\rangle\ , \qquad (3.3)$$

where $f_{CS}=\tilde{f}_T=(3\tilde{3})=(222)$ and $Y_{CS}=\tilde{Y}_T$ because

$\langle f_{CS}Y_{CS},(6)_O[111111]_O \mid \tilde{f}_T\tilde{Y}_T\rangle = \delta_{f_{CS}\tilde{f}_T}\delta_{Y_{CS}\tilde{Y}_T}$.

Since we know from the Clebsch-Gordan series of S_6 that the possible values of f_S are (6) and (42), we have two unknown coefficients $C^{(222)}_{(6)(6)}$ and $C^{(222)}_{(6)(42)}$ which must satisfy the normalization condition

$$(C^{(222)}_{(6)(6)})^2+(C^{(222)}_{(6)(42)})^2=1\ . \qquad (3.4)$$

Because of the normalization condition (3.4), one can define two eigensolutions in terms of one mixing angle θ. The corresponding eigenvalues are

$$\begin{aligned} e&=e_1 \quad \text{when } C^{(222)}_{(6)(6)}=\cos\theta,\ C^{(222)}_{(6)(42)}=\sin\theta\ ,\\ e&=e_2 \quad \text{when } C^{(222)}_{(6)(6)}=-\sin\theta,\ C^{(222)}_{(6)(42)}=\cos\theta\ . \end{aligned} \qquad (3.5)$$

Furthermore, the kernel K has the factorized color factor corresponding to one-gluon exchange,

$$K=\sum_{i\neq j}\left[\frac{\lambda i}{2}\cdot\frac{\lambda j}{2}\right]V_{ij}\ , \qquad (3.6)$$

where each term represents the kernel given by the interaction between the ith and jth quark and each component of the eight-dimensional vector $\boldsymbol{\lambda}$ is the Gell-Mann matrix of the $SU(3)_C$ group. If we sandwich Eq. (3.5) between two color states which have $Y_C=\alpha$ and $Y_C=\beta$, respectively, we can define a 5×5 matrix representation of K whose elements are given by

$$K_{\alpha\beta}\equiv\langle (222)_C\alpha \mid K \mid (222)_C\beta\rangle=\sum_{i\neq j}^{6} C_{\alpha\beta}(i,j)V_{ij}\ , \qquad (3.7)$$

where

$$C_{\alpha\beta}(i,j)=\left\langle (222)_C\alpha\left|\frac{\lambda_i}{2}\cdot\frac{\lambda_j}{2}\right|(222)_C\beta\right\rangle\ . \qquad (3.8)$$

The most important observation in this formulation is that the kernel K is a linear combination of the operators Θ_{fY} in color space, each of which has a definite Young symmetry f with Yamanouchi labels Y:

$$K=\sum_{fY}K_{fY}\Theta_{fY}\ .$$

Therefore the kernel element $K_{\alpha\beta}$ can be rewritten in terms of the kernel K_{fY},

$$\begin{aligned} K_{\alpha\beta}&=\left\langle (222)_C\alpha\left|\sum_{fY}K_{fY}\Theta_{fY}\right|(222)_C\beta\right\rangle\\ &=\sum_f\sum_Y\langle (222)_C\alpha,fY \mid (222)_C\beta\rangle K_{fY}\ , \end{aligned} \qquad (3.9)$$

where the possible f which gives the nonzero Clebsch-Gordan coefficient $\langle (222)_C\alpha,fY \mid (222)_C\beta\rangle$ is only (6) or (42). One can rewrite K_{fY} in terms of color factors $C_{\alpha\beta}(i,j)$ and V_{ij}:

$$K_{(6)[111111]}=\tfrac{1}{5}\sum_\alpha\sum_{i\neq j}C_{\alpha\alpha}(i,j)V_{ij}=-\tfrac{1}{5}C_F\sum_{i\neq j}V_{ij}\quad (C_F=\tfrac{4}{3})\ , \qquad (3.10)$$

$$K_{(42)Y}=\tfrac{9}{5}\sum_\alpha\sum_\beta\langle (222)_C\alpha,(42)Y \mid (222)_C\beta\rangle C_{\alpha\beta}(i,j)V_{ij}\quad (Y=1,\ldots,9)\ . \qquad (3.11)$$

In Appendix B, we present details of the derivation of Eqs. (3.10) and (3.11) and the color factors of Eq. (3.8). Note there are five possible labels Y_{CS} in correspondence with the labels $\beta=1,\ldots,5$ of Y_C. If we project Eq. (3.3) with $Y_{CS}=\beta$ by a color-singlet state $Y_C=\alpha$, then we obtain a set of two equations using Eqs. (3.5) through (3.11). One of them is given by

$$\begin{aligned}
&\cos\theta\left[\delta_{\alpha\beta}K_{(6)[111111]}+\sum_{Y}\langle(222)_C\alpha,(42)Y\,|\,(222)_{CS}\beta\rangle K_{(42)Y}\right]|(6)_S[111111]_S\rangle\,|(6)_O[111111]_O\rangle\\
&\quad+\sin\theta\left[\sum_{Y_S}\langle(222)_C\alpha,(42)_SY_S\,|\,(222)_{CS}\beta\rangle K_{(6)[111111]}\right.\\
&\qquad\left.+\sum_{Y_S}\sum_{Y}\sum_{\gamma}\langle(222)_C\gamma,(42)_SY_S\,|\,(222)_{CS}\beta\rangle\langle(222)_C\alpha,(42)Y\,|\,(222)_C\gamma\rangle K_{(42)Y}\right]|(42)_SY_S\rangle\,|(6)_O[111111]_O\rangle\\
&\qquad\qquad=e_1\left[\cos\theta\delta_{\alpha\beta}\,|(6)_S[111111]_S\rangle\,|(6)_O[111111]_O\rangle\right.\\
&\qquad\qquad\qquad\left.+\sin\theta\sum_{Y_S}\langle(222)_C\alpha,(42)_SY_S\,|\,(222)_{CS}\beta\rangle\,|(42)_SY_S\rangle\,|(6)_O[111111]_O\rangle\right], \qquad (3.12)
\end{aligned}$$

and another is given by substitutions $\theta\to\theta+\pi/2$ and $e_1\to e_2$ in Eq. (3.12).

