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NOVEL EFFECTS IN DEEP INELASTIC SCATTERING FROM SPIN-ONE HADRONS[1]

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Deep inelastic scattering from a polarized spin-one target yields qualitatively new information which is not available in the spin-half case. Among several new structure functions, one, $b_1(x)$, is leading twist in QCD. It can be measured with an unpolarized beam. $b_1(x)$ is small and calculable for a weakly bound collection of nucleons, and therefore its measurement would provide a clear signature for exotic components in a spin-one nucleus.

1. Introduction

Deep inelastic electron scattering from polarized targets has attracted much attention recently. In particular, the measurement of $g_1^{ep}(x)$ by the European Muon Collaboration [1] has stimulated several proposals to investigate spin effects in other nuclear targets. Several potentially polarizable nuclei have spin one, including deuterium, ^{6}Li and ^{14}N. In this paper we describe the effects which may be observed in deep inelastic scattering from a polarized spin-one hadron, in practice a nucleus.

We find new features in scattering from a spin-one target not found in the spin-half case. The new effects all reside in a single new structure function $b_1(x)$ which is of leading twist (twist two). $b_1(x)$ can be determined by measuring the deep inelastic cross section for an *unpolarized* lepton beam to scatter from a target polarized along the beam and subtracting the same cross section for an unpolarized target. (If the beam is unavoidably polarized (e.g. the muons at CERN) then it is necessary to average the cross sections for target-spin parallel and antiparallel to the beam, and subtract the unpolarized cross section.) The function b_1 vanishes if the spin-one target is made up of spin-half constituents at rest, or in a relative s-wave. b_1 is non-zero but very small for a target made up of spin-half particles moving

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non-relativistically in higher angular-momentum states. For the deuteron, we expect $b_1 \approx 0$. On the other hand, for a spin-one hadron (e.g. the ρ) consisting of relativistic spin-half constituents (e.g. quarks) we expect $b_1 \sim O(F_1)$. Thus $b_1(x)$ measures the extent to which a target nucleus deviates from a trivial bound state of protons and neutrons.

The electroproduction cross section off a hadronic target can be calculated in terms of the hadronic tensor

$$W^{\mu\nu}(p,q,\lambda,\lambda') = \frac{1}{4\pi}\int d^4x\, e^{iq\cdot x}\langle p,\lambda'|[j^\mu(x), j^\nu(0)]|p,\lambda\rangle, \tag{1}$$

which is the imaginary part of a forward current–hadron scattering amplitude. λ and λ' are components of the target spin along a quantization axis, and j^μ is the electromagnetic current. $W^{\mu\nu}$ can be decomposed into linearly independent structure functions. If the target has spin-$\frac{1}{2}$, there are four such structure functions: W_1, W_2, G_1 and G_2. W_1 and W_2 are spin independent, whereas G_1 and G_2 depend on the target spin. It is well known that G_1 and G_2 do not contribute to deep inelastic scattering unless the beam is polarized. The analogous tensor for a spin-one target depends on eight invariant structure functions, four in addition to W_1, W_2, G_1 and G_2. The four new structure functions, which we denote b_{1-4}, all appear in the part of $W_{\mu\nu}$ symmetric in $\mu \leftrightarrow \nu$ and therefore contribute to scattering with an unpolarized beam.

Using standard methods of the operator product expansion, we find that the structure functions b_1 and b_2 are leading twist and therefore scale (modulo logarithms) in the Bjorken limit. Furthermore, they obey a relation $b_2(x) = 2xb_1(x)$, which has the same physical basis as the Callan–Gross relation [2] and is violated in a calculable way by radiative corrections in QCD. The other structure functions, b_3 and b_4, do not receive contributions at leading twist. They presumably appear at twist-4 and therefore vanish faster, by one power of Q^2 as $Q^2 \to \infty$, than expected from dimensional analysis alone.

There is a simple parton-model interpretation of the new structure function which is discussed in sect. 4. For a spin-one target, there are three quark probability distributions, $q^1(x)$, $q^0(x)$ and $q^{-1}(x)$, for the 1, 0 and -1 helicity states of the target. Parity invariance of the strong interactions implies that $q^1(x) = q^{-1}(x)$. There are, therefore, two independent distributions, the difference of which is proportional to $b_1(x)$.

In sect. 2, we discuss kinematics, and helicity amplitudes for spin-$\frac{1}{2}$ and spin-one targets, and the decomposition of $W_{\mu\nu}$ for spin one. In sect. 3, we study the structure functions using the operator product expansion, and derive moment sum rules for the structure functions. Sect. 4 contains a naive parton-model discussion of the leading-twist structure functions. b_1 is calculated in various models in sect. 5. Sect. 6 discusses experimental issues surrounding the determination of the new structure function b_1.

2. Kinematics

In this section we classify the independent structure functions for a spin-one target. To be certain we have been complete, and to expose the physical significance of the various structure functions, it is very useful to describe virtual Compton scattering in terms of helicity amplitudes. This approach has the further advantage of generalizing easily to spin greater than one. It is instructive to introduce the classification for the familiar case of a spin-$\frac{1}{2}$ target, the results of which will be required in sect. 5. After this warmup, the spin-one case is not difficult.

The helicity amplitude for $\gamma_{h_1} + \text{target}_{H_1} \to \gamma_{h_2} + \text{target}_{H_2}$ will be denoted by $A_{h_1 H_1, h_2 H_2}$. Choosing the photon incident along the z-axis, h_i, H_i are the spin components along this quantization axis. We are interested only in forward scattering. Angular momentum conservation requires $h_1 + H_1 = h_2 + H_2$. There are ten independent helicity amplitudes for a spin-$\frac{1}{2}$ target. Time-reversal invariance requires $A_{h_1 H_1, h_2 H_2} = A_{h_2 H_2, h_1 H_1}$ and parity requires $A_{h_1 H_1, h_2 H_2} = A_{-h_1 -H_1, -h_2 -H_2}$. Imposing P and T invariance leaves four independent helicity amplitudes which we choose to be $A_{+\uparrow,+\uparrow}$, $A_{+\downarrow,+\downarrow}$, $A_{0\uparrow,0\uparrow}$ and $A_{+\downarrow,0\uparrow}$. The standard decomposition of $W_{\mu\nu}$ for a spin-$\frac{1}{2}$ target involves four Lorentz (and P and T) invariant structure functions W_1, W_2, G_1 and G_2. It is more convenient to use the scaling forms $F_1 \equiv W_1$, $F_2 \equiv \nu W_2/M^2$, $g_1 \equiv \nu G_1/M^2$ and $g_2 \equiv \nu^2 G_2/M^4$ (where $\nu = p \cdot q$, $Q^2 = -q^2 > 0$). In writing $W_{\mu\nu}$, we may drop terms proportional to q_μ and/or q_ν because the leptonic current is conserved

$$W_{\mu\nu} = -F_1 g_{\mu\nu} + \frac{F_2}{\nu} p_\mu p_\nu + \frac{i g_1}{\nu} \epsilon_{\mu\nu\lambda\sigma} q^\lambda s^\sigma + \frac{i g_2}{\nu^2} \epsilon_{\mu\nu\lambda\sigma} q^\lambda (p \cdot q s^\sigma - s \cdot q p^\sigma), \quad (2)$$

