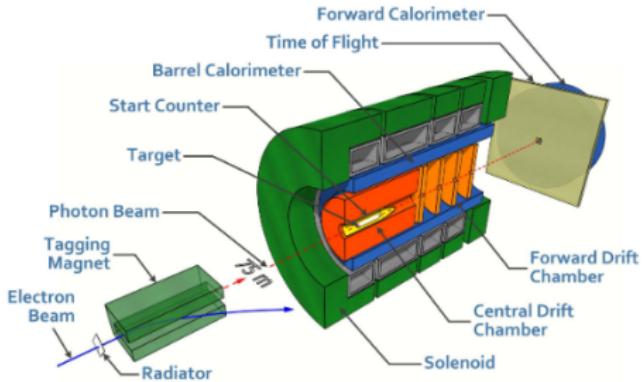
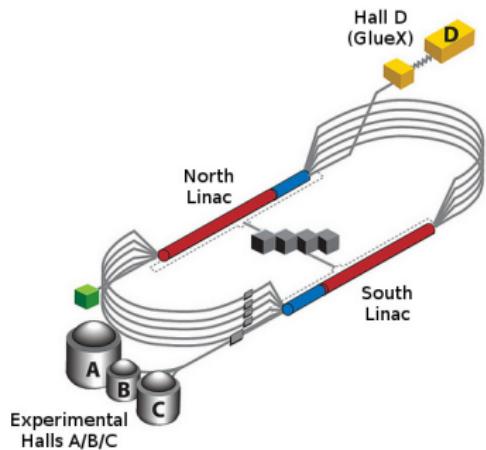


The GlueX Kinematic Fitter in ~ 20 min

Daniel Lersch

May 23, 2022

The Gluonic Excitation Experiment - GlueX

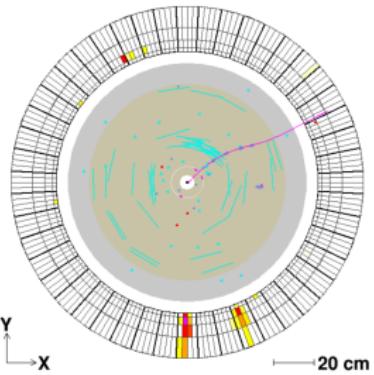
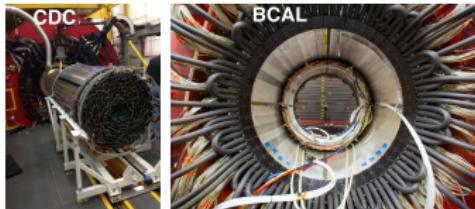
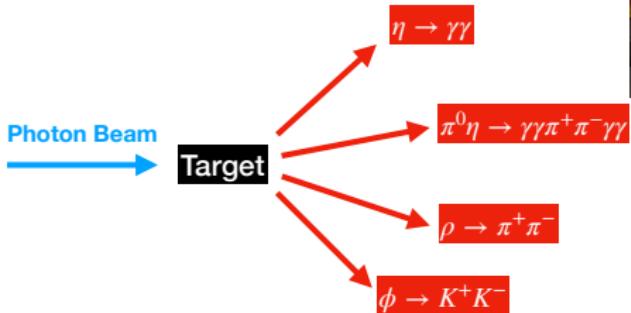


Experimental Hall D:

- Over 130 scientists from:
 - ▶ 30 Institutions
 - ▶ 10 Countries
- Experiments with polarized photon beam

Phase	Run Period	Raw Data [PB]
GlueX-I	Spring 2017	0.9
	Spring 2018	1.9
	Fall 2018	1.1
GlueX-II	Spring 2020	2.8
	Summer 2020	1.7

Event Reconstruction



Charge q	Reconstructed via	Observables
$>0, <0$	FDC/CDC + mag. field	Momenta, track parameters, dE/dx , ...
0	BCAL/FCAL/CompCAL	Clusters, shower info, E, ...

- **Goal:** Reconstruct entire reaction from final state particles
- Many final states particles
 - ▶ Overlapping topologies, combinatorics, ... ([See talk by Beni Zihlman](#))
 - ▶ Poorly reconstructed events
→ Need an efficient filter
- **Kinematic fit algorithm:** Information from detector + reaction hypothesis

Constrained Least Squares Fitting

- Backbone of kinematic fit algorithm

Constrained Least Squares Fitting

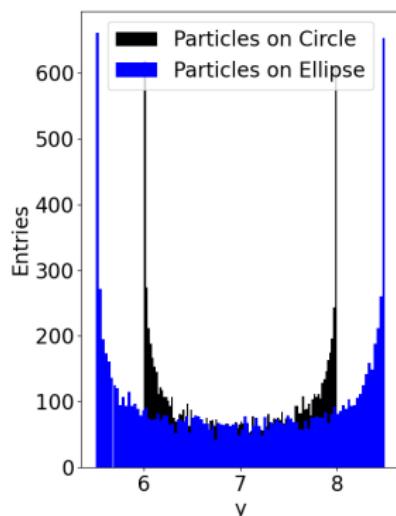
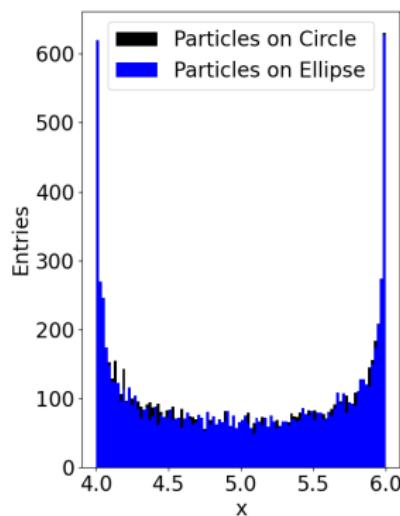
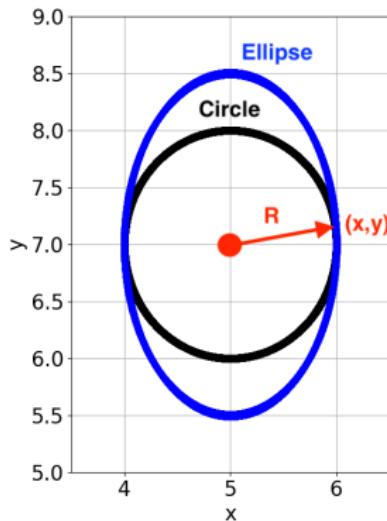
- Backbone of kinematic fit algorithm
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Constrained Least Squares Fitting

- Backbone of kinematic fit algorithm
- Requires knowledge about underlying data
- Introduce method via a simple example: Particles in a 2D world...

