

# The Deuteron Tensor Structure Function $b_1$

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A Proposal to Jefferson Lab PAC-38  
(Update to LOI-11-003)

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## Abstract

The EMC experiment revealed that only a small fraction of the nucleon spin is carried by quarks. Two decades later, the spin crisis remains an open issue. Quark orbital angular momentum is now considered to be one of the principal contributions in generating the nucleon spin, but a precise determination of this critical piece has remained elusive. The leading twist tensor structure function  $b_1$  of spin-1 hadrons can provide new insight into this puzzle, since it is directly related to effects arising from orbital angular momentum, which differ from the case in a spin-1/2 target. For this reason, it provides a unique tool to study partonic effects, while also being sensitive to coherent nuclear properties in the simplest nuclear system.

At low  $x$ , shadowing effects are expected to dominate  $b_1$ , while at larger values,  $b_1$  provides a clean probe of exotic QCD effects, such as hidden color due to 6-quark configuration. Since the deuteron wave function is relatively well known, any novel effects are expected to be readily observable. All available models predict a small or vanishing value of  $b_1$  at moderate  $x$ . However, the first pioneer measurement of  $b_1$  at HERMES revealed a crossover to an anomalously large negative value in the region  $0.2 < x < 0.5$ , albeit with relatively large experimental uncertainty.

We will perform an inclusive measurement of the deuteron polarized cross sections in the region  $0.30 < x < 0.50$ , for  $1.5 < Q^2 < 3.6 \text{ GeV}^2$ . With 28 days of 11 GeV incident beam, we can determine  $b_1$  with sufficient precision to discriminate between conventional nuclear models, and the more exotic behaviour which is hinted at by the HERMES data. The UVa solid polarized ND<sub>3</sub> target will be used, along with the Hall C spectrometers, and an unpolarized 115 nA beam. An additional 11.8 days will be needed for overhead. This measurement will provide access to the tensor quark polarization, and allow a test of the Close-Kumano sum rule, which vanishes in the absence of tensor polarization in the quark sea. Until now, tensor structure has been largely unexplored, so the study of these quantities holds the potential of initiating a new field of spin physics at Jefferson Lab.

# Foreword

This proposal follows the letter of intent LOI-11-003 which was submitted to PAC 37. For convenience we reproduce the draft PAC report comments below. We note that we plan to use Hall C's HMS/SHMS spectrometers, an option that we briefly explored in the appendix of the LOI. We have significantly revised our experimental method, as discussed in Sec. 2, which now makes the Hall C option compelling. ND<sub>3</sub> has been selected as the target material instead of LiD in order to simplify the analysis, and to take advantage of the extensive experience using ND<sub>3</sub> within the collaboration and in the JLab Target group. We have also expanded our discussion in Sec. 1.3 of the expected behaviour of  $b_1(x)$ , in order to strengthen the justification for measuring in the region  $0.30 < x < 0.50$ , where most models predict very small or vanishing values for  $b_1(x)$ , in contrast to the HERMES data.

## LOI-11-003 “The Deuteron Tensor Structure Function $b_1$ ”

**Motivation:** *The collaboration proposes to measure the deuteron tensor structure function  $b_1$  by measuring deep inelastic scattering from a tensor polarized deuterium. This structure function would be zero for a deuteron with constituents in a relative  $s$ -wave. The structure function  $b_1$  can be compared with conventional calculations of quark distribution functions convoluted with nucleon momentum distributions in the deuteron including the  $d$ -state admixture. Departures from such approach, as hinted at in pioneering data at HERMES, is conjectured to be sensitive to orbital angular momentum effects.*

**Measurement and Feasibility:** *The letter of intent proposes such experiment in Hall A using an 11 GeV beam and the SoLID spectrometer. The polarized target proposed is a <sup>6</sup>LiD target. The rates in the proposal only assume tensor polarizations that have been demonstrated previously. The projected precision on the tensor structure function using SoLID is compelling to improve on the HERMES measurement at small  $x$  and extend it into the large  $x$  region. The proposed measurement will allow to map out the qualitative behavior of  $b_1$ , which will serve as a benchmark for theoretical interpretations. In the appendix to the LOI, a feasibility study has also been performed for a measurement in Hall C using the HMS/SHMS spectrometers. Given the projected precision obtained, such measurement using HMS/SHMS does not seem to be compelling at this stage.*

**Issues:** *The main issue is on the theoretical interpretation of such experiment. The authors are urged to consult some theorists to provide at least some qualitative behavior of  $b_1$  when making their physics case for a proposal.*

**Recommendation:** *The PAC encourages the submission of a fully developed proposal that addresses the issue raised above.*

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# 1 Background and Motivation

The deuteron is the simplest nuclear system, and in many ways it is as important to understanding bound states in QCD as the hydrogen atom was to understanding bound systems in QED. Unlike its atomic analogue, our understanding of the deuteron remains unsatisfying both experimentally and theoretically. A deeper understanding of the deuteron's tensor structure will help to clarify how the gross properties of the nucleus arise from the underlying partons. This provides novel information about nuclear structure, quark angular momentum, and the polarization of the quark sea that is not accessible in spin-1/2 targets.

A measurement of the tensor structure function  $b_1$  is of considerable interest since it provides a clear measure of possible exotic effects in nuclei, i.e. the extent to which the nuclear ground state deviates from being a composite of nucleons only [1]. Jefferson Lab is the ideal place to investigate tensor structure in a deuteron target at intermediate and large  $x$ . We describe such a measurement in this proposal.

## 1.1 Tensor Structure of the Deuteron

When a spin 1 system such as the deuteron is subjected to a magnetic field along the z-axis, the Zeeman interaction gives rise to three magnetic sublevels  $I_z = +1, 0, -1$  with population fractions  $p_+, p_-, p_0$ , respectively. These populations are described by both a vector polarization,

$$\begin{aligned} P_z &= \langle I_z/I \rangle \\ &= (p_+ - p_0) + (p_0 - p_-) = p_+ - p_- \end{aligned} \quad (1)$$

and a tensor polarization [2]:

$$\begin{aligned} P_{zz} &= \langle 3I_z^2 - I(I+1) \rangle / I^2 \\ &= (p_+ - p_0) - (p_0 - p_-) = 1 - 3p_0 \end{aligned} \quad (2)$$

which are subject to the overall normalization  $p_+ + p_- + p_0 = 1$ .

Fig. 1 graphically demonstrates the dependence of the two nucleon distribution on the spin projection. If the two nucleons are in a relative  $m = 0$  state, the surface of constant density is toroidal, while if they are in the  $m = \pm 1$  state, the surface has a dumbbell shape.

In the case of deuteron spins in thermal equilibrium with the solid lattice, and neglecting the small quadrupole interaction [2], the tensor polarization is related to the vector polarization via:

$$P_{zz} = 2 - \sqrt{4 - 3P_z^2} \quad (3)$$

The maximum absolute value of  $P_{zz} = -2$  occurs only for vanishing populations in the  $m = \pm 1$  states. If, on the other hand, only the  $m = 1$  or  $m = -1$  state are occupied, the vector polarization reaches its maximum value of  $\pm 1$ , and  $P_{zz} = \pm 1$ .