where $\epsilon_{0123} = 1$, and s^μ is the spin four-vector, $s^\mu p_\mu = 0$, normalized so that $s^2 = -M^2$. The helicity amplitudes are given by

$$A_{h_1 H_1, h_2 H_2} \equiv \epsilon_{h_2}^{*\mu} \epsilon_{h_1}^{\nu} W_{\mu\nu}(s),$$

where $s = M \mathscr{U}_{H_2}^\dagger \sigma \mathscr{U}_{H_1}$. Here ϵ_h^μ is the polarization vector of an helicity-h photon, and $\mathscr{U}_H$ is the Pauli spinor for a spin-$\frac{1}{2}$ particle with spin H along the z-axis. Choosing a coordinate frame where the target momentum is $p^\mu = (M,0,0,0)$ and the photon momentum is $q^\mu = (q^0, 0, 0, q^3)$, we have $\sqrt{2}\,\epsilon_+^\mu = (0, -1, -i, 0)$, $\sqrt{2}\,\epsilon_-^\mu = (0, 1, -i, 0)$, $\sqrt{-q^2}\,\epsilon_0^\mu = (q^3, 0, 0, q^0)$, and $\mathscr{U}_\uparrow = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\mathscr{U}_\downarrow = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. We can then calculate the helicity amplitudes by substituting these vectors into eq. (2) and obtain

$$\begin{aligned}
A_{+\uparrow,+\uparrow} &= F_1 - g_1 + (\kappa - 1) g_2, \\
A_{+\downarrow,+\downarrow} &= F_1 + g_1 - (\kappa - 1) g_2, \\
A_{0\uparrow,0\uparrow} &= -F_1 + \frac{\kappa}{2x} F_2, \\
A_{+\downarrow,0\uparrow} &= \sqrt{2(\kappa - 1)}\,(g_1 + g_2),
\end{aligned} \quad (3)$$

where

$$\kappa \equiv 1 + \frac{4x^2M^2}{Q^2} = 1 + \frac{M^2Q^2}{\nu^2}. \tag{4}$$

In the scaling limit, $Q^2 \to \infty$, x fixed, the functions $F_i(x, Q^2)$, $g_i(x, Q^2)$ approach a finite limit $F_i(x)$, $g_i(x)$, modulo logarithms from QCD radiative corrections. In this limit, the "diagonal" helicity amplitudes $A_{+\uparrow,+\uparrow}$, $A_{+\downarrow,+\downarrow}$ and $A_{0\uparrow,0\uparrow}$ approach finite limits, whereas the helicity-flip amplitude $A_{+\downarrow,0\downarrow}$ vanishes as $1/\sqrt{Q^2}$.

For a spin-one target there are eight independent helicity amplitudes after requiring P and T invariance. We choose them to be $A_{++,++}$, $A_{+0,+0}$, $A_{+0,0+}$, $A_{+-,+-}$, $A_{+-,-+}$, $A_{+-,00}$, $A_{0+,0+}$ and $A_{00,00}$. Note that as for spin $\frac{1}{2}$, the first label is the photon helicity and the second label is the target spin $(+,0,-)$. Since there are eight independent helicity amplitudes, $W_{\mu\nu}$ for a spin-1 target can be decomposed into eight independent structure functions. We will drop the terms proportional to q^μ and q^ν to simplify the expression

$$W_{\mu\nu} = -F_1 g_{\mu\nu} + F_2 \frac{p_\mu p_\nu}{\nu} - b_1 r_{\mu\nu} + \tfrac{1}{6} b_2 (s_{\mu\nu} + t_{\mu\nu} + u_{\mu\nu}) + \tfrac{1}{2} b_3 (s_{\mu\nu} - u_{\mu\nu})$$
$$+ \tfrac{1}{2} b_4 (s_{\mu\nu} - t_{\mu\nu}) + i\frac{g_1}{\nu}\epsilon_{\mu\nu\lambda\sigma} q^\lambda s^\sigma + i\frac{g_2}{\nu^2}\epsilon_{\mu\nu\lambda\sigma} q^\lambda (p\cdot q s^\sigma - s\cdot q p^\sigma), \tag{5}$$

where

$$r_{\mu\nu} = \frac{1}{\nu^2}\left(q\cdot E^* q\cdot E - \tfrac{1}{3}\nu^2\kappa\right) g_{\mu\nu},$$

$$s_{\mu\nu} = \frac{2}{\nu^2}\left(q\cdot E^* q\cdot E - \tfrac{1}{3}\nu^2\kappa\right)\frac{p_\mu p_\nu}{\nu},$$

$$t_{\mu\nu} = \frac{1}{2\nu^2}\left(q\cdot E^* p_\mu E_\nu + q\cdot E^* p_\nu E_\mu + q\cdot E p_\mu E_\nu^* + q\cdot E p_\nu E_\mu^* - \tfrac{4}{3}\nu p_\mu p_\nu\right),$$

$$u_{\mu\nu} = \frac{1}{\nu}\left(E_\mu^* E_\nu + E_\nu^* E_\mu + \tfrac{2}{3}M^2 g_{\mu\nu} - \tfrac{2}{3}p_\mu p_\nu\right),$$

where

$$s^\sigma \equiv \frac{-i}{M^2}\epsilon^{\sigma\alpha\beta\tau} E_\alpha^* E_\beta p_\tau .$$

E is the polarization of the target, $p\cdot E = 0$, normalized so that $E^2 = -M^2$. s^σ is analogous to the spin vector of a spin-$\frac{1}{2}$ particle. The reason for this identification will become clearer in sect. 4. The functions F_1 and F_2 and g_1 and g_2 are analogous to the scaling structure functions of a spin-$\frac{1}{2}$ target with the same name. They are measured in precisely the same way for spin one as for spin $\frac{1}{2}$. The tensors multiplying b_{1-4} have been chosen so that they vanish upon spin averaging

$$\langle E_\mu E_\nu^* \rangle_{\text{avg}} = \tfrac{1}{3}\left(-g_{\mu\nu}M^2 + p_\mu p_\nu\right).$$

The b_1 and b_2 terms are totally symmetric under the interchange of p, E and E^*, whereas the b_3 and b_4 terms have no totally symmetric part. This property will simplify the operator product analysis in sect. 4. The spin-dependent structure functions b_{1-4}, g_1, g_2 can be divided into two groups: b_{1-4}, which are symmetric under $\mu \leftrightarrow \nu$ and under $E \leftrightarrow E^*$, and g_1, g_2 which are antisymmetric under $\mu \leftrightarrow \nu$ and under $E \leftrightarrow E^*$. The cross section for deep inelastic lepton scattering is obtained by contracting $W_{\mu\nu}$ with the lepton current tensor

$$l_{\mu\nu}^{\pm} = \langle j_\mu j_\nu \rangle = \operatorname{Tr} \not{k}\gamma_\mu(\not{k} - \not{q})\gamma_\nu(1 \pm \gamma_5), \tag{6}$$

for helicity-$\pm\frac{1}{2}$ leptons of initial (final) momentum k_μ (k'_μ). Only the symmetric part of $l_{\mu\nu}$ survives averaging over the lepton helicity. Therefore, a polarized beam is required to measure g_1 and g_2, but b_{1-4} can be measured using an unpolarized beam.