...each particle is characterized by: $\vec{v} = \begin{pmatrix} x \\ y \end{pmatrix}$

...particles either live on a circle or on an ellipse

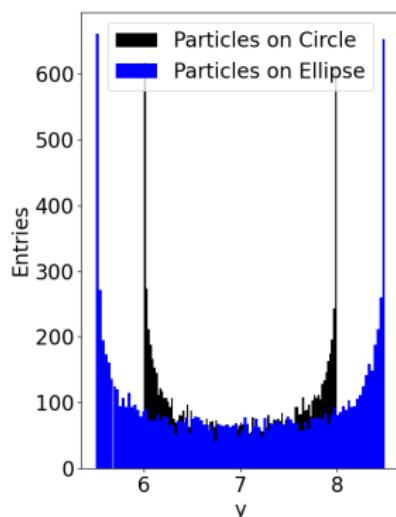
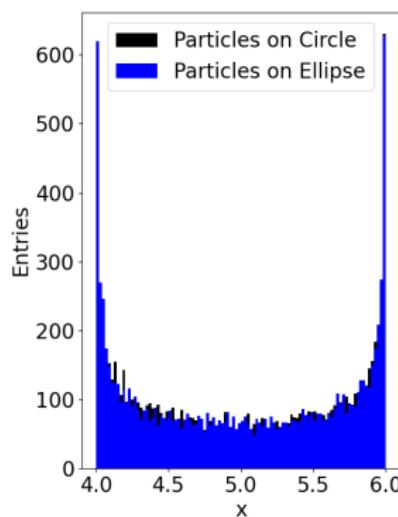
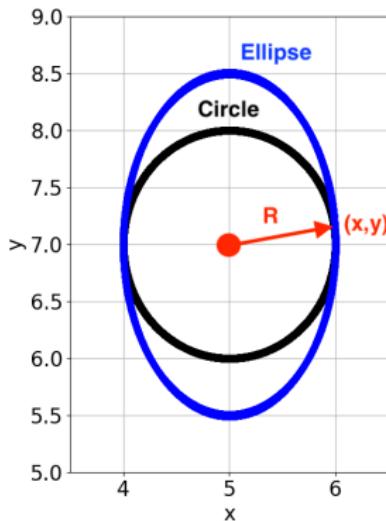


Constrained Least Squares Fitting

- Backbone of kinematic fit algorithm
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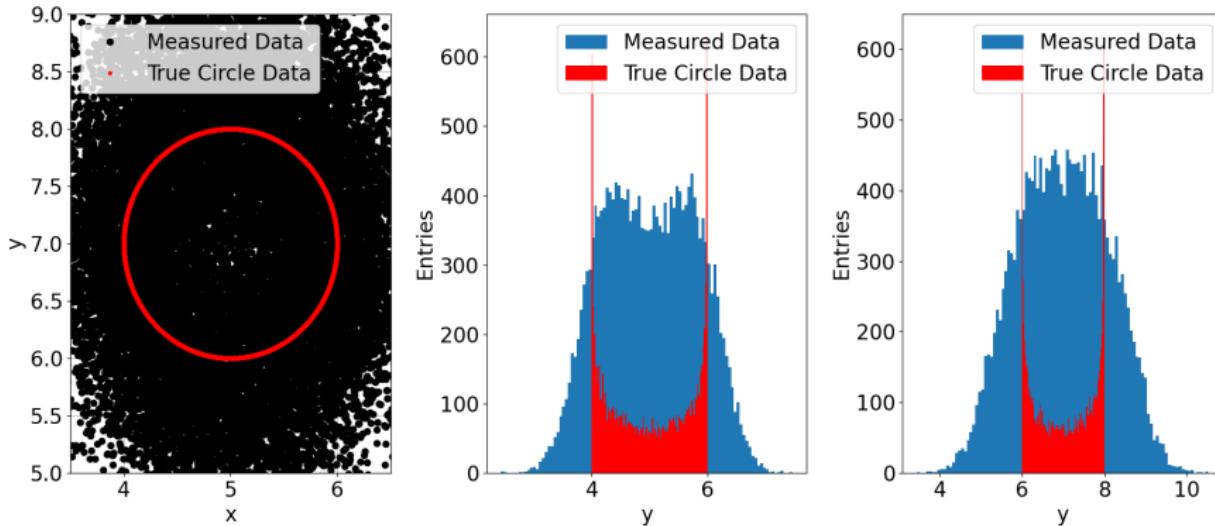
...each particle is characterized by: $\vec{v} = \begin{pmatrix} x \\ y \end{pmatrix}$

...particles either live on a circle or on an ellipse



- Measure particles with imperfect apparatus
→ finite precision: $\sigma_x \approx 0.4$ and $\sigma_y \approx 0.6$

Measuring 2D Particles



- Measurement not very precise
 - ▶ Altered particle distributions
 - ▶ Can not distinguish between circular and elliptic particles
- Approach(es):
 1. Update / buy new apparatus → Requires money and time
 2. Recover true particle information via Least Squares Fit → Requires knowledge about experiment
- Your budget got cut → Go for option 2

Least Squares Fit

- Knowing the uncertainties on x and y , one may formulate:

$$\chi^2 = \left(\frac{x - x_{fit}}{\sigma_x} \right)^2 + \left(\frac{y - y_{fit}}{\sigma_y} \right)^2 \quad (1)$$

Least Squares Fit

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- Just find new coordinates $\vec{v}_{fit} = (x_{fit}, y_{fit})$ which minimize Eq. 1 within the given uncertainties
- This does not help much \rightarrow Just set: $x = x_{fit}$ and $y = y_{fit} \rightarrow$ No improvement
- We need more information:
 - New coordinates need to reflect physics of the measured particles
 - Particles live on a circle: $(x_{fit} - x_c)^2 + (y_{fit} - y_c)^2 \stackrel{!}{=} R^2$

Least Squares Fit with Constraints

- Knowing the uncertainties on x and y AND the underlying physics, one may formulate:

$$\chi^2 = \left(\frac{x - x_{fit}}{\sigma_x} \right)^2 + \left(\frac{y - y_{fit}}{\sigma_y} \right)^2 + 2\lambda \cdot F(x_{fit}, y_{fit}) \quad (2)$$

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with: $F(x_{fit}, y_{fit}) = (x_{fit} - x_c)^2 + (y_{fit} - y_c)^2 - R^2$

$F(x_{fit}, y_{fit}) = 0$, if new particle coordinates are on a circle

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Least Squares Fit with Constraints

