Amplitude Analysis Outline

- Idea #1: information about the production of resonances and resonance properties is reflected in the distributions of final state particles, which we detect
  - peaks in invariant mass suggest resonances
  - angular distributions provide information about spin
- Idea #2: the method of maximum likelihood estimation provides a mechanism to estimate parameters of a probability density function
  - without having to bin phase space or count events in a bin
  - independent of the number of dimensions spanned by the p.d.f.
- Amplitude analysis connects these two ideas: the parameters of the p.d.f. we fit to the observed events are often connected to fundamental parameters of nature whose values and uncertainties we want to estimate from the observed data

Amplitude Fundamentals

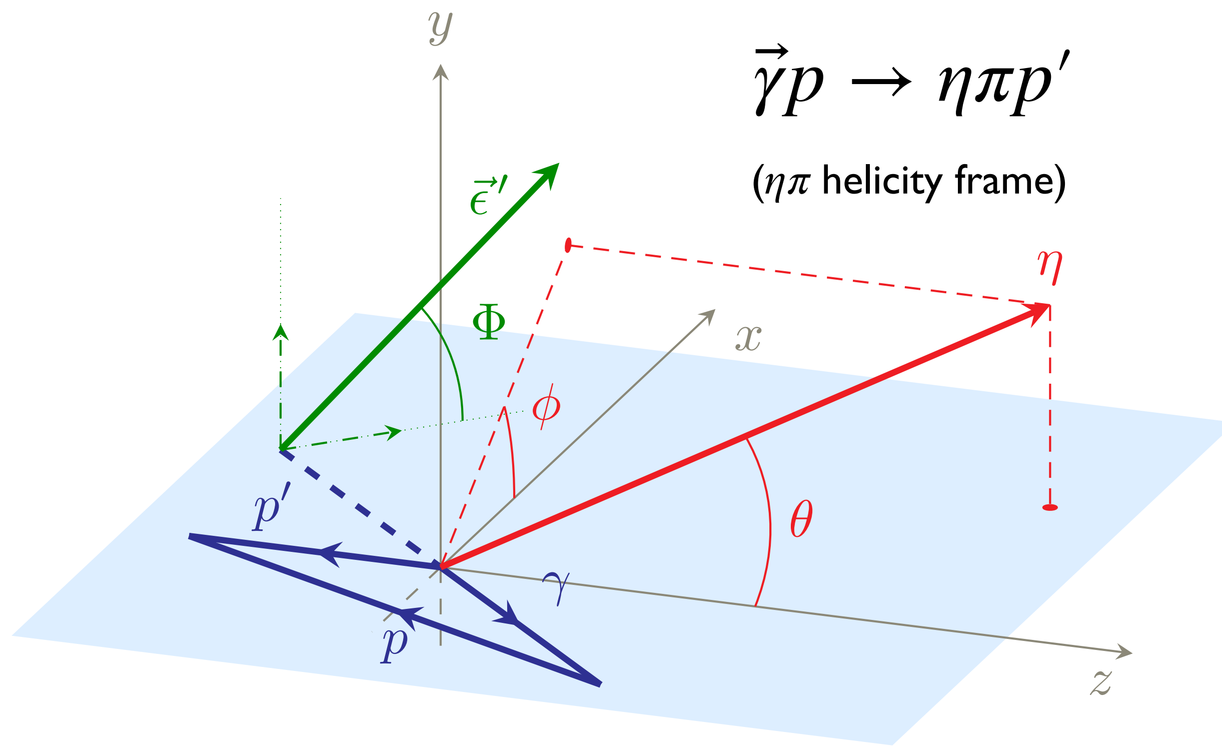
- Quantum mechanics: amplitudes are complex-valued functions that describe the various indistinguishable paths one can take to go between some distinguishable incoming and outgoing particles
  - amplitudes coherently add to form an intensity
  - one may need multiple coherent sums to completely describe a system, especially one where incoming and outgoing particles have spin
- Writing equations for the amplitudes generally involves making two types of assumptions:
  - kinematic: assumes conservation of parity, angular momentum, etc. (pretty safe assumptions) -- e.g., using $Y_\ell^m(\theta, \phi)$
  - dynamic: assumes some model for the underlying physics (validity depends on model and application) -- e.g., using a BreitWigner to describe the lineshape and phase of the $a_2(1320)$
- The existence of an exotic and its mass and width is a question about dynamics -- where to build that dynamics into the analysis is a matter of strategy
  - collaboration with theory is essential