The helicity amplitudes can be calculated in terms of the structure functions using methods identical to the spin-$\frac{1}{2}$ case. The results are

$$
\begin{aligned}
A_{++,++} &= F_1 - \tfrac{1}{3}\kappa b_1 + \frac{M^2}{6\nu}a_3 - g_1 + (\kappa-1)g_2, \\
A_{+0,+0} &= F_1 + \tfrac{2}{3}\kappa b_1 - \frac{M^2}{3\nu}a_3, \\
A_{+0,0+} &= \sqrt{\kappa-1}\,(g_1+g_2) + \frac{M}{2\sqrt{Q^2}}a_3 + \frac{\kappa M}{4\sqrt{Q^2}}a_4, \\
A_{+-,+-} &= F_1 - \tfrac{1}{3}\kappa b_1 + \frac{M^2}{6\nu}a_3 + g_1 - (\kappa-1)g_2, \\
A_{+-,00} &= \sqrt{\kappa-1}\,(g_1+g_2) - \frac{M}{2\sqrt{Q^2}}a_3 - \frac{\kappa M}{4\sqrt{Q^2}}a_4, \\
A_{+-,-+} &= \frac{M^2}{\nu}a_3, \\
A_{0+,0+} &= -F_1 + \frac{\kappa F_2}{2x} + \tfrac{1}{3}\kappa b_1 - \frac{1}{18x}(\kappa^2+\kappa+1)b_2 + \frac{1}{6x}(1-\kappa^2)b_3 + \frac{\kappa}{6x}(1-\kappa)b_4, \\
A_{00,00} &= -F_1 + \frac{\kappa F_2}{2x} - \frac{2\kappa b_1}{3} + \frac{1}{9x}(\kappa^2+\kappa+1)b_2 - \frac{1}{3x}(1-\kappa^2)b_3 - \frac{\kappa}{3x}(1-\kappa)b_4,
\end{aligned}
\tag{7}
$$

where $a_3 = (\frac{1}{3}b_2 - b_3)$, $a_4 = (\frac{1}{3}b_2 - b_4)$ and κ is defined by eq. (4).

In the scaling limit the helicity diagonal amplitudes $A_{++,++}$, $A_{+0,+0}$, $A_{+-,+-}$, $A_{0+,0+}$ and $A_{00,00}$ approach finite limiting values. The single helicity-flip amplitudes $A_{+0,0+}$ and $A_{+-,00}$ fall off like $1/\sqrt{Q^2}$. The double helicity-flip amplitude $A_{+-,-+}$ falls off like $1/Q^2$. If QCD radiative corrections are ignored in the scaling limit, $F_2 = 2xF_1$ and $b_2 = 2xb_1$ which implies that $A_{0+,0+}$ and $A_{00,00}$ vanish too.

3. Operator product expansion analysis

The structure functions can be calculated using an operator product expansion for the time-ordered product of two currents

$$T_{\mu\nu} \equiv i\int \mathrm{d}^4x\, \mathrm{e}^{iq\cdot x} T\big(j_\mu(x) j_\nu(x)\big). \tag{8}$$

For deep inelastic scattering, the leading operators in the expansion are twist two. To zeroth order in QCD (i.e. free field theory)

$$\begin{aligned} T_{\mu\nu} = &\sum_{n=2,4,\dots}^{\infty} 2C_n^{(1)}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\frac{2^n q_{\mu_1}\cdots q_{\mu_n}}{(-q^2)^n} O_{\mathrm{V}}^{\mu_1\cdots\mu_n} \\ &+ \sum_{n=2,4,\dots}^{\infty} 2C_n^{(2)}\left(g_{\mu\mu_1} - \frac{q_\mu q_{\mu_1}}{q^2}\right)\left(g_{\nu\mu_2} - \frac{q_\nu q_{\mu_2}}{q^2}\right)\frac{2^n q_{\mu_3}\cdots q_{\mu_n}}{(-q^2)^{n-1}} O_{\mathrm{V}}^{\mu_1\cdots\mu_n} \\ &+ \sum_{n=1,3,\dots}^{\infty} 2C_n^{(3)} i\epsilon_{\mu\nu\lambda\mu_1} q^\lambda \frac{2^n q_{\mu_2}\cdots q_{\mu_n}}{(-q^2)^n} O_{\mathrm{A}}^{\mu_1\cdots\mu_n}, \end{aligned} \tag{9}$$

where $C_n^{(1)} = C_n^{(2)} = C_n^{(3)} = 1 + \mathrm{O}(\alpha_s)$

$$O_{\mathrm{V}}^{\mu_1\cdots\mu_n} = \tfrac{1}{2}\left(\tfrac{1}{2}i\right)^{n-1} S\left\{\bar\psi\gamma^{\mu_1}\overleftrightarrow{D}^{\mu_2}\dots\overleftrightarrow{D}^{\mu_n}\mathcal{Q}^2\psi\right\}, \tag{10}$$

$$O_{\mathrm{A}}^{\mu_1\cdots\mu_n} = \tfrac{1}{2}\left(\tfrac{1}{2}i\right)^{n-1} S\left\{\bar\psi\gamma^{\mu_1}\overleftrightarrow{D}^{\mu_2}\dots\overleftrightarrow{D}^{\mu_n}\gamma_5\mathcal{Q}^2\psi\right\}, \tag{11}$$

$\mathcal{Q}$ is the quark change matrix. S symmetrizes the subsequent operator and removes all traces in $\mu_1 \dots \mu_n$.

To calculate the structure functions, one takes the matrix element of $T_{\mu\nu}$ between target states. For a spin-1 target

$$\begin{aligned} \langle p, E|O_{\mathrm{V}}^{\mu_1\cdots\mu_n}|p, E\rangle &= S\left[a_n p^{\mu_1}\dots p^{\mu_n} + d_n\left(E^{*\mu_1}E^{\mu_2} - \tfrac{1}{3}p^{\mu_1}p^{\mu_2}\right)p^{\mu_3}\dots p^{\mu_n}\right], \\ \langle p, E|O_{\mathrm{A}}^{\mu_1\cdots\mu_n}|p, E\rangle &= S\left[r_n\epsilon^{\lambda\sigma\tau\mu_1}E_\lambda^* E_\sigma p_\tau p^{\mu_2}\dots p^{\mu_n}\right], \end{aligned} \tag{12}$$

which defines a_n, d_n and r_n. The new feature for a spin-1 target is the appearance of the tensor structure $E^{*\mu_1}E^{\mu_2}$ for a polarized target. (The $\frac{1}{3}p^{\mu_1}p^{\mu_2}$ piece cancels the spin-averaged value of $E^{*\mu_1}E^{\mu_2}$.) The structure functions b_1 and b_2 depend on the coefficients d_n.