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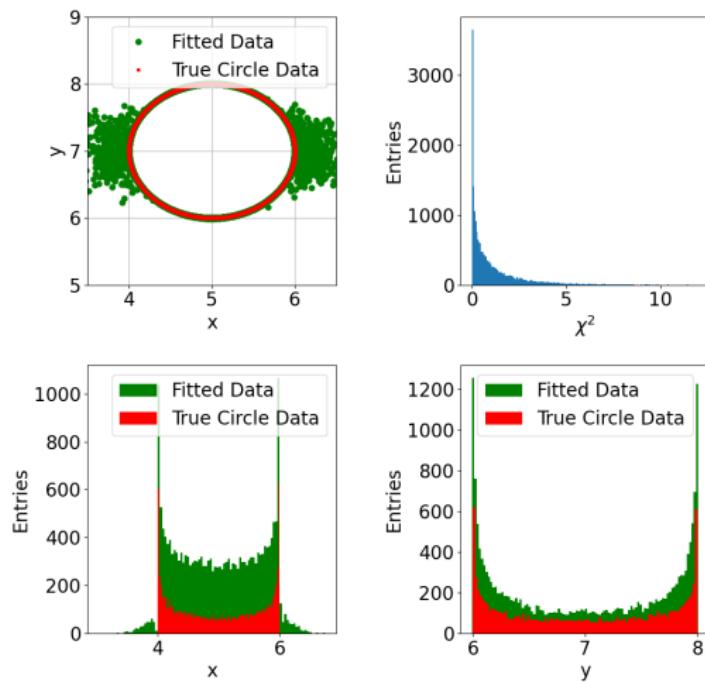
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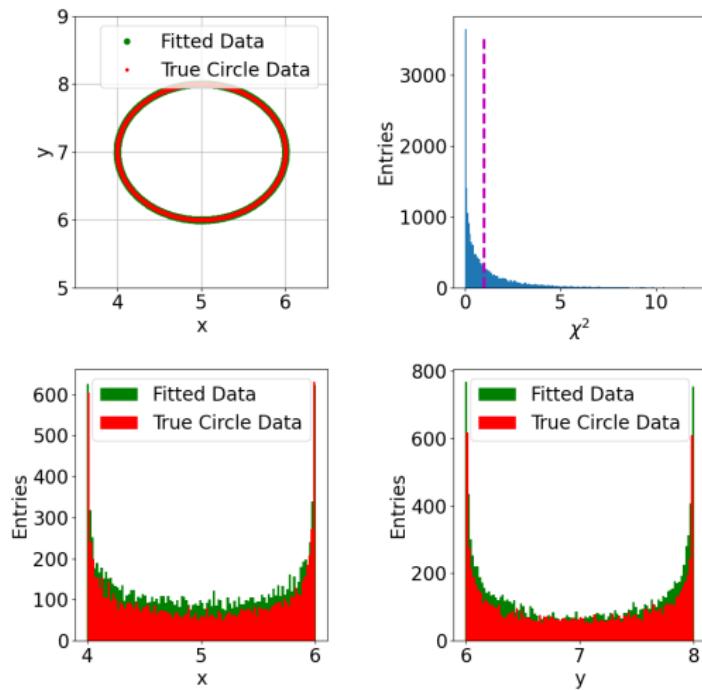
- The details of the (iterative) minimization routine can be found in the backup slides of this talk

Results from Least Squares Fit



- Fitted particle coordinates resemble a circle
- Recovered true information via constrained least squares fit

Results from Least Squares Fit



- Fitted particle coordinates resemble a circle
- Recovered true information via constrained least squares fit
- Further background removal via: $\chi^2 \leq 1.0$

From Circles to entire Physics Reactions

- Constrained least squares fitting is very successful, just needed:
 - Uncertainties related to measurements
 - Constraints, i.e. physics
- Use this technique to find particle coordinates for an entire physics reaction!

Particles	Coordinates	Uncertainties	Constraints
on a circle	(x, y)	σ_x, σ_y	$(x_{fit} - x_c)^2 + (y_{fit} - y_c)^2 - R^2 = 0$
from a reaction	(p, θ, ϕ, m)	$\sigma(p), \sigma(\theta), \sigma(\phi)$	Energy and momentum conservation
	(X, Y, Z)	$\sigma(X), \sigma(Y), \sigma(Z)$	Production vertex
	M	Γ	Mass of decaying particle

Kinematic Fitting at GlueX

- Master equation

$$\chi^2 = \boldsymbol{\epsilon}^T \hat{V}^{-1} \boldsymbol{\epsilon} + 2\boldsymbol{\lambda}^T \boldsymbol{F} \quad (4)$$

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- Residuals: $\boldsymbol{\epsilon} \equiv \begin{pmatrix} p_{fit,1} - p_{meas,1} \\ \theta_{fit,1} - \theta_{meas,1} \\ \vdots \\ \phi_{fit,N} - \phi_{meas,N} \end{pmatrix} \in \mathbb{R}^{3N}$

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- Uncertainties from experiment:

$$\hat{V} = \begin{pmatrix} \sigma^2(p_1) & \rho_{p_1, \theta_1} \sigma(p_1) \sigma(\theta_1) & \cdots & \rho_{p_1, \phi_{3N}} \sigma(p_1) \sigma(\phi_{3N}) \\ \rho_{p_1, \theta_1} \sigma(p_1) \sigma(\theta_1) & \sigma^2(\theta_1) & \cdots & \rho_{\theta_1, \phi_{3N}} \sigma(\theta_1) \sigma(\phi_{3N}) \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{p_1, \phi_{3N}} \sigma(p_1) \sigma(\phi_{3N}) & \rho_{\theta_1, \phi_{3N}} \sigma(\theta_1) \sigma(\phi_{3N}) & \cdots & \sigma^2(\phi_{3N}) \end{pmatrix}$$

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- Reaction specific constraints: \boldsymbol{F}

Kinematic Fitting at GlueX

- Master equation

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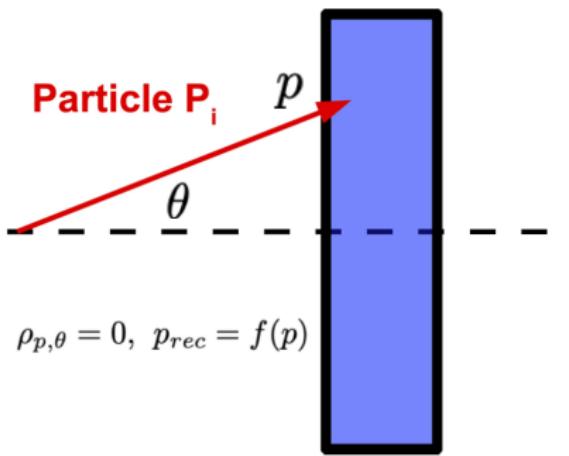
- Reaction specific constraints: F

