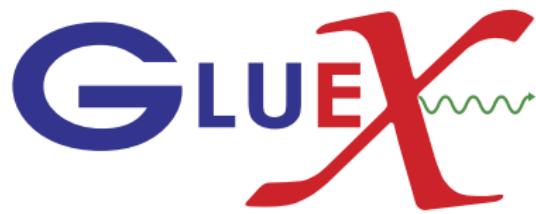


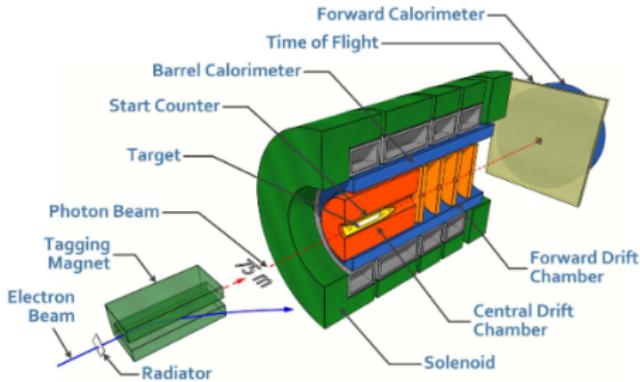
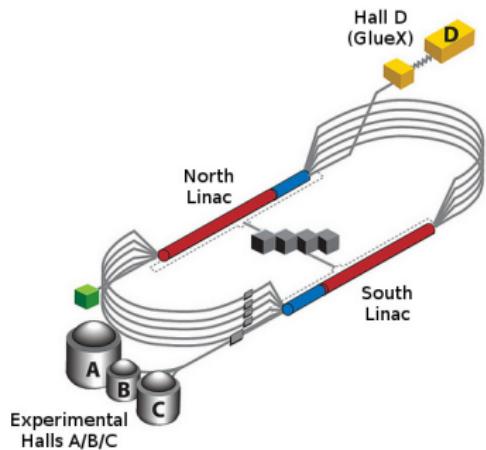
The GlueX Kinematic Fitter in ~ 20 min

Daniel Lersch

May 23, 2022



The Gluonic Excitation Experiment - GlueX

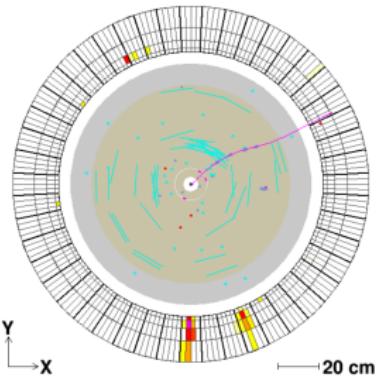
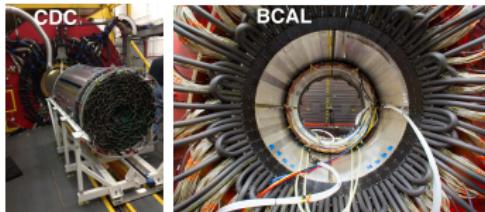
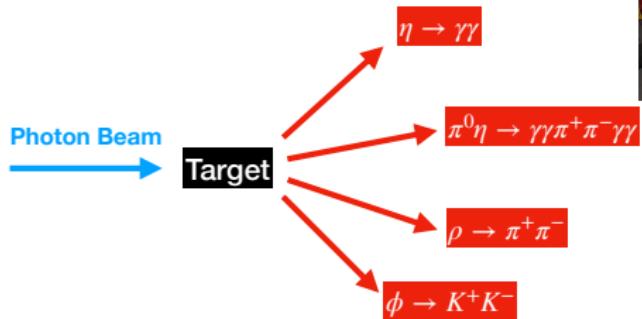


Experimental Hall D:

- Over 130 scientists from:
 - 30 Institutions
 - 10 Countries
- Experiments with polarized photon beam

Phase	Run Period	Raw Data [PB]
GlueX-I	Spring 2017	0.9
	Spring 2018	1.9
	Fall 2018	1.1
GlueX-II	Spring 2020	2.8
	Summer 2020	1.7

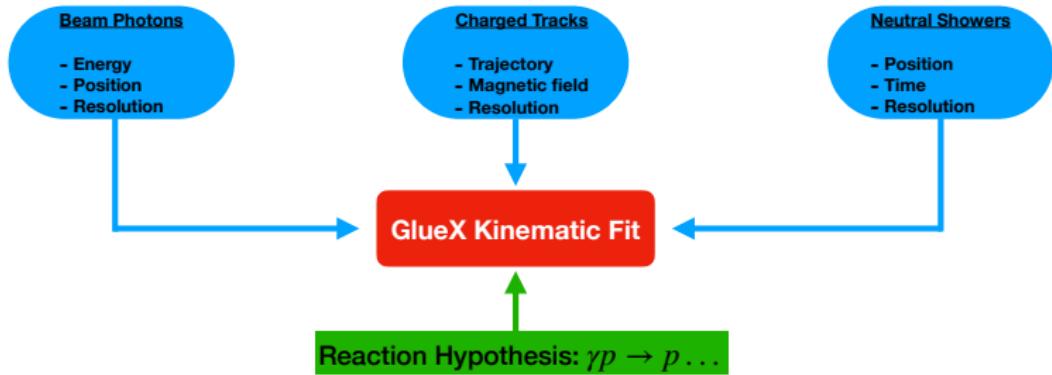
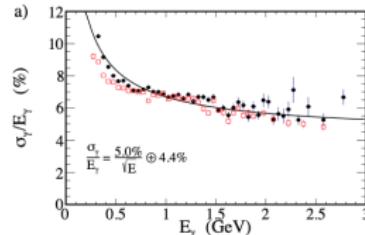
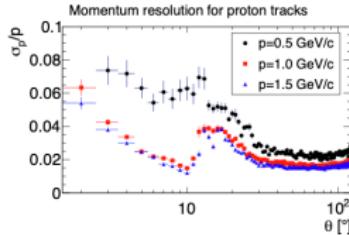
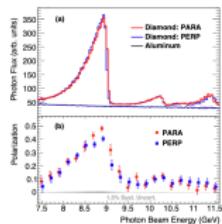
Event Reconstruction



Charge q	Reconstructed via	Observables
$>0, <0$	FDC/CDC + mag. field	Momenta, track parameters, $dE/dx, \dots$
0	BCAL/FCAL/CompCAL	Clusters, shower info, E, \dots

- **Goal:** Reconstruct entire reaction from final state particles
- Many final state particles
 - ▶ Overlapping topologies, combinatorics, ... ([See talk by Beni Zihlman](#))
 - ▶ Poorly reconstructed events
→ Need an efficient filter
- **Kinematic fit algorithm:** Information from detector + reaction hypothesis