Combining the two equations and using properties of Clebsch-Gordan coefficients, we obtain the following set of evolution equations (we drop the trivial orbital factor $|(6)_O[111111]_O\rangle$):

$$K_{(6)[111111]}\,|(6)_S[111111]_S\rangle=(e_1\cos^2\theta+e_2\sin^2\theta)\,|(6)_S[111111]_S\rangle\ , \qquad (3.13)$$

$$\tfrac{1}{9}\sum_{Y}K_{(42)Y}\,|K_{(42)S}Y\rangle=(e_1-e_2)\cos\theta\sin\theta\,|(6)_S[111111]_S\rangle\ , \qquad (3.14)$$

$$K_{(42)Y}\,|(6)_S[111111]_S\rangle=(e_1-e_2)\cos\theta\sin\theta\,|(42)_SY\rangle\ , \qquad (3.15)$$

$$\begin{aligned}
K_{(6)[111111]}\,|(42)_SY\rangle+\tfrac{9}{5}\sum_{\alpha}\sum_{\beta}\sum_{\gamma}\sum_{Y_S}\sum_{Y_K}&\langle(222)_C\beta,(42)_SY\,|\,(222)_{CS}\alpha\rangle\langle(222)_C\gamma,(42)_SY_S\,|\,(222)_{CS}\beta\rangle\\
&\times\langle(222)_C\alpha,(42)Y_K\,|\,(222)_C\gamma\rangle K_{(42)Y_K}\,|(42)_SY_S\rangle\\
&=(e_1\sin^2\theta+e_2\cos^2\theta)\,|(42)_SY\rangle\ . \qquad (3.16)
\end{aligned}$$

Since the operator order of the K_{fY} is irrelevant, $[K_{(6)[111111]},K_{(42)Y}]=0$, we can see, from Eqs. (3.13) and (3.15), that

$$K_{(6)[111111]}\,|(42)_SY\rangle=(e_1\cos^2\theta+e_2\sin^2\theta)\,|(42)_SY\rangle\ . \qquad (3.17)$$

Furthermore, we can prove the following property of Clebsch-Gordan coefficients:

$$\begin{aligned}
\sum_{\alpha}\sum_{\beta}\sum_{\gamma}&\langle(222)_C\beta,(42)_SY\,|\,(222)_{CS}\alpha\rangle\langle(222)_C\gamma,(42)_SY_S\,|\,(222)_{CS}\beta\rangle\\
&\times\langle(222)_C\alpha,(42)Y_K\,|\,(222)_C\gamma\rangle=\frac{5\sqrt{53}}{108}\langle(42)_SY_S,(42)Y_K\,|\,(42)_SY\rangle\ . \qquad (3.18)
\end{aligned}$$

Thus, if we combine Eqs. (3.16) through (3.18), then we obtain

$$\sum_{Y_S}\sum_{Y_K}\frac{\sqrt{53}}{12}\langle(42)_SY_S,(42)Y_K\,|\,(42)_SY\rangle K_{(42)Y_K}\,|(42)_SY_S\rangle=(e_1-e_2)(\sin^2\theta-\cos^2\theta)\,|(42)_SY\rangle\ . \qquad (3.19)$$

Equations (3.13)–(3.15) and (3.19) appear to be independent. However, we can see that only three combinations of e_1, e_2, and θ can be determined in the above equations. So we need to solve three equations [for example, Eqs. (3.13), (3.15), and (3.19)] and the other equation, (3.14), can be used to check the results. We will describe a convenient way to solve these equations in the next section.

IV. SYMMETRY OF KERNEL EQUATIONS

The symmetry of the left- and right-hand sides of any of the equations derived in the preceding section should be conserved. This can be easily checked since each kernel and eigenstate has definite symmetry. If the kernel is symmetric, then the states of left- and right-hand side have the same symmetry. For the simple case where the

x_i dependence of the orbital distribution amplitude is given by the lowest power, such as the example presented in the preceding section, all the equations become an eigenvalue equation where the eigenvalue is determined by S_Z. This explains why Eqs. (3.13) and (3.17) have the same eigenvalue $e_1\cos^2\theta+e_2\sin^2\theta$. For the symmetric kernel case, we can fix the color factor as $-C_F/5$ [see Eq. (3.10)]. We can generalize this procedure for any spin-orbit state and find the general matrix representation of the kernel in the basis of polynomials. This will be done in the next section.

However, the eigensolution of the symmetric kernel equation (3.13) is not a true solution of the whole system in general, because there could be mixing, i.e., $\theta\neq 0$ in our example. We need to use other mixed symmetric kernel equations and find more constraints to determine e_1, e_2, and θ. In our example, we can find relations between three combinations of e_1, e_2, and θ by counting spin annihilation or surviving terms. (In the general case one must consider the x_i-dependent orbital distribution functions and their integration. One can find relations between combinations of eigenvalues and mixing angles case by case by counting the number of terms which survive or are annihilated after the kernel operation. The equation which we need to solve actually is the symmetric kernel equation presented in the next section.)

By counting spin terms, we find the following results for the leading anomalous dimension. If the symmetric kernel equation is given by

$$K_{(6)[111111]}\,|(6)_S[111111]_S\rangle=\gamma\,|(6)_S[111111]_S\rangle\ , \tag{4.1}$$

then

$$K_{(6)[111111]}\,|(42)_S Y\rangle=\gamma\,|(42)_S Y\rangle\ , \tag{4.2}$$

$$K_{(42)Y}\,|(6)_S[111111]_S\rangle=\frac{3\sqrt{6}}{16}\gamma\,|(42)_S[111111]_S\rangle\ , \tag{4.3}$$

$$\tfrac{1}{9}K_{(42)Y}\,|(42)_S Y\rangle=\frac{3\sqrt{6}}{16}\gamma\,|(6)_S[111111]_S\rangle\ , \tag{4.4}$$

and

$$\sum_{Y_S}\sum_{Y_K}\frac{\sqrt{53}}{12}\langle (42)_S Y_S,(42)Y_K\,|\,(42)_S Y\rangle \times K_{(42)Y_K}\,|(42)_S Y_S\rangle=\tfrac{5}{48}\gamma\,|(42)_S Y\rangle\ . \tag{4.5}$$

Upon comparison with Eqs. (3.13)—(3.15), (3.17), and (3.19), we can solve e_1, e_2, and θ in terms of γ. We find two solutions:

$$\begin{aligned}&\text{(i)}\ \ e_1=\tfrac{25}{16}\gamma,\ \ e_2=\tfrac{5}{8}\gamma,\ \ \tan\theta=\frac{\sqrt{6}}{2}\ ,\\ &\text{(ii)}\ \ e_1=\tfrac{11}{8}\gamma,\ \ e_2=\tfrac{7}{16}\gamma,\ \ \tan\theta=-\frac{\sqrt{6}}{3}\ .\end{aligned} \tag{4.6}$$

However, if $\gamma>0$ then only solution (i) is valid. The value of γ will be determined in the next section.

