NUCLEAR GLUONOMETRY ☆

R.L. JAFFE and Aneesh MANOHAR
Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

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We identify a new leading twist structure function in QCD which can be measured in deep elastic scattering from polarized targets (such as nuclei) with spin $\geqslant 1$. The structure function measures a gluon distribution in the target and vanishes for a bound state of protons and neutrons, thereby providing a clear signature for exotic gluonic components in the target.

1. Introduction

The physical photon has four structure functions [1,2]. Three are familiar: F_1^γ, $F_L^\gamma = F_2^\gamma - 2xF_1^\gamma$ and g_1^γ. The fourth, called F_3^γ by Ahmed and Ross [2], corresponds to the imaginary part of the double helicity flip Compton amplitude, $A_{+-,-+}$ in the notation of ref. [3]. The other three are proportional to helicity conserving Compton amplitudes, $\binom{F_1}{g_1} \propto (A_{++,++} \pm A_{-+,-+})$, $F_L \propto A_{0+,0+}$. In parton models both F_L^γ and F_3^γ would be expected to vanish in the Bjorken limit since massless quarks do not couple to longitudinal photons, nor flip the photon helicity by two units. In QCD both F_L^γ and F_3^γ get contributions from the box graph [1,2,4] which persist in the scaling limit because the short-distance behavior of the box graph violates parton model assumptions.

Witten [5] pointed out that these contributions to F_L^γ are associated with towers of *photon* operators which appear in the operator product expansion (OPE) of two electromagnetic currents. Their coefficient functions have been calculated from the box graph. Recently, one of us [6] identified the tower of photon operators which contribute to F_3^γ. By analogy it is evident that there must be a tower of *gluon* operators in QCD, with coefficient functions of order $\alpha_s(Q^2)$ obtained from the box graph, which generate a double helicity flip Compton amplitude on a hadronic target. These operators belong to different representations of the Lorentz group than the other operators which appear in the OPE and therefore do not mix under renormalization with quark operators and the other gluon operators. These operators have vanishing matrix elements in any state with spin less than one and appear to have been overlooked in all QCD analyses in the past. We name the hadronic structure function associated with this tower of operators $\Delta(x, Q^2)$ (to avoid confusion with the parity-violating structure function $F_3(x, Q^2)$ of neutrino scattering). $\Delta(x, Q^2)$ can be measured by scattering an *unpolarized* electron beam from a target *aligned* ((that is, polarized either along or against) perpendicular to the beam. [Actually any direction not exactly parallel to the beam will do, but perpendicular is best.] The only targets with $J \geqslant 1$ are nuclei. $\Delta(x, Q^2)$ vanishes identically for a nucleus made up of protons, neutrons and pions regardless of Fermi motion or binding corrections in the approximation in which the nucleons or pions scatter independently. It is therefore an unambiguous probe of the gluonic components of the nuclear wavefunction which cannot be identified with individual nucleons or pions.

If the scattering cross section is measured as a function of the usual variables, $x = Q^2/2\nu$, $y = \nu/ME$ and the azimuthal angle ϕ between the plane formed by the beam and the alignment axis and the plane formed by the beam and the scattered electron (fig. 1), then in the scaling limit (Q^2, $\nu \to \infty$, x fixed),

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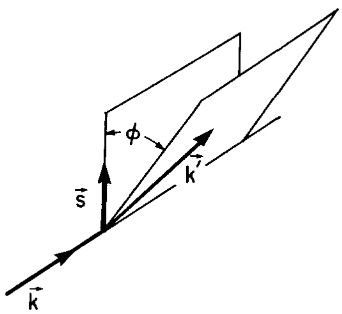

Fig. 1. Definition of the azimuthal scattering angle ϕ.

$$\frac{2\pi(\mathrm{d}\sigma/\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\phi)}{(\mathrm{d}\sigma/\mathrm{d}x\,\mathrm{d}y)}=1-\frac{1}{2}\frac{x(1-y)}{xy^2F_1(x,Q^2)+(1-y)F_2(x,Q^2)}\times\Delta(x,Q^2)\cos 2\phi\,, \tag{1}$$

where $F_1(x, Q^2)$ and $F_2(x, Q^2)$ are the usual spin averaged structure functions of the target.