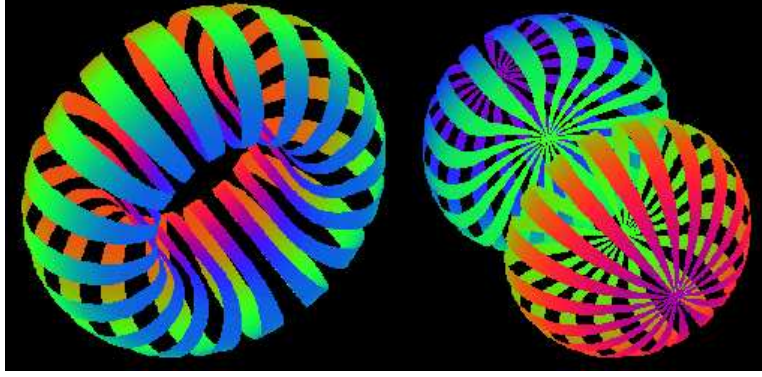


Figure 1: Nucleon densities of the deuteron in its two spin projections,  $I_z = \pm 1$  and  $I_z = 0$ , respectively. *Reproduced from [3, 4].*

## 1.2 Deep Inelastic Scattering from Spin-1 Targets

Four independent helicity amplitudes are sufficient to describe virtual Compton scattering from a spin-1/2 target, after requiring parity and time reversal invariance. This number doubles for a spin-1 target, as the spin can be in three states (+, 0, -). This gives rise to a tensor structure which was first discussed for the deuteron for the real photon case by Pais [5], and later in the virtual photon case, by Frankfurt and Strikman [6]. Hoodbhoy, Jaffe and Manohar [7] introduced the notation which we now follow, whereby the tensor structure is described by the four functions  $b_1$ ,  $b_2$ ,  $b_3$  and  $b_4$ . To summarize, the hadronic tensor can be decomposed as:

$$\begin{aligned}
 W_{\mu\nu} = & -F_1 g_{\mu\nu} + F_2 \frac{P_\mu P_\nu}{\nu} \\
 & -b_1 r_{\mu\nu} + \frac{1}{6} b_2 (s_{\mu\nu} + t_{\mu\nu} + u_{\mu\nu}) \\
 & + \frac{1}{2} b_3 (s_{\mu\nu} - u_{\mu\nu}) + \frac{1}{2} b_4 (s_{\mu\nu} - t_{\mu\nu}) \\
 & + i \frac{g_1}{\nu} \epsilon_{\mu\nu\lambda\sigma} q^\lambda s^\sigma + i \frac{g_2}{\nu^2} \epsilon_{\mu\nu\lambda\sigma} q^\lambda (p \cdot q s^\sigma - s \cdot q p^\sigma)
 \end{aligned} \tag{4}$$

where the purely kinematic expressions  $r_{\mu\nu}$ ,  $s_{\mu\nu}$ ,  $t_{\mu\nu}$  and  $u_{\mu\nu}$  can be found in [7]. The terms are all proportional to the polarization of the target  $E$ . The spin-1 structure functions  $F_1$ ,  $F_2$ ,  $g_1$  and  $g_2$  have the same expressions and are measured the same way as for a spin-1/2 target. The spin-dependent structure functions  $b_1$ ,  $b_2$ ,  $b_3$ ,  $b_4$  are symmetric under  $\mu \leftrightarrow \nu$  and  $E \leftrightarrow E^*$  and therefore can be isolated from  $F_1$  and  $g_1$  by unpolarized beam scattering from a polarized spin-1 target.

### 1.2.1 Interpretation in the Operator Product Expansion

In the Operator Product Expansion (OPE) framework, the leading operators  $O_V^{\mu_1 \dots \mu_n}$  and  $O_A^{\mu_1 \dots \mu_n}$  in the expansion are twist two. For a spin-1 target, the matrix elements of the time-ordered product

of two currents  $T_{\mu\nu}$  have the following expressions:

$$\begin{aligned} \langle p, E | O_V^{\mu_1 \dots \mu_n} | p, E \rangle &= S[a_n p^{\mu_1} \dots p^{\mu_n} + d_n (E^{*\mu_1} E^{\mu_2} - \frac{1}{3} p^{\mu_1} p^{\mu_2}) p^{\mu_3} \dots p^{\mu_n}], \\ \langle p, E | O_A^{\mu_1 \dots \mu_n} | p, E \rangle &= S[r_n \epsilon^{\lambda \sigma \tau \mu_1} E_\lambda^* E_\sigma p_\tau p^{\mu_2} \dots p^{\mu_n}] \end{aligned} \quad (5)$$

The non-zero value of  $b_1$  arises from the fact that, in a spin-1 target, the  $\frac{1}{3} p^{\mu_1} p^{\mu_2}$  term doesn't cancel the tensor structure  $E^{*\mu_1} E^{\mu_2}$ . The coefficient  $d_n$  can be extracted from the comparison of  $T_{\mu\nu}$  expansion and the spin-1 target hadronic tensor Eq. 4 as follows:

$$\begin{aligned} b_1(\omega) &= \sum_{n=2,4,\dots}^{\infty} 2C_n^{(1)} d_n \omega^n, \\ b_2(\omega) &= \sum_{n=2,4,\dots}^{\infty} 4C_n^{(2)} d_n \omega^{n-1}, \end{aligned} \quad (6)$$

for  $1 \leq |\omega| \leq \infty$  (where  $\omega = 1/x$ ). A Callan-Gross-type relation exists for the two leading order tensor structure functions:

$$2xb_1 = b_2 \quad (7)$$

valid at lowest order of QCD, where  $C_n^{(1)} = C_n^{(2)}$ . At higher orders, Eq. 7 is violated.

Sum rules can be extracted from the moments of the tensor structure functions:

$$\begin{aligned} \int_0^1 x^{n-1} b_1(x) dx &= \frac{1}{2} C_n^{(1)} d_n, \\ \int_0^1 x^{n-2} b_2(x) dx &= C_n^{(2)} d_n, \end{aligned} \quad (8)$$

where  $n$  is even.

The OPE formalism is based on QCD and is target-independent. However, a target dependence is generated by Eq. 5, and spin-1 structure functions are subject to the same QCD corrections and their moments have the same anomalous dimensions as for a spin-1/2 target. In addition, the tensor structure functions should exhibit the same scaling behavior as  $F_1$  and  $F_2$ , since they are generated from the same matrix element  $O_V^{\mu_1 \dots \mu_n}$ .

We focus in this document on the leading twist structure function  $b_1$ . A Callan-Gross type relation allows access to  $b_2$  once  $b_1$  is determined, and  $b_3$  and  $b_4$  do not contribute at leading twist.

### 1.2.2 Interpretation in the Parton Model

In the infinite momentum frame<sup>‡</sup> of the parton model, the scattering of the virtual photon from a free quark with spin up (or down), which carries a momentum fraction  $x$  of the spin- $m$  hadron, can be expressed through the hadronic tensor  $W_{\mu\nu}^{(m)}$ :

$$W_{\mu\nu}^{(1)} = \left( -\frac{1}{2} g_{\mu\nu} + \frac{x}{\nu} P_\mu P_\nu \right) (q_\uparrow^1(x) + q_\downarrow^1(x)) + \frac{i\epsilon_{\mu\nu\lambda\sigma} q^\lambda s^\sigma}{2\nu} (q_\uparrow^1(x) - q_\downarrow^1(x)),$$

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<sup>‡</sup>All spins and momenta are along the  $z$ -axis.



for a target of spin projection equal to 1 along the  $z$ -direction, and:

$$W_{\mu\nu}^{(0)} = \left( -\frac{1}{2}g_{\mu\nu} + \frac{x}{\nu}P_\mu P_\nu \right) 2q_\uparrow^0(x) \quad (9)$$

for a target of spin projection equal to zero along the  $z$ -direction. The tensor structure functions  $b_1$  and  $b_2$  can be expressed from the comparison of  $W_{\mu\nu}^{(1)} - W_{\mu\nu}^{(0)}$  with Eq. 4 as follows:

$$b_1(x) = \frac{1}{2} \left( 2q_\uparrow^0(x) - q_\uparrow^1(x) - q_\downarrow^1(x) \right) \quad (10)$$

$$b_2(x) = 2xb_1(x) \quad (11)$$

where  $q_\uparrow^m$  ( $q_\downarrow^m$ ) represents the probability to find a quark with momentum fraction  $x$  and spin up (down) in a hadron which is in helicity state  $m$ . The tensor structure function  $b_1$  depends only on the spin-averaged parton distributions<sup>§</sup>

$$\begin{aligned} q^1(x) &= q_\uparrow^1(x) + q_\downarrow^1(x) \\ q^0(x) &= q_\uparrow^0(x) + q_\downarrow^0(x) = 2q_\uparrow^0(x) \end{aligned}$$

so it can be expressed as:

$$b_1(x) = \frac{q^0(x) - q^1(x)}{2} \quad (12)$$

Explicitly,  $b_1$  measures the difference in partonic constituency in an  $|m|=1$  target and an  $m=0$  target. From this we see that while  $b_1$  is defined in terms of quark distributions, it interestingly depends also on the spin state of the nucleus as a whole.