The target matrix element of $T_{\mu\nu}$ may be expanded in terms of the independent tensors $\tilde{F}_{1,2}$, $\tilde{b}_{1-4}$, $\tilde{g}_{1,2}$ defined in analogy to eq. (5) for $W_{\mu\nu}$. Substituting eq. (12) in eq. (9) and comparing with the expansion for $T_{\mu\nu}$ gives

$$\tilde{F}_1(\omega) = \sum_{n=2,4,\dots}^{\infty} 2C_n^{(1)} a_n \omega^n ,$$

$$\tilde{F}_2(\omega) = \sum_{n=2,4,\dots} 4C_n^{(2)} a_n \omega^{n-1} ,$$

$$\tilde{b}_1(\omega) = \sum_{n=2,4,\dots}^{\infty} 2C_n^{(1)} d_n \omega^n ,$$

$$\tilde{b}_2(\omega) = \sum_{n=2,4,\dots}^{\infty} 4C_n^{(2)} d_n \omega^{n-1} ,$$

$$\tilde{g}_1(\omega) = \sum_{n=1,3,\dots}^{\infty} 2C_n^{(3)} r_n \omega^n , \tag{13}$$

where $\omega = 1/x$. The functions $\tilde{b}_3$, $\tilde{b}_4$ and $\tilde{g}_2$ do not get contributions at leading order in the twist expansion. The functions $\tilde{F}(\omega)$, $\tilde{b}(\omega)$, $\tilde{g}(\omega)$ have a discontinuous imaginary part for ω real, $1 \le |\omega| \le \infty$. This discontinuity gives the functions $F(\omega), b(\omega)$, $g(\omega)$ in the expansion for $W_{\mu\nu}$ (viz. eq. (5))

$$F_i(\omega) = \frac{1}{2\pi} \operatorname{Im} \tilde{F}_i(\omega + i0), \text{ etc.},$$

and are non-zero for $1 \le |\omega| \le \infty$. From eq. (13), we get the Callan–Gross relation

$$2xF_1 = F_2 ,$$

and a new relation

$$2xb_1 = b_2 ,$$

both of which are satisfied to lowest order in QCD because $C_n^{(1)} = C_n^{(2)}$. In higher orders, $C_n^{(1)} \neq C_n^{(2)}$ and both relations are violated, but we still have the relations

$$M_n(2xb_1) M_n(F_2) = M_n(b_2) M_n(2xF_1), \quad n \quad \text{odd},$$

where $M_n(f)$, the nth moment of $f(x)$, is defined by

$$M_n(f) \equiv \int_0^1 x^{n-1} f(x)\, dx .$$

One also gets a series of sum rules for the moments of the structure functions

$$
\begin{aligned}
2M_n(F_1) &= C_n^{(1)} a_n, && n \quad \text{even},\\
M_{n-1}(F_2) &= C_n^{(2)} a_n, && n \quad \text{even},\\
2M_n(b_1) &= C_n^{(1)} d_n, && n \quad \text{even},\\
M_{n-1}(b_2) &= C_n^{(2)} d_n, && n \quad \text{even},\\
2M_n(g_1) &= C_n^{(3)} r_n, && n \quad \text{odd}.
\end{aligned}
\tag{14}
$$

The operator product expansion (9) is calculated using QCD, and clearly does not depend on the nature of the target. The target dependence of $W_{\mu\nu}$ is due only to the target dependence of the matrix elements of $O_V^{\mu_1 \cdots \mu_n}$ and $O_A^{\mu_1 \cdots \mu_n}$. Therefore, QCD corrections to eq. (9) are precisely the same as for electroproduction off nucleons. In particular, the anomalous dimensions of the operators $O_{V,A}$ and the coefficient functions $C_n^{(1,2,3)}$ are the standard ones. Thus the moments of F_1 and F_2 and g_1 have the same anomalous dimensions as for a spin-$\frac{1}{2}$ target. Since b_1, b_2 are obtained from the same operator tower as F_1, F_2, they obey the same scaling equations as F_1 and F_2. The anomalous dimensions of the moments of g_2 are determined by axial vector operators of twist-3, and of those of b_3 and b_4 by vector operators of twist-4.

4. Parton model calculation*

The twist-two structure functions F_1, F_2, b_1, b_2 and g_1 can be calculated using a parton-model description. Let $q_\uparrow^m(x)$ ($q_\downarrow^m(x)$) be the probability to find a quark with momentum fraction x and spin up (down) along the z-axis in a hadron moving with infinite momentum along the z-axis, with spin m along the z-axis. Reflection symmetry in the xz plane implies

$$
\begin{aligned}
&q_\uparrow^1(x) = q_\downarrow^{-1}(x), \qquad q_\downarrow^1(x) = q_\uparrow^{-1}(x),\\
&q_\uparrow^0(x) = q_\downarrow^0(x).
\end{aligned}
$$

There are only three independent parton distributions, which we choose to be $q_\uparrow^1(x)$, $q_\downarrow^1(x)$ and $q_\uparrow^0(x)$. Let $W_{\mu\nu}^{(m)}$ be $W_{\mu\nu}$ for a target with spin m along the

*For simplicity, we omit the flavor indices, the charge-squared, and the anti-quark distributions for each structure function.

z-direction. $W_{\mu\nu}^{(m)}$ can be calculated for photon-free quark scattering to be

$$W_{\mu\nu}^{(1)} = \left(-\tfrac{1}{2}g_{\mu\nu} + \frac{x}{\nu}p_\mu p_\nu\right)\left(q_\uparrow^1(x) + q_\downarrow^1(x)\right) + \frac{i\epsilon_{\mu\nu\lambda\sigma}q^\lambda s^\sigma}{2\nu}\left(q_\uparrow^1(x) - q_\downarrow^1(x)\right),$$

$$W_{\mu\nu}^{(0)} = \left(-\tfrac{1}{2}g_{\mu\nu} + \frac{x}{\nu}p_\mu p_\nu\right)2q_\uparrow^0(x),$$

where $s^\sigma = (0,0,0,M) = -i/M^2 e^{\sigma\alpha\beta\tau}E_\alpha^* E_\beta p_\tau$. Therefore

$$W_{\mu\nu}^{(1)} - W_{\mu\nu}^{(0)} = \left(-\tfrac{1}{2}g_{\mu\nu} + \frac{x}{\nu}p_\mu p_\nu\right)\left(q_\uparrow^1(x) + q_\downarrow^1(x) - 2q_\uparrow^0(x)\right)$$
$$+ \frac{i\epsilon_{\mu\nu\lambda\sigma}q^\lambda s^\sigma}{2\nu}\left(q_\uparrow^1(x) - q_\downarrow^1(x)\right),$$

$$W_{\mu\nu}^{\text{avg}} = \left(-\tfrac{1}{2}g_{\mu\nu} + \frac{x}{\nu}p_\mu p_\nu\right)\tfrac{2}{3}\left(q_\uparrow^1(x) + q_\downarrow^1(x) + q_\uparrow^0(x)\right), \tag{15}$$

where $W_{\mu\nu}^{\text{avg}}$ is the spin-averaged value of $W_{\mu\nu}$. Comparing the spin-averaged value of $W_{\mu\nu}$ in eq. (15) with eq. (5) we find

$$F_1(x) = \tfrac{1}{3}\left[q_\uparrow^1(x) + q_\downarrow^1(x) + q_\uparrow^0(x)\right],$$

$$F_2(x) = 2xF_1(x). \tag{16}$$

Comparing $W_{\mu\nu}^{(1)} - W_{\mu\nu}^{(0)}$ with the value obtained from eq. (5) by substituting the $m=1$ and $m=0$ polarization vectors, we find

$$b_1(x) = \tfrac{1}{2}\left(2q_\uparrow^0(x) - q_\uparrow^1(x) - q_\downarrow^1(x)\right),$$

$$b_2(x) = 2xb_1(x),$$

$$g_1(x) = \tfrac{1}{2}\left(q_\uparrow^1(x) - q_\downarrow^1(x)\right), \tag{17}$$

$g_1(x)$ depends on the difference $q_\uparrow^1(x) - q_\downarrow^1(x)$. Since $q_\uparrow^0 = q_\downarrow^0$, g_1 does not contribute to the scattering cross-section for an $m=0$ target. b_1 and b_2 depend only on the spin-averaged distributions $q^1(x) = (q_\uparrow^1 + q_\downarrow^1)$ and $q^0(x) = (q_\uparrow^0 + q_\downarrow^0) = 2q_\uparrow^0(x)$; $b_1(x) = \frac{1}{2}(q^0(x) - q^1(x))$. $b_1(x)$ is a measure of the difference in parton distributions of an $m=1$ and $m=0$ target.