$\Delta(x, Q^2)$ has a parton interpretation. Consider the target in an infinite momentum frame with its spin aligned along the $\hat{x}$ direction, which is perpendicular to its momentum. Define $g_{\hat{x},\hat{y}}(x, Q^2)$ to be the probability (per unit x) to find a gluon with momentum fraction x and linearly polarized along the $\hat{x}$, $\hat{y}$ direction and let $a(x, Q^2)=g_{\hat{x}}(x, Q^2)-g_{\hat{y}}(x, Q^2)$. Then

$$\Delta(x,Q^2)=\frac{\alpha_s(Q^2)}{2\pi}\,\mathrm{Tr}\,\mathcal{Q}^2x^2\int_x^1\frac{\mathrm{d}y}{y^3}a(y,Q^2)\,. \tag{2}$$

$\mathcal{Q}$ is the quark charge matrix, $\mathcal{Q}=\mathrm{diag}\,(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$ for u, d, s. The integral in eq. (2) reflects the non-local character of the box graph and occurs for all gluon operators in deep inelastic scattering. Eq. (2) is easier to interpret if it is rewritten,

$$\Delta(x,Q^2)=\frac{\alpha_s(Q^2)}{2\pi}\times\mathrm{Tr}\,\mathcal{Q}^2\int_0^1\mathrm{d}y\,\mathrm{d}z\,\delta(yz-x)z^2a(y,Q^2)\,, \tag{3}$$

which can be viewed (à la Altarelli–Parisi [7]) as the convolution of the transverse aligned gluon distribution $a(y, Q^2)$ with a "splitting function" $\alpha_s(Q^2)\mathrm{Tr}\,\mathcal{Q}^2z^2/2\pi$ from the box graph, subject to the kinematic constraint $x=yz$.

A measurement of $\Delta(x, Q^2)$ would be very interesting. In the remainder of this letter we first discuss the kinematics of the double helicity flip contribution to deep inelastic scattering, next we present the operator product expansion and project out the operators which contribute to double helicity flip. Finally we develop the parton model interpretation of our results and close with a brief summary. We make the presentation for the case of a spin-one target, but it applies without modification to a spin-$\frac{3}{2}$ target as well. If the target has spin-two or greater there is more than one double helicity flip Compton amplitude and changes are necessary, though the basic idea is unchanged. We summarize the results for arbitrary spin in an appendix at the end of the paper.

2. Kinematics

We study inelastic scattering of an electron (or muon) of initial four-momentum k^μ and final four-momentum $k'^\mu=k^\mu-q^\mu$ from a target of four-momentum p^μ. We will generally work in the rest frame of the target, $p^\mu=(M,\mathbf{0})$, and orient the virtual photon along the $\hat{e}_3(\hat{z})$-direction, $q^\mu=(q^0, 0, 0, q^3)$. We define the azimuthal angle of the final lepton (with respect to the x-axis) in the xy-plane to be ϕ. As usual, we define $Q^2\equiv-q^2=(q^3)^2-(q^0)^2$, $\nu=p\cdot q=Mq^0$, and q^3 is given by $q^3=\sqrt{\kappa}q^0$, where

$$\kappa=1+M^2Q^2/\nu^2\,. \tag{4}$$

The cross section for inelastic lepton scattering from a polarized spin-one target is proportional to the imaginary part of the amplitude for forward virtual Compton scattering,

$$W_{\mu\nu}(E',E)=\frac{1}{4\pi}\int\mathrm{d}^4x\exp(\mathrm{i}q\cdot x)\times\langle p,E'\,|\,[j_\mu(x),j_\nu(0)]\,|p,E\rangle=\frac{1}{2\pi}\mathrm{Im}\,T_{\mu\nu}(E',E)\,, \tag{5}$$

where E_μ is the polarization vector describing the spin orientation of the target, with $p\cdot E=0$, $E^2=-1$. If we were only interested in targets polarized along $\hat{e}_3$ we could set $E_\mu=E'_\mu$. However, targets polarized perpendicular to the incident photon are linear superpositions of states polarized along $\hat{e}_3$ and terms with $H\neq H'$ occur. In fact they are the subject of this letter.