Event Reconstruction



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Constrained Least Squares Fitting

- Backbone of kinematic fit algorithm

Constrained Least Squares Fitting

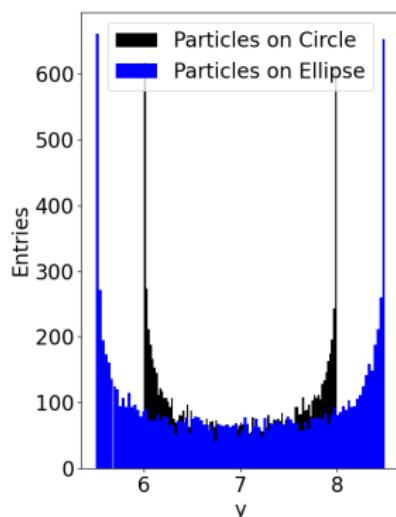
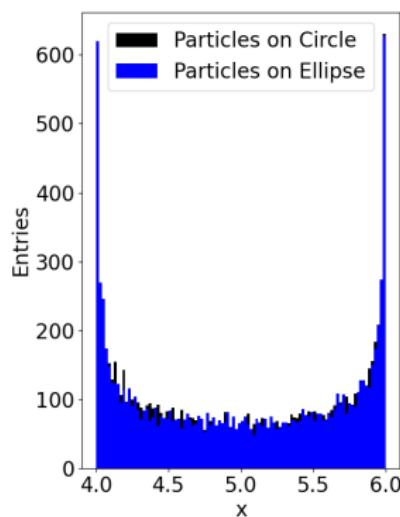
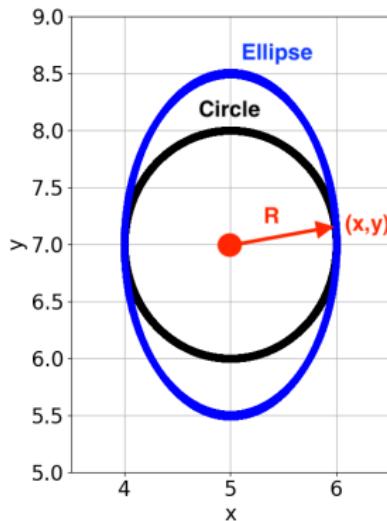
- Backbone of kinematic fit algorithm
- Requires knowledge about underlying data

Constrained Least Squares Fitting

- Backbone of kinematic fit algorithm
- Requires knowledge about underlying data
- Introduce method via a simple example: Particles in a 2D world...

...each particle is characterized by: $\vec{v} = \begin{pmatrix} x \\ y \end{pmatrix}$

...particles either live on a circle or on an ellipse

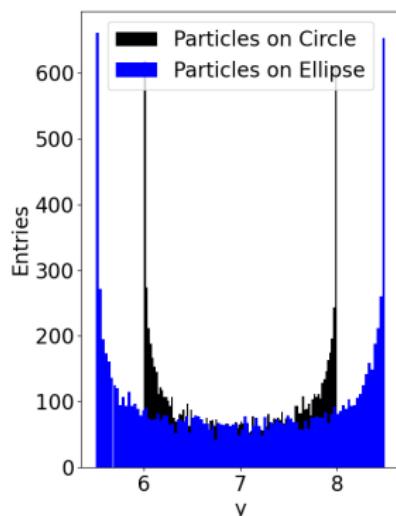
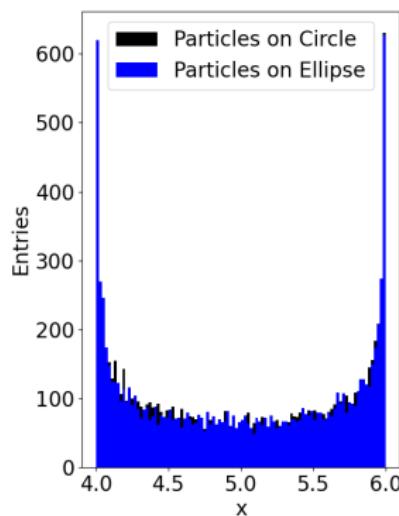
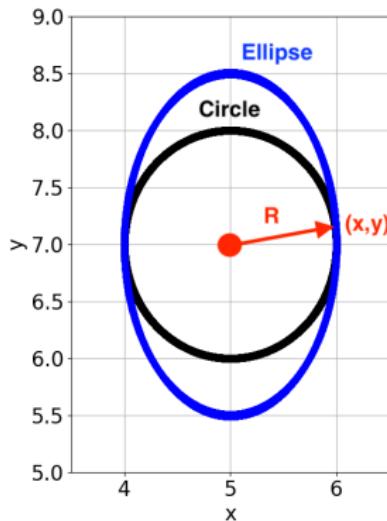


Constrained Least Squares Fitting

- Backbone of kinematic fit algorithm
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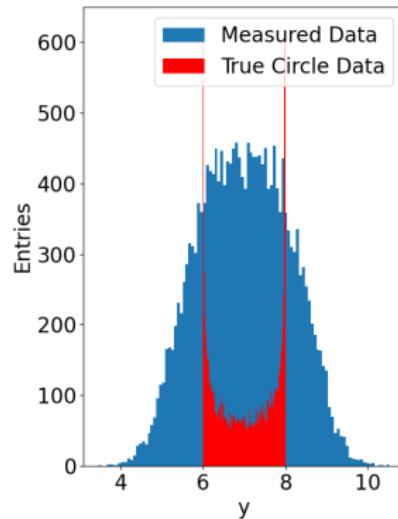
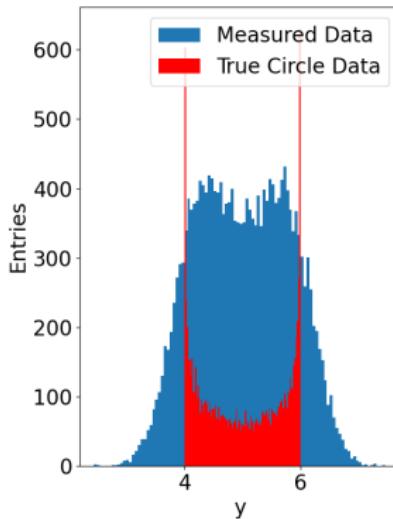
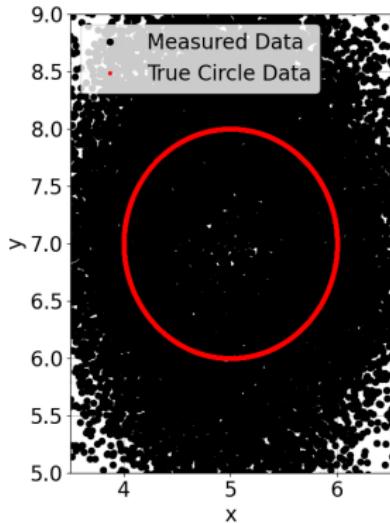
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...particles either live on a circle or on an ellipse