V. SYMMETRIC KERNEL EQUATION AND SOLUTION

The evolution equation with the symmetric kernel has the same spin-orbital symmetry f_{SO} and Y_{SO} in the left- and right-hand sides of the equation:

$$K_{(6)[111111]}\,|f_{SO}Y_{SO}\rangle=\Gamma\,|f_{SO}Y_{SO}\rangle\ , \tag{5.1}$$

where Γ is the eigenvalue of the symmetric kernel equation. In order to give a more explicit expression of Eq. (5.1), we project both sides of Eq. (5.1) to the light-cone momentum space of six quarks as illustrated in Sec. II. We then obtain a kernel equation for the spin-orbital wave function $\psi_{SO}(x_i,q_\perp^i)$:

$$\psi_{SO}(x_i,q_\perp^i)=C_d\int[dy]\int[d^2k_\perp^i]\left\langle x_i,q_\perp^i\left|\frac{1}{\Gamma}\sum_{k\neq l}V_{kl}\right|y_i,k_\perp^i\right\rangle\psi_{SO}(y_i,k_\perp^i)\ , \tag{5.2}$$

where $C_d=-C_F/5$, and

$$[dy]=\delta\left[1-\sum_{i=1}^{6}y_i\right]\prod_{i=1}^{6}dy_i,\ \ [d^2k_\perp^i]=\prod_{i=i}^{6}\left[\frac{d^2k_\perp^i}{16\pi^3}\right]16\pi^3\delta^2\left[\sum_i k_\perp^i\right]\ . \tag{5.3}$$

Thus $\psi_{SO}(x_i,q_\perp^i)$ is a linear combination of some orbital wave functions $\phi_O(x_i,q_\perp^i)$ [see Eq. (2.5)] with some coefficients of spin tensor representations. By calculating the leading-order diagrams shown in Fig. 1, one obtains an explicit expression of

$$\left\langle x_i,q_\perp^i\left|\frac{1}{\Gamma}\sum_{k\neq l}V_{kl}\right|y_i,k_\perp^i\right\rangle$$

for the spin-orbital wave function $\psi(x_i,q_\perp^i)$ (the spin-orbit index SO is dropped here since each possible f_{SO} and Y_{SO} satisfies the same equation):

$$\psi(x_i,q_\perp^i)=-2C_d\sum_{i\neq j}\frac{4\pi\alpha_s((q_\perp^i)^2)}{(q_\perp^i)^2}\prod_{k=1}^{6}x_k\int[dy]\theta(y_i-x_i)\prod_{l\neq i,j}^{6}\delta(y_l-x_l)\frac{y_j}{x_j}\left[\frac{\delta_{h_i\bar{h}_j}}{x_i+x_j}+\frac{1}{y_i-x_i}\right]\int[d^2k_\perp^i]\frac{\psi(y_i,k_\perp^i)}{\prod_{k=1}^{6}y_k}\,, \tag{5.4}$$

where

$$\alpha_s((q_\perp^i)^2)\ (\cong(4\pi/\beta)[\ln(q_\perp^i)^2/\Lambda^2]^{-1})$$

is the QCD running coupling constant ($\beta=11-\frac{2}{3}n_f$, n_f being the effective number of flavors) and $\delta_{h_i\bar{h}_j}=1$ (0) when the constituents' $\{i,j\}$ helicities are antiparallel (parallel). Equation (5.4) has an infrared singularity at $x_i=y_i$. Thus, in order to obtain a well-defined evolution equation of the six-quark system, we consider the quark distribution amplitude $\phi(x_i,Q)$, which is the amplitude for finding constituents with longitudinal momenta x_i in the deuteron which are collinear up to the scale Q^2:

$$\phi(x_i,Q)\equiv\int^{Q^2}[d^2q_\perp^{(i)}]\psi^{(Q)}(x_i,q_\perp^{(i)})\,. \tag{5.5}$$

This definition is the same as Eq. (2.6), except that the only spin-orbital wave function $\psi^{(Q)}(x_i,q_\perp^{(i)})$ is integrated instead of the total wave function $\Psi_d^{(Q)}(x_i,q_\perp^{(i)})$. By differentiating both sides of Eq. (5.5) with respect to Q^2, and combining with Eq. (5.4), we obtain the evolution equation of the six-quark system:

$$\prod_{k=1}^{6}x_k\left[\frac{\partial}{\partial\xi}+\frac{3C_F}{\beta}\right]\tilde{\phi}(x_i,Q)=-\frac{C_d}{\beta}\int_0^1[dy]V(x_i,y_i)\tilde{\phi}(y_i,Q)\,, \tag{5.6}$$

where we use the definition $\tilde{\phi}(x_i,Q)$ as

$$\phi(x_i,Q)=\prod_{k=1}^{6}x_k\tilde{\phi}(x_i,Q)\,, \tag{5.7}$$

and the variable

$$\xi(Q^2)=\frac{\beta}{4\pi}\int_{Q_0^2}^{Q^2}\frac{dk^2}{k^2}\alpha_S(k^2)\sim\ln\left[\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0{}^2/\Lambda^2)}\right], \tag{5.8}$$

and $V(x_i,y_i)$ is given by

$$\begin{aligned}V(x_i,y_i)=2\prod_{k=l}^{6}x_k\sum_{i\neq j}^{6}\theta(y_i-x_i)\\ \times\prod_{l\neq i,j}^{6}\delta(x_l-y_l)\frac{y_j}{x_j}\\ \times\left[\frac{\delta_{h_i\bar{h}_j}}{x_i+x_j}+\frac{\Delta}{y_i-x_i}\right]\\ =V(y_i,x_i)\,.\end{aligned} \tag{5.9}$$

By definition, the factor Δ in Eq. (5.9) means

$$\Delta\tilde{\phi}(y_i,Q)=\tilde{\phi}(y_i,Q)-\tilde{\phi}(x_i,Q)\,, \tag{5.10}$$

so we can see that the infrared singularity in Eq. (5.4) at $x_i=y_i$ is completely canceled by that in Eq. (5.5). We notice that this cancellation happens only when the correct value of deuteron color factor $C_d=-C_F/5$, so that this is a good check of the correctness of C_d. We calculated C_d explicitly from the definition in Appendix B.

Since the six-quark evolution equation (5.6) has a form similar to that of the three-quark evolution equation,[4] we can solve this equation following the similar methods of the three-quark case. First, we separate the variables of $\tilde{\phi}(x_i,Q)$ such as

$$\begin{aligned}\tilde{\phi}(x_i,Q)&=\tilde{\phi}(x_i)e^{-\gamma\xi}\\ &=\tilde{\phi}(x_i)\left[\ln\left[\frac{Q^2}{\Lambda^2}\right]\right]^{-\gamma},\end{aligned} \tag{5.11}$$

and substitute into Eq. (5.6) so that Eq. (5.6) becomes (note that $C_d=-C_F/5$)

$$\begin{aligned}\prod_{k=1}^{6}x_k\left[-\gamma+\frac{3C_F}{\beta}\right]\tilde{\phi}(x_i)&=\frac{C_F}{5\beta}\int_0^1[dy]V(x_i,y_i)\tilde{\phi}(y_i)\\ &\equiv\frac{C_F}{5\beta}V\,|\,\tilde{\phi}\rangle\,,\end{aligned} \tag{5.12}$$

where the equation is simply redefined by the quantum-mechanical notation $V\,|\,\tilde{\phi}\rangle$, and $V(x_i,y_i)$ is given by Eq. (5.9). Next we expand $\tilde{\phi}(x_i)$ in terms of eigenfunctions $\tilde{\phi}_n(x_i)$, so that the general solution of $\tilde{\phi}(x_i,Q)$ is of the form