Since $W_{\mu\nu}$ depends linearly on E and E' we may define a target-polarization-independent tensor $W_{\mu\nu,\alpha\beta}$,

$$W_{\mu\nu}(E',E)=E'^{*\alpha}E^{\beta}W_{\mu\nu,\alpha\beta}\,. \qquad (6)$$

$W_{\mu\nu,\alpha\beta}$ can be related to the imaginary part of helicity amplitudes for virtual Compton scattering. Define helicity projection operators $P(hH,h'H')_{\mu\nu,\alpha\beta}$, by

$$P(hH,h'H')_{\mu\nu,\alpha\beta}\equiv\epsilon^{*}_{\mu h'}E^{*}_{\alpha H'}E_{\beta H}\epsilon_{\nu h}\,, \qquad (7)$$

where $\epsilon_{\mu h}$ are photon polarization vectors,

$$\epsilon^{\mu}_{\pm}=\mp(1/\sqrt{2})(0,1,\pm\mathrm{i},0)\,,$$

$$\epsilon_0=(1/\sqrt{Q^2})(q^3,0,0,q^0)\,, \qquad (8)$$

and $E_{\mu H}$ are target polarization vectors,

$$E^{\mu}_{\pm}=\epsilon^{\mu}_{\pm}\,,$$

$$E_0=(0,0,0,1)\,, \qquad (9)$$

in the target rest frame. In terms of $\{P(hH,h'H')\}$

$$W_{\mu\nu,\alpha\beta}=\sum_{hH,h'H'}P(hH,h'H')_{\mu\nu,\alpha\beta}A_{hH,h'H'}\,, \qquad (10)$$

where $A_{hH,h'H'}$ is $(2\pi)^{-1}$ times the imaginary part of the forward Compton helicity amplitude $(T_{hH,h'H'})$ for $\gamma_h+\text{target}_H\to\gamma_{h'}+\text{target}_{H'}$. The sum in eq. (10) is constrained by $h+H=h'+H'$.

We are interested only in the double helicity flip amplitudes $A_{+-,-+}$ and $A_{-+,+-}$ which are equal by parity invariance. If we denote the double helicity flip contribution to $W_{\mu\nu,\alpha\beta}$ as $W^{\Delta=2}_{\mu\nu,\alpha\beta}$, and define $\Delta(x,Q^2)\equiv A_{+-,-+}$, then

$$W^{\Delta=2}_{\mu\nu,\alpha\beta}=[P(+-,-+)_{\mu\nu,\alpha\beta}$$
$$+P(-+,+-)_{\mu\nu,\alpha\beta}]\Delta(x,Q^2)\,. \qquad (11)$$

The necessary projection operators can be constructed from the following identity which is obtained from eqs. (7) and (8):

$$\epsilon_{\pm\alpha}\epsilon^{*}_{\pm\nu}$$
$$=\frac{1}{2}\left(-g_{\alpha\nu}+\frac{q_\alpha p_\nu}{\kappa\nu}\pm\frac{\mathrm{i}}{\sqrt{\kappa\nu}}\epsilon_{\alpha\nu\lambda\kappa}p^{\lambda}q^{\kappa}\right)+\dots, \qquad (12)$$

where the omitted terms are proportional to q_ν or p_α do not contribute to Compton scattering because $q\cdot\epsilon=P\cdot E=0$. After some algebra we find