### 1.2.3 First Measurement of $b_1(x)$ by the HERMES Collaboration

The HERMES collaboration made the first measurement [8, 9] of  $b_1$  in 2005. The experiment explored the low  $x$  region of  $0.001 < x < 0.45$  for  $0.5 < Q^2 < 5 \text{ GeV}^2$ . An atomic beam source was used to generate a deuterium gas target with high tensor polarization. The HERA storage ring provided 27.6 GeV positrons incident on the internal gas target.

As displayed in Fig. 2, the tensor asymmetry  $A_{zz}$  was found to be non-zero at about the two sigma level, with an apparent zero crossing around  $x = 0.3$ . The tensor structure function  $b_1$  exhibits a steep rise as  $x \rightarrow 0$ , which is qualitatively in agreement with the predictions of coherent double-scattering models. See for example Ref. [10]. The authors of Ref. [9] interpret the rapid rise at low  $x$  in terms of the same mechanism that leads to nuclear shadowing in unpolarized scattering, i.e. double scattering of the lepton, first from the proton, then from the neutron, with sensitivity to the spatial alignment of the two nucleons.

As is often the case with a pioneer measurement, the precision of the results leaves some room for ambiguity. Despite the surprisingly large magnitude and interesting trend of the data, all points

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<sup>§</sup>since, by parity,  $q_\uparrow^m = q_\downarrow^m$

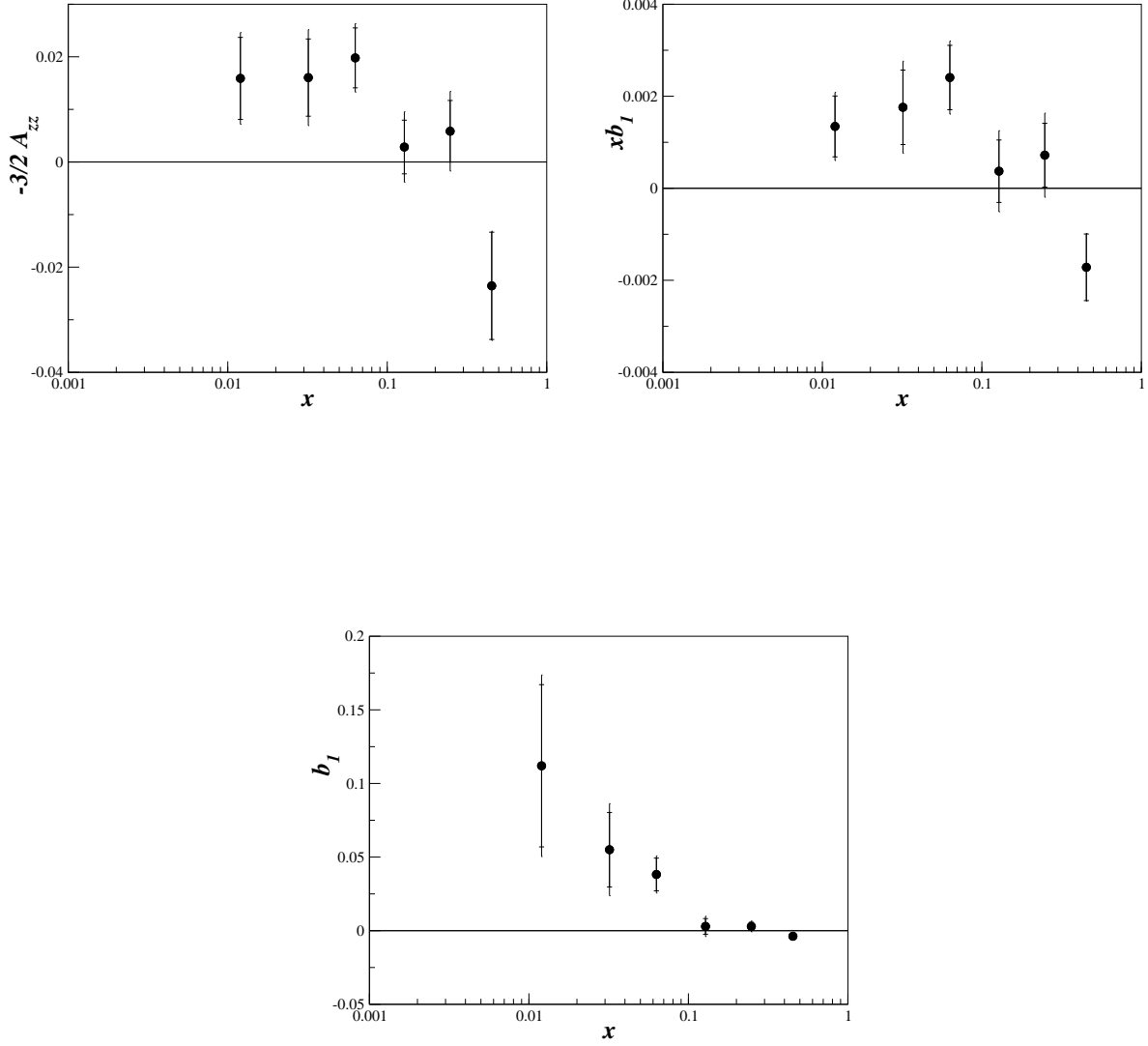


Figure 2: **Top:** HERMES [8] measurement of the inclusive tensor asymmetry  $A_{zz}(x)$  and  $xb_1(x)$  of the deuteron. **Bottom :** The tensor structure function  $b_1(x)$  without  $x$ -weighting, which reveals a steep rise as  $x \rightarrow 0$ .

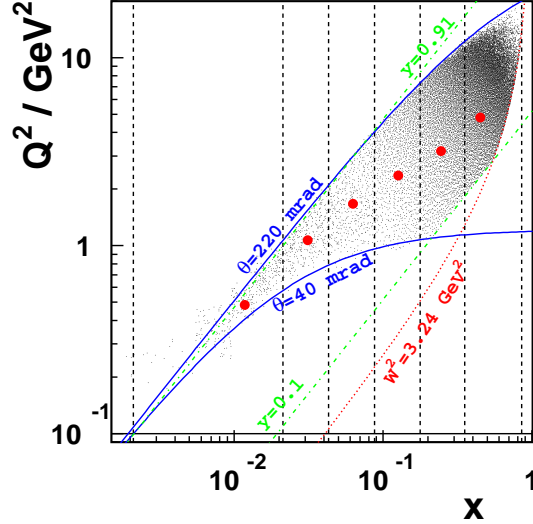


Figure 3: Kinematic coverage of the HERMES measurement. The dashed vertical lines indicate the borders of the bins in  $x$ , the dots their centers of gravity. The solid curves indicate the vertical acceptance of the spectrometer, defined by its aperture. In addition, the kinematic cuts imposed on the variables  $Q^2$ ,  $y$  and  $W^2$  are shown. *Reproduced from [8].*

are roughly within two sigma from zero, which calls for a higher precision measurement. Another issue is that some of the HERMES momentum transfer values are low (see Fig. 3), so that quark structure functions may not be the correct language. The  $Q^2$  variation in each  $x$ -bin is also quite wide ( $\approx 10 \text{ GeV}^2$  for  $x \sim 0.3$ ), which complicates the interpretation of this data, since several models predict significant  $Q^2$ -dependence of  $b_1$ . See for example Fig. 4.

### 1.3 Predictions for the Tensor Structure Function $b_1(x)$

The leading twist tensor structure function  $b_1$  quantifies effects not present in the case of spin-1/2 hadrons. However, tensor effects only exist in nuclear targets, so the study of  $b_1$  serves as a very interesting bridge between nucleon and nuclear physics. On the one hand, deep inelastic scattering (DIS), clearly probes partonic degrees of freedom, i.e. quarks, but on the other hand,  $b_1$  depends solely on the deuteron (nuclear) spin state as seen in Eq. 11. We discuss now several predictions for the  $x$  dependence of  $b_1$ .