$$\tilde{\phi}(x_i,Q)=\sum_{n=0}^{\infty}a_n\tilde{\phi}_n(x_i)\left[\ln\left[\frac{Q^2}{\Lambda^2}\right]\right]^{-\gamma_n}, \tag{5.13}$$

where a_n are the coefficients and eigenvalues γ_n correspond to the anomalous dimensions of the six-quark system. Since $V(x_i,y_i)$ is both real and symmetric $[V(x_i,y_i)=V(y_i,x_i)]$, the γ_n are real. The $\{\tilde{\phi}_n(x_i)\}_{n=0}^{\infty}$ are orthogonal with weight $\omega(x_i)=x_1x_2x_3x_4x_5x_6$:

$$\int_0^1[dx]\omega(x_i)\tilde{\phi}_n^*(x_i)\tilde{\phi}_m(x_i)=K_n\delta_{nm}\,, \tag{5.14}$$

where K_n are the normalizations. Since the $\{\tilde{\phi}_n(x_i)\}_{n=0}^{\infty}$ form a completely orthonormal basis

$$\left[\prod_{i=1}^{5}x_i{}^{m_i}\right]_{m_1,m_2,\ldots,m_5=0}^{\infty},$$

where $\sum_{i=1}^{5}m_i=n$, we expand V on this polynomial basis. After some calculation, we obtain[12]

$$\frac{1}{2}\frac{V\left|\prod_{i=1}^{5} x_i^{m_i}\right\rangle}{\omega(x_i)} = \left[\prod_{i=1}^{5} x_i^{m_i}\right]\left[\delta_{h_6\bar{h}}\sum_{i=1}^{3}\left[\frac{1}{m_i+1}-\frac{1}{m_i+2}\right]+\sum_{i=4}^{5}\delta_{h_6\bar{h}_i}\left[\frac{1}{m_i+1}-\frac{1}{m_i+2}\right]+\sum_{i=4}^{5}\delta_{h_i\bar{h}}\sum_{j=1}^{3}\frac{1}{m_i+m_j+2}\right.$$

$$\left.+\delta_{h_4\bar{h}_5}\frac{1}{m_4+m_5+2}-6\sum_{i=1}^{5}\sum_{k=2}^{m_i+1}\frac{1}{k}\right]$$

$$+\sum_{i=1}^{3}\sum_{l=1}^{m_i}\left[\frac{m_i-l+1}{l(m_i+1)}+\frac{\delta_{h_6\bar{h}}}{(m_i+1)(m_i+2)}\right]x_i^{m_i-l}\sum_{\{j_{k\neq i}\}}\frac{l!}{\left[l-\sum_{k\neq 1}j_k\right]!\prod_{k\neq i}^{5}(j_k!)}\prod_{k\neq i}^{5}(-1)^{\sum_{k\neq i}j_k}x_k^{m_k+j_k}$$

$$+\sum_{i=4}^{5}\sum_{l=1}^{m_i}\left[\frac{m_i-l+1}{l(m_i+1)}+\frac{\delta_{h_6\bar{h}_i}}{(m_i+1)(m_i+2)}\right]x_i^{m_i-l}\sum_{\{j_k\}}\frac{l!}{\left[l-\sum_{k\neq i}j_k\right]!\prod_{k\neq i}^{5}(j_k!)}\prod_{k\neq i}^{5}(-1)^{\sum_{k\neq i}j_k}x_k^{m_k+j_k}$$

$$+\sum_{i<j}^{5}\left[\sum_{l=1}^{m_j}x_i^{m_i+l}x_j^{m_j-l}\frac{m_j(m_j-1)\cdots(m_j-l+1)}{l(m_i+2)(m_i+3)\cdots(m_i+l+1)}\right.$$

$$\left.+\sum_{l=1}^{m_i}x_i^{m_i-l}x_j^{m_j+l}\frac{m_i(m_i-1)\cdots(m_i-l+1)}{l(m_j+2)(m_j+3)\cdots(m_j+l+1)}\right]\prod_{k\neq i,j}^{5}x_k^{m_k}$$

$$+\sum_{i=4}^{5}\delta_{h_i\bar{h}}\sum_{j=1}^{3}\frac{1}{m_i+m_j+2}\left[\sum_{l=1}^{m_j}x_i^{m_i+l}x_j^{m_j-l}\frac{m_j(m_j-1)\cdots(m_j-l+1)}{(m_i+2)(m_i+3)\cdots(m_i+l+1)}\right.$$

$$\left.+\sum_{l=1}^{m_i}x_i^{m_i-l}x_j^{m_j+l}\frac{m_i(m_i-1)\cdots(m_i-l+1)}{(m_j+2)(m_j+3)\cdots(m_j+l+1)}\right]\prod_{k\neq i,j}^{5}x_k^{m_k}$$

$$+\delta_{h_4h_5}\frac{1}{m_4+m_5+2}\left[\sum_{l=1}^{m_5}x_4^{m_4+l}x_5^{m_5-l}\frac{m_5(m_5-1)\cdots(m_5-l+1)}{(m_4+2)(m_4+3)\cdots(m_4+l+1)}\right.$$

$$\left.+\sum_{l=1}^{m_4}x_4^{m_4-l}x_5^{m_5+l}\frac{m_4(m_4-1)\cdots(m_4-l+1)}{(m_5+2)(m_5+3)\cdots(m_5+l+1)}\right]\prod_{k=1}^{3}x_k^{m_k}$$

$$=\tfrac{1}{2}\sum_{\{n_i\}}\left|\prod_{k=1}^{5}x_i^{n_i}\right\rangle U_{\{n_i\},\{m_i\}}\ , \tag{5.15}$$

where particles 1, 2, and 3 have helicity parallel to the hadron's helicity h, and the particles 4, 5, and 6 have helicities h_4, h_5, and h_6. The notation $\sum_{\{j_{k\neq i}\}}$ means the summation over all possible integer $j_k(k\neq i)$ as long as $\sum_{k\neq i}^{5} j_k \lesssim l$ is satisfied. $U_{\{n_i\},\{m_i\}}$ is a matrix representation of the linear operator $\omega^{-1}V$ on the basis $\{x_i^{m_i}\}$. Since $U_{\{n_i\},\{m_i\}}=0$ when $\sum_{i=1}^{5} n_i > \sum_{i=1}^{5} m_i$, the eigenfunctions are polynomials of degree $n=\sum_{i=1}^{5} m_i=0,1,2,\ldots$. The corresponding eigenvalues are obtained by diagonalizing the matrix $U_{\{n_i\},\{m_i\}}$ with $\sum_{i=1}^{5} n_i=\sum_{i=1}^{5} m_i=n$. Several leading eigenvalues and the eigenfunctions are given in Table I.