$$P(+-,-+)_{\mu\nu,\alpha\beta}+P(-+,+-)_{\mu\nu,\alpha\beta}$$
$$=\frac{1}{2}\left[\left(g_{\mu\alpha}-\frac{p_\mu q_\alpha}{\kappa\nu}\right)\left(g_{\nu\beta}-\frac{p_\nu q_\beta}{\kappa\nu}\right)+(\mu\leftrightarrow\nu)\right]$$
$$-\frac{1}{2}\left(g_{\mu\nu}+\frac{q^2}{\nu^2\kappa}p_\mu p_\nu\right)\left(g_{\alpha\beta}+\frac{M^2}{\nu^2\kappa}q_\alpha q_\beta\right)$$
$$+\dots, \qquad (13)$$

where, again, the omitted terms are proportional to q_μ, q_ν, p_α or p_β.

We now combine eqs. (6), (10), (11) and (13) to obtain an expression for the double helicity flip term in $W_{\mu\nu}(E',E)$:

$$W^{\Delta=2}_{\mu\nu}(E',E)$$
$$=\frac{1}{2}\left(\left\{\left[E'^{*}_{\mu}-\frac{q\cdot E'^{*}}{\kappa\nu}\left(p_\mu-\frac{M^2}{\nu}q_\mu\right)\right]\right.\right.$$
$$\left.\times\left[E_\nu-\frac{q\cdot E}{\kappa\nu}\left(p_\nu-\frac{M^2}{\nu}q_\nu\right)\right]+(\mu\leftrightarrow\nu)\right\}$$
$$-\left[g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}+\frac{q^2}{\kappa\nu^2}\left(p_\mu-\frac{\nu}{q^2}q_\mu\right)\left(p_\nu-\frac{\nu}{q^2}q_\nu\right)\right]$$
$$\left.\times\left(E'^{*}\cdot E+\frac{M^2}{\kappa\nu^2}q\cdot E'^{*}\,q\cdot E\right)\right)\Delta(x,Q^2)\,. \qquad (14)$$

Here we have restored the terms proportional to q_μ or q_ν required by gauge invariance. It is easy to verify that $W^{\Delta=2}_{\mu\nu}$ has the following properties: (1) It vanishes if $E=E'=E_0^{\pm}$; (2) It vanishes if averaged over spin; (3) It does not vanish if $E'=E_\pm$ and $E=E_\mp$ in which case it is proportional to $\epsilon_{\mp\mu}\epsilon_{\pm\nu}$. $W^{\Delta=2}_{\mu\nu}$ simplifies considerably in the Bjorken limit.

Next we find the contribution of $\Delta(x,Q^2)$ to deep inelastic scattering from a target polarized transverse to the photon beam. We work in the Bjorken limit where the virtual photon is nearly parallel to the incident electron so that the target may be regarded as polarized perpendicular to the incident electron beam. We take $E_\mu=E'_\mu\equiv E^{x}_{+\mu}\equiv-(1/\sqrt{2})\,(0,0,1,\mathrm{i})$ and use

$$E^{x}_{+\mu}=\tfrac{1}{2}(E_{+\mu}+\sqrt{2}E_{0\mu}+E_{-\mu})\,, \qquad (15)$$

where $E_{\pm,0}$ are defined in eq. (9). From eq. (2.6) of ref. [8] we obtain

$$\lim_{\substack{Q^2\to\infty \\ x \text{ fixed}}} \frac{d\sigma}{dx\,dy\,d\phi} = \frac{e^4 ME}{4\pi^2 Q^4}$$

$$\times [xy^2 F_1(x, Q^2) + (1-y) F_2(x, Q^2)$$

$$-\tfrac{1}{2}x(1-y)\Delta(x, Q^2)\cos 2\phi] \ . \qquad (16)$$

In eq. (16) we have dropped both higher twist structure functions and kinematic corrections which vanish like powers of M_N^2/Q^2 in the Bjorken limit. The same cross section is obtained if the target is polarized in the opposite sense, $E = E_-^x$, which verifies that the effect is sensitive only to the alignment rather than the polarization of the target.