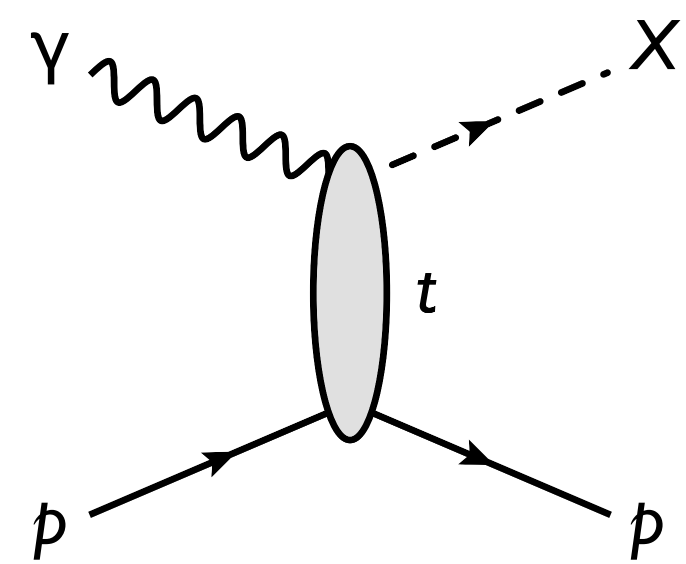
$\eta\pi$ Polarized Photoproduction Kinematics

- Write amplitudes with definite "reflectivity": $\epsilon = \pm 1$, which have a non-trivial distribution in Φ
- Defining naturality: $P(-1)^J$
  - natural parity: $P(-1)^J = +1$; $J^P = 0^+, 1^-, 2^+, \ldots$
  - unnatural parity: $P(-1)^J = -1$; $J^P = 0^-, 1^+, 2^-, \ldots$
- High energy *t*-channel picture: the reflectivity fixes the product of the naturalities of the exchange particle and the produced resonance
- Make use of this additional dimension (Φ) in the amplitude analysis to gain insight into production mechanisms