One can use the parton-model description to calculate the independent structure functions at leading twist for a spin-j target. There are $2(2j+1)$ quark distributions $q_\uparrow^m$ and $q_\downarrow^m$, $-j \le m \le j$. By parity $q_\uparrow^m = q_\downarrow^{-m}$. This leaves the distributions $q_\uparrow^m$, $q_\downarrow^m$,

$m > 0$ and $q_\uparrow^0$ if j = integer, a total of $2j+1$ distributions. If $2j$ is odd, there are $j+\frac{1}{2}$ distributions $q_\uparrow^m + q_\downarrow^m$, $m > 0$, that contribute to $W^{\mu\nu} + W^{\nu\mu}$, and $j+\frac{1}{2}$ distributions $q_\uparrow^m - q_\downarrow^m$, $m > 0$, that contribute to $W^{\mu\nu} - W^{\nu\mu}$. If $2j$ is even, there are $j+1$ distributions $q_\uparrow^m + q_\downarrow^m$, $m \geq 0$ that contribute to $W^{\mu\nu} + W^{\nu\mu}$ and j distributions $q_\uparrow^m - q_\downarrow^m$, $m > 0$ that contribute to $W^{\mu\nu} - W^{\nu\mu}$.

5. Contributions to $b_1(x)$ from bound nucleons

Unfortunately, nuclei are the only spin-1 hadrons available for use as targets in deep inelastic scattering. To a very good approximation, nuclei can be regarded as composed of nucleons bound by meson exchange, perhaps with properties modified slightly by the nuclear medium. There has been considerable interest recently in signatures for more exotic hadronic components of nuclei. $b_1(x)$ may be such a signature. In this section, we show that $b_1(x)$ vanishes for a nucleus made of nucleons at rest. Next, we study nucleons moving in a central potential. We show that nucleons with total angular momentum $j = \frac{1}{2}$ (either $l = 0$ or $l = 1$) make no contribution to $b_1(x)$. Higher angular-momentum states, $j \geq \frac{3}{2}$, contribute to $b_1(x)$, but the contribution is small and calculable. We adapt this discussion to the d-state admixture in the deuteron. Finally, to demonstrate that $b_1(x)$ is not suppressed at the quark level, we calculate the contribution to $b_1(x)$ from a massless relativistic quark in a $j = \frac{3}{2}$ state within a spin-1 hadron.

5.1. TWO SPIN-$\frac{1}{2}$ NUCLEONS AT REST

If we treat the spin-1 target as made up of two spin-$\frac{1}{2}$ non-interacting nucleons at rest, we can calculate the eight spin-1 structure functions in terms of the structure functions of the spin-$\frac{1}{2}$ constituents. The helicity amplitudes are trivially related to each other. Using $|+\rangle = |\uparrow\uparrow\rangle$, $\sqrt{2}\,|0\rangle = |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$ and $|-\rangle = |\downarrow\downarrow\rangle$ we find

$$\begin{aligned}
A_{++,++} &= A^{(1)}_{+\uparrow,+\uparrow} + A^{(2)}_{+\uparrow,+\uparrow}\,, \\
A_{+-,+-} &= A^{(1)}_{+\downarrow,+\downarrow} + A^{(2)}_{+\downarrow,+\downarrow}\,, \\
\sqrt{2}\,A_{+-,00} &= A^{(1)}_{+\downarrow,0\uparrow} + A^{(2)}_{+\downarrow,0\uparrow}\,, \\
A_{+-,-+} &= 0, \\
2A_{+0,+0} &= A^{(1)}_{+\uparrow,+\uparrow} + A^{(1)}_{+\downarrow,+\downarrow} + A^{(2)}_{+\uparrow,+\uparrow} + A^{(2)}_{+\downarrow,+\downarrow}\,, \\
A_{00,00} &= A^{(1)}_{0\uparrow,0\uparrow} + A^{(2)}_{0\uparrow,0\uparrow}\,, \\
\sqrt{2}\,A_{+0,-+} &= A^{(1)}_{+\downarrow,0\uparrow} + A^{(2)}_{+\downarrow,0\uparrow}\,, \\
A_{0+,0+} &= A^{(1)}_{0\uparrow,0\uparrow} + A^{(2)}_{0\uparrow,0\uparrow}\,,
\end{aligned} \tag{18}$$

where $A^{(1)}$ and $A^{(2)}$ are the helicity amplitudes for scattering off constituents 1 and 2, respectively. Since eight helicity amplitudes have been expressed in terms of four, there are four relations among the spin-1 helicity amplitudes

$$
\begin{aligned}
A_{+-,-+} &= 0\,, \\
2A_{+0,+0} &= A_{++,++} + A_{+-,+-}\,, \\
A_{00,00} &= A_{0+,0+}\,, \\
A_{+0,0+} &= A_{+-,00}\,.
\end{aligned}
\tag{19}
$$

Converting these to relations among structure functions, we find

$$
\begin{aligned}
F_1 &= F_1^{(1)} + F_1^{(2)}\,, \\
F_2 &= F_2^{(1)} + F_2^{(2)}\,, \\
g_1 &= g_1^{(1)} + g_1^{(2)}\,, \\
g_2 &= g_2^{(1)} + g_2^{(2)}\,, \\
b_1 &= b_2 = b_3 = b_4 + 0\,.
\end{aligned}
$$

5.2. NUCLEON MOVING NON-RELATIVISTICALLY IN A POTENTIAL

Next, we consider the structure functions of a nucleon moving non-relativistically in some central potential. For a two-body bound state, momentum conservation is important and changes our results somewhat. This will be discussed in subsect. 5.3. As before, we use the photon helicity, target-spin amplitudes $A_{h_1H_1,\,h_2H_2}$ with

$$b_1(x) = \tfrac{1}{2}\left(2A_{+0,+0} - A_{++,++} - A_{+-,+-}\right). \tag{20}$$

Each of these diagonal amplitudes scales and is given by a sum over photon–nucleon helicity amplitudes modified by the target motion. This modification is easily incorporated using the convolution formalism developed in ref. [3]. In the scaling limit each diagonal amplitude $A_{hH,hH}$ at $Q^2/2M_{\mathrm{N}}q^0 \equiv x$ is a convolution of the null-plane probability to find a nucleon with a light-like momentum fraction y and spin s ($\uparrow$ or $\downarrow$) with the appropriate spin-$\frac{1}{2}$ nucleon helicity amplitude ($A_{h\uparrow,h\uparrow}$ or $A_{h\downarrow,h\downarrow}$) with the scaling argument z, constrained so $x = yz$,

$$A_{hH,hH}(x) = \int_0^{M_{\mathrm{A}}/M} \mathrm{d}y \int_0^1 \mathrm{d}z\, \delta(x-yz) \sum_{s=\uparrow\downarrow} f_s^H(y) A_{hs,hs}(z)\,. \tag{21}$$