3. Operator product expansion and parton model

The structure functions can be calculated using an operator product expansion for the time-ordered product of two currents,

$$\mathcal{T}_{\mu\nu}(q) \equiv i \int d^4x \exp(iq\cdot x)\, \mathrm{T}(j_\mu(x) j_\nu(0)) \ . \qquad (17)$$

In ref. [6] it was shown that in addition to the usual contributions at leading twist, there is another tower of gluonic operators which is usually omitted,

$$\tfrac{1}{2}\mathcal{T}_{\mu\nu}(q)$$

$$= \ldots + \sum_{n=2,3} \frac{2^n q^{\mu_1} \ldots q^{\mu_n}}{(Q^2)^n} C_n(Q^2) O_{\mu\nu\ \mu_1\ldots\nu_n}, \qquad (18)$$

where

$$C_n(Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \mathrm{Tr}\, \mathcal{Q}^2 \frac{2}{n+2}, \qquad (19)$$

and

$$O_{\mu\nu\ \mu_1\ldots\mu_n} \equiv \tfrac{1}{2}(\tfrac{1}{2}i)^{n-2} S\{G^a_{\mu\mu_1} \overleftrightarrow{D}_{\mu_3} \ldots \overleftrightarrow{D}_{\mu_n} G^a_{\nu\mu_2}\} \ , \qquad (20)$$

where G^a is the gluon field strength, S symmetrizes and removes traces in the indices $\mu_1 \ldots \mu_n$, and $O_{\mu\nu\ \mu_1\ldots\mu_n}$ is understood to be renormalized at a scale Q^2.

The terms omitted from eq. (18) are the standard ones, all of which vanish when contracted with $P(\pm\mp, \mp\pm)_{\mu\nu,\alpha\beta}$ and therefore do not contribute to the double helicity flip Compton amplitude. The reason the operator defined in eq. (20) can generate a double helicity flip is its antisymmetry in $\mu \leftrightarrow \mu_1$ and $\nu \leftrightarrow \mu_2$. Its target matrix element has terms proportional to $(p_\mu E^*_{\mu_1} - p_{\mu_1} E^*_\mu)(p_\nu E_{\mu_2} - p_{\mu_2} E_\nu)$, which, when contracted with $q^{\mu_1} q^{\mu_2}$, yield the helicity structure of eq. (14). All of the other gluon operators appearing in the expansion of $\mathrm{T}(j_\mu(0) j_\nu(0))$ have a contracted Lorentz index on each gluon field strength tensor – e.g. $O^\alpha_{\alpha\mu_1\ldots\mu_n}$ is the standard gluon operator contributing to F_1 and F_2 at $O(\alpha_s)$ – and cannot generate the helicity structure of eq. (14).

To proceed, we take the matrix element of $O_{\mu\nu\ \mu_1\ldots\mu_n}$ in a spin-one target with polarization E and E':

$$\langle pE' | O_{\mu\nu\ \mu_1\ldots\mu_n} | pE \rangle$$

$$= \tfrac{1}{2} S\{[(p_\mu E'^*_{\mu_1} - p_{\mu_1} E'^*_\mu)(p_\nu E_{\mu_2} - p_{\mu_2} E_\nu)$$

$$+ (\mu \leftrightarrow \nu)] p_{\mu_3} \ldots p_{\mu_n}\} A_n(Q^2) \ldots \ , \qquad (21)$$

where the omitted terms (traces) are lower twist. We project out the double helicity flip amplitude by contracting $\mathcal{T}_{\mu\nu}$ with $\epsilon_+^{\mu *} \epsilon_-^\nu$:

$$\tfrac{1}{2} T_{-+,+-}(x, Q^2)$$

$$= \sum_{n=2,4} \left(\frac{2p\cdot q}{Q^2}\right)^n C_n(Q^2) A_n(Q^2) \qquad (22)$$