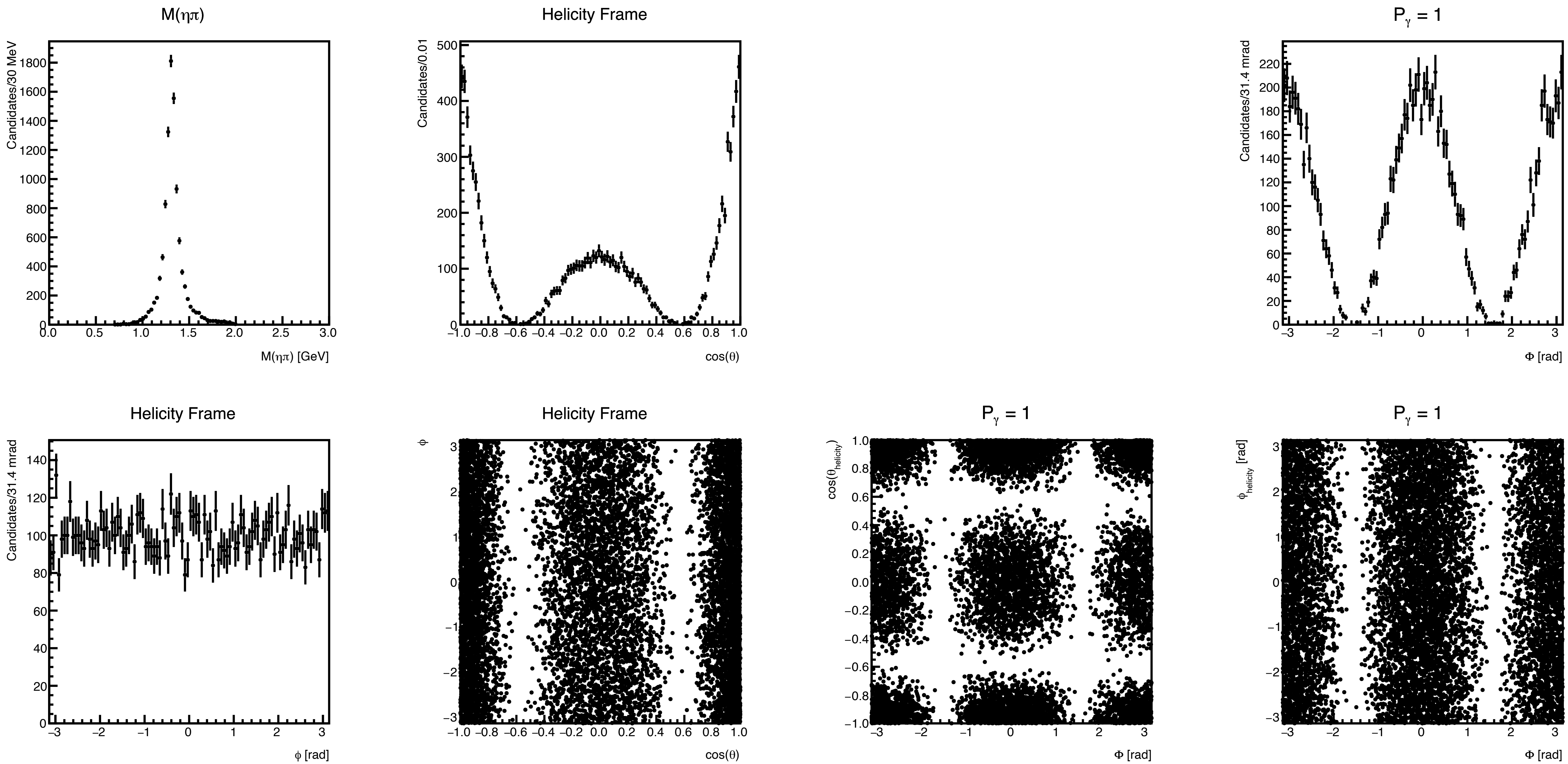
$\vec{\gamma} p \to \eta\pi p'$

($\eta\pi$ helicity frame)