#### 1.3.1 Conventional Nuclear Effects

In Ref. [7], the authors note that  $b_1(x)$  is small and calculable for a weakly bound system like the deuteron, and that its measurement would provide a clear signature for exotic components in a spin one nucleus. In effect,  $b_1(x)$  measures the extent to which a target nucleus deviates from a trivial bound state of protons and neutrons. The authors evaluate the value of  $b_1$  in three conventional

scenarios for the deuteron constituents and their dynamics:

- I. If the deuteron is composed of two spin-1/2 non-interacting nucleons at rest, then the eight helicity amplitudes characteristic of a spin-1 target are expressed in terms of the four helicity amplitudes of each spin-1/2 nucleons, and therefore the total number of independent amplitudes is reduced from eight to four. All structure functions of the deuteron are then the simple sum of the structure functions of the two nucleons, and the tensor structure functions vanish:  $b_1 = b_2 = b_3 = b_4 = 0$ .
- II. If instead, the deuteron is composed of two spin-1/2 nucleons moving non-relativistically in a central potential, then the target motion modifies the helicity amplitudes. Using the convolution formalism, it was found that the contribution of these moving nucleons to  $b_1$  is small and is dominated by the lower component of the nucleon's Dirac wave function.
- III. In the final scenario considered, the deuteron contains a  $D$ -state admixture. Because the proton and the neutron are moving in opposite directions, an additional term due to the  $S - D$  interference appears in the convolution procedure. This extra contribution to  $b_1$  is predicted to be even smaller than in the previous case.

All three scenarios predict a small or vanishing  $b_1$ , leading the authors to predict that  $b_1 \approx 0$  for the deuteron.

As an interesting counter example for which  $b_1$  could be significant, the authors consider a model of a massless relativistic quark with  $j = 3/2$  moving in a central potential. In this calculation, a meson in the  $j = 1$  state is formed from the coupling of a  $P_{3/2}$  massless quark with a spin-1/2 spectator. This crude model predicts that  $b_1(x)$  exhibits large negative values peaked around  $x = 0.5$  [7]. Curiously, this behavior is possibly mirrored by the existing HERMES data (see Fig. 4), but there is only a single data point with large uncertainty in this region.

### 1.3.2 Nuclear Pions

In 1988, Miller also examined the tensor structure function  $b_1$  [13]. The basic mechanism is that the virtual photon hits an exchanged pion which is responsible for the binding of the deuteron. In this early calculation, the convention used by Miller for  $b_1$  was different from that used in the HERMES results and in Ref. [12]. A recent update to this calculation [14], which uses a consistent convention and the pion structure function from [15], is shown in Fig. 4. The spread of the curve originates from the parameter  $A_s = (.9 \pm 0.3)$  which governs the strength of the sea in the pion. Miller's calculation, similar to other 'non-exotic' models, is unable to reproduce the trend of the HERMES data, and predicts very small values of  $b_1(x)$  at intermediate and large  $x$ .

### 1.3.3 Convolution Model

Khan and Hoodbhoy [1] evaluated  $b_1(x)$  in a convolution model with relativistic and binding energy corrections. They use this to evaluate the effect of nuclear Fermi motion and binding on the deuteron structure functions. They observe that for zero Fermi motion and binding  $b_1^D(x) = 0$ . They also predict a small enhancement of  $b_1$  in the region of  $x \sim 0.3$ , as seen in Fig. 5. Note

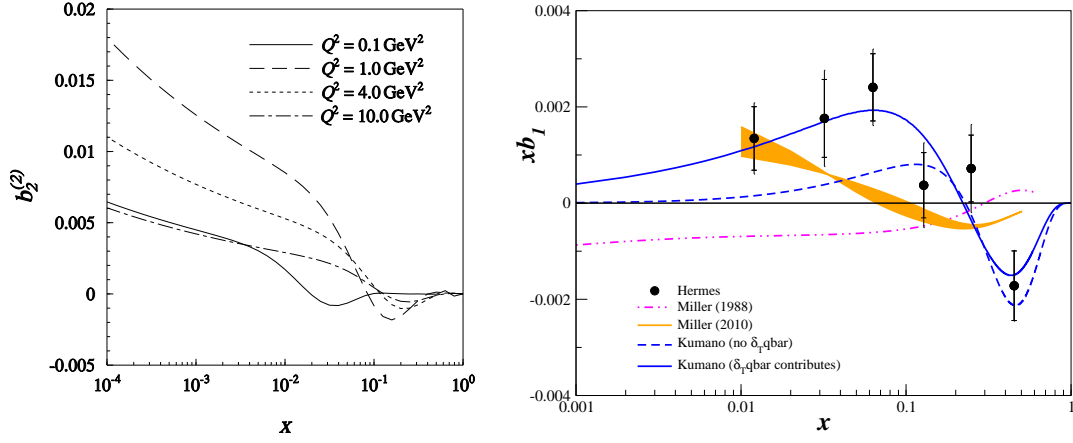


Figure 4: Theoretical predictions. **Left plot:** Double-scattering contribution to  $b_2(x, Q^2)$  as a function of  $x$  [11]. Note the strong  $Q^2$  dependence at low  $x$ . **Right plot:** HERMES results [9] compared to calculations from S. Kumano [12] and from the one-pion exchange effects of G. Miller [13, 14].

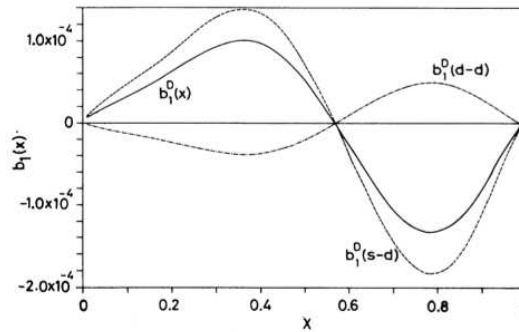


Figure 5: Prediction for  $b_1^D(x)$  (solid curve) from Ref. [1], the S-D contribution to  $b_1^D(x)$  (dashed curve), and the D-D contribution to  $b_1^D(x)$  (dot-dashed curve). Note the vertical scale which would make the curve mostly indiscernible from zero in Fig. 4 (right). *Reproduced from Ref. [1].*

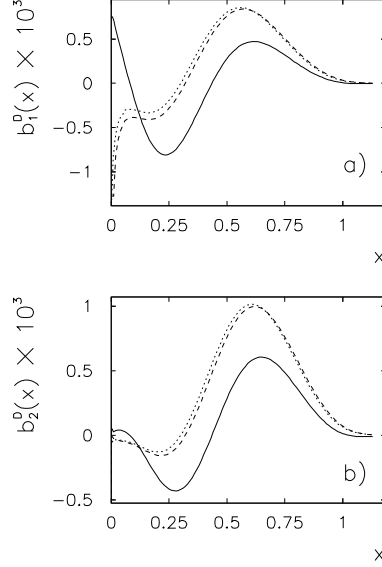


Figure 6: Relativistic convolution calculation of  $b_1^D(x)$  and  $b_2^D(x)$ . Curves: BS - solid, Bonn - dotted, Bonn with cut -dashed. *Reproduced from Ref. [16].*

however, that the absolute scale of this predicted  $b_1$  is  $\mathcal{O}(10^{-4})$ , while the HERMES data implies that the scale is more than an order of magnitude larger than this.

### 1.3.4 Relativistic Calculation

Umnikov [16] calculated  $b_1(x)$  and  $b_2(x)$  within a covariant approach, based on the relativistic convolution formalism for DIS and the Bethe-Salpeter formalism for the deuteron bound state. Fig. 6 sets the scale for  $b_1(x)$  at the  $10^{-3}$  level. Both the relativistic and non-relativistic calculations are consistent with the CK sum rule (see Sec. 1.3.8), although the nonrelativistic convolution model results in an incorrect behaviour of at low  $x$ .