- Measure particles with imperfect apparatus
→ finite precision: $\sigma_x \approx 0.4$ and $\sigma_y \approx 0.6$

Measuring 2D Particles



- Measurement not very precise
 - ▶ Altered particle distributions
 - ▶ Can not distinguish between circular and elliptic particles
- Approach(es):
 1. Update / buy new apparatus → Requires money and time
 2. Recover true particle information via Least Squares Fit → Requires knowledge about experiment
- Your budget got cut → Go for option 2

Least Squares Fit

- Knowing the uncertainties on x and y , one may formulate:

$$\chi^2 = \left(\frac{x - x_{fit}}{\sigma_x} \right)^2 + \left(\frac{y - y_{fit}}{\sigma_y} \right)^2 \quad (1)$$

Least Squares Fit

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- This does not help much \rightarrow Just set: $x = x_{fit}$ and $y = y_{fit} \rightarrow$ No improvement
- We need more information:
 - New coordinates need to reflect physics of the measured particles
 - Particles live on a circle: $(x_{fit} - x_c)^2 + (y_{fit} - y_c)^2 \stackrel{!}{=} R^2$

Least Squares Fit with Constraints

- Knowing the uncertainties on x and y AND the underlying physics, one may formulate:

$$\chi^2 = \left(\frac{x - x_{fit}}{\sigma_x} \right)^2 + \left(\frac{y - y_{fit}}{\sigma_y} \right)^2 + 2\lambda \cdot F(x_{fit}, y_{fit}) \quad (2)$$

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Least Squares Fit with Constraints

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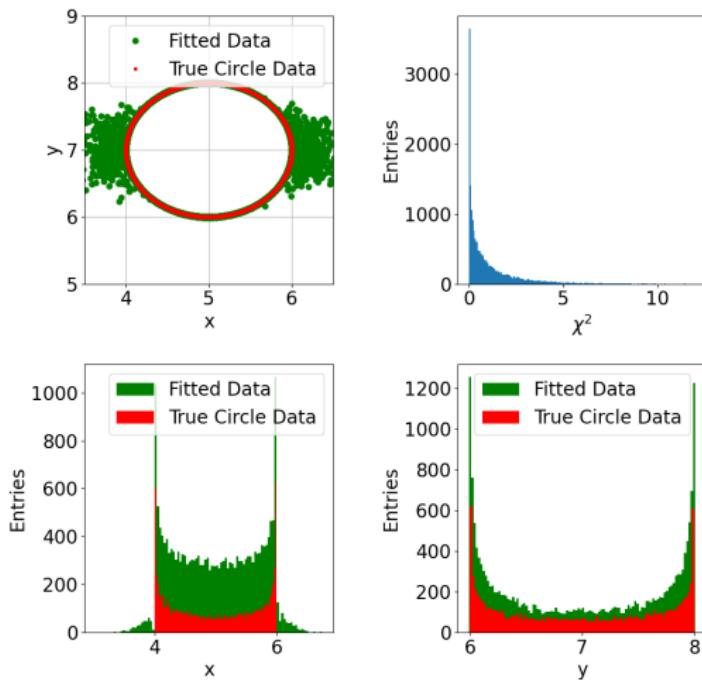
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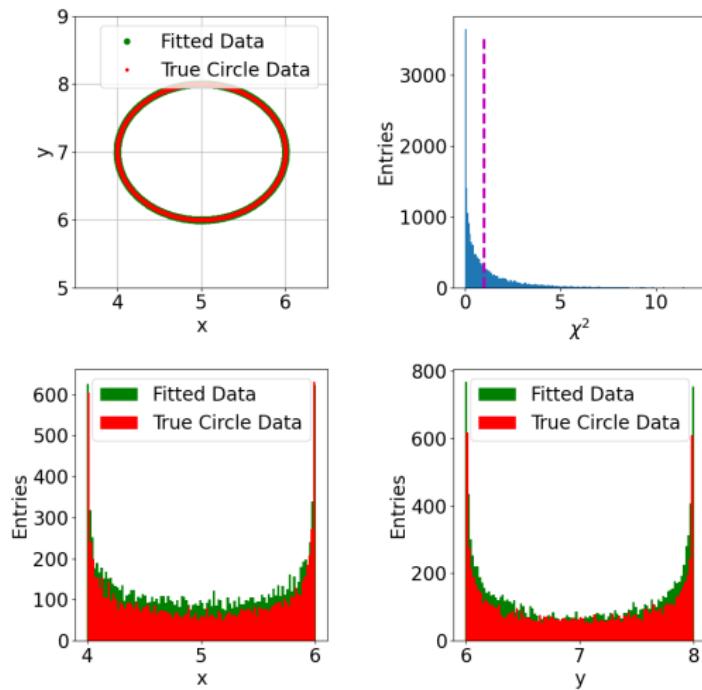
- The details of the (iterative) minimization routine can be found in the backup slides of this talk

Results from Least Squares Fit



- Fitted particle coordinates resemble a circle
- Recovered true information via constrained least squares fit

Results from Least Squares Fit



- Fitted particle coordinates resemble a circle
- Recovered true information via constrained least squares fit
- Further background removal via: $\chi^2 \leq 1.0$

From Circles to entire Physics Reactions

- Constrained least squares fitting is very successful, just needed:
 - Uncertainties related to measurements
 - Constraints, i.e. physics
- Use this technique to find particle coordinates for an entire physics reaction!

Particles	Coordinates	Uncertainties	Constraints
on a circle	(x, y)	σ_x, σ_y	$(x_{fit} - x_c)^2 + (y_{fit} - y_c)^2 - R^2 = 0$
from a reaction	(p, θ, ϕ, m)	$\sigma(p), \sigma(\theta), \sigma(\phi)$	Energy and momentum conservation
	(X, Y, Z)	$\sigma(X), \sigma(Y), \sigma(Z)$	Production vertex
	M	$\sigma(M)$	Mass of decaying particle

Kinematic Fitting at GlueX

- Master equation

$$\chi^2 = \boldsymbol{\epsilon}^T \hat{V}^{-1} \boldsymbol{\epsilon} + 2\boldsymbol{\lambda}^T \boldsymbol{F} \quad (4)$$

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- Residuals: $\boldsymbol{\epsilon} \equiv \begin{pmatrix} p_{fit,1} - p_{meas,1} \\ \theta_{fit,1} - \theta_{meas,1} \\ \vdots \\ \phi_{fit,N} - \phi_{meas,N} \end{pmatrix} \in \mathbb{R}^{3N}$