$a_2(1320) \to \eta\pi$ in the $L_m^\epsilon = D_0^+$ amplitude



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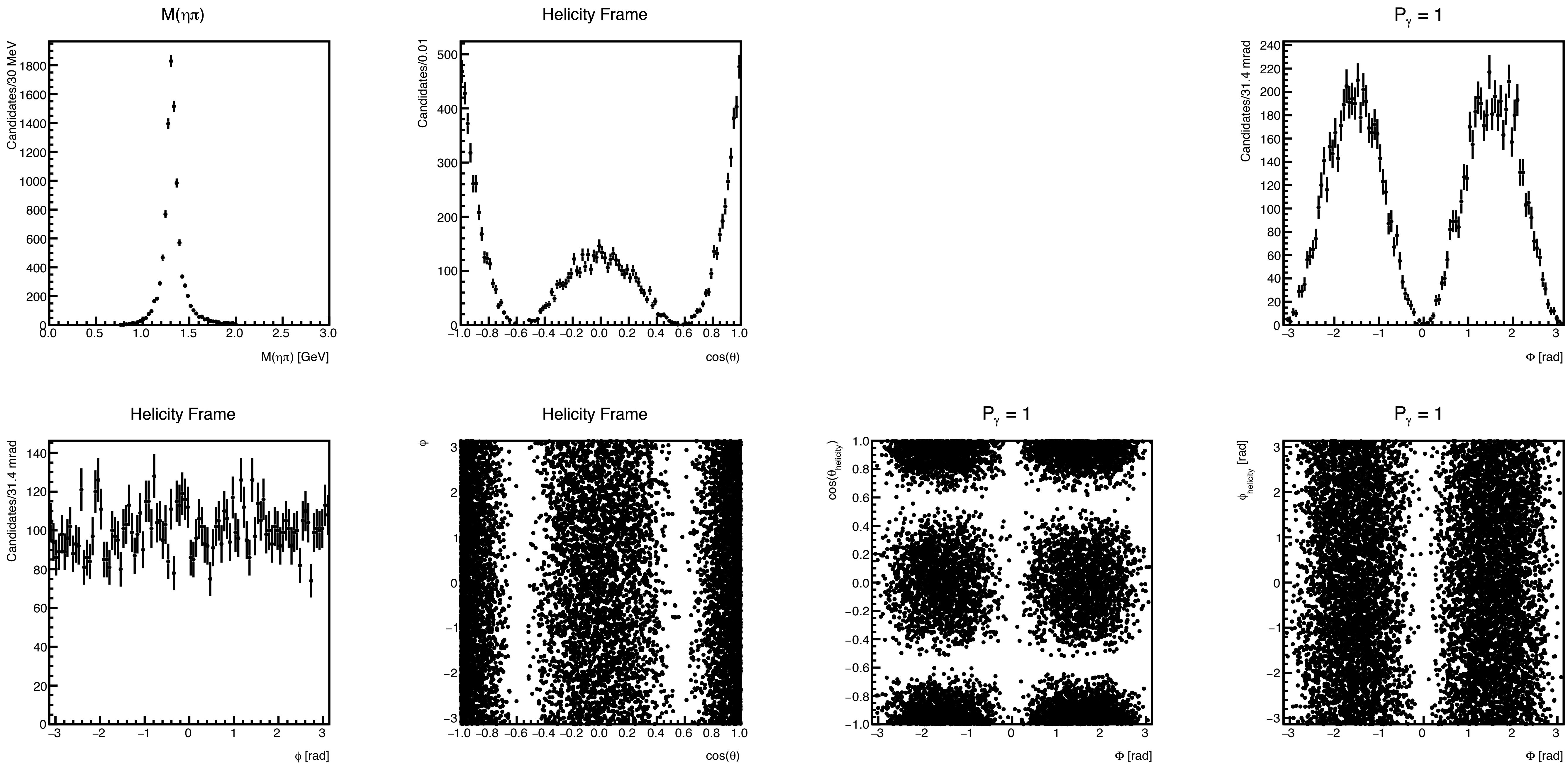
GlueX Software Tutorial
May 23, 2022

