Sum rule for the spin-dependent structure function $b_1(x)$ for spin-one hadrons

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We show that the spin-dependent structure function of spin-one hadrons, $b_1(x)$, is related to the electric quadrupole moment of the target, and obtain $\int dx\, b_1(x) = \lim_{(t\to 0)} -\frac{5}{3}(t/4M^2)F_Q(t) = 0$ for isoscalar targets if the sea of quarks and antiquarks is unpolarized. We show how this sum rule is modified in the presence of a polarized sea.

The recent measurement by the European Muon Collaboration[1] of the spin-dependent structure function of the proton $[g_1(x)]$ has stimulated intense interest in the details of spin structures in the proton and neutron.[2] Measurements of $g_1(x)$ for the neutron are planned[3] and will require the existence of polarized nuclear targets, such as the deuteron. Nuclear targets with $J \geq 1$, such as the deuteron, are also interesting in their own right.[4] In particular there is a new effect which does not exist for spin-$\frac{1}{2}$ hadrons, namely, the existence of a further spin-dependent structure function $b_1(x)$ which could be measured by polarizing the spin-one target. For real photons, this structure function is essentially that discussed by Pais[5] in 1967.

The only available fixed targets with $J \geq 1$ are nuclei, and so early discussions of $b_1(x)$ have tended to be based upon models where the nucleus consists of nonrelativistic nucleons. For nucleons in an S state, $b_1(x) \equiv 0$. For nucleons in a D state,[4,6] $b_1(x) \neq 0$ in general. However, we note that these models have the property $\int dx\, b_1(x) = 0$. We find that in a quark-parton model this sum rule is generally true if the sea of quarks and antiquarks is unpolarized. After completing this work, we learned that Mankiewicz[7] has studied $b_1(x)$ for the ρ meson and noticed empirically that $\int dx\, b_1(x) = 0$ in his model. He noticed also that this need not be the case on the light plane where the probability for the $q\bar{q}$ to have $S_z = +1$ is independent of that for $S_z = 0$. Our sum rule confirms this and quantifies the effect of sea polarization.

The lepton scattering cross section from a hadron target involves the hadron tensor shown in Fig. 1(a):

$$W_{\mu\nu}(p,q,H_1,H_2) = \frac{1}{4\pi}\int d^4\xi\, e^{iq\cdot\xi}\langle p,H_2|[J_\mu(\xi),J_\nu(0)]|p,H_1\rangle \ , \qquad (1)$$

where the H_1 and H_2 are z components of the target spin. Parity and time-reversal invariances lead to eight independent structure functions. We may write a general expression for $W_{\mu\nu}$ of a spin-one hadron by considering current conservation[4]

$$\begin{aligned} W_{\mu\nu} = {} & -F_1 g_{\mu\nu} + F_2 \frac{p_\mu p_\nu}{\nu} - b_1 r_{\mu\nu} \\ & + \tfrac{1}{6}b_2(s_{\mu\nu}+t_{\mu\nu}+u_{\mu\nu}) + \tfrac{1}{2}b_3(s_{\mu\nu}-u_{\mu\nu}) \\ & + \tfrac{1}{2}b_4(s_{\mu\nu}-t_{\mu\nu}) + i\frac{g_1}{\nu}\epsilon_{\mu\nu\lambda\sigma}q^\lambda s^\sigma \\ & + i\frac{g_2}{\nu^2}\epsilon_{\mu\nu\lambda\sigma}q^\lambda(p\cdot q\, s^\sigma - s\cdot q\, p^\sigma) \ , \end{aligned} \qquad (2)$$

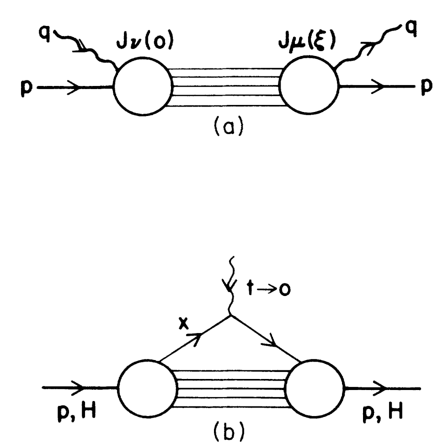

FIG. 1. (a) The imaginary part of the forward virtual Compton amplitude and (b) elastic form factor in the quark-parton model.