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- Uncertainties from experiment:

$$\hat{V} = \begin{pmatrix} \sigma^2(p_1) & \rho_{p_1,\theta_1} \sigma(p_1) \sigma(\theta_1) & \cdots & \rho_{p_1,\phi_{3N}} \sigma(p_1) \sigma(\phi_{3N}) \\ \rho_{p_1,\theta_1} \sigma(p_1) \sigma(\theta_1) & \sigma^2(\theta_1) & \cdots & \rho_{\theta_1,\phi_{3N}} \sigma(\theta_1) \sigma(\phi_{3N}) \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{p_1,\phi_{3N}} \sigma(p_1) \sigma(\phi_{3N}) & \rho_{\theta_1,\phi_{3N}} \sigma(\theta_1) \sigma(\phi_{3N}) & \cdots & \sigma^2(\phi_{3N}) \end{pmatrix}$$

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- Reaction specific constraints: \boldsymbol{F}

Kinematic Fitting at GlueX

- Master equation

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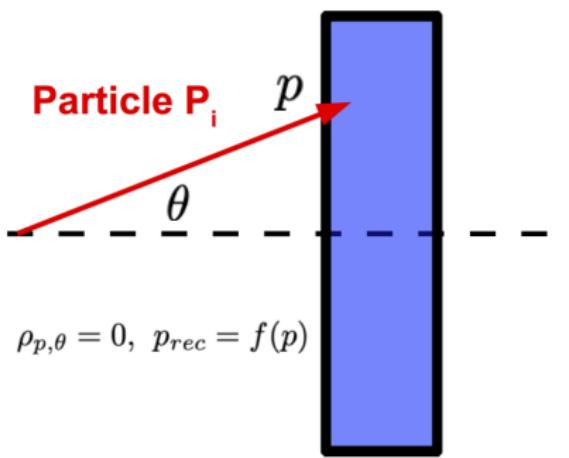
- Reaction specific constraints: F

- Find ϵ by solving:

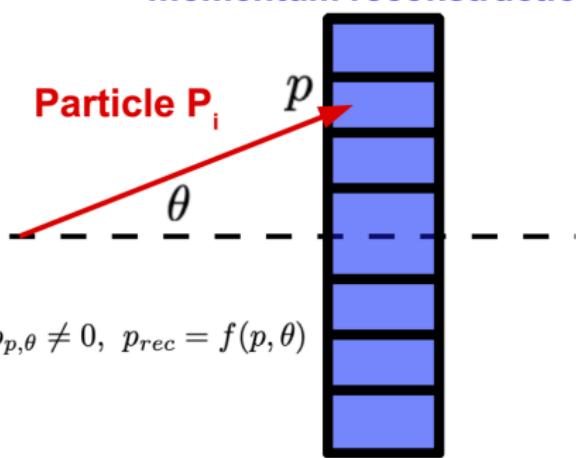
$$\frac{d\chi^2}{d\epsilon} = 0 \quad (5)$$

Know your Detector - Know your Covariance Matrix \hat{V}

sub-detector for momentum reconstruction



segmented sub-detector for momentum reconstruction



- **Left:** Detector response is uniform with respect to $\theta \rightarrow$ Reconstructed momentum p_{rec} does not depend on θ
- **Right:** Detector response is not uniform with respect to $\theta \rightarrow$ Reconstructed momentum p_{rec} depends on θ

Covariance Matrix, Constraints and NDF

- Covariance matrix \hat{V} is determined for every event and every particle

Particle Charge	Track Info	Dimension of \hat{V}
$q \neq 0$	$P_x, P_y, P_z, X_x, X_y, X_z, t$	7×7
$q = 0$	X_x, X_y, X_z, E, t	5×5

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- Number of degrees of freedom: $NDF = N_m - 3N + N_c$

N : Number of all particles in the reaction

N_m : Number of measured observables

N_c : Number of constraints

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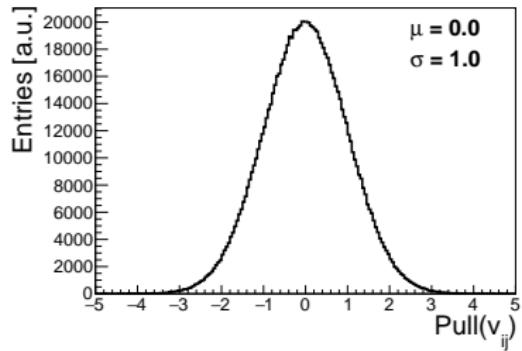
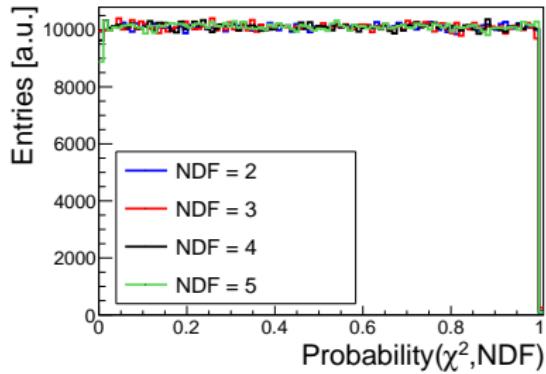
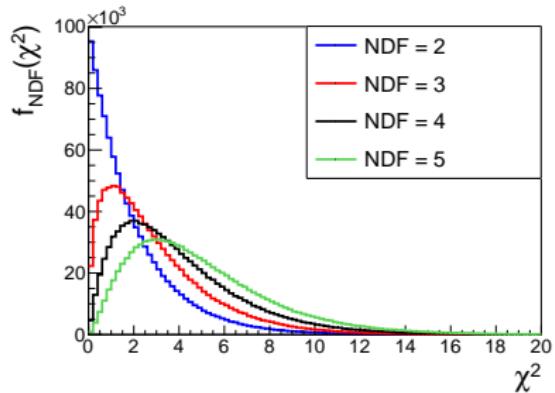
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- **Example:** All particles measured + P4-Fit