$a_2(1320) \to \eta\pi$ in the $L^{\epsilon}_{m} = D^{-}_{0}$ amplitude



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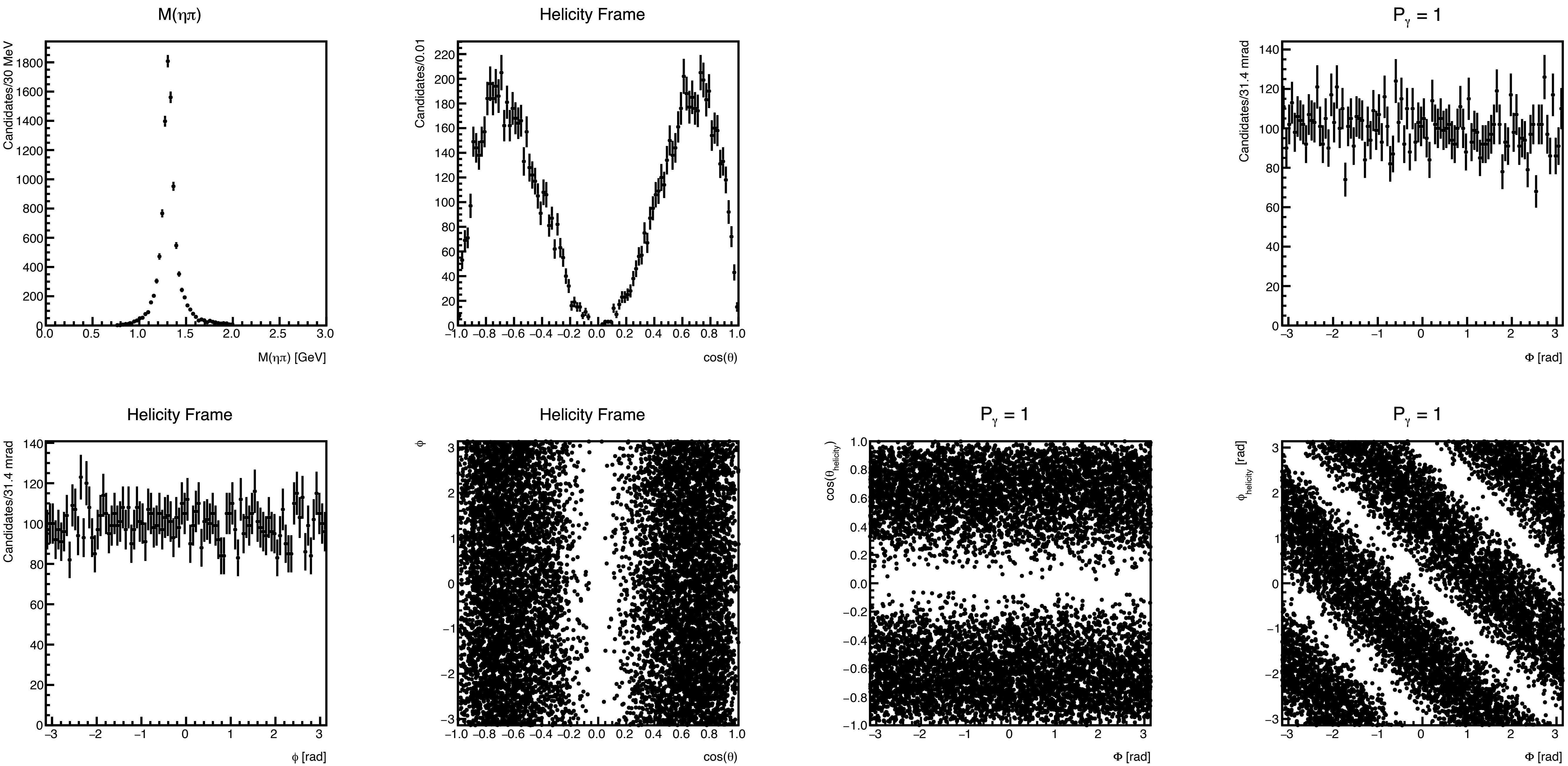
$a_2(1320) \to \eta\pi$ in the $L_m^{\epsilon} = D_1^{-}$ amplitude