M_{A} is the mass of the "nucleus," i.e. nucleon plus core. The limits on the y and z integrations and the normalization and interpretation of $f_s(y)$ are discussed in refs. [3, 4]. $f_s^H(y)$ is given by a modified nucleon-momentum distribution

$$f_s^H(y) \equiv \int \mathrm{d}^3p\, \varphi_s^{H\dagger}(\boldsymbol{p})(1+\alpha^3)\varphi_s^H(\boldsymbol{p})\,\delta\left(y - \frac{p\cos\theta + E}{M}\right), \tag{22}$$

where $\varphi_s^H(\boldsymbol{p})$ is obtained by decomposing the state $|H\rangle$ according to whether the nucleon spin is $\uparrow$ or $\downarrow$ along the axis defined by the virtual photon (from which θ is measured)

$$\varphi_s^H(\boldsymbol{p}) = \langle \boldsymbol{p}s|H\rangle ;$$

α^3 is the Dirac matrix $\alpha^3 = \begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{pmatrix}$. The structure $(1+\alpha^3) = \sqrt{2}\,\gamma^0\gamma^+$ reflects the underlying null-plane kinematics of deep inelastic scattering [4]. Frequently, the term proportional to α^3 is dropped as a "relativistic correction". However, if it is kept it is seen to be the same order $(\mathrm{O}(p^2/M^2))$ as the rest of the expression. $f_s^H(y)$ is normalized to $\Sigma_{s=\pm 1/2} \int_0^{M_{\mathrm{A}}/M} \mathrm{d}y f_s^H(y) = 1$.

$b_1(x)$ can be calculated from $A_{hH,hH}(x)$ making use of the symmetries of the A's and the f's ($f_\uparrow^H = f_\downarrow^{-H}$) and eq. (3) relating the A's to F_1 and g_1 in the scaling limit

$$b_1(x) = \int_0^{M_{\mathrm{A}}/M} \mathrm{d}y \int_0^1 \mathrm{d}z\, \delta(yz - x)\Delta f(y) F_1(z),$$

where

$$\Delta f(y) = \tfrac{1}{2}\left(2f^0(y) - f^1(y) - f^{-1}(y)\right)$$

$$= \int \mathrm{d}^3p \Delta\left(\varphi^\dagger(1+\alpha^3)\varphi\right)\delta\left(y - \frac{p\cos\theta + E}{M}\right), \tag{23}$$

$$\Delta\left(\varphi^\dagger(1+\alpha^3)\varphi\right) \equiv \tfrac{1}{2}\left(2\varphi^{0\dagger}(1+\alpha^3)\varphi^0 - \varphi^{1\dagger}(1+\alpha^3)\varphi^1 - \varphi^{-1\dagger}(1+\alpha^3)\varphi^{-1}\right).$$

We have defined $f^H(y) = f_\uparrow^H(y) + f_\downarrow^H(y)$ and $\varphi^H = \varphi_\uparrow^H + \varphi_\downarrow^H$. It is clear from eq. (23) that $b_1(x)$ does not depend on the nucleon spin, but instead measures only the dependence of the nucleon momentum-distribution on the nucleus' spin. In line with this, note that all dependence on $g_1(z)$ has cancelled.

It remains merely to compute $\Delta(\varphi^\dagger(1+\alpha^3)\varphi)$ using angular-momentum algebra. The result is proportional to $Y_{20}(\Omega_p)$ because of the quadrupole nature of $\Delta(\varphi^\dagger(1+\alpha^3)\varphi)$. As a first, pedagogical, example, consider a *spinless* nucleon in a p-state, with $\varphi^H(\boldsymbol{p}) = \xi(p)Y_{1H}(\Omega_p)$. Then, an elementary calculation yields $\Delta = (3/8\pi)(3\cos^2\theta - 1)[(E + p\cos\theta)/M]\xi^2(p)$. More generally, consider a *spinless* nucleon with angular momentum l_1 coupled to a "core" with angular momentum l_2 to give $L=J=1$ and $M_J=H$. Then it is easy to show (using the Wigner–Eckart theorem) that

$$\Delta = \frac{45}{8\pi}(2l_1+1)\begin{pmatrix} 2 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 2 & l_1 & l_1 \\ 0 & 0 & 0 \end{pmatrix}\begin{Bmatrix} l_1 & 1 & l_2 \\ 1 & l_1 & 2 \end{Bmatrix}$$

$$\times(-1)^{l_2}(3\cos^2\theta - 1)\xi^2(p).$$

Finally, we consider the physically most interesting case of a Dirac nucleon with total angular momentum j,

$$\varphi(\boldsymbol{p}) = \begin{pmatrix} \xi(p) \\ (\boldsymbol{\sigma}\cdot\boldsymbol{p}/M)\eta(p) \end{pmatrix}\varphi_{jlm}(\Omega_p),$$

coupled to a "core" of angular momentum j' to give total angular momentum $J=1$ and $M_J=H$. Then

$$\Delta\left(\varphi^{\dagger}(1+\alpha^3)\varphi\right) = \frac{\beta_{ljj'}}{4\pi}\left(\xi^2(p) + \frac{p^2\eta^2(p)}{M^2} + 2\frac{p}{M}\xi(p)\eta(p)\cos\theta\right)(3\cos^2\theta - 1).$$

The coefficient $\beta_{ljj'}$ contains all the angular-momentum dependence. A general formula for it may be readily derived through the use of the algebra of Wigner $3-j$ and $6-j$ coefficients. The result is

$$\beta_{ljj'} = \tfrac{45}{4}(-1)^{j'-1/2}(2j+1)(2l+1)\begin{pmatrix} 2 & l & l \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 2 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\times\begin{Bmatrix} 2 & j & j \\ j' & 1 & 1 \end{Bmatrix}\begin{Bmatrix} 2 & l & l \\ \frac{1}{2} & j & j \end{Bmatrix}.$$

For $l=0$, $j=1/2$, $\Delta(\varphi^{\dagger}(1+\alpha^3)\varphi)=0$ and $b_1(x)$ remains zero although the nucleon is in motion. From the above formula it is clear that for $l=1$, j and j' may be $\frac{1}{2}$ or $\frac{3}{2}$. If $j=\frac{1}{2}$ ($p_{1/2}$ nucleon) then $\Delta(\varphi^{\dagger}(1+\alpha^3)\varphi)=0$ regardless of j'. If $j=\frac{3}{2}$, ($p_{3/2}$ nucleon) and $j'=\frac{1}{2}$, $\beta_{1(3/2)(1/2)}=\frac{3}{4}$, for $j'=\frac{3}{2}$, $\beta_{1(3/2)(3/2)}=-\frac{3}{5}$. For $l=2$, j may be $\frac{3}{2}$ or $\frac{5}{2}$ and depending on j, j' may range from $\frac{1}{2}$ to $\frac{7}{2}$. As an example, we have computed for $j=\frac{3}{2}$ and $j'=\frac{1}{2}$, $\beta_{2(3/2)(1/2)}=\frac{3}{4}$.