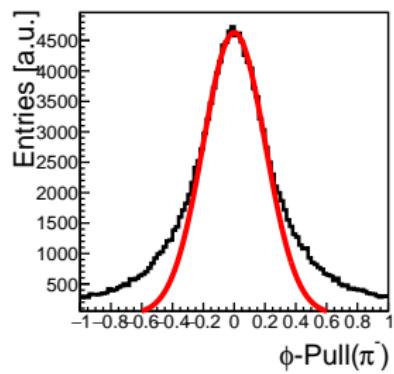
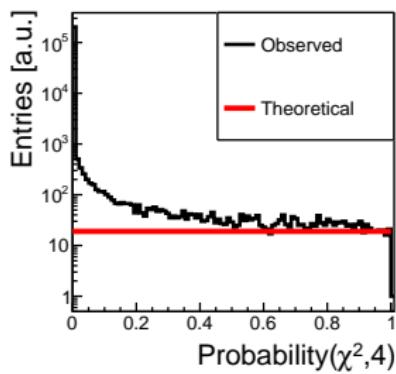
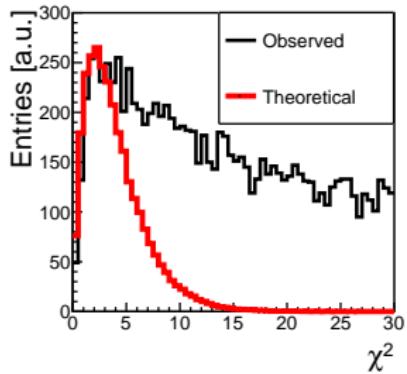
$$\Rightarrow NDF = N_m - 3N + N_c = 3N - 3N + 4 = 4$$

Performance Monitoring Tools



- Expected distributions for gaussian errors and independent measurements
- $$f_{NDF}(x) = \frac{x^{(NDF/2-1)}}{2^{(NDF/2)}\Gamma(NDF/2)} \cdot e^{-x/2}$$
- $$\text{Probability}(\chi^2, NDF) = \frac{\chi^2}{\sqrt{2^{NDF}\Gamma(NDF/2)}}$$
- $$\text{Pull}(v_{ij}) = \frac{v_{meas,ij} - v_{fit,ij}}{\sqrt{|\sigma_{meas,ij}^2 - \sigma_{fit,ij}^2|}}$$

Impact of Error Estimation in \hat{V}



mean[ϕ -Pull(π^-)]

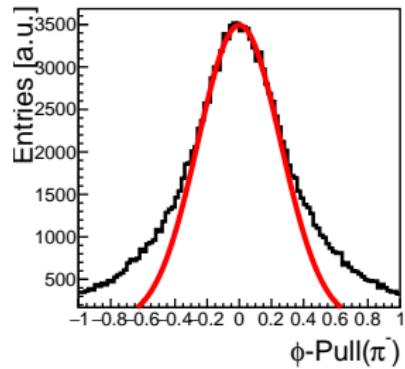
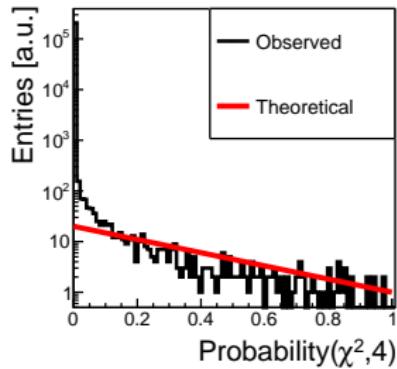
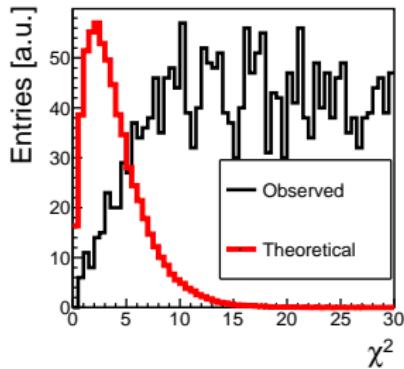
≈ 0

$\sigma[\phi\text{-Pull}(\pi^-)]$

0.201

- Look at toy data set:
 $pp \rightarrow pp\eta [\eta \rightarrow \pi^+\pi^-\pi^0 [\pi^0 \rightarrow \gamma\gamma]]$
- Measured particle observables are correlated
- Realistic error estimation for \hat{V}

Impact of Error Estimation in \hat{V}



mean[$\phi\text{-Pull}(\pi^-)$]

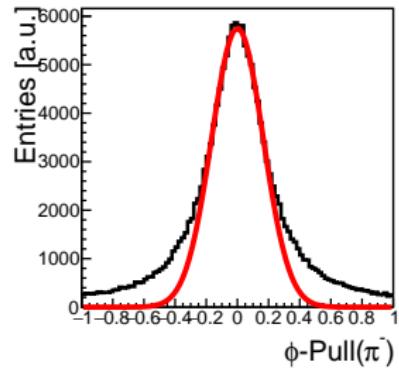
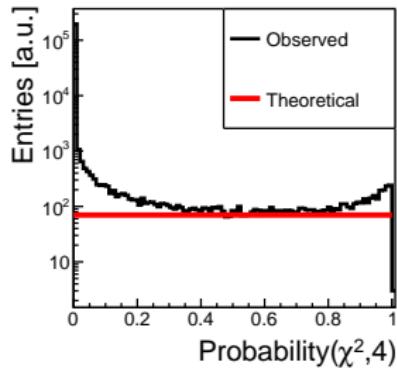
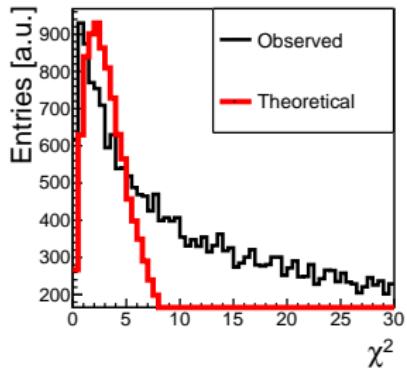
≈ 0

$\sigma[\phi\text{-Pull}(\pi^-)]$

0.257

- Look at toy data set:
 $pp \rightarrow pp\eta[\eta \rightarrow \pi^+\pi^-\pi^0[\pi^0 \rightarrow \gamma\gamma]]$
- Measured particle observables are correlated
- Underestimated errors by $\sim 25\%$

Impact of Error Estimation in \hat{V}



mean[$\phi\text{-Pull}(\pi^-)$]

≈ 0

$\sigma[\phi\text{-Pull}(\pi^-)]$

0.165

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 $pp \rightarrow pp\eta[\eta \rightarrow \pi^+\pi^-\pi^0[\pi^0 \rightarrow \gamma\gamma]]$
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In a Nutshell

$$\chi^2 = \boxed{\epsilon^T \hat{V}^{-1} \epsilon} + \boxed{2\lambda^T F} = 0$$

Inputs from experiment Inputs from physics

Kinematic Fit provides:

1. Fitted particle 4-momenta
 2. Possibility to reject events which do not match the reaction hypothesis

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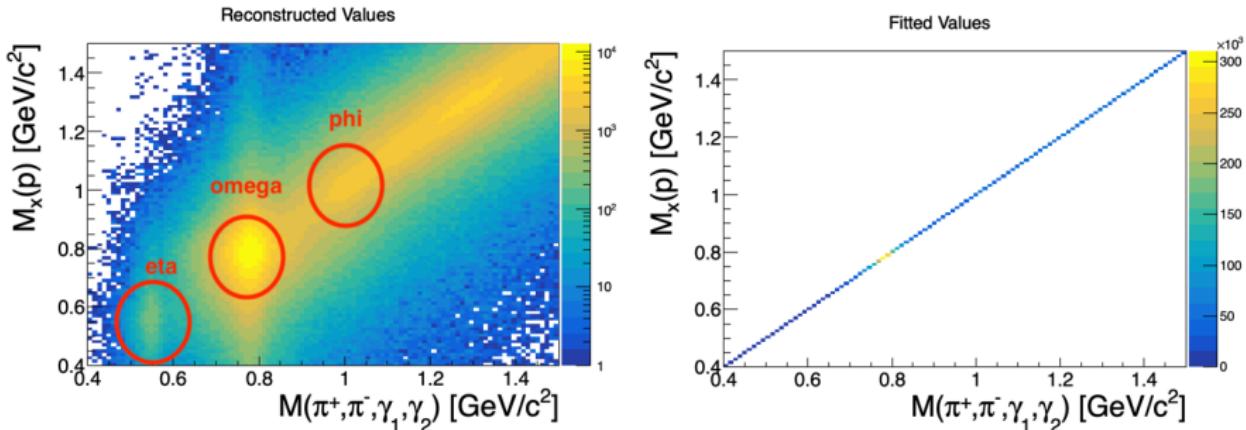
Kinematic Fit provides:

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 2. Possibility to reject events which do not match the reaction hypothesis

- Kinematic fitter is a least squares fit with constraints
 - Use probability / χ^2 and pull-distributions to monitor and diagnose algorithm

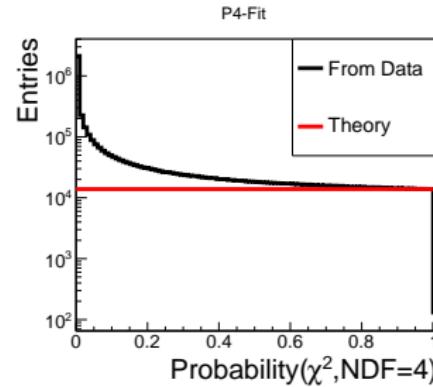
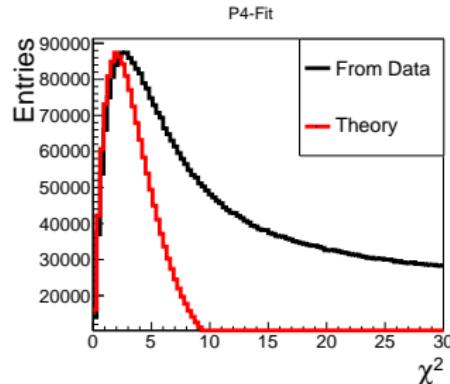
$\text{mean}[\text{Pull}(v_{ij})]$	$\sigma[\text{Pull}(v_{ij})]$	Scenario
0	1.0	everything is fine
0	< 1.0	$\sigma_{\text{meas},ij}$ is overestimated
0	> 1.0	$\sigma_{\text{meas},ij}$ is underestimated
$\neq 0$	$\in [0, 1]$	introduced bias
n.a.	n.a.	non-gaussian pulls \rightarrow you are in trouble

Analysis of $\gamma p \rightarrow \pi^+ \pi^- \pi^0 [\pi^0 \rightarrow \gamma\gamma]$ with the GlueX Kinematic Fit - Mass Spectra

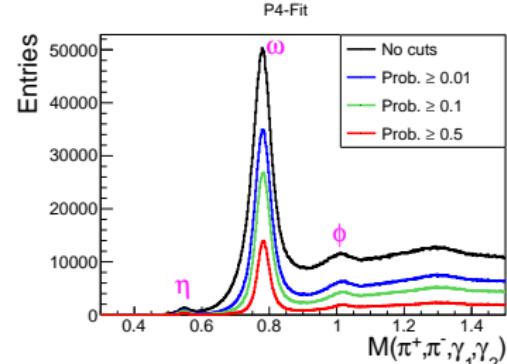


- Analyzed sub-sample of 2017 data
- Used a P4-Fit with no additional constraints
- Missing Mass:** $M_x(p) \equiv \|P_{beam} + P_{target} - P_p\|$
- Invariant Mass:** $M(\pi^+, \pi^-, \gamma_1, \gamma_2) \equiv \|P_{\pi^+} + P_{\pi^-} + P_{\gamma_1} + P_{\gamma_2}\|$
- Ideally:** $M_x(p) = M(\pi^+, \pi^-, \gamma_1, \gamma_2)$ (\rightarrow Fulfilled via kinematic fit)

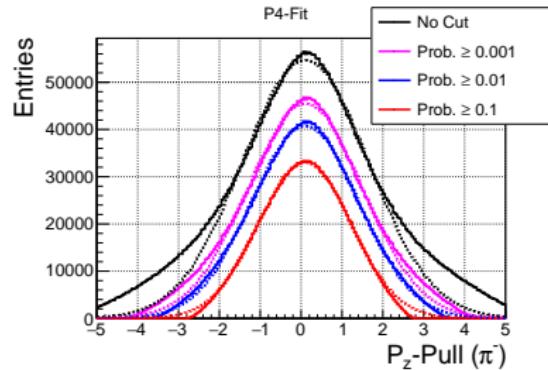
Analysis of $\gamma p \rightarrow \pi^+ \pi^- \pi^0 [\pi^0 \rightarrow \gamma\gamma]$ with the GlueX Kinematic Fit - Performance Plots



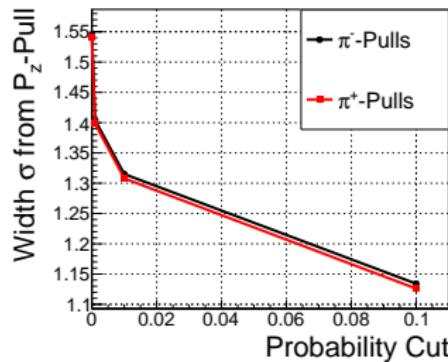
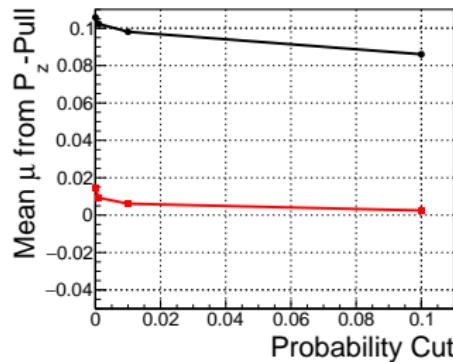
- Enhancement at small probabilities / large χ^2
 - ▶ Purely reconstructed events
 - ▶ Tracks with resolution outside of \hat{V}
 - ▶ Background reactions that do not match reaction hypothesis
- Check impact of different probability cuts