### 1.3.5 Double-Scattering Effects

Using Vector Meson Dominance (VMD), the authors of Ref. [11] isolate the double-scattering contribution to  $b_1$ . The existence time of a vector meson can be described by the coherence length:

$$\lambda = \frac{Q^2}{Mx(M_v^2 + Q^2)} \quad (13)$$

which is the length over which the vector meson propagates during the time  $\Delta t = 1/\Delta E$ . For significant shadowing or double scattering to occur, a minimum coherence length of  $\approx 1.7$  fm (the inter-nucleon separation) is required. At  $x > 0.3$ , the coherence length is only about the size of the nucleon, so double scattering contributions are anticipated to be negligible. However, for  $x \leq 0.1$ , double-scattering should be significant in  $b_1$  behaving as  $(1-x)^{2\delta}/x^{1+2\delta}$ , where  $\delta$  is

determined from the soft pomeron intercept  $\alpha_P(t = 0) = 1 + \delta$ . The authors predicted a significant enhancement of  $b_1$  at low  $x$  ( $\leq 0.01$ ) due to the quadrupole deformation of the deuteron, which is qualitatively confirmed by the HERMES data. See Fig. 2.

### 1.3.6 Virtual Nucleon Approximation

M. Sargsian [17] recently calculated the tensor asymmetry  $A_{zz}$  for deep inelastic scattering. See Fig. 8. In the approximation in which only proton-neutron component of the deuteron is taken into account and nuclear parton distributions are generated through the convolution of partonic distribution of nucleon and deuteron density matrix (see e.g. Refs. [18, 19]), the deuteron structure function  $b_1$  is related directly to the d-partial wave of the deuteron wave function [17, 18]. As a result, this approximation predicts negligible magnitude for  $b_1$  for  $x \leq 0.6$  due to small Fermi momenta involved in the convolution integral. However, the predicted magnitude of  $b_1$  is large at  $x \geq 0.7$  where one expects substantial contribution from the d-waves due to high momentum component of the deuteron wave function involved in the convolution picture of DIS scattering off the deuteron. In this case,  $b_1$  is very sensitive to the relativistic description of the deuteron and its measurement can be used for checking the different approximations of high momentum component of deuteron wave function.

In the calculation presented, two Virtual Nucleon and Light-Cone approximations are used to calculate the tensor polarization for DIS scattering off the deuteron. In both approximations only the proton-neutron component of the deuteron is taken into account. In the Virtual Nucleon approximation, the covariant scattering amplitude is reduced by estimating the spectator nucleon propagator at its on-energy shell in the lab frame of the deuteron. Within this approximation the baryonic sum rule is satisfied while the momentum sum rule is not. The latter is due to the fact that part of the light cone momentum of the bound virtual nucleon is lost to the unaccounted non-nucleonic degrees of freedom in the deuteron wave function. In the light cone approximation the scattering amplitude is estimated the  $E + p_z$  pole of the spectator nucleon on the light cone. In this case the wave function is defined on the light-cone reference frame and it satisfies both baryon number and momentum sum rules. For the detailed comparison of these approximations, see Ref. [19].

### 1.3.7 Fit to HERMES Data

Kumano [12] points out that the twist-2 structure functions  $b_1$  and  $b_2$  can be used to probe orbital angular momentum. He then extracts the tensor polarized quark and anti-quark distributions from a fit to the HERMES data [9]. He finds that a non-negligible tensor polarization of the sea is necessary to reproduce the trend of the data, as shown in Fig. 4. However, this conclusion has to be considered with caution due to the extended  $Q^2$  coverage (Fig. 3), and large uncertainty of each HERMES data point. In particular, the author calls for better measurements of  $b_1$  at large  $x$  ( $> 0.2$ ), and further investigation of the tensor structure functions in general.

### 1.3.8 The Close-Kumano Sum Rule

Following the formalism from the parton model in [7], Close and Kumano [20] related the tensor structure function  $b_1$  to the electric quadrupole form factor of the spin-1 target through a sum rule<sup>¶</sup>:

$$\begin{aligned}\int_0^1 dx b_1(x) &= -\frac{5}{12M^2} \lim_{t \rightarrow 0} t F_Q(t) + \frac{1}{9} (\delta Q + \delta \bar{Q})_s \\ &= \frac{1}{9} (\delta Q + \delta \bar{Q})_s = 0\end{aligned}\tag{14}$$

where  $F_Q(t)$  is the electric quadrupole form factor of a spin-1 hadron at the momentum squared  $t$ . The Close Kumano (CK) sum rule is satisfied in the case of an unpolarized sea. The authors note that in nucleon-only models, the integral of  $b_1$  is not sensitive to the tensor-polarization of the sea, and consequently the sum rule is always true, even when the deuteron is in a  $D$ -state.

The authors of Ref. [1] calculated the first moment of  $b_1(x)$  in a version of the convolution model that incorporates relativistic and binding energy corrections. They found a value of  $-6.65 \cdot 10^{-4}$ , and emphasize that deviations from this will serve as a good signature of exotic effects in the deuteron wave function. Similarly, Ref. [16] predicts  $5 \cdot 10^{-4}$  and  $3 \cdot 10^{-5}$  for the relativistic and nonrelativistic calculation of Eq. 14, respectively.

A truncated version of Eq. 14 was evaluated by the HERMES [8, 9] experiment and found to be:

$$\int_{0.0002}^{0.85} b_1(x) dx = 0.0105 \pm 0.0034 \pm 0.0035\tag{15}$$

which possibly indicates a breaking of the Close-Kumano sum rule, and consequently a tensor-polarized quark sea. However, since the comparison is only at the two sigma level, more precise data is needed for a true test.

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<sup>¶</sup>Efremov and Teryaev evidently proposed the same relation for mesons in Ref. [21].



## 1.4 Interest from Theorists

During the preparation of this proposal, we contacted several theorists to gauge interest in a precision measurement of  $b_1$ . The response was uniformly positive. We provide some of their feedback for context.

*It is known that  $b_1$  is sensitive to dynamical aspects of constituents with angular momenta. Measurements of  $b_1$  could open a new field of spin physics because this kind of spin physics has not been explored anywhere else. The only experimental information came from the HERMES collaboration; however, their data are not accurate enough to find the  $x$  dependence of  $b_1$ , especially at large  $x$ .*

*It is an unique opportunity at JLab to develop this new field of spin physics.*

**S. Kumano (KEK)**

*I'm glad to hear that  $b_1$  is not forgotten in all the excitement about other spin dependent effects.*

**R. Jaffe (MIT)**

*I am particularly interested in signatures of novel QCD effects in the deuteron. The tensor charge could be sensitive to hidden color (non-nucleonic) degrees of freedom at large  $x$ . It is also interesting that antishadowing in DIS in nuclei is not universal but depends on the quark flavor and spin. One can use counting rules from PQCD to predict the  $x \rightarrow 1$  dependence of the tensor structure function.*

**S. Brodsky (SLAC)**

*I am certainly interested in the experimental development to find the novel QCD phenomena from the hidden color component of deuteron.*

**Chueng-Ryong Ji (NCSU)**

*You have finally piqued my interest in this subject...Surely this is of real interest the spin community! I hope I might be able to say something coherent about the partonic interpretation at some point—this of course is where my real interest lays.*

**Leonard Gamberg (Penn State Berks)**

*I find the proposal well written, well justified, sound, and exciting.*

**Alessandro Bacchetta (Universita di Pavia)**

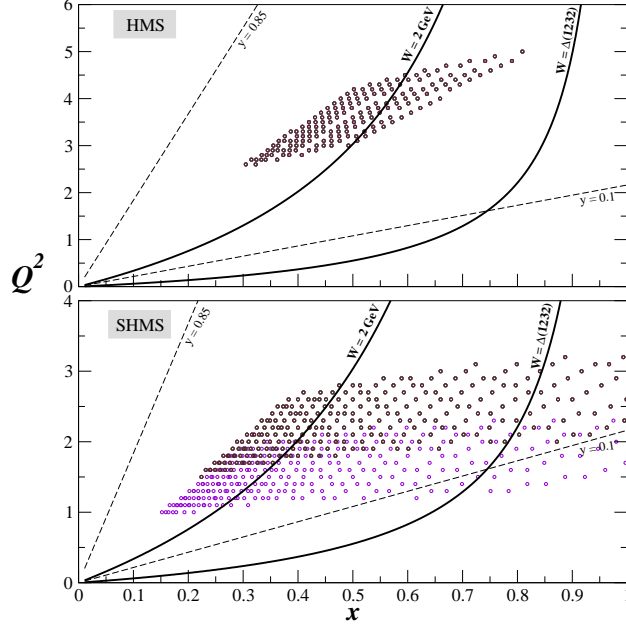


Figure 7: Kinematic coverage for 11 GeV beam in Hall C using the HMS and SHMS. A cut will be applied for  $W \geq 1.8 \text{ GeV}$ .