From Table I, we see that the leading anomalous dimension obtained from the symmetric kernel equation is given by

$$\gamma_0=\frac{6}{5}\frac{C_F}{\beta} \quad \text{for } S_Z=0, \quad \frac{7}{5}\frac{C_F}{\beta} \quad \text{for } S_Z=\pm 1 \; . \tag{5.16}$$

Therefore, $\gamma_0>0$ and the true leading anomalous dimension is given by

$$\gamma_0^{\text{real}}=\frac{3}{4}\frac{C_F}{\beta} \quad \text{for } S_Z=0, \quad \frac{7}{8}\frac{C_F}{\beta} \quad \text{for } S_Z=\pm 1 \; , \tag{5.17}$$

from the value of e_2 in the case (i) of Eq. (4.6).

VI. DISCUSSIONS AND CONCLUSIONS

Harvey[10] has classified the color-singlet six-quark states in terms of a physical cluster decomposition. Using his classification, the physical deuteron state (i.e., a bound state of two color-singlet clusters) is represented as a linear combination of several different kinds of totally antisymmetric color-singlet six-quark states. For example, the two well-separated nucleons $|NN\rangle$ are given by[10]

$$|NN\rangle=(\tfrac{1}{9})^{1/2}\,|[6]\{33\}\rangle+(\tfrac{4}{9})^{1/2}\,|[42]\{33\}\rangle - \frac{\sqrt{4}}{9}\,|[42]\{51\}\rangle \; , \tag{6.1}$$

where [ ] and { } represent the orbital and spin-isospin symmetry (i.e., f_O and f_{TS} in our notations) and color symmetry (222) is abbreviated. However, this classification by itself does not include the dynamics of strong interactions between the constituents. In other words, the dynamics between the quarks inside the deuteron is not included.

Thus far in this paper we have formulated the dynamical evolution equation of six-quark systems and solved it to give the general form of the quark distribution amplitude $\phi_d(x_i,Q)$:

$$\phi_d(x_i,Q)=(CTS)\phi(x_i,Q) \; , \tag{6.2}$$

TABLE I. Solutions of the evolution equation (5.12) for total helicity $|3h+h_4+h_5+h_6|=0$ ($\phi^{\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow}$), 1 ($\phi^{\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow}$, $\phi^{\uparrow\uparrow\uparrow\downarrow\uparrow\downarrow}$, $\phi^{\uparrow\uparrow\uparrow\downarrow\downarrow\uparrow}$), 2 ($\phi^{\uparrow\uparrow\uparrow\uparrow\uparrow\downarrow}$, $\phi^{\uparrow\uparrow\uparrow\uparrow\downarrow\uparrow}$, $\phi^{\uparrow\uparrow\uparrow\downarrow\uparrow\uparrow}$), and 3 ($\phi^{\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow}$) cases. The procedure for the systematic derivation of the $\tilde{\phi}_n$ is given in Sec. V.

	$\gamma_n \Big/ \left[\frac{C_F}{\beta}\right]$	N	$a^{(0)}_{00000}$	$a^{(1)}_{10000}$	$a^{(1)}_{01000}$	$a^{(1)}_{00100}$	$a^{(1)}_{00010}$	$a^{(1)}_{00001}$
$\phi^{\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow}$	$\frac{6}{5}$	$11!$						
	$\frac{13}{5}$	$\frac{13!}{4}$		1	-1			
	$\frac{13}{5}$	$\frac{13!}{12}$		1	1	-2		
	$\frac{13}{5}$	$\frac{13!}{4}$					1	-1
	$\frac{13}{5}$	$\frac{13!}{12}$	2	-2	-2	-2	-3	-3
	$\frac{14}{5}$	$\frac{13!}{12}$	1	-2	-2	-2		
$\phi^{\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow}$								
$\phi^{\uparrow\uparrow\uparrow\downarrow\uparrow\downarrow}$	$\frac{7}{5}$	$11!$						
$\phi^{\uparrow\uparrow\uparrow\downarrow\downarrow\uparrow}$								
$\phi^{\uparrow\uparrow\uparrow\uparrow\uparrow\downarrow}$								
$\phi^{\uparrow\uparrow\uparrow\uparrow\downarrow\uparrow}$	2	$11!$						
$\phi^{\uparrow\uparrow\uparrow\downarrow\uparrow\uparrow}$								
$\phi^{\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow}$	3	$11!$						

$$\tilde{\phi}_n(x_i)=\sqrt{N}\sum_{\{m_i\}} a^{(n)}_{m_1\cdots m_5}\prod_{i=5}^{5} x_i^{m_i}\left[n=\sum_{i=1}^{5} m_i\right]$$

where (CTS) is a tensor representation obtained from the Young symmetry of $SU(3)_C$, $SU(2)_T$, and $SU(2)_S$ [one example is given by Eq. (2.7)], and the orbital distribution amplitude $\phi(x_i,Q)$ is given by

$$\phi(x_i,Q)=x_1x_2x_3x_4x_5x_6 \times \sum_{n=0}^{\infty} a_n\tilde{\phi}_n(x_i)\left[\ln\left[\frac{Q^2}{\Lambda^2}\right]\right]^{-\gamma_n} . \tag{6.3}$$

We project Eq. (6.1) to momentum space:

$$\begin{aligned}\phi_{NN}(x_i,Q)=&(\tfrac{1}{9})^{1/2}\phi_{[6]\{33\}}(x_i,Q)\\ &+(\tfrac{4}{9})^{1/2}\phi_{[42]\{33\}}(x_i,Q)\\ &-(\tfrac{4}{9})^{1/2}\phi_{[42]\{33\}}(x_i,Q) .\end{aligned} \tag{6.4}$$

In the limit $Q\rightarrow\infty$ the dependence of Q is determined by the leading anomalous dimension; all other terms which have nonleading anomalous dimensions are suppressed by logarithmic damping factors. However, as we can see from Table I, the orbital symmetry of the eigensolution which has the leading anomalous dimension cannot be [42] but is [6]. This means only the first term of Eq. (6.4) survives at the large-Q limit. The NN amplitude itself is not sufficient. One can show that an 80% hidden-color state is necessary to saturate the normalization of the six-quark amplitude when six quarks approach the same position in impact space $b_\perp\rightarrow 0$. We have called this new degree of freedom an anomalous state since it does not correspond to the usual nucleonic degrees of freedom of the nucleus. The physical implication of the anomalous state is discussed in our toy model analysis.[7]

The asymptotic behavior of the deuteron distribution amplitude is given by

$$\phi_D(x_i,Q)=a_0x_1x_2x_3x_4x_5x_6\left[\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0{}^2/\Lambda^2)}\right]^{-\gamma_0} , \tag{6.5}$$

where $\gamma_0=\frac{3}{4}(C_F/\beta)$ for the $S_Z=0$ deuteron. The QCD predictions for high-Q^2 behavior of the deuteron form factor and the form of the deuteron distribution amplitude at short distances are given in Ref. 3. The fact that the six-quark state is 80% hidden color at small transverse separation implies that the deuteron form factors cannot be described at large Q^2 by meson-nucleon degrees of freedom alone, and that the nucleon-nucleon potential is repulsive at short distances.[3,7,13]

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APPENDIX A

In this appendix we describe a general method for finding the color-singlet representations for multihadron systems. This leads to the explicit representation of Eq. (2.7).