$$= \frac{\alpha_s(Q^2)}{2\pi} \mathrm{Tr}\, \mathcal{Q}^2 \sum_{n=2,4\ldots} \left(\frac{1}{x}\right)^n \frac{2}{n+2} A_n(Q^2) \ , \qquad (23)$$

where we have submitted for $C_n(Q^2)$ from eq. (19). To relate this expansion to $\Delta(x, Q^2)$ we write the subtracted dispersion relation for $T_{-+,+-}(x, Q^2)$

$$T_{-+,+-}(x, Q^2) = T^0_{-+,+-}(Q^2)$$

$$+ 4 \int_0^1 x'\, dx' \frac{\Delta(x', Q^2)}{x^2 - x'^2} \ . \qquad (24)$$

In the unphysical region $|x| > 1$ the Taylor expansion of $T_{-+,+-}$ in x^{-2} is guaranteed to converge (T is analytic for $|x| > 1$). Comparing this expansion with eq. (23) we obtain

$$M_n(\Delta) = \frac{\alpha_s(Q^2)}{2\pi} \mathrm{Tr}\, \mathcal{Q}^2 \frac{A_n(Q^2)}{n+2} \ ,$$

$$n = 2, 4, 6, \ldots \ , \qquad (25)$$

where the nth moment of Δ, $M_n(\Delta)$, is defined by

$$M_n(\Delta) = \int_0^1 \mathrm{d}x\, x^{n-1} \Delta(x, Q^2) \;. \tag{26}$$

Note the absence of a sum rule for $n=0$. Had we written an *unsubtracted* dispersion relation for $T_{-+,+-}$, we would have obtained an $n=0$ sum rule: $M_0(\Delta)=0$. Pomeronchuk's theorem [9] suggests that our unsubtracted dispersion relation for $T_{-+,+-}$ would converge; however, the sum rule is likely ruined in QCD by the appearance of a constant real piece in $T_{-+,+-}(x, Q^2)$ at high energies (a $J=0$ fixed pole in the archaic language of Regge theory). That is,

$$\lim_{x\to 0} T_{-+,+-}(x, Q^2) \neq -4 \int_0^1 \frac{\mathrm{d}x'}{x'} \Delta(x', Q^2) . \tag{27}$$

Exactly the same effect ruins the Schwinger term sum rule in QCD [10] and the lowest order graph which violates the sum rule is the box graph. So, although the x-integral in eq. (27) is likely to converge, it does not converge to zero or to any quantity presently calculable in QCD.

Returning to the $n \geqslant 2$ sum rules we cast them in a more familiar form by defining inverse Mellin transforms,

$$A_n(Q^2) \equiv \int_0^1 \mathrm{d}y\, y^{n-1} a(y, Q^2) \;, \tag{28a}$$

$$\frac{1}{n+2} = \int_0^1 \mathrm{d}z\, z^{n-1} z^2 \;, \tag{28b}$$

and inverting eq. (25) we obtain the result quoted in eqs. (2) and (3):

$$\Delta(x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \operatorname{Tr} \mathscr{Q}^2 x^2 \int_x^1 \frac{\mathrm{d}y}{y^3} a(y, Q^2) . \tag{2'}$$

The $n=2$ sum rule is particularly simple:

$$\int_0^1 \mathrm{d}x\, x\Delta(x, Q^2)$$
$$= \frac{\alpha_s(Q^2)}{8\pi} \operatorname{Tr} \mathscr{Q}^2 \frac{1}{(p^+)^2}$$
$$\times \langle p, E_x | (E_2 - B_1)^2 - (E_1 - B_2)^2 | p, E_x \rangle \;, \tag{29}$$

where $E^a_k \equiv G^a_{0k}$ and $B^a_k \equiv \frac{1}{2}\epsilon_{kij} G^a_{ij}$ are cartesian components of the color electric and magnetic fields (the color sum is suppressed in eq. (29)), $E^\mu_x = -(1/\sqrt{2})\ (0, 0, 1, \mathrm{i})$, and $p^+ = (1/\sqrt{2})\ (p^0 + p^3)$.