Analysis of $\gamma p \rightarrow \pi^+ \pi^- \pi^0 [\pi^0 \rightarrow \gamma\gamma]$ with the GlueX Kinematic Fit - Pull Distributions

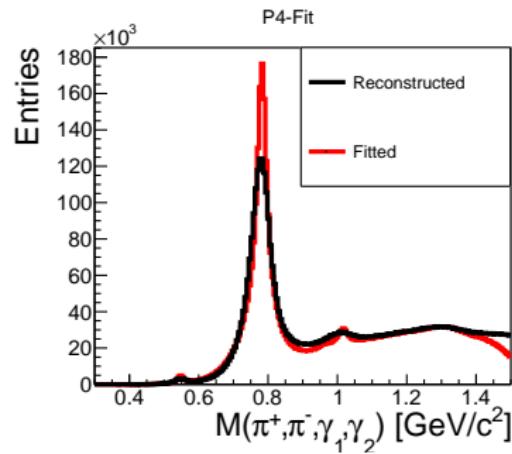
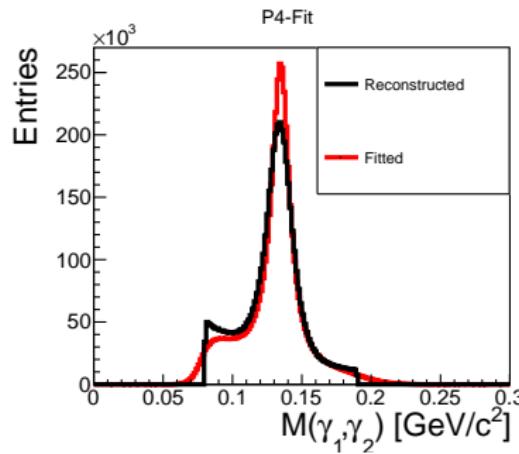


- Look at P_z -pulls for π^- and π^+
- Determine μ and σ for different probability cuts
- Fitter seems to "treat" oppositely charged pions differently
- Observe: $\lim_{\text{prob} \rightarrow 1} \mu = 0$ and: $\lim_{\text{prob} \rightarrow 1} \sigma = 1$ (\rightarrow As it should)



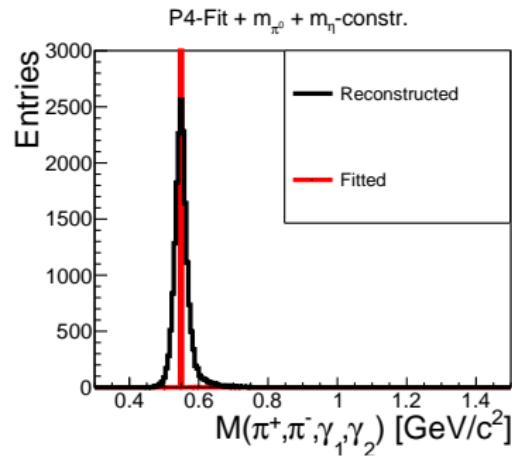
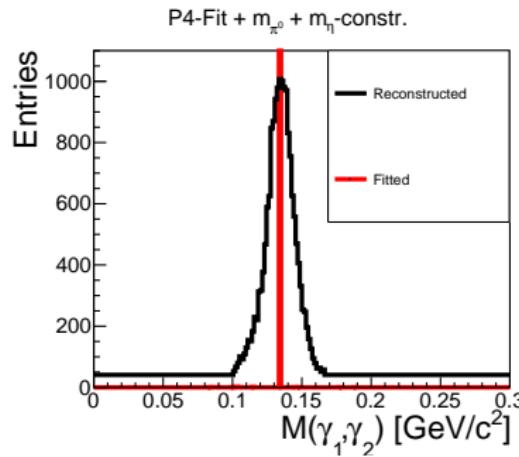
Analysis of $\gamma p \rightarrow \pi^+ \pi^- \pi^0 [\pi^0 \rightarrow \gamma\gamma]$ with the GlueX Kinematic Fit - Mass Constraints

- Kinematic fit helps to improve the resolution
(→ Combine information from all sub-detectors)
- Wish to reconstruct: $\eta \rightarrow \pi^+ \pi^- \pi^0$
- Why not using mass constraints ?
 $M(\gamma_1, \gamma_2) = m(\pi^0)$ and $M(\pi^+, \pi^-, \gamma_1, \gamma_2) = m(\eta)$

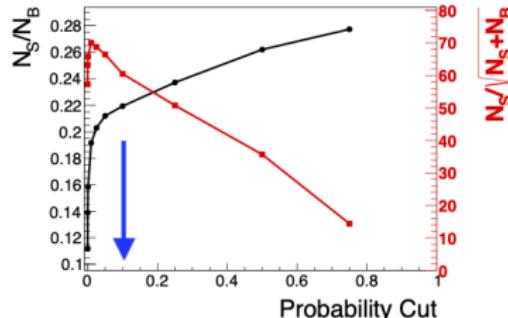
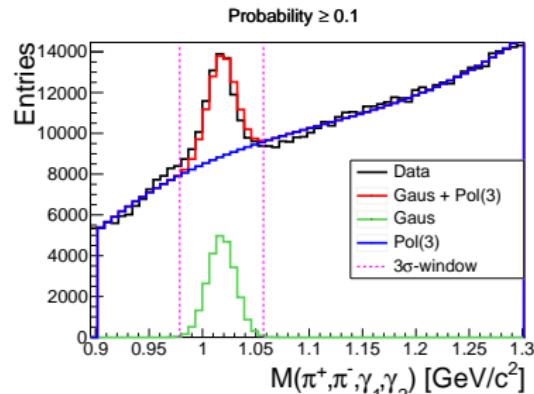
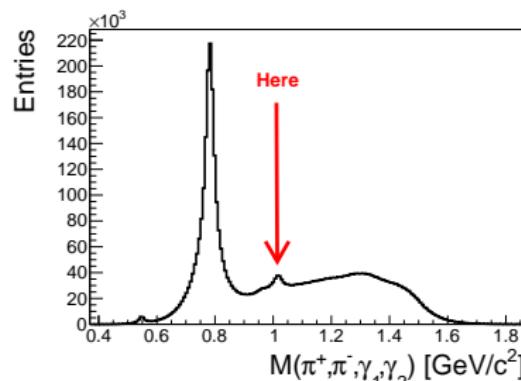


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- Additional mass constraints are helpful, but make it impossible to judge the involved background