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Maximum Likelihood

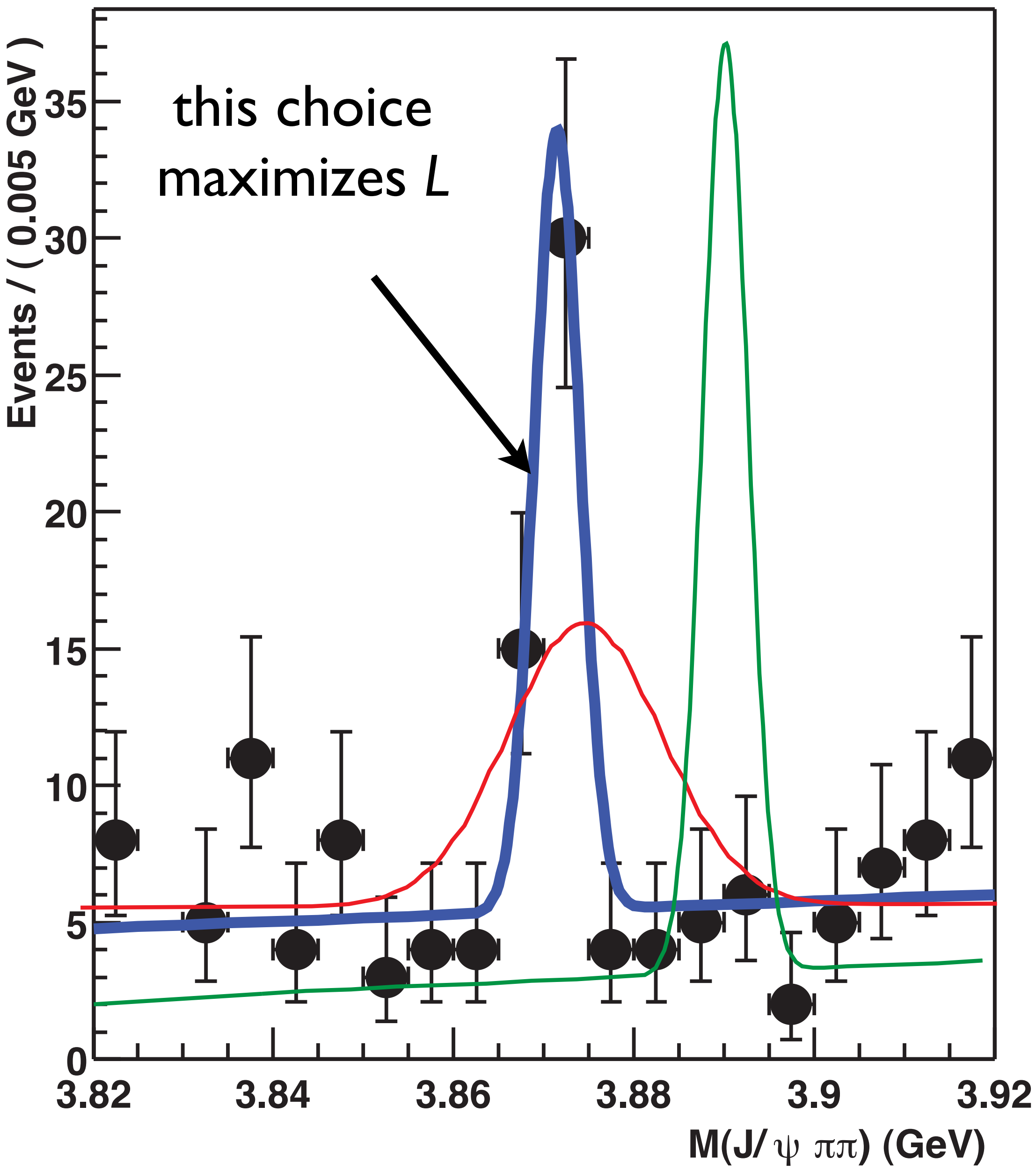
- Amplitude analysis is built around the (extended) maximum likelihood method
- Start with a model that contains free parameters (θ) and predicts the probability of having an event with a particular set of kinematic variables x (angles, invariant mass, etc.)

$$\mathcal{P}(\vec{x};\vec{\theta})$$

- Vary the free parameters to maximize the probability for the entire data set

$$\mathcal{L} = \prod_{i=1}^{N_{\text{events}}} \mathcal{P}(\vec{x}_i;\vec{\theta})$$