Generally, the inverse Mellin transform of the tower of twist-two operator matrix elements is identified with a parton probability distribution in the parton model version of perturbative QCD. To make this connection for $a(x, Q^2)$ we view the target as a collection of quarks, antiquarks and gluons moving colinearly in an infinite momentum frame. We take the target to be polarized along the $\hat{x}$-direction ($E_\mu = E^x_{+\mu}$, note $E{\cdot}p=0$ in this frame) and define $g_{\hat{x},\hat{y}}(\xi, Q^2)$ to be the (unknown) probability to find a gluon with momentum $p_\mu \equiv \xi p_\mu$, $0<\xi<1$ linearly polarized along the $\hat{x}$, $\hat{y}$ direction.

Repeating the steps which led from eq. (21) to eq. (23) we obtain

$$A_n(Q^2) = \int_0^1 \mathrm{d}\xi\, \xi^{n-1} [g_{\hat{x}}(\xi, Q^2) - g_{\hat{y}}(\xi, Q^2)] \;, \tag{30}$$

and comparing this with eq. (28a) we find

$$a(y, Q^2) = g_{\hat{x}}(y, Q^2) - g_{\hat{y}}(y, Q^2) \;. \tag{31}$$

Of course we do not know how to calculate $g_{\hat{x},\hat{y}}(x, Q^2)$ for an elementary spin-one hadron like the ρ-meson, much less for exotic gluonic components in a nuclear target. Nevertheless, our analysis assures us that this quantity, if measured, has a direct and simple interpretation in QCD.

4. Discussion and conclusions

Precise probes of the quark and gluon substructure of the nucleus are not easy to find. The EMC effect and the higher spin structure function discussed in refs. [3,8] can receive contributions from the motion and binding of nucleons in the nucleus, as well as from familiar nuclear effects like pion exchange currents. The double helicity flip structure function we have described is free from these conventional sources. It receives no contributions from nucleons or pions bound in nuclei. This is obvious because neither nucleons nor pions can transfer two units of helicity to the nuclear target. The simplest "conventional" source we can find would be from gluon distributions in ρ or Δ admixtures in the nuclear wavefunction. Generically, $\Delta(x, Q^2)$ receives contributions from gluons not associated with individual nucleons in the nucleus. The $n=2$ sum rule, eq. (29) could be combined with simple quark models to obtain a simple estimate of the magnitude of $\Delta(x, Q^2)$ for a hadron with $J \geqslant 1$ (e.g. a ρ, Δ, or $n>3$ quark correlation in a nucleus). The color-spin dependent forces responsible for hadron spin splittings (e.g. the N–Δ and π–ρ splittings) can be attributed to color magnetic fields correlated to the quarks' spins. These $\boldsymbol{B}^a$ fields will generate a calculable non-zero contribution to $\langle p, E_x | B_1^2 - B_2^2 | p, E_x \rangle$ provided the target spin is greater than $\frac{1}{2}$.

$\Delta(x, Q^2)$ is relatively simple to measure provided it is possible to polarize the target transverse to the beam. A polarized lepton beam is not required. Such a measurement might be undertaken in its own right or as a by-product of a measurement of the transverse spin-dependent structure function g_2 on a target with spin one or greater in a polarized beam experiment.