- Find ϵ by solving:

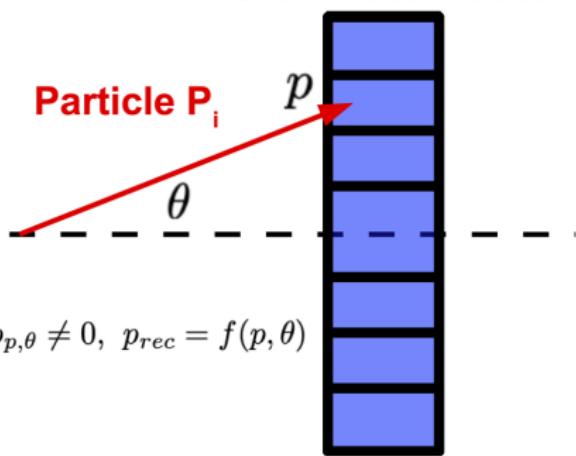
$$\frac{d\chi^2}{d\epsilon} = 0 \quad (5)$$

Know your Detector - Know your Covariance Matrix \hat{V}

sub-detector for momentum reconstruction



segmented sub-detector for momentum reconstruction



- **Left:** Detector response is uniform with respect to $\theta \rightarrow$ Reconstructed momentum p_{rec} does not depend on θ
- **Right:** Detector response is not uniform with respect to $\theta \rightarrow$ Reconstructed momentum p_{rec} depends on θ

Covariance Matrix, Constraints and NDF

- Covariance matrix \hat{V} is determined for every event and every particle

Particle Charge	Track Info	Dimension of \hat{V}
$q \neq 0$	$P_x, P_y, P_z, X_x, X_y, X_z, t$	7×7
$q = 0$	X_x, X_y, X_z, E, t	5×5

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- Basic constraints:

Fit Type	Constraint	Observable used
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- Number of degrees of freedom: $NDF = N_m - 3N + N_c$

N : Number of all particles in the reaction

N_m : Number of measured observables

N_c : Number of constraints

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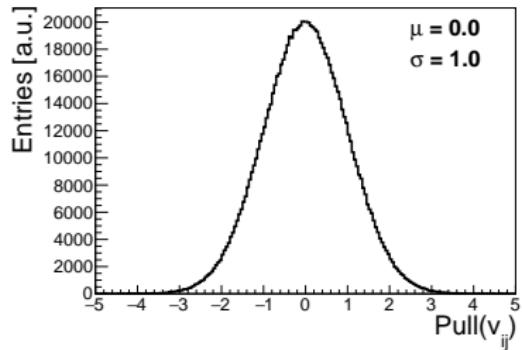
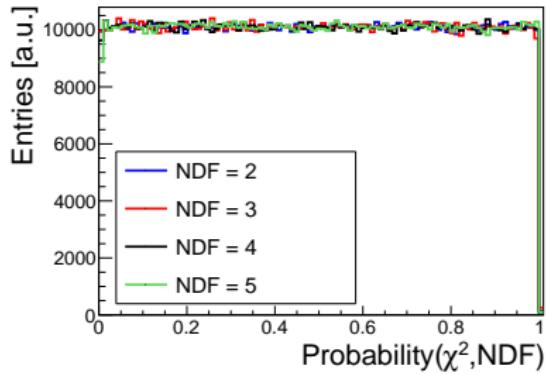
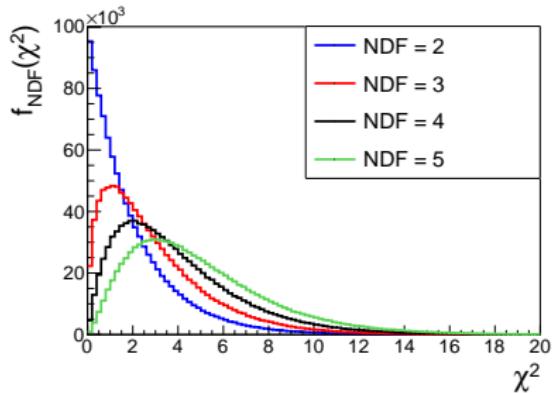
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- **Example:** All particles measured + P4-Fit

$$\Rightarrow NDF = N_m - 3N + N_c = 3N - 3N + 4 = 4$$

Performance Monitoring Tools



- Expected distributions for gaussian errors and independent measurements

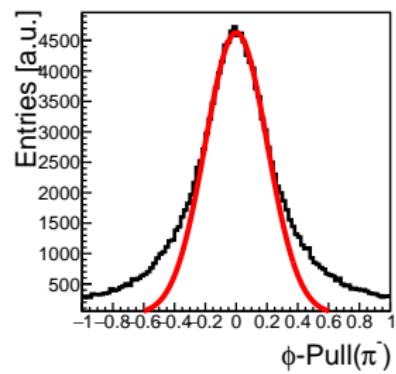
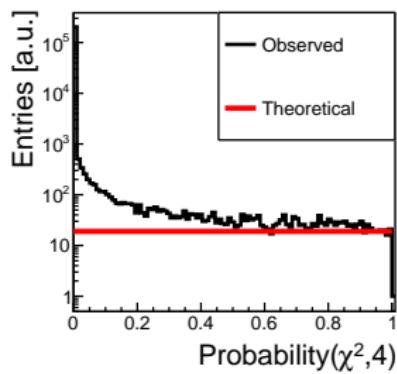
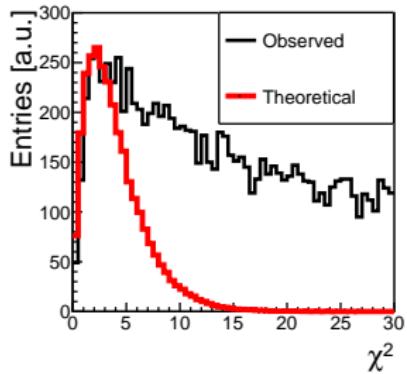
$$f_{NDF}(x) = \frac{x^{(NDF/2-1)}}{2^{(NDF/2)}\Gamma(NDF/2)} \cdot e^{-x/2}$$

$$\int_0^{\infty} t^{NDF/2-1} e^{-t/2} dt$$

$$\text{Probability}(\chi^2, NDF) = \frac{\chi^2}{\sqrt{2^{NDF}\Gamma(NDF/2)}}$$

$$\text{Pull}(v_{ij}) = \frac{v_{meas,ij} - v_{fit,ij}}{\sqrt{|\sigma_{meas,ij}^2 - \sigma_{fit,ij}^2|}}$$