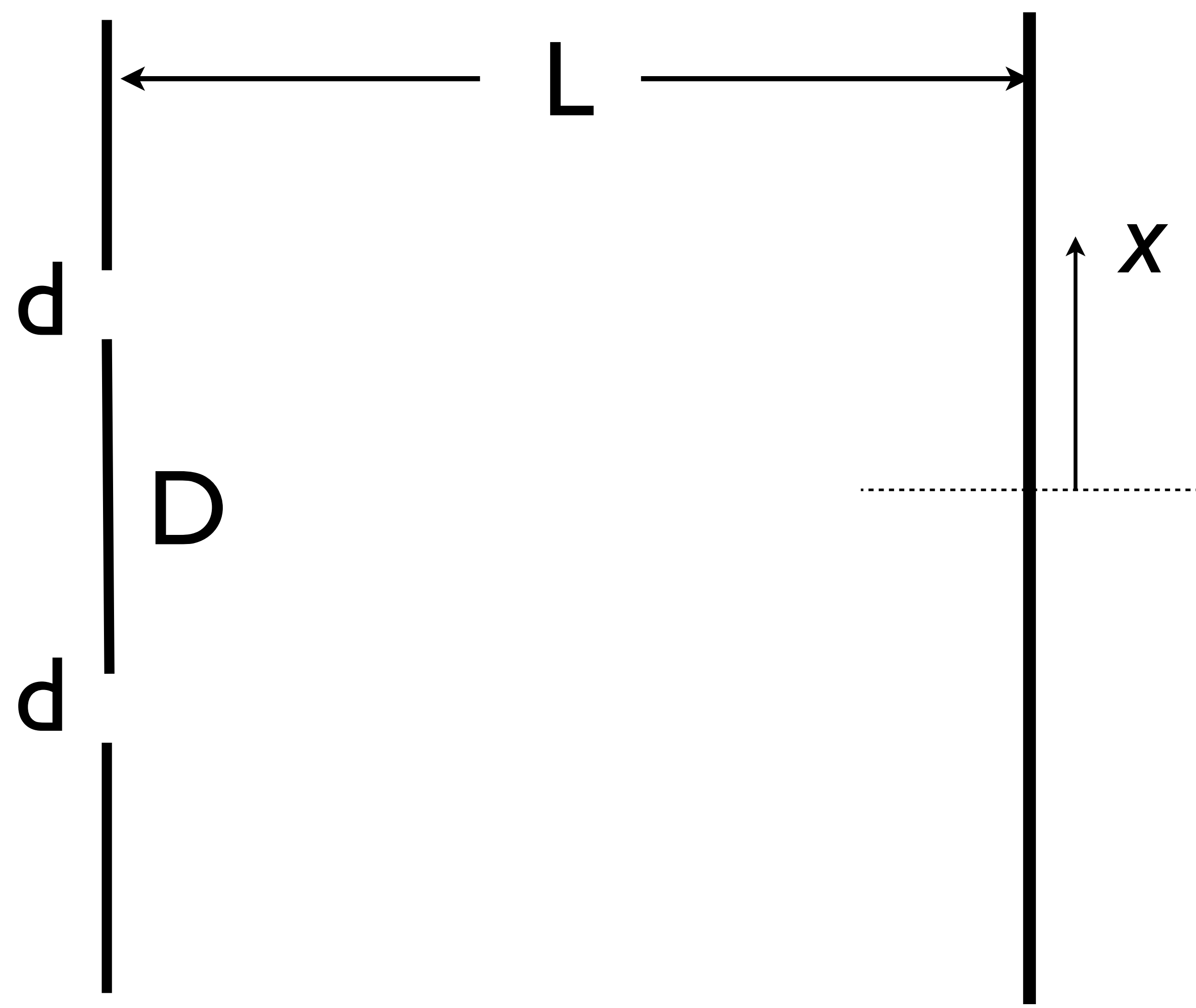
Example: 1D in x

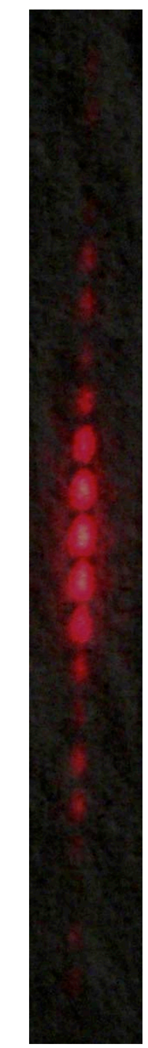


Experiment Application

Step 2: For each particle record location *x* where it was detected

Step 1: Shoot particles at slits





Probe
Beam of Particles
wavelength λ

Goal: determine the values of *d* and *D*

Physical System Under Study
Two Slits: width d, separation D

Detector
Measures location x_i
for each arriving particle

The Fit Procedure

- Our "theoretical model" that parametrizes the intensity of the particles in the detector is given by

$$I(x) = I_0 \left(\frac{\sin(d\pi x/\lambda L)}{d\pi x/\lambda L} \right)^2 \cos^2(2D\pi x/\lambda L)$$

- Start with a guess for values for *d* and *D*
- Convert *I*(*x*) into a properly normalized PDF -- multiple techniques are available for evaluating the integral

$$\mathcal{P}(x) = \frac{I(x)}{\int_{x_{\min}}^{x_{\max}} I(x)dx}$$

- Compute the likelihood by taking the product over all detected events

$$\mathcal{L} = \prod_{i=1}^{N} \mathcal{P}(x_i)$$

- Iterate with a new choice of *d* and *D* until the likelihood is maximized