## 2 The Proposed Experiment

We will measure the leading twist tensor structure function  $b_1$  and tensor asymmetry  $A_{zz}$  for  $0.30 < x < 0.50$ ,  $1.5 < Q^2 < 3.6 \text{ GeV}^2$  and  $W \geq 1.8 \text{ GeV}$ . Fig. 7 shows the kinematic coverage available at JLab utilizing an 11 GeV beam, and the Hall C HMS and SHMS spectrometers at forward angle.

The polarized  $\text{ND}_3$  target is discussed in section 2.2. The vector polarization, packing fraction and dilution factor used in the estimate of the rates are 35%, 0.55 and 0.30 respectively. With an incident electron beam current of 115 nA, the expected deuteron luminosity is  $2 \times 10^{35} / \text{cm}^2 \cdot \text{s}^{-1}$ . The momentum bite and the acceptance were assumed to be  $\Delta P = \pm 8\%$  and  $\Delta\Omega = 6.5 \text{ msr}$  for the HMS, and  $\Delta P = {}^{+20\%}_{-8\%}$  and  $\Delta\Omega = 4.4 \text{ msr}$  for the SHMS. For the choice of the kinematics, special attention was taken onto the angular and momentum limits of the spectrometers: for the HMS,  $10.5^\circ \leq \theta \leq 85^\circ$  and  $1 \leq P_0 \leq 7.3 \text{ GeV}/c$ , and for the SHMS,  $5.5^\circ \leq \theta \leq 40^\circ$  and  $2 \leq P_0 \leq 11 \text{ GeV}/c$ . In addition, the opening angle between the spectrometers is physically constrained to be larger than  $17.5^\circ$ . The invariant mass  $W$  was kept to  $W \geq 1.8 \text{ GeV}$  for all settings. The projected uncertainties for  $b_1$  and  $A_{zz}$  are summarized in Table 1 and displayed in Fig. 8.

A total of 28 days of beam time is requested for production data, with an additional 11.8 days of expected overhead.

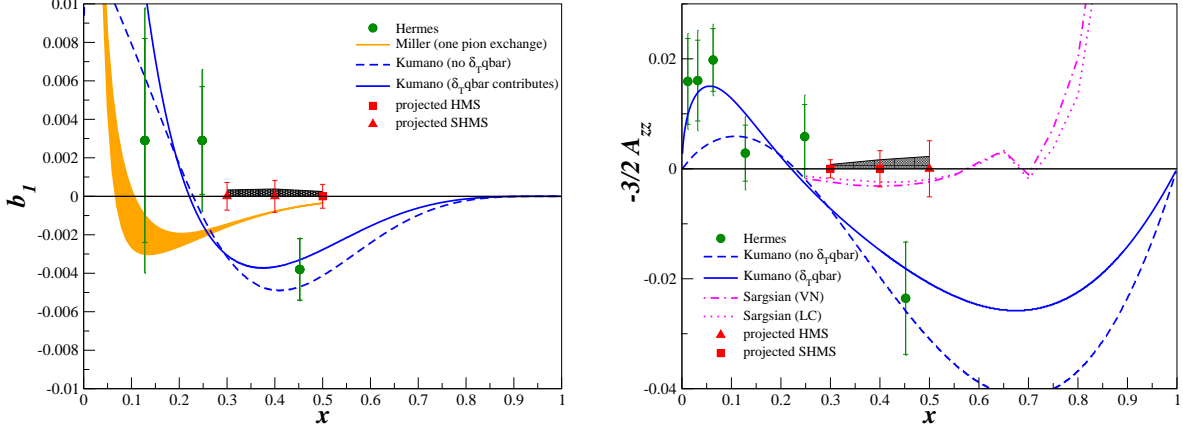


Figure 8: **Left:** Projected precision of the tensor structure function  $b_1$  with 28 days of beam time. **Right:** Corresponding projected precision of the tensor asymmetry  $A_{zz}$ . The black band represents the systematic uncertainty. Also shown are the HERMES data [9], and the calculations from Kumano [12], Miller [13, 14], and Sargsian [17].

	$\bar{x}$	$\overline{Q^2}$ (GeV <sup>2</sup> )	$\overline{W}$ (GeV)	$P_0$ (GeV)	$\theta$ (deg.)	Rates (kHz)	$A_{zz}$ $\times 10^{-2}$	$\delta A_{zz}^{stat}$ $\times 10^{-2}$	$b_1$ $\times 10^{-2}$	$\delta b_1^{stat}$ $\times 10^{-2}$	time (days)
SHMS	0.30	1.5	2.11	8.46	7.3	0.48	0.48	0.11	-0.33	0.072	15.7
SHMS	0.40	2.2	2.07	8.20	9.0	0.14	0.99	0.22	-0.38	0.083	12.5
HMS	0.50	3.5	2.11	7.30	12.2	0.03	1.40	0.34	-0.25	0.062	28.1

Table 1: Summary of the kinematics and physics rates using Hall C spectrometers.

## 2.1 Experimental Method

Following Ref. [7], we will extract  $b_1(x)$  from the difference of parallel and perpendicular polarized target cross sections, with an unpolarized<sup>‡</sup> incident electron beam. The polarized cross sections<sup>§</sup> can be extracted from data collected by scattering an unpolarized electron beam off a spin-1 target polarized longitudinally and perpendicularly to the electron beam direction.

For this configuration, it can be shown that:

$$\sigma_{\perp} - \sigma_{\parallel} = \frac{K}{6}(2P_z^{\parallel} + P_z^{\perp})xb_1 \quad (16)$$

where  $P_z^{\parallel}$  and  $P_z^{\perp}$  are the vector polarization achieved in the longitudinal and transverse configurations respectively. The tensor asymmetry can then be extracted from the structure function  $b_1$  via:

$$A_{zz} = -\frac{2}{3} \frac{b_1}{F_1} \quad (17)$$

The time necessary to achieve the desired precision  $\delta b_1$  is:

$$T = \frac{N}{R_D} = \frac{2}{\epsilon^2 R_D (\delta b_1 / b_1)^2} \quad (18)$$

where  $R_D$  is the deuteron rate, which is estimated from Ref. [22], and  $\epsilon$  is given by Eq. 32. Full details of the statistical error calculation are provided in Appendix A, for reference.

### 2.1.1 Systematic Uncertainties

Table 2 summarizes the systematic error estimate for the cross section measurement. The unpolarized structure function  $F_1$  is used to evaluate  $A_{zz}$  from the measured  $b_1$  using Eq. 17. This contributes an additional 5% relative, which raises the total systematic for  $A_{zz}$  to 10.6%.

### 2.1.2 Overhead

Table 3 summarizes the expected overhead, which sums to 11.8 days. incident energy of 2.2 GeV. Measurements of the dilution from the unpolarized materials contained in the target, and of the packing fraction due to the granular composition of the target material will be performed with a carbon target. Target annealing will be performed approximately once per day, and target material changes will be performed slightly more than once a week. Configuration changes include rotation of the magnetic field of the target from parallel to perpendicular and vice versa.

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<sup>‡</sup>Polarized beam is not required for this experiment, but would enable a simultaneous measurement of  $g_1$ . The tensor structure function  $b_1$  can then be isolated by averaging the results of data with beam polarized parallel and anti-parallel. Any contribution from residual vector polarization can be eliminated by grouping together two sets of data of opposing beam helicity. False asymmetries will be monitored in a similar fashion by periodically flipping the target spin direction.

<sup>§</sup>For simplicity, we will use  $\sigma_{\parallel}$  for  $\frac{d\sigma_{\parallel}^H}{dx dy}$  and  $\sigma_{\perp}$  for  $\frac{d\sigma_{\perp}^H}{dx dy}$ .