The oscillation of $\Delta(\varphi^{\dagger}(1+\alpha^3)\varphi)$ with $\cos\theta$ translates directly (via eq. (23)) into an oscillation of $\Delta f(y)$ with y. For a non-relativistic nucleon ($\boldsymbol{p}^2/M^2 \ll 1$), $\Delta f(y)$ is concentrated near $y=1$ and can be approximated by a distribution at $y=1$ [3]. Each component, $f_s^H(y)$, has a δ-function term in its distribution, but it cancels from the difference $\Delta f(y)$. To $\mathrm{O}(p^2/M^2)$ all the effects of nucleon motion are expressible in terms of

$$\left\langle \frac{p^2}{M^2} \right\rangle_1 \equiv \int_0^\infty p^2 \, \mathrm{d}p \, \xi^2(p) \frac{p^2}{M^2},$$

$$\left\langle \frac{p^2}{M^2} \right\rangle_2 \equiv \int_0^\infty p^2 \, \mathrm{d}p \, \xi(p) \eta(p) \frac{p^2}{M^2}.$$

Then

$$\Delta f(y) = \beta_{ljj'} \left\{ -\frac{8}{15} \left\langle \frac{p^2}{M^2} \right\rangle_2 \delta'(y-1) + \frac{2}{15} \left\langle \frac{p^2}{M^2} \right\rangle_1 \delta''(y-1) \right\} + \mathrm{O}(p^4/M^4).$$

For comparison, the function $f(y)$ ($f_s^H(y)$ averaged over H and s) which related the nuclear structure function $F_1(x)$ to the *nucleon* structure function $F_1(z)$, is given by

$$f(y) = \delta(y-1-\eta) + \frac{1}{6} \left\langle \frac{p^2}{M^2} \right\rangle_1 \delta''(y-1-\eta) + \mathrm{O}(p^4/M^4),$$

where

$$\eta = \left\langle \frac{\varepsilon}{M} \right\rangle + \frac{2}{3} \left\langle \frac{p^2}{M^2} \right\rangle_2,$$

$$\left\langle \frac{\varepsilon}{M} \right\rangle = \int_0^\infty p^2 \, \mathrm{d}p \, \xi^2(p) \frac{\varepsilon}{M},$$

and $\varepsilon \equiv E - M$ is the nucleon's energy in the nucleus which may be p dependent and is of order p^2/M. The shift, η, in the argument of δ has been chosen so that no term proportional to δ' occurs.

$b_1(x)$ is suppressed because $\Delta f(y)$ is $\mathrm{O}(p^2/M^2)$ and because $\int \mathrm{d}y \Delta f(y) = 0$. Naively, one would expect effects at $\mathrm{O}(\sqrt{p^2/M^2})$, since the width of $f(y)$ is easily seen to be of this order. However, the coefficient of $\sqrt{p^2/M^2}$ in the generalized function-expansion of both $f(y)$ and $\Delta f(y)$ vanishes because of the orthonormality properties of Legendre polynomials and the relative order (in p^2/M^2) of the upper and lower components of Dirac wavefunctions. Quantitative estimates of these effects for anticipated nuclear targets can and should be made. It appears that the contribution to $b_1(x)$ from nucleons in motion is small and is influenced significantly by the lower component of the nucleon's Dirac wave function.

5.3. *D*-STATE ADMIXTURE IN THE DEUTERON

The formalism developed in subsect.5.2 must be modified slightly for the deuteron. The proton and neutron move in opposite directions with momentum

$\pm\frac{1}{2}\boldsymbol{p}$, so one must replace $p\cos\theta$ by $\pm\frac{1}{2}p\cos\theta$ in the arguments of the δ-functions which define $f(y)$ and $\Delta f(y)$. Since the integrand is even under $\cos\theta\to-\cos\theta$, the net effect is merely to replace p by $\frac{1}{2}p$ in the δ-function. Also, there is a new term due to s–d interference. A calculation analogous to subsect. 5.2 gives for the deuteron

$$b_1(x)=\sum_{k=p,n}\int \mathrm{d}y\,\mathrm{d}z\,\delta(yz-x)\left\{\sin^2\alpha\,\Delta f_{dd}(y)-4\sqrt{\tfrac{2}{5}}\,\sin\alpha\cos\alpha\,\Delta f_{sd}(y)\right\}F_1^k(z),$$

where $\sin\alpha$ measures the d-state admixture. ($|^2\mathrm{H}\rangle=\cos\alpha|s\rangle+\sin\alpha|d\rangle$) and the functions Δf_{dd} and Δf_{sd} are defined by

$$\Delta f_{dd}(y)=-\frac{1}{16\pi}\int \mathrm{d}^3p\left(\xi_d^2(p)+\frac{p^2}{M^2}\eta_d^2(p)+2\frac{p}{M}\xi_d(p)\eta_d(p)\cos\theta\right)$$

$$\times(3\cos^2\theta-1)\,\delta\left(\frac{\frac{1}{2}p\cos\theta+E}{M}-y\right)$$

$$\Delta f_{sd}(y)=\frac{\sqrt{5}}{8\pi}\int \mathrm{d}^3p\left(\xi_s(p)\xi_d(p)+\frac{p^2}{M^2}\eta_s(p)\eta_d(p)\right.$$

$$\left.+\frac{p}{M}(\eta_s(p)\xi_d(p)+\xi_s(p)\eta_d(p))\cos\theta\right)$$

$$\times(3\cos^2\theta-1)\,\delta\left(\frac{\frac{1}{2}p\cos\theta+E}{M}-y\right).$$

The contributions are further suppressed by the small value of p^2/M^2 for the deuteron.

5.4. $b_1(x)$ FOR A MASSLESS RELATIVISTIC QUARK

Lest the reader conclude that $b_1(x)$ is always negligible, we present a case where it is not. Consider a massless quark with $j=\frac{3}{2}$ moving in a central potential. The best known example is the MIT bag model where the potential is approximated by a confining boundary condition. This model is known to give a reasonable zeroth-order approximation to the valence component of hadron structure functions [5]. The nucleon structure functions in such a model peak near $x\approx\frac{1}{3}$ because the quarks carry roughly one-third of the nucleon's energy. The spread about $x\approx\frac{1}{3}$ is a result of the quark motion within the nucleon and is large because the quarks are relativistic. We form a state with total $j=1$ by coupling to another quark with $j=\frac{1}{2}$. The methods for extracting the structure functions of bag eigenstates are now well known [6]. We take the quark wavefunction in momentum space to be

$$\psi_{jlm}(p) = N\begin{pmatrix} f(p) \\ g(p)\boldsymbol{\sigma}\cdot\hat{p} \end{pmatrix}\varphi_{jlm},$$

where φ_{jlm} is a spinor spherical harmonic, we have chosen $j=\frac{3}{2}$ and, for definiteness, we take $l=1$. We take the quark to be on energy shell with energy ε in a hadron of mass M. The spin $+1$, 0 and -1 states of the target are linear superpositions of $j=\frac{3}{2}$ quark states with $m_j=\pm\frac{1}{2}$ and $\pm\frac{3}{2}$, each of which contributes incoherently to the structure function $b_1(x)$. The quark distributions are (once again) light-cone probability distributions

$$f(x) = \int \mathrm{d}^3p\, \bar{\psi}(\boldsymbol{p})\gamma^+\psi(\boldsymbol{p})\,\delta\left(\frac{p\cos\theta+\varepsilon}{M}-x\right),$$