Impact of Error Estimation in \hat{V}



mean[$\phi\text{-Pull}(\pi^-)$]

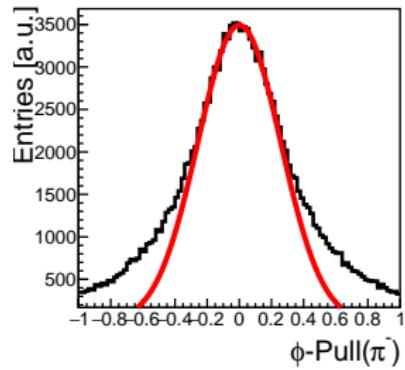
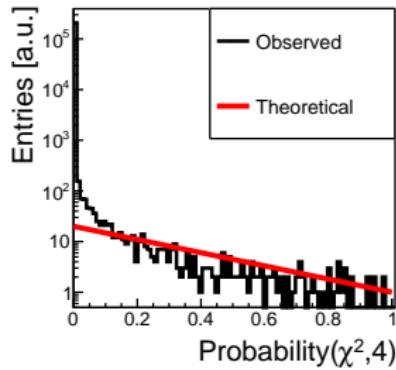
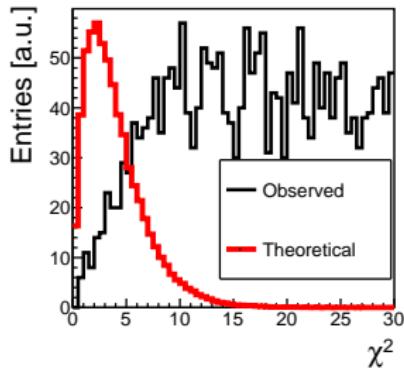
≈ 0

$\sigma[\phi\text{-Pull}(\pi^-)]$

0.201

- Look at toy data set:
 $pp \rightarrow pp\eta[\eta \rightarrow \pi^+\pi^-\pi^0[\pi^0 \rightarrow \gamma\gamma]]$
- Measured particle observables are correlated
- Realistic error estimation for \hat{V}

Impact of Error Estimation in \hat{V}



mean[$\phi\text{-Pull}(\pi^-)$]

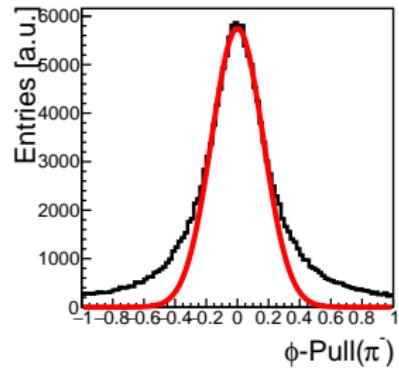
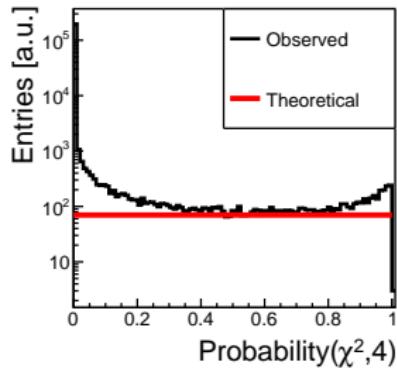
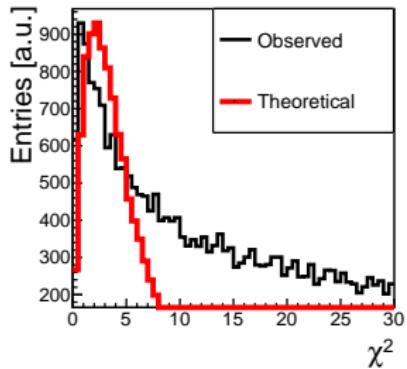
≈ 0

$\sigma[\phi\text{-Pull}(\pi^-)]$

0.257

- Look at toy data set:
 $pp \rightarrow pp\eta[\eta \rightarrow \pi^+\pi^-\pi^0[\pi^0 \rightarrow \gamma\gamma]]$
- Measured particle observables are correlated
- Underestimated errors by $\sim 25\%$

Impact of Error Estimation in \hat{V}



mean[$\phi\text{-Pull}(\pi^-)$]

≈ 0

$\sigma[\phi\text{-Pull}(\pi^-)]$

0.165

- Look at toy data set:
 $pp \rightarrow pp\eta[\eta \rightarrow \pi^+\pi^-\pi^0[\pi^0 \rightarrow \gamma\gamma]]$
- Measured particle observables are correlated
- Overestimated errors by $\sim 25\%$

In a Nutshell

$$\chi^2 = \boxed{\epsilon^T \hat{V}^{-1} \epsilon} + \boxed{2\lambda^T F} = 0$$

Inputs from experiment Inputs from physics

Kinematic Fit provides:

- 1. Fitted particle 4-momenta
 - 2. Possibility to reject events which do not match the reaction hypothesis

In a Nutshell

$$\chi^2 = \boxed{\epsilon^T \hat{V}^{-1} \epsilon} + \boxed{2\lambda^T F} = 0$$

Inputs from experiment Inputs from physics

Kinematic Fit provides:

1. Fitted particle 4-momenta
 2. Possibility to reject events which do not match the reaction hypothesis

- Kinematic fitter is a least squares fit with constraints

In a Nutshell

$$\chi^2 = \boxed{\epsilon^T \hat{V}^{-1} \epsilon} + \boxed{2\lambda^T F} = 0$$

Inputs from experiment Inputs from physics

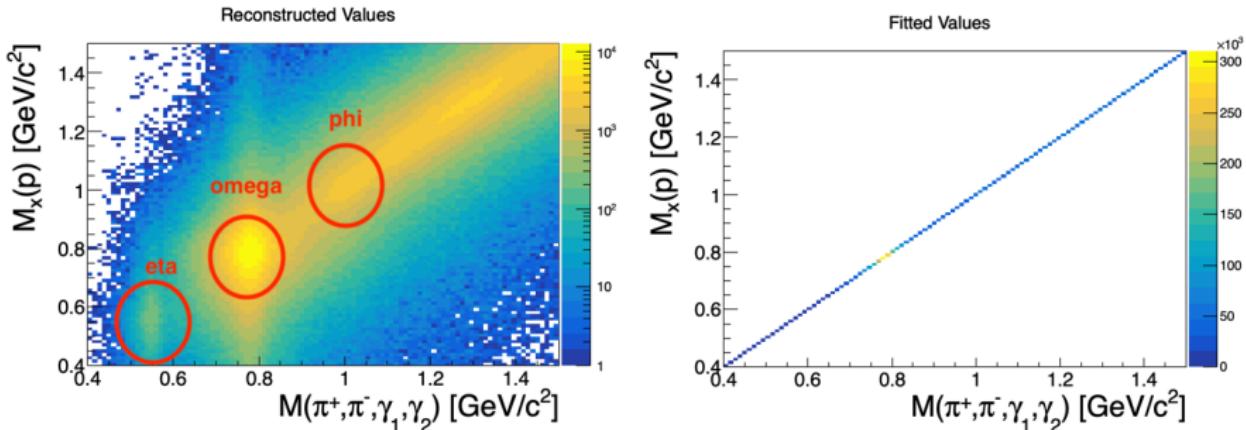
Kinematic Fit provides:

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- Kinematic fitter is a least squares fit with constraints
 - Use probability / χ^2 and pull-distributions to monitor and diagnose algorithm

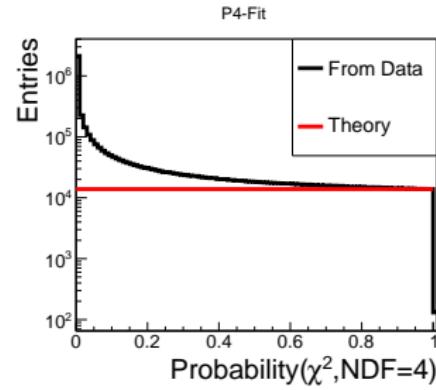
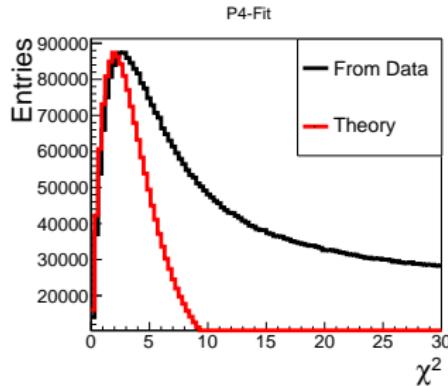
$\text{mean}[\text{Pull}(v_{ij})]$	$\sigma[\text{Pull}(v_{ij})]$	Scenario
0	1.0	everything is fine
0	< 1.0	$\sigma_{\text{meas},ij}$ is overestimated
0	> 1.0	$\sigma_{\text{meas},ij}$ is underestimated
$\neq 0$	$\in [0, 1]$	introduced bias
n.a.	n.a.	non-gaussian pulls \rightarrow you are in trouble

Analysis of $\gamma p \rightarrow \pi^+ \pi^- \pi^0 [\pi^0 \rightarrow \gamma\gamma]$ with the GlueX Kinematic Fit - Mass Spectra

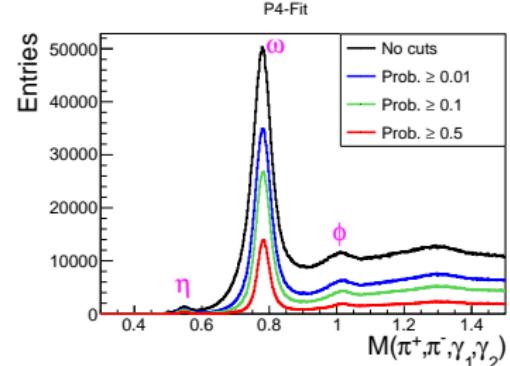


- Analyzed sub-sample of 2017 data
- Used a P4-Fit with no additional constraints
- Missing Mass:** $M_x(p) \equiv \|P_{beam} + P_{target} - P_p\|$
- Invariant Mass:** $M(\pi^+, \pi^-, \gamma_1, \gamma_2) \equiv \|P_{\pi^+} + P_{\pi^-} + P_{\gamma_1} + P_{\gamma_2}\|$
- Ideally:** $M_x(p) = M(\pi^+, \pi^-, \gamma_1, \gamma_2)$ (\rightarrow Fulfilled via kinematic fit)

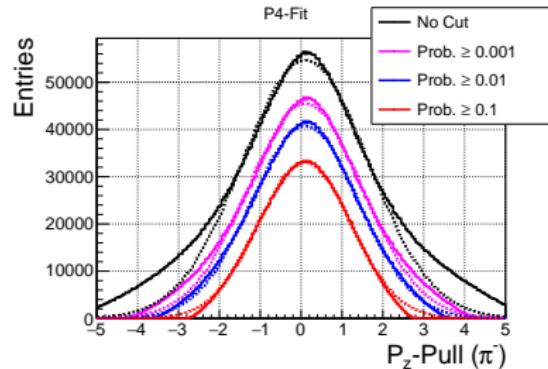
Analysis of $\gamma p \rightarrow \pi^+ \pi^- \pi^0 [\pi^0 \rightarrow \gamma\gamma]$ with the GlueX Kinematic Fit - Performance Plots



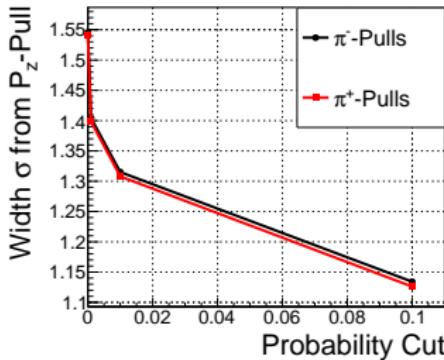
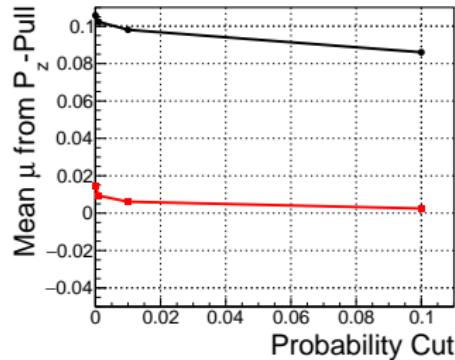
- Enhancement at small probabilities / large χ^2
 - ▶ Purely reconstructed events
 - ▶ Tracks with resolution outside of \hat{V}
 - ▶ Background reactions that do not match reaction hypothesis
- Check impact of different probability cuts