Group theoretically, color states of every quark and/or antiquark systems are represented by the multiple outer products

$$(3)^p\times(3^*)^q \tag{A1}$$

for the system of p quarks and q antiquarks. The reducible tensor representations of (A1) are decomposed into irreducible tensor representations and the resulting singlet representations provide the physical quark system.

Since the color singlet is invariant under SU(3) transformation, the only tensor representations[14] which are invariant under SU(3) transformation are the Kronecker delta and the completely antisymmetric Cartesian tensors:

$$\delta^i_j,\ \ \epsilon_{ijk},\ \ \epsilon^{ijk} , \tag{A2}$$

where the lower (upper) indices correspond to 3 (3^*) representations. This comes from the fact that only operations of contraction and antisymmetrization commute with the SU(3) transformations on the mixed tensors. Thus, every color-singlet representation can be represented by the products of the three tensors (A2).

From this observation, we describe the rules to construct the color-singlet tensor representations for arbitrary quark and/or antiquark systems such as (A1).

(1) Give each index to every 3 and 3^* representations. For example, p lower indices and q upper indices will be given to (A1).

(2) Assemble one possible product of Kronecker deltas and completely antisymmetric Cartesian tensors to use up all indices considered in rule (1). For example, the $p=6$ and $q=0$ case needs the product of two antisymmetric Cartesian tensors.

(3) Permute the upper and lower indices separately. For example, the $p=6$ and $q=0$ case will give 10 possible different representations. However, note that they are not all independent.

(4) To construct all the independent (orthogonal) representations, follow the method of Schmidt's orthogonalization, where the inner product is defined as contraction. For example, in the $p=6$ and $q=0$ case, five independent representations are obtained:

$$\begin{aligned}S^1_{ijklmn}&=\frac{1}{12\sqrt{2}}(\epsilon_{ikm}\epsilon_{jln}+\epsilon_{ikn}\epsilon_{jlm}+\epsilon_{jkm}\epsilon_{iln}+\epsilon_{jkn}\epsilon_{ilm}) ,\\ S^2_{ijklmn}&=\frac{1}{4\sqrt{6}}(\epsilon_{ijm}\epsilon_{kln}+\epsilon_{ijn}\epsilon_{klm}) ,\\ S^3_{ijklmn}&=\frac{1}{4\sqrt{6}}(\epsilon_{ikl}\epsilon_{jmn}+\epsilon_{jkl}\epsilon_{imm}) ,\\ S^4_{ijklmn}&=\frac{1}{4\sqrt{2}}(\epsilon_{ijl}\epsilon_{kmn}-\tfrac{1}{3}\epsilon_{ijk}\epsilon_{lmn}) ,\\ S^5_{ijklmn}&=\tfrac{1}{6}\epsilon_{ijk}\epsilon_{lmn} ,\end{aligned} \tag{A3}$$

where S^α_{ijklmn} $(\alpha=1,2,\ldots,5)$ are the five independent color-singlet representations.

The correspondence between Eq. (A3) and the five different Young tableaus (or Yamanouchi labels) is as follows: If we denote $i=1, j=2, \ldots, n=6$, then the corresponding Young tableaus with Eq. (A3) are

$$\alpha = \begin{array}{|c|c|}\hline 1&2\\\hline 3&4\\\hline 5&6\\\hline\end{array}\;1\;,\quad \begin{array}{|c|c|}\hline 1&3\\\hline 2&4\\\hline 5&6\\\hline\end{array}\;2\;,\quad \begin{array}{|c|c|}\hline 1&2\\\hline 3&5\\\hline 4&6\\\hline\end{array}\;3\;,\quad \begin{array}{|c|c|}\hline 1&3\\\hline 2&5\\\hline 4&6\\\hline\end{array}\;4\;,\quad \begin{array}{|c|c|}\hline 1&4\\\hline 2&5\\\hline 3&6\\\hline\end{array}\;5$$

A similar method can be applied to construct the isospin-singlet representations in SU(2). The five independent isospin-singlet tensors corresponding to the dual of the color-singlet Young tableau shown in (A3) are represented by

$$\begin{aligned}
\begin{array}{|c|c|c|}\hline 1&3&5\\\hline 2&4&6\\\hline\end{array} &= \frac{1}{2\sqrt{2}}\epsilon_{ab}\epsilon_{cd}\epsilon_{ef}\,,\\
\begin{array}{|c|c|c|}\hline 1&2&5\\\hline 3&4&6\\\hline\end{array} &= \frac{1}{2\sqrt{6}}(\epsilon_{ac}\epsilon_{bd}\epsilon_{ef}+\epsilon_{ad}\epsilon_{bc}\epsilon_{ef})\,,\\
\begin{array}{|c|c|c|}\hline 1&3&4\\\hline 2&5&6\\\hline\end{array} &= \frac{1}{2\sqrt{6}}(\epsilon_{ab}\epsilon_{ce}\epsilon_{df}+\epsilon_{ab}\epsilon_{cf}\epsilon_{de})\,,\\
\begin{array}{|c|c|c|}\hline 1&2&4\\\hline 3&5&6\\\hline\end{array} &= \frac{1}{6\sqrt{2}}(\epsilon_{ac}\epsilon_{be}\epsilon_{df}+\epsilon_{ac}\epsilon_{bf}\epsilon_{de}+\epsilon_{ae}\epsilon_{bc}\epsilon_{df}+\epsilon_{af}\epsilon_{bc}\epsilon_{de})\,,\\
\begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4&5&6\\\hline\end{array} &= \frac{1}{6}(\epsilon_{ad}\epsilon_{be}\epsilon_{cf}+\epsilon_{ae}\epsilon_{bf}\epsilon_{cd}+\epsilon_{af}\epsilon_{bd}\epsilon_{ce})\,,
\end{aligned} \tag{A4}$$

where the ϵ_{ab}'s are the antisymmetric Cartesian tensors of $SU(2)_T$ and the Young tableaux are denoted by $a=1$, $b=2,\ldots,f=6$. If we multiply the color- and isospin-singlet tensors given by Eqs. (A3) and (A4), respectively, taking care of the phase factor[8] η_T for the Yamanouchi labels, we get the special case of the asymptotic deuteron representation, Eq. (2.7).