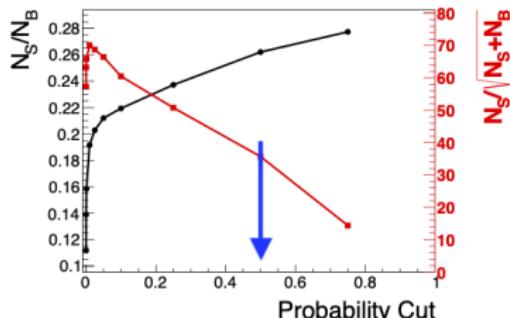
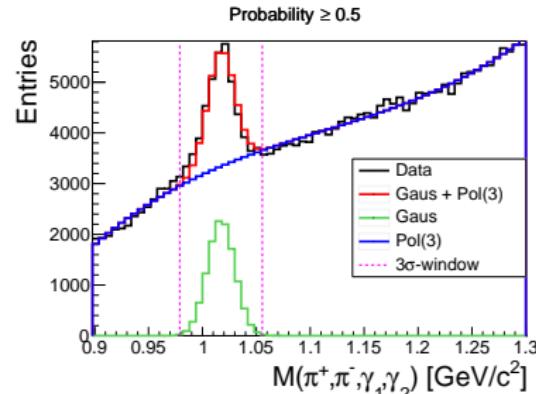
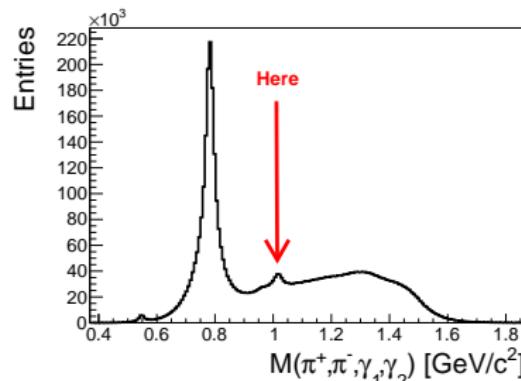


Analysis of $\gamma p \rightarrow \pi^+ \pi^- \pi^0 [\pi^0 \rightarrow \gamma\gamma]$ with the GlueX Kinematic Fit - Cut Optimization



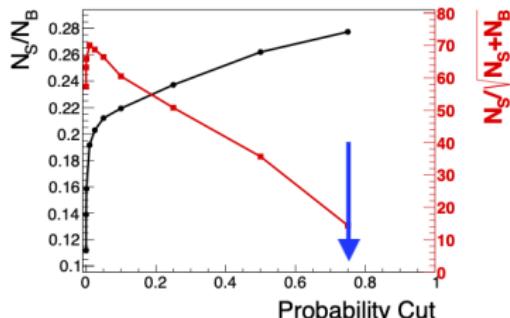
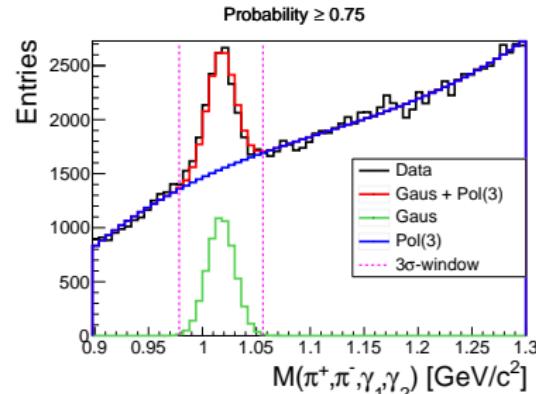
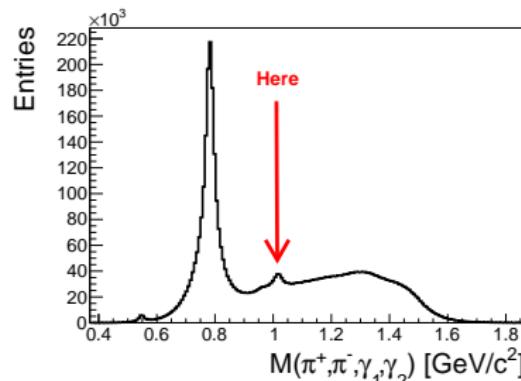
- **Goal:** Find optimum probability cut to select $\phi \rightarrow \pi^+ \pi^- \pi^0$
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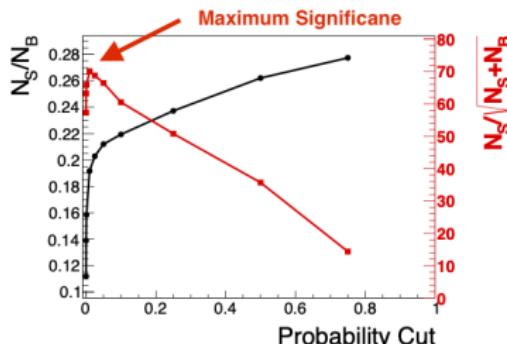
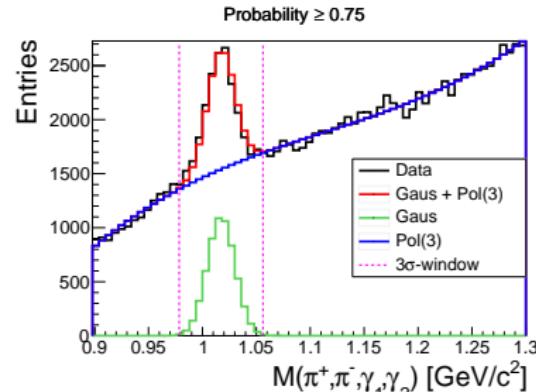
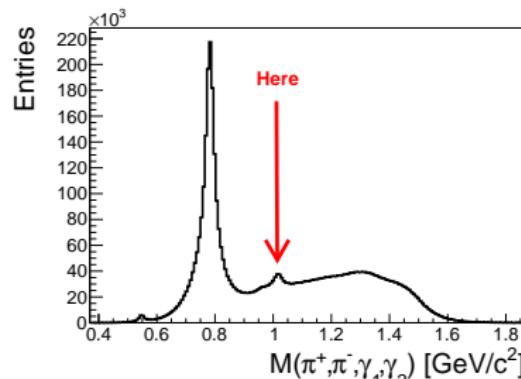
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- Goal: Find optimum probability cut to select $\phi \rightarrow \pi^+ \pi^- \pi^0$
- Vary probability cut and determine number of ϕ / background events
- Significance at maximum for 0.01
⇒ Sweet spot between background rejection and signal loss

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 - ▶ Check and compare pull distributions (**mid-level**)
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- Use the kinematic fit in your own analysis

(See following talks by: Justin Stevens, Beni Zihlman, Lawrence Ng)