Analysis of $\gamma p \rightarrow \pi^+ \pi^- \pi^0 [\pi^0 \rightarrow \gamma\gamma]$ with the GlueX Kinematic Fit - Pull Distributions

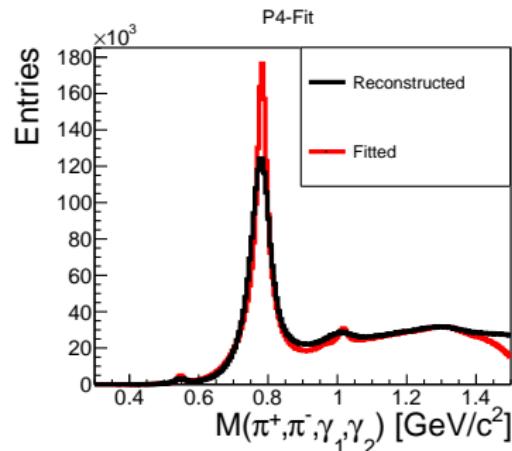
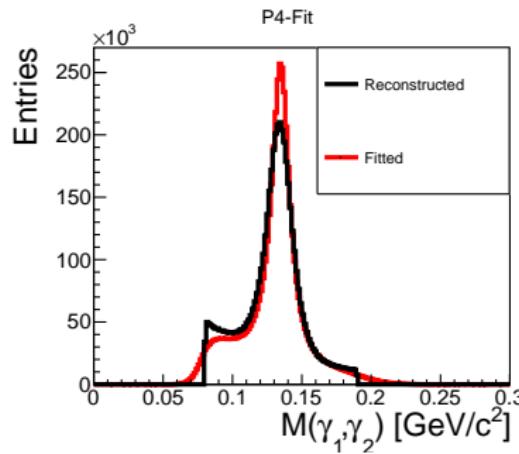


- Look at P_z -pulls for π^- and π^+
- Determine μ and σ for different probability cuts
- Fitter seems to "treat" oppositely charged pions differently
- Observe: $\lim_{\text{prob} \rightarrow 1} \mu = 0$ and: $\lim_{\text{prob} \rightarrow 1} \sigma = 1$ (\rightarrow As it should)



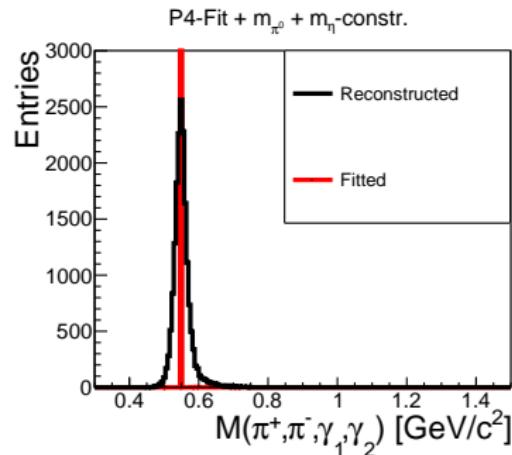
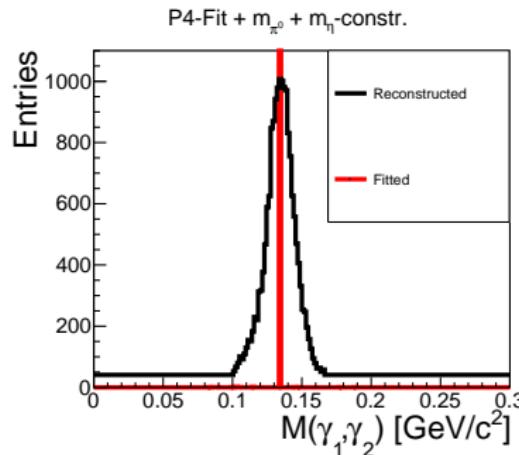
Analysis of $\gamma p \rightarrow \pi^+ \pi^- \pi^0 [\pi^0 \rightarrow \gamma\gamma]$ with the GlueX Kinematic Fit - Mass Constraints

- Kinematic fit helps to improve the resolution
(→ Combine information from all sub-detectors)
- Wish to reconstruct: $\eta \rightarrow \pi^+ \pi^- \pi^0$
- Why not using mass constraints ?
 $M(\gamma_1, \gamma_2) = m(\pi^0)$ and $M(\pi^+, \pi^-, \gamma_1, \gamma_2) = m(\eta)$

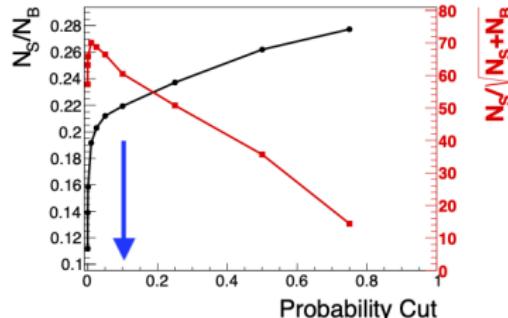
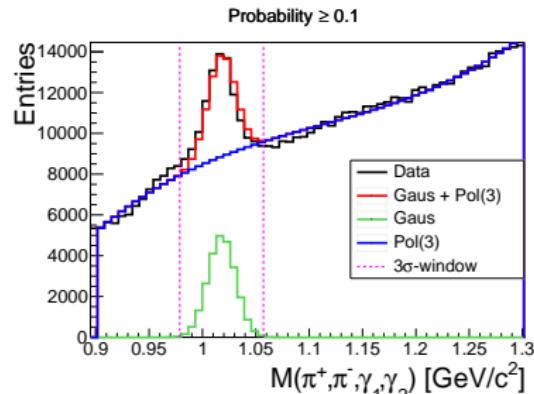
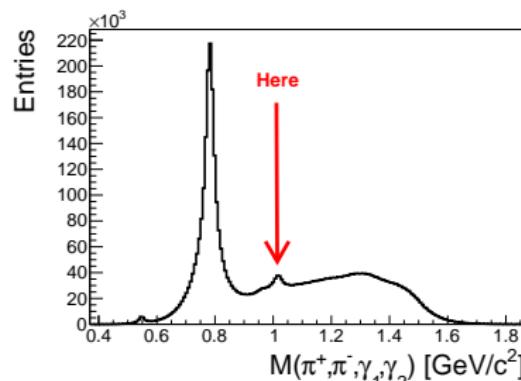


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- Additional mass constraints are helpful, but make it impossible to judge the involved background