Source	(%)
Target Polarization	5.0
Dilution/Packing fraction	5.0
Radiative Corrections	4.0
Acceptance	4.0
Charge determination	$\leq 1$
VDC efficiency	$\leq 1$
PID detector efficiencies	$\leq 1$
Software cut efficiency	$\leq 1$
Energy	0.5
Total	9.3

Table 2: Major contributions to the cross section systematic.

Overhead	Number	Time Per (hr)	(hr)
Target anneal	30	2.0	60.0
Target field rotation	3	12.0	36.0
Beamline survey	2	8.0	16.0
Target material change	5	8.0	40.0
Target T.E.	16	4.0	64.0
Packing Fraction	6	2.0	12.0
Linac change	2	8.0	16.0
Momentum/angle change	1	2.0	2.0
Moller measurement	6	2.0	12.0
Optics	3	4.0	12.0
Arc Energy Meas.	3	2.0	6.0
BCM calibration	2	2.0	8.0
			11.8 days

Table 3: Major contributions to the overhead.

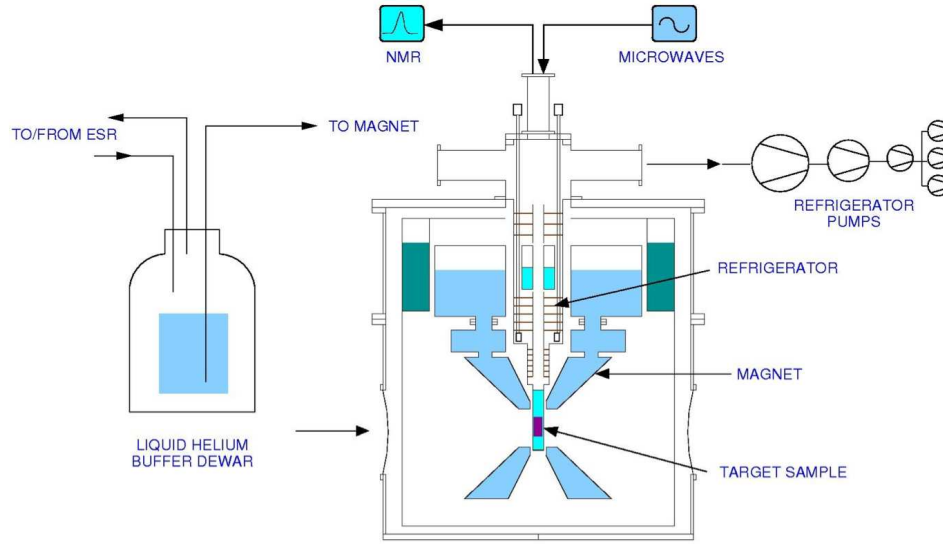


Figure 9: Cross section view of the JLab/UVa polarized target. Figure courtesy of C. Keith.

## 2.2 Polarized Target

This experiment will require the installation of the JLab/UVa polarized target operated in longitudinal and also transverse mode. Transverse polarization requires operation of an upstream chicane to ensure proper transport through the target magnetic field. The target is typically operated with a specialized slow raster, and beamline instrumentation capable of characterizing the low current 50-100 nA beam. All of these requirements have been met previously in Hall C. The polarized target (see Fig. 9), has been successfully used in experiments E143, E155, and E155x at SLAC, and E93-026, E01-006 and E07-003 at JLab. The same target will be utilized in experiments E08-027 and E08-007 in late 2011. A similar target was used in Hall B for the EG1, EG4 and DVCS experiments, although Hall B does not at present have the facilities necessary to operate a transversely polarized target with an electron beam.

The target is in the process of undergoing significant renovation and improvement [23]. The superconducting coils were refurbished by Oxford instruments. A new 1 K refrigerator and target insert were designed and constructed by the JLab target group. The cryogenic pumping system has been overhauled. In particular, the older Alcatel 2060H rotary vane pumps have been replaced with new Pfeiffer DU065 magnetically coupled rotary vane pumps, and the pump controls are being refurbished. The target motion system has been rebuilt from scratch. And now, the magnet and vacuum jacket rotate independently of the refrigerator and target insert, which simplifies rotation from parallel to perpendicular magnetic field orientations.

The target operates on the principle of Dynamic Nuclear Polarization, to enhance the low temperature (1 K), high magnetic field (5 T) polarization of solid materials by microwave pumping. The polarized target assembly contains several target cells of 3.0 cm length that can be selected individually by remote control to be located in the uniform field region of a superconducting Helmholtz pair. The permeable target cells are immersed in a vessel filled with liquid Helium

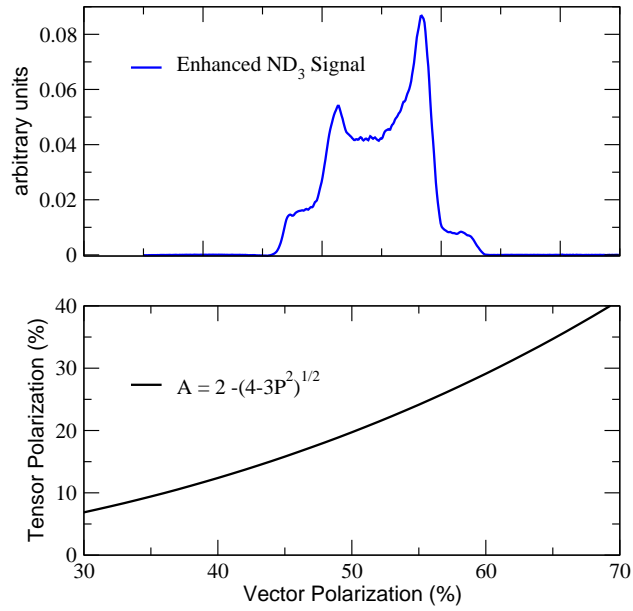


Figure 10: **Top:** NMR signal for  $\text{ND}_3$  with a vector polarization of approximately 50% from the GeN experiment. The average polarization in beam for that experiment was 35%. **Bottom:** Relationship between vector and tensor polarization in equilibrium, and neglecting the small quadrupole interaction.

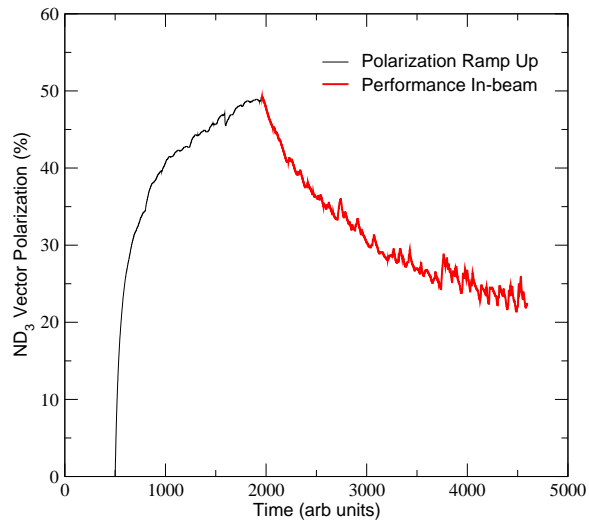


Figure 11: Performance of the  $\text{ND}_3$  target during the GeN experiment.

and maintained at 1 K by use of a high power evaporation refrigerator. The coils have a 50° conical shaped aperture along the beam axis which allow for unobstructed forward scattering.

The target material is exposed to microwaves to drive the hyperfine transition which aligns the nucleon spins. The heating of the target by the beam causes a drop of a few percent in the polarization, and the polarization slowly decreases with time due to radiation damage. Most of the radiation damage can be repaired by periodically annealing the target, until the accumulated dose reached is greater than about  $0.5 \times 10^{17} \text{ e}^-/\text{cm}^2$ , at which time the target material needs to be replaced.

### 2.2.1 Polarization Analysis

The three Zeeman sublevels of the deuteron system ( $m = -1, 0, 1$ ) are shifted unevenly due to the quadrupole interaction [2]. This shift depends on the angle between the magnetic field and the electrical field gradient, and gives rise to two separate transition energies. Hence, the unique double peaked response displayed in Fig. 10. When the system is at thermal equilibrium with the solid lattice, the deuteron polarization is known from:

$$P_z = \frac{4 + \tanh \frac{\mu B}{2kT}}{3 + \tanh^2 \frac{\mu B}{2kT}} \quad (19)$$

where  $\mu$  is the magnetic moment, and  $k$  is Boltzmann's constant. The vector polarization can be determined by comparing the enhanced signal with that of the TE signal (which has known polarization). This polarimetry method is typically reliable to about 5% relative.