where

$$r_{\mu\nu}=\frac{1}{\nu^2}(q\cdot E^* q\cdot E-\tfrac{1}{3}\nu^2\kappa)g_{\mu\nu}\ , \tag{3}$$

$$s_{\mu\nu}=\frac{2}{\nu^2}(q\cdot E^* q\cdot E-\tfrac{1}{3}\nu^2\kappa)\frac{p_\mu p_\nu}{\nu}\ , \tag{4}$$

$$t_{\mu\nu}=\frac{1}{2\nu^2}(q\cdot E^* p_\mu E_\nu+q\cdot E^* p_\nu E_\mu+q\cdot E\, p_\mu E^*_\nu + q\cdot E\, p_\nu E^*_\mu-\tfrac{4}{3}\nu p_\mu p_\nu)\ , \tag{5}$$

$$u_{\mu\nu}=\frac{1}{\nu}(E^*_\mu E_\nu+E^*_\nu E_\mu+\tfrac{2}{3}M^2 g_{\mu\nu}-\tfrac{2}{3}p_\mu p_\nu)\ , \tag{6}$$

$$s^\sigma=-\frac{i}{M^2}\epsilon^{\sigma\alpha\beta\tau}E^*_\alpha E_\beta p_\tau\ , \tag{7}$$

and ν and κ are defined by $\nu=p\cdot q$ and $\kappa=1+M^2Q^2/\nu^2$. E^μ is the polarization of the target and it is normalized as $E^2=-M^2$.

The helicity amplitudes are defined by

$$A_{h_1H_1,h_2H_2}=\epsilon^{*\mu}_{h_2}\epsilon^{\nu}_{h_1}W_{\mu\nu}(p,q,H_1,H_2)\ , \tag{8}$$

where $\epsilon_h{}^\mu$ is the photon polarization vector with helicity h. The relations between the structure functions $b_1(x)$ and $F_1(x)$ in the scaling limit and these helicity amplitudes are

$$F_1(x)=\tfrac{1}{3}(A_{+0,+0}+A_{++,++}+A_{+-,+-})\ , \tag{9a}$$

$$b_1(x)=A_{+0,+0}-\frac{A_{++,++}+A_{+-,+-}}{2}\ . \tag{9b}$$

Measurement of $b_1(x)$ requires that the target be polarized.

In the parton model, we define $q^m_\uparrow(x)$ $[q^m_\downarrow(x)]$ as the probability to find a quark with momentum fraction x and spin up [down] in the spin-one hadron with the z component of spin m moving with infinite momentum along the z axis. This gives

$$F_1(x)=\tfrac{1}{2}\sum_i e_i^2[q_i(x)+\bar{q}_i(x)]\ , \tag{10a}$$

$$b_1(x)=\sum_i e_i^2[\delta q_i(x)+\delta\bar{q}_i(x)]\ , \tag{10b}$$

where $q_i(x)$ and $\delta q_i(x)$ are defined by

$$q_i(x)=\tfrac{2}{3}[q^0_{\uparrow i}(x)+q^1_{\uparrow i}(x)+q^1_{\downarrow i}(x)]\ , \tag{11a}$$

$$\delta q_i(x)=q^0_{\uparrow i}(x)-\frac{q^1_{\uparrow i}(x)+q^1_{\downarrow i}(x)}{2}\ , \tag{11b}$$

and analogously for $\bar{q}_i(x)$ and $\delta\bar{q}_i(x)$.

To form sum rules in the parton model, one may calculate the dependence of amplitudes on Bjorken x for a spin-one target moving fast in the z direction, Fig. 1(a), and compare with static properties integrated over x at zero-momentum transfer, Fig. 1(b). For example, in the spin-averaged case one has the Gottfried sum rule

$$\int dx[F_1^{\gamma p}(x)-F_1^{\gamma n}(x)]=\tfrac{1}{6}[F_C^p(0)-F_C^n(0)]=\tfrac{1}{6}\ , \tag{12}$$

where $F_C(0)$ is the charge of the target, which follows if the number of $\bar{u}$ and $\bar{d}$ in the sea are equal (see Ref. 8). Analogously one can form a sum rule for the deuteron

$$\int dx\, F_1^{\gamma d}(x)=\tfrac{5}{6}F_C^d(0)+\tfrac{1}{18}(Q+\bar{Q})_s\ , \tag{13}$$

where $F_C^d(0)$ is the deuteron's charge and $(Q+\bar{Q})_s$ is the number of charge weighted partons in the sea

$$(Q+\bar{Q})_s\equiv 5(U+\bar{U}+D+\bar{D})_s+2(S+\bar{S})\ . \tag{14}$$

This quantity is infinite and so the sum rule (13) has no utility, but we exhibit it in order to facilitate comparison with our sum rule for $b_1(x)$.