Appendix

Spin J

The analysis of the previous sections can be readily extended to targets of arbitrary spin J using the methods developed in ref. [8]. The double helicity flip amplitudes are $A^J_{+H,-H'}$ and $A^J_{-H,+H'}$. The (unpolarized beam) cross section in the deep inelastic limit can be written in terms of these amplitudes as [8]

$$\frac{d\sigma^{JH'H}}{dx\,dy\,d\phi} = \frac{e^4 EMx}{4\pi^2 Q^4}$$
$$\times \{2(1-y)[A^J_{0H,0H'}$$
$$-\tfrac{1}{2}A^J_{+H,-H'}\exp(-2i\phi) - \tfrac{1}{2}A^J_{-H,+H'}\exp(2i\phi)]$$
$$+[(1-y)+\tfrac{1}{2}y^2](A^J_{+H,+H'} + A^J_{-H,-H'})\} .$$

It is simple to evaluate the helicity amplitudes in terms of the general structure function decomposition of $T^{JH'H}_{\mu\nu}$ given in [8]

$$A^J_{+H-H'} \equiv \epsilon^{\mu *}_- \epsilon^\nu_+ W^{JH'H}_{\mu\nu} = \epsilon^\mu_+ \epsilon^\nu_+ W^{JH'H}_{\mu\nu} ,$$

using $(\epsilon^\nu_-)^* = \epsilon^\nu_+$. In ref. [8] we developed a method for expressing helicity amplitudes in terms of multipole structure functions with simple rotational properties. Thus

$$A^J_{+H,-H'}$$
$$= - \sum_{L=2,4}^{2J} \frac{{}^J_L b_3}{\nu^{L+1}} \epsilon^\mu_+ \epsilon^\nu_+$$
$$\times \langle p, JH' | \mathcal{S}_{\mu\nu\,\mu_3 \ldots \mu_L} | p, JH \rangle p_{\mu_1} p_{\mu_2} q^{\mu_1} \ldots q^{\mu_L}$$
$$= - \sum_{L=2,4}^{2J} {}^J_L b_3 \frac{M^2}{\nu} (-1)^{J-H} (JHJ-H' | L-2) ,$$

where ${}^J_L b_3$ are multipole structure functions defined in ref. [8] and the brackets are Clebsch–Gordan coefficients for $J \otimes J \to L$. The tensor $\mathcal{S}_{\mu_1 \ldots \mu_L}$ is an irreducible tensor of spin L which characterizes the polarization of the target. The equation can be inverted to give the structure functions in terms of the helicity amplitudes [#1]

$${}^J_L b_3 = - \frac{\nu}{M^2} \sum_{H,H'} A^J_{+H,-H'} (-1)^{J-H}$$
$$\times (JHJ-H' | L-2) .$$

The structure function ${}^J_L b_3$ can be computed in terms of the matrix elements of the gluon operators of eq. (20),

$$\langle JH' | O^{\mu\nu\,\mu_1 \ldots \mu_n} | JH \rangle$$
$$= S \sum_{L=2}^{n} {}^J_L C_n(Q^2) \theta^{J\mu\nu\,\mu_3 \ldots \mu_L}_{HH'}(p) p^{\mu_1} p^{\mu_2} p^{\mu_{L+1}} \ldots p^{\mu_n} ,$$

which defines the coefficients ${}^J_L C_n$. Here S antisymmetrizes in $\mu \leftrightarrow \mu_1$, $\nu \leftrightarrow \mu_2$, and symmetrizes and removes traces in $\mu_1 \ldots \mu_n$, and $\langle p, J H' | \mathcal{S}^{\mu_1 \ldots \mu_L} | p, J H \rangle \equiv \theta^{J\mu_1 \ldots \mu_L}_{HH'}(p)$ defines θ. Combining this with the operator product expansion as for spin one gives a series of sum rules,

$$M_n\left(-\frac{{}^J_L b_3}{\nu}\right) = {}^J_L C_n(Q^2) \frac{\alpha_s(Q^2)}{2\pi} \operatorname{Tr} \mathcal{Q}^2 \frac{1}{n+2}$$

for $L=2, 4, \ldots, n \geqslant L$,

and

$$M_n\left(-\frac{{}^J_L b_3}{\nu}\right) = 0, \quad 0<n<L, \; n \text{ even} .$$

[#1] In ref. [8] we defined b_3 with the prejudice that it was a twist-four structure function, hence the factor of ν relating ${}^J_L b_3$ to $A^J_{+H,-H'}$.

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