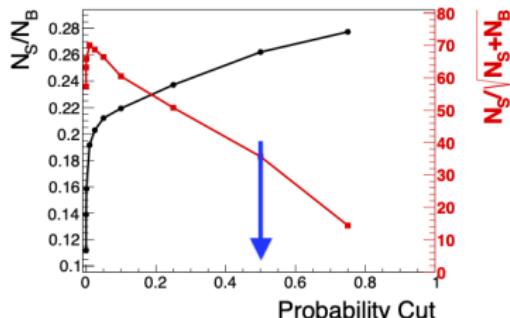
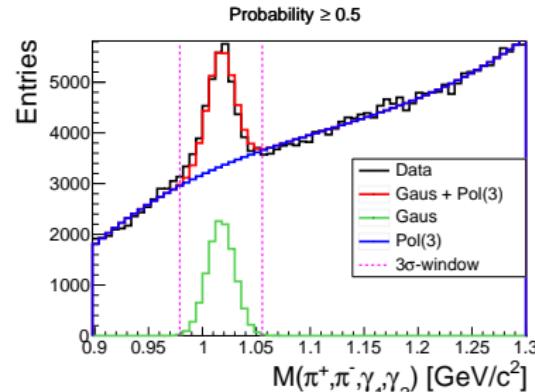
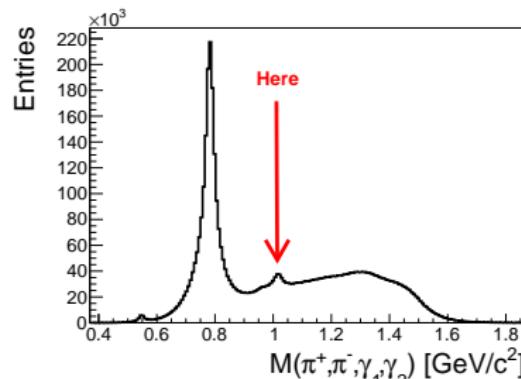


Analysis of $\gamma p \rightarrow \pi^+ \pi^- \pi^0 [\pi^0 \rightarrow \gamma\gamma]$ with the GlueX Kinematic Fit - Cut Optimization



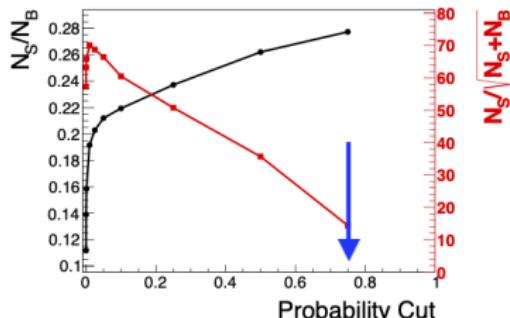
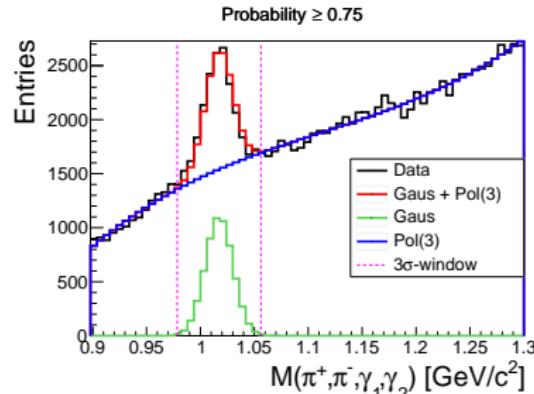
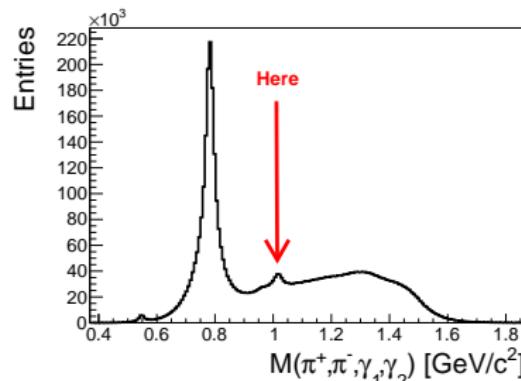
- Goal: Find optimum probability cut to select $\phi \rightarrow \pi^+ \pi^- \pi^0$
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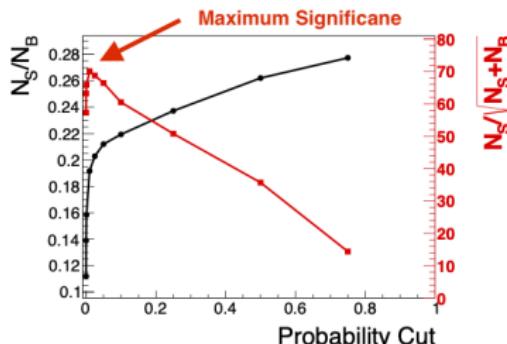
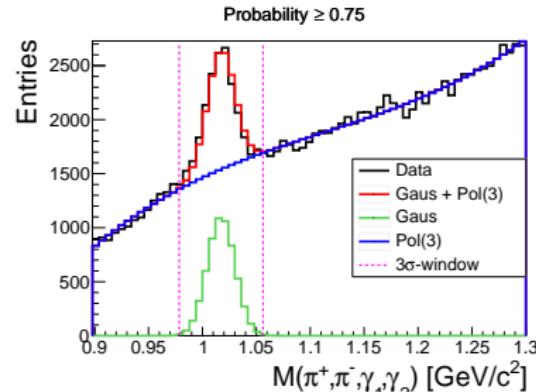
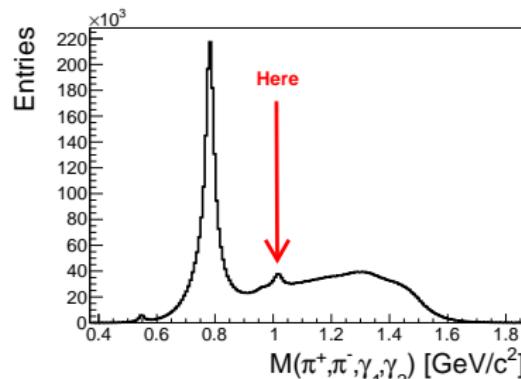
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- Vary probability cut and determine number of ϕ / background events
- Significance at maximum for 0.01
⇒ Sweet spot between background rejection and signal loss

Summary and Outlook

- The kinematic fit is a very powerful tool \Rightarrow **But no silver bullet!**
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 - ▶ Try not to solve everything with mass constraints
- Use the kinematic fit in your own analysis (**See following talk by Justin Stevens**)
 - ▶ Define fitter settings in config file for reaction filter plugin
 - ▶ Use analysis-launch webpage