up to an irrelevant normalization. From their definition in terms of target-helicity amplitudes we compute $b_1(x)/F_1(x)$ and find

$$\frac{b_1}{F_1} = \frac{3\int \mathrm{d}^3p\left(f^2(p)g^2(p)+2\cos\theta f(p)g(p)\right)(3\cos^2\theta-1)\,\delta([(p\cos\theta+\varepsilon)/M]-x)}{4\int \mathrm{d}^3p\left(f^2(p)+g^2(p)+2(\cos\theta)f(p)g(p)\right)\delta([(p\cos\theta+\varepsilon)/M]-x)}$$

which resembles the distribution $\Delta f(y)$ for a $p_{3/2}$ nucleon except for the crucial difference that the motion is relativistic. To obtain a numerical estimate, we have taken $\varepsilon=\frac{1}{2}M$, $f(p)=\exp-2p^2/\varepsilon^2$ and $g(p)=apf(p)/\varepsilon$ with a estimated from the bag boundary conditions as $a\approx 0.36$. The results are shown in fig. 1; $b_1(x)$ is not small. The fact that $b_1(x)/F_1(x)$ changes sign twice and that it fails to vanish for $x>1$ are artifacts of this crude model. Better estimates could be attempted but a real understanding of $b_1(x)$ at the quark level is not yet available.

6. Comments on experiments and conclusions

The cross section for deep inelastic lepton scattering with an unpolarized beam off a spin-one target can be calculated from eqs. (5) and (6) (neglecting higher twist effects due to b_3, b_4 and g_2, and using $F_2=2xF_1$, $b_2=2xb_1$). The most general target polarization may be parameterized by a density matrix, ρ, with $\rho^\dagger=\rho$ and $\mathrm{Tr}\,\rho=1$. In terms of ρ we find

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}y}(\rho) = \frac{e^4ME}{2\pi Q^4}\left[1+(1-y)^2\right]\left[xF_1(x)-\frac{1}{\sqrt{3}}xb_1(x)\mathrm{Tr}\,\rho\lambda_8\right],$$

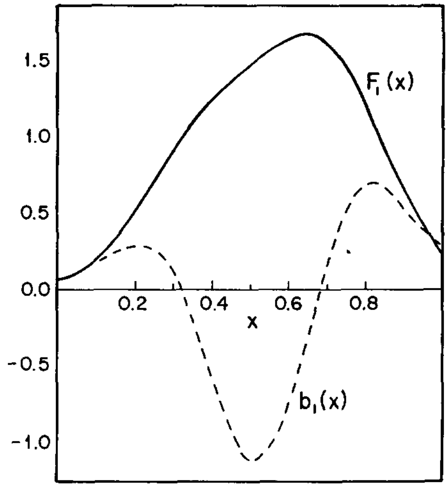

Fig. 1. $F_1(x)$ and $b_1(x)$ for a $P_{3/2}$ massless quark coupled to a spin-$\frac{1}{2}$ spectator to form a $j=1$ state.

where $\lambda_8 \equiv \sqrt{\frac{1}{3}}\,\mathrm{diag}(1,1,-2)$ and ρ is referred to a cartesian basis in which the beam defines the z-axis. Important special cases are first, the target polarized with spin component H along the beam

$$\frac{\mathrm{d}\sigma_{\parallel}^{H}}{\mathrm{d}x\,\mathrm{d}y}=\frac{e^4ME}{2\pi Q^4}\left[1+(1-y)^2\right]\left[xF_1(x)+\left(\tfrac{2}{3}-H^2\right)xb_1(x)\right],$$

and second, the target polarized with spin component H perpendicular to the beam

$$\frac{\mathrm{d}\sigma_{\perp}^{H}}{\mathrm{d}x\,\mathrm{d}y}=\frac{e^4ME}{2\pi Q^4}\left[1+(1-y)^2\right]\left[xF_1(x)-\left(\tfrac{1}{3}-\tfrac{1}{2}H^2\right)xb_1(x)\right].$$

A simple way of determining the structure functions F_1 and b_1 is to measure the cross section for an unpolarized target, which determines F_1, and to measure the cross section for a target polarized along the beam direction, which determines $F_1-\frac{1}{3}b_1$. If the beam is polarized, the structure function g_1 also contributes to the cross section. g_1 can be determined as for spin $\frac{1}{2}$, by taking the difference in cross sections for target polarized parallel and antiparallel to the beam. To avoid g_1 contaminating the measurement of b_1 outlined above, it is preferable to average the cross sections for a target polarized parallel and antiparallel to the beam direction to determine $F_1-\frac{1}{3}b_1$.

In this paper we have concentrated on the case of a spin-one target. Similar effects exist for targets of higher spin, as discussed in sect. 4. The parton-model and helicity-amplitude descriptions of the leading twist structure functions are relatively simple for higher spin targets. However, the tensor decomposition of $W_{\mu\nu}$ is more complicated. The cross sections for a spin-j target polarized parallel to the beam determines the quark distribution function $q^j(x)$*. Measuring the cross section for an unpolarized target determines the averaged quark distribution

$$q(x)=\frac{1}{2j+1}\sum_{m=-j}^{j} q^m(x).$$

In general, this is not enough to determine all the quark distribution functions. However, it is sufficient in two experimentally interesting cases, those where the target spin j is 1 or $\frac{3}{2}$. The unpolarized target measurement determines $\frac{1}{2}q^{3/2}+\frac{1}{2}q^{1/2}$, and the longitudinally polarized target measurement determines $q^{3/2}$.

We believe light nuclear targets are preferable for measuring b_1. Heavy nuclei contain many spin-paired nucleons which contribute to $F_1(x)$ but not to $b_1(x)$, thereby decreasing the signal to background. Among stable light nuclei, we have noted that ^{6}Li and ^{14}N are abundant, and have spin one. LiH and NH_3 are possible targets. The hydrogen is benign because it does not contribute to $b_1(x)$. ^{7}Li, ^{9}Be and ^{11}B are potential spin-$\frac{3}{2}$ nuclear targets. We note in passing that ^{10}B, with spin three, poses an interesting challenge to both theorists and experimenters.

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Note added in proof

After completion of this work we learned that $b_1(x)$ has been computed for conventional deuteron wavefunctions by Frankfurt and Strickman [7].

References

[1] EMC Collaboration, J. Ashman et al., Phys. Lett. B206 (1988) 364
[2] C. Callan and D.J. Gross, Phys. Rev. Lett. 22 (1969) 156
[3] R.L. Jaffe, *in* Relativistic dynamics and quark nuclear physics, ed. M.B. Johnson and A. Picklesimer (Wiley, New York, 1986)
[4] R.L. Jaffe, Nucl. Phys. A478 (1988) 3c
[5] R.L. Jaffe, Phys. Rev. D11 (1975) 1953;
R.L. Jaffe and G.G. Ross, Phys. Lett. B93 (1980) 313
[6] R.J. Hughes, Nucl. Phys. B138 (1978) 319
[7] L.I. Frankfurt and M.S. Strickman, Physics Rep. 76 (1981) 216; Nucl. Phys. A405 (1983) 557

* Recall that $q^m(x)$ is the probability to find a quark with momentum fraction x in a target with spin m along the z-axis, and that $q^m(x)=q^{-m}(x)$.