APPENDIX B

The color factor defined in Eq. (3.8) can be explicitly expressed in terms of tensor notation, since we know the tensor representation for the multiplication of Gell-Mann matrices in Eq. (3.8) as[14]

$$\left[\frac{\lambda}{2}\right]^{i}_{i'}\left[\frac{\lambda}{2}\right]^{j}_{j'}=\tfrac{1}{2}(\delta^{i}_{j'}\delta^{j}_{i'}-\tfrac{1}{3}\delta^{i}_{i'}\delta^{j}_{j'})\,, \tag{B1}$$

where the indices i and i' designate the color i'th quark before and after a gluon exchange. Using the notation of Eq. (A3), the generalized color matrices (5×5) given by Eq. (3.8) can be obtained by

$$C_{\alpha\beta}(i,j)=S^{\alpha}_{\ldots i\ldots j\ldots}\left[\frac{\lambda}{2}\right]^{i}_{i'}\left[\frac{\lambda}{2}\right]^{j}_{j'}S^{\beta}_{\ldots i'\ldots j'\ldots}\,, \tag{B2}$$

and Eq. (B1). There are 15 such matrices and they are given by

$$C(1,2)=\begin{bmatrix}\frac{1}{3}&0&0&0&0\\0&-\frac{2}{3}&0&0&0\\0&0&\frac{1}{3}&0&0\\0&0&0&-\frac{2}{3}&0\\0&0&0&0&-\frac{2}{3}\end{bmatrix},\quad C(1,3)=\begin{bmatrix}-\frac{5}{12}&-\frac{\sqrt{3}}{4}&0&0&0\\-\frac{\sqrt{3}}{4}&\frac{1}{12}&0&0&0\\0&0&-\frac{5}{12}&-\frac{\sqrt{3}}{4}&0\\0&0&-\frac{\sqrt{3}}{4}&\frac{1}{12}&0\\0&0&0&0&-\frac{2}{3}\end{bmatrix},$$

$$C(1,4)=\begin{bmatrix}-\frac{5}{12}&\frac{\sqrt{3}}{4}&0&0&0\\\frac{\sqrt{3}}{4}&\frac{1}{12}&0&0&0\\0&0&-\frac{5}{12}&-\frac{1}{4\sqrt{3}}&\frac{1}{\sqrt{6}}\\0&0&-\frac{1}{4\sqrt{3}}&-\frac{7}{12}&-\frac{1}{3\sqrt{2}}\\0&0&\frac{1}{\sqrt{6}}&-\frac{1}{3\sqrt{2}}&0\end{bmatrix},\quad C(1,5)=\begin{bmatrix}-\frac{5}{12}&0&0&-\frac{1}{4}&-\frac{1}{2\sqrt{2}}\\0&-\frac{5}{12}&-\frac{1}{4}&-\frac{1}{2\sqrt{3}}&\frac{1}{2\sqrt{6}}\\0&-\frac{1}{4}&-\frac{5}{12}&\frac{1}{2\sqrt{3}}&-\frac{1}{2\sqrt{6}}\\-\frac{1}{4}&-\frac{1}{2\sqrt{3}}&\frac{1}{2\sqrt{3}}&-\frac{1}{12}&\frac{1}{6\sqrt{2}}\\-\frac{1}{2\sqrt{2}}&\frac{1}{2\sqrt{6}}&-\frac{1}{2\sqrt{6}}&\frac{1}{6\sqrt{2}}&0\end{bmatrix},$$

$$C(1,6)=\begin{bmatrix}-\frac{5}{12}&0&0&\frac{1}{4}&\frac{1}{2\sqrt{2}}\\0&-\frac{5}{12}&\frac{1}{4}&\frac{1}{2\sqrt{3}}&-\frac{1}{2\sqrt{6}}\\0&\frac{1}{4}&-\frac{5}{12}&\frac{1}{2\sqrt{3}}&-\frac{1}{2\sqrt{6}}\\\frac{1}{4}&\frac{1}{2\sqrt{3}}&\frac{1}{2\sqrt{3}}&-\frac{1}{12}&\frac{1}{6\sqrt{2}}\\\frac{1}{2\sqrt{2}}&-\frac{1}{2\sqrt{6}}&-\frac{1}{2\sqrt{6}}&\frac{1}{6\sqrt{2}}&0\end{bmatrix},\quad C(2,3)=\begin{bmatrix}-\frac{5}{12}&\frac{\sqrt{3}}{4}&0&0&0\\\frac{\sqrt{3}}{4}&\frac{1}{12}&0&0&0\\0&0&-\frac{5}{12}&\frac{\sqrt{3}}{4}&0\\0&0&\frac{\sqrt{3}}{4}&\frac{1}{12}&0\\0&0&0&0&-\frac{2}{3}\end{bmatrix},$$

$$C(2,4)=\begin{pmatrix} -\frac{5}{12} & -\frac{\sqrt{3}}{4} & 0 & 0 & 0 \\ -\frac{\sqrt{3}}{4} & \frac{1}{12} & 0 & 0 & 0 \\ 0 & 0 & -\frac{5}{12} & \frac{1}{4\sqrt{3}} & -\frac{1}{\sqrt{6}} \\ 0 & 0 & \frac{1}{4\sqrt{3}} & -\frac{7}{12} & -\frac{1}{3\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{6}} & -\frac{1}{3\sqrt{2}} & 0 \end{pmatrix}, \quad C(2,5)=\begin{pmatrix} -\frac{5}{12} & 0 & 0 & \frac{1}{4} & \frac{1}{2\sqrt{2}} \\ 0 & -\frac{5}{12} & \frac{1}{4} & -\frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{6}} \\ 0 & \frac{1}{4} & -\frac{5}{12} & -\frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{6}} \\ \frac{1}{4} & -\frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{3}} & -\frac{1}{12} & \frac{1}{6\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{6}} & \frac{1}{6\sqrt{2}} & 0 \end{pmatrix},$$

$$C(2,6)=\begin{pmatrix} -\frac{5}{12} & 0 & 0 & -\frac{1}{4} & -\frac{1}{2\sqrt{2}} \\ 0 & -\frac{5}{12} & -\frac{1}{4} & \frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{6}} \\ 0 & -\frac{1}{4} & -\frac{5}{12} & -\frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{6}} \\ -\frac{1}{4} & \frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{3}} & -\frac{1}{12} & \frac{1}{6\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{6}} & \frac{1}{6\sqrt{2}} & 0 \end{pmatrix}, \quad C(3,4)=\begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & -\frac{2}{3} & 0 & 0 & 0 \\ 0 & 0 & -\frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{3} & \frac{\sqrt{2}}{3} \\ 0 & 0 & 0 & \frac{\sqrt{2}}{3} & 0 \end{pmatrix}, \tag{B3}$$