We may rederive these familiar spin-averaged sum rules, keeping target polarizations explicit throughout, and thereby immediately see the relation to our new sum rule. Defining [Fig. 1(b)]

$$\Gamma_{H,H}=\langle p,H|J_0(0)|p,H\rangle\ , \tag{15}$$

then in the parton model we have

$$\tfrac{1}{3}(\Gamma_{00}+\Gamma_{11}+\Gamma_{-1-1})=\sum_i e_i\int dx\, q^A_{i,v}(x) =\frac{A}{6}\int dx[u_v(x)+d_v(x)]\ , \tag{16a}$$

$$\tfrac{1}{2}(\Gamma_{00}-\Gamma_{11})=\sum_i e_i\int dx\,\delta q^A_{i,v}(x) =\frac{A}{6}\int dx[\delta u_v(x)+\delta d_v(x)]\ , \tag{16b}$$

where the subscript v refers to the valence quarks, and where we restrict ourselves to $I=0$ targets. The Γ_{HH} amplitudes in Eqs. (15) and (16) are related to the electric charge and electric quadrupole form factors as[9–11]

$$\Gamma_{00}=\lim_{t\to 0}\left[F_C(t)-\frac{t}{3M^2}F_Q(t)\right]\ , \tag{17a}$$

$$\Gamma_{11}=\Gamma_{-1-1}=\lim_{t\to 0}\left[F_C(t)+\frac{t}{6M^2}F_Q(t)\right]\ , \tag{17b}$$

where the form factors F_C and F_Q are measured in the units of e and e/M^2. Thus,

$$\tfrac{1}{3}(\Gamma_{00}+\Gamma_{11}+\Gamma_{-1-1})=F_C(0)\ , \tag{18a}$$

$$\tfrac{1}{2}(\Gamma_{00}-\Gamma_{11})=\lim_{t\to 0}\left[-\frac{t}{4M^2}F_Q(t)\right]=0\ . \tag{18b}$$

Substituting Eq. (18a) into Eq. (16a) and Eqs. (10a) and (11a) for isoscalar targets leads to Eq. (13). In a similar way, we use Eqs. (18b), (16b), (10b), and (11b) to obtain a polarized analog of Eq. (13):

$$\int dx\, b_1^{\gamma d}(x)=\lim_{t\to 0}\left[-\frac{5}{3}\frac{t}{4M^2}F_Q(t)\right]+\tfrac{1}{9}(\delta Q+\delta\bar{Q})_s =\tfrac{1}{9}(\delta Q+\delta\bar{Q})_s\ . \tag{19}$$

Thus we see that the sum rule for the structure function $b_1(x)$ is closely related to the electric quadrupole structure of the target and that the integral vanishes in any model with an unpolarized sea.

This quark-model sum rule provides insight into the $b_1(x)$ calculated in various models. Models involving nucleons alone, even in a D state, must give vanishing $\int dx\, b_1(x)$ since there need be no nonzero $\delta Q_s, \delta \bar{Q}_s$. This can be verified explicitly by integrating the equations given in Refs. 4 and 6:

$$\int dx\, b_1(x) = \sum_{k=p,n} \left[\sin^2\alpha \int dy\, \Delta f_{dd}(y) - 4\sqrt{2/5} \sin\alpha \cos\alpha \int dy\, \Delta f_{sd}(y) \right] \int dz\, F_1^k(z) = 0 \,, \tag{20}$$

because $\int dy\, \Delta f_{dd}(y) = 0$ and $\int dy\, \Delta f_{sd}(y) = 0$.

Models with π exchange generate a tensor force and thereby a quadrupole moment for the target. However, with vanishing δQ they still preserve $\int dx\, b_1(x) = 0$ as can be seen by inspection of the explicit $b_1(x)$ in Refs. 4 and 6. Models involving ρ exchange could give a nonvanishing integral as ρ can effectively transport a nonzero δQ. This is essentially noted in Ref. 7 where in the light cone, or in $SU(6)_W$, the longitudinal and transverse ρ components are, in principle, independent and hence $\delta Q + \delta \bar{Q}$ need not vanish.

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