Similarly, the tensor polarization is given by:

$$P_{zz} = \frac{4 + \tanh^2 \frac{\mu B}{2kT}}{3 + \tanh^2 \frac{\mu B}{2kT}} \quad (20)$$

From Eqs. 19 and 20, we find:

$$P_{zz} = 2 - \sqrt{4 - 3P_z^2}$$

In addition to the TE method, polarizations can be determined by analyzing NMR lineshapes as described in [24] with a typical 7% relative uncertainty. At high polarizations, the intensities of the two transitions differ, and the NMR signal shows an asymmetry  $R$  in the value of the two peaks, as shown in Fig. 10. The vector polarization is then given by:

$$P_z = \frac{R^2 - 1}{R^2 + R + 1} \quad (21)$$

and the tensor polarization is given by:

$$P_{zz} = \frac{R^2 - 2R + 1}{R^2 + R + 1} \quad (22)$$



The DNP technique produces deuteron vector polarizations of up to 60% in ND<sub>3</sub> and 64% in LiD [25], which corresponds to tensor polarizations of approximately 30%. The target polarization decays while in beam, so that the average vector polarization can be expected to be about 35%, as seen in Fig. 11.

An average polarization of 35 percent enables a significant measurement of  $b_1(x)$ , as shown in Fig. 8. Any improvement to the expected polarization, although not strictly necessary, would allow the addition of kinematic points, and/or improved statistical accuracy. With this in mind, we are pursuing techniques to enhance the tensor polarization by directly stimulating transitions to/from the  $M_s = 0$  state, as discussed in Ref. [2]. D. Crabb from the UVa group had some success in obtaining enhanced tensor polarizations via RF saturation of one of the Zeeman transitions, otherwise known as ‘hole-burning’. The method was not pursued due to the lack of need for tensor polarized targets at the time of the study. Another method to enhance tensor polarization entails simultaneously pumping the sample with two independent microwave frequencies, which requires careful isolation of the respective cavities. We reiterate that the rates in this proposal assume only polarizations that have been demonstrated previously during typical operation at JLab.

### 3 Summary

We request 39.8 days of beam time in order to perform a precision measurement of  $b_1^d$  using a longitudinally polarized deuteron (ND<sub>3</sub>) target, together with the Hall C HMS and SHMS spectrometers. All existing theoretical predictions for  $b_1$  in the region of interest predict small or vanishing values for  $b_1$  at intermediate values of  $x$ , in contrast to the apparent large negative result of the only existing measurement from HERMES. Tensor structure measurements provide information not available from spin-1/2 targets. This measurement will help clarify the role quark orbital angular momentum plays in the nucleon spin, and open a new avenue of spin structure studies at Jefferson Lab.

## A Statistical error calculations of $A_{zz}$ and $b_1^d$

Full details of the error calculation can be found in Ref. [26]. From section 6 of Ref. [7], we have:

$$\frac{d\sigma_{\parallel}^H}{dxdy} = K \left[ xF_1(x) + \left( \frac{2}{3} - H^2 \right) x b_1(x) \right] \quad (23)$$

$$\frac{d\sigma_{\perp}^H}{dxdy} = K \left[ xF_1(x) - \left( \frac{1}{3} - \frac{1}{2} H^2 \right) x b_1(x) \right] \quad (24)$$

with  $K = \frac{e^4 M E}{2\pi Q^4} [1 + (1 - y)^2]$ .

For simplicity, we will use  $\sigma_{\parallel}$  for  $\frac{d\sigma_{\parallel}^H}{dxdy}$  and  $\sigma_{\perp}$  for  $\frac{d\sigma_{\perp}^H}{dxdy}$ .

We know that  $H^2 = (P + 2)/3$ , where  $P$  is the vector polarization of the target. And the tensor polarization  $P_{zz}$  is related to  $P_z$  via Eq. 3.

The tensor asymmetry  $A_{zz}$  depends on  $b_1$  and  $F_1$ :

$$\frac{b_1}{F_1} = -\frac{3}{2} A_{zz} \quad (25)$$

### A.1 Cross section method

Working with the equations of  $\sigma_{\parallel}$  and  $\sigma_{\perp}$ , we can isolate  $b_1$ :

$$\sigma_{\parallel} - \sigma_{\perp} = \frac{-K}{6} (2P_z^{\parallel} + P_z^{\perp}) x b_1 \quad (26)$$

where  $P_z^{\parallel}$  and  $P_z^{\perp}$  are the vector polarization achieved in the longitudinal and transverse configurations respectively.

$$\frac{\delta b_1}{b_1} = \frac{\sqrt{\delta\sigma_{\perp}^2 + \delta\sigma_{\parallel}^2}}{\sigma_{\perp} - \sigma_{\parallel}} \quad (27)$$

In the valence region,  $b_1 < 0$  which implies  $\sigma_{\perp} < \sigma_{\parallel}$ . We can define the difference between  $\sigma_{\perp}$  and  $\sigma_{\parallel}$  as:

$$\sigma_{\parallel} = (1 + \epsilon) \sigma_{\perp} \quad (28)$$

It is safe to assume that we will need  $\delta\sigma_{\perp} \approx \delta\sigma_{\parallel}$ . We obtain:

$$\frac{\delta b_1}{b_1} = -\frac{\sqrt{2} \delta\sigma_{\perp}}{\epsilon \sigma_{\perp}} \quad (29)$$

Now to evaluate the time necessary to perform a significant measurement of  $b_1$ , we need an estimate of the value of  $\epsilon$ . So we start from:

$$\sigma_{\parallel} = \sigma_u \left(1 - \frac{1}{3} P_z^{\parallel} \frac{b_1}{F_1}\right) \quad (30)$$

$$\sigma_{\perp} = \sigma_u \left(1 + \frac{1}{6} P_z^{\perp} \frac{b_1}{F_1}\right) \quad (31)$$

with  $\sigma_u$  the unpolarized cross section. Which leads to:

$$\epsilon = \frac{1 - \frac{1}{3} P_z^{\parallel} \frac{b_1}{F_1}}{1 + \frac{1}{6} P_z^{\perp} \frac{b_1}{F_1}} \quad (32)$$

We use for  $b_1^d$  the fit from Kumano [12] and for  $F_1^d$  MSTW [22] (no EMC effect or smearing included). For  $x$ -values of 0.30, 0.40 and 0.50,  $\epsilon$  is equal to 0.0013, 0.0026 and 0.0037 respectively, assuming  $P_z^{\perp} = P_z^{\parallel} = 0.35$

$$\frac{\delta b_1}{b_1} = -\frac{\sqrt{2}}{\epsilon} \frac{1}{\sqrt{N}} \quad (33)$$

The number of events needed in each parallel and perpendicular kinematics are  $N/2$  with:

$$N = \frac{2}{\epsilon^2} \frac{1}{(\delta b_1/b_1)^2} \quad (34)$$

The time necessary to achieve this statistics is:

$$T = \frac{N}{R_D} = \frac{2}{\epsilon^2 R_D (\delta b_1/b_1)^2} \quad (35)$$

The deuterium rates are estimated from the unpolarized deuteron cross section [22]  $\sigma_D$ :

$$R_D = \sigma_D dp d\Omega L = \sigma_D dp d\Omega \rho_D \frac{I}{e} \quad (36)$$

with  $\rho_D = \rho_{ND3} \cdot f_{ND3} \cdot PF_{ND3}$ , where  $f_{ND3}$  is the dilution and  $PF_{ND3}$  is the packing fraction. To estimate the physics rates, the spectrometer acceptance and momentum bite were reduced to  $d\Omega = 6.5$  msr and  $dp = \pm 8\%$  for the HMS and  $d\Omega = 4.4$  msr and  $dp = {}^{+20\%}_{-8\%}$  for SHMS and a cut on  $W \geq 1.8$  GeV was required.

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