$$C(3,5)=\begin{pmatrix} -\frac{5}{12} & 0 & -\frac{\sqrt{3}}{4} & 0 & 0 \\ 0 & -\frac{5}{12} & 0 & \frac{1}{4\sqrt{3}} & -\frac{1}{\sqrt{6}} \\ -\frac{\sqrt{3}}{4} & 0 & \frac{1}{12} & 0 & 0 \\ 0 & \frac{1}{4\sqrt{3}} & 0 & -\frac{7}{12} & -\frac{1}{3\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{3\sqrt{2}} & 0 \end{pmatrix}, \quad C(3,6)=\begin{pmatrix} -\frac{5}{12} & 0 & \frac{\sqrt{3}}{4} & 0 & 0 \\ 0 & -\frac{5}{12} & 0 & -\frac{1}{4\sqrt{3}} & \frac{1}{\sqrt{6}} \\ \frac{\sqrt{3}}{4} & 0 & \frac{1}{12} & 0 & 0 \\ 0 & -\frac{1}{4\sqrt{3}} & 0 & -\frac{7}{12} & -\frac{1}{3\sqrt{2}} \\ 0 & \frac{1}{\sqrt{6}} & 0 & -\frac{1}{3\sqrt{2}} & 0 \end{pmatrix},$$

$$C(4,5)=\begin{pmatrix} -\frac{5}{12} & 0 & \frac{\sqrt{3}}{4} & 0 & 0 \\ 0 & -\frac{5}{12} & 0 & \frac{\sqrt{3}}{4} & 0 \\ \frac{\sqrt{3}}{4} & 0 & \frac{1}{12} & 0 & 0 \\ 0 & \frac{\sqrt{3}}{4} & 0 & \frac{1}{12} & 0 \\ 0 & 0 & 0 & 0 & -\frac{2}{3} \end{pmatrix}, \quad C(4,6)=\begin{pmatrix} -\frac{5}{12} & 0 & -\frac{\sqrt{3}}{4} & 0 & 0 \\ 0 & -\frac{5}{12} & 0 & -\frac{\sqrt{3}}{4} & 0 \\ -\frac{\sqrt{3}}{4} & 0 & \frac{1}{12} & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{4} & 0 & \frac{1}{12} & 0 \\ 0 & 0 & 0 & 0 & -\frac{2}{3} \end{pmatrix},$$

$$C(5,6)=\begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & -\frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & -\frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & -\frac{2}{3} \end{pmatrix}.$$

TABLE II. The coefficients A_{ij} for all (i,j) pairs and the normalization factor N_{fY} defined in Eq. (B6). Since color symmetry is fixed by (222), the possible symmetry of the kernel is $f=(6)$ or (42).

K_{fY}	N_{fY}	(56)	(46)	(45)	(36)	(35)	(26)	(25)	(16)	(15)	(23)	(24)	(34)	(14)	(13)	(12)
$K_{(6)[111111]}$	$-\frac{4}{15}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$K_{(42)[221111]}$	$\frac{\sqrt{6}}{20}$	12	-3	-3	-3	-3	-3	-3	-3	-3	2	2	2	2	2	2
$K_{(43)[212111]}$	$\frac{\sqrt{10}}{20}$		9	-3	-3	1	-3	1	-3	1	1	-2	-2	-2	2	2
$K_{(42)[211211]}$	$\frac{\sqrt{5}}{10}$				6	-2	-3	1	-3	1	-1	1	-2	1	-1	-2
$K_{(43)[211121]}$	$\frac{\sqrt{15}}{10}$						3	-1	-3	1	-1	-1	0	1	1	
$K_{(42)[122111]}$	$\frac{1}{\sqrt{5}}$			3		-1		-1		-1	1	-1	-1	-1	1	1
$K_{(42)[121211]}$	$\frac{1}{\sqrt{10}}$					4		-2		-2	-1	1	-2	1	-1	2
$K_{(42)[121121]}$	$(\frac{3}{10})^{1/2}$							2		-2	-1	-1		1	1	
$K_{(42)[112211]}$	$(\frac{2}{10})^{1/2}$										-1	-1	2	-1	-1	2
$K_{(42)[112121]}$	$(\frac{3}{10})^{1/2}$										-1	1		-1	1	

Using the results (B3) and the formulas (3.10) and (3.11), we can find expressions for the kernels in terms of V_{ij}. Since $C_{\alpha\alpha}(i,j)$ is independent of i and j and given by

$$C_{\alpha\alpha}(i,j)=-\tfrac{2}{3}\times 3+\tfrac{1}{3}\times 2=-C_F\ , \tag{B4}$$

the symmetric kernel is given by

$$K_{(6)[111111]}=C_d\sum_{i\neq j}V_{ij} \tag{B5}$$

where $C_d=-C_F/5$.

The other kernels can also be obtained by using the Clebsch-Gordan coefficients of S_6 and (B3). The results are given by

$$K_{fY}=N_{fY}\sum_{i\neq j}A_{ij}V_{ij}\ , \tag{B6}$$

where N_{fY} and A_{ij} are summarized in Table II.

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[9][654321] is the Yamanouchi label of (1^6) symmetry; see Ref. 8.

[10]M. Harvey, Nucl. Phys. **A352**, 301 (1981); **A352**, 326 (1981); **A424**, 428 (1984).

[11]Detailed calculations and the origin of the logarithmic Q dependence are given in Ref 4.

[12]In Eq. (5.15) we use a more compact form for the coefficient of $x_i^{m_i+l}x_j^{m_j-l}$ than that given in Ref. 4, since we can prove that

$$-\sum_{j=0}^{i}\begin{bmatrix} m \\ j\end{bmatrix}\begin{bmatrix} m-j \\ m-i\end{bmatrix}(-1)^j\sum_{k=2}^{n+j+1}\frac{1}{k}=\frac{m(m-1)\cdots(m-i+1)}{i(n+2)(n+3)\cdots(n+i+1)}\ .$$

Also, we correct a typographical error in Appendix D of Ref. 4:

$$\frac{m-i+1+\delta_{h_2\bar{h}}}{i(m+2)}$$

should read

$$\frac{m-i+1}{i(m+1)}+\frac{\delta_{h_2\bar{h}}}{(m+1)(m+2)}\ .$$

[13]Qualitative QCD-based arguments for a repulsive NN potential at short distances are given in C. Detar, in *Asymptotic Realms of Physics (Essays in Honor of F. E. Low)*, edited by A. Guth, K. Huang, and R. Jaffe (MIT, Cambridge, Massachusetts, 1983), pp. 118–127; see M. Harvey, Ref. 10; K. Maltman and N. Isgur, Phys. Rev. Lett. **50**, 1827 (1983); R. L. Jaffe, *ibid.* **24**, 228 (1983); G. E. Brown, in *Quarks and the Nucleus,* proceedings of the International School of Nuclear Physics, 5th Course, Erice, Italy, edited by D. Wilkinson (Progress in Particle and Nuclear Physics, Vol. 8) (Pergamon, Oxford, England, 1982), p. 147. The possibility that the deuteron form factor is dominated at large momentum transfer by hidden-color components is discussed in V. A. Matveev and P. Sorba, Nuovo Cimento **45A**, 257 (1978); Lett. Nuovo Cimento **20**, 435 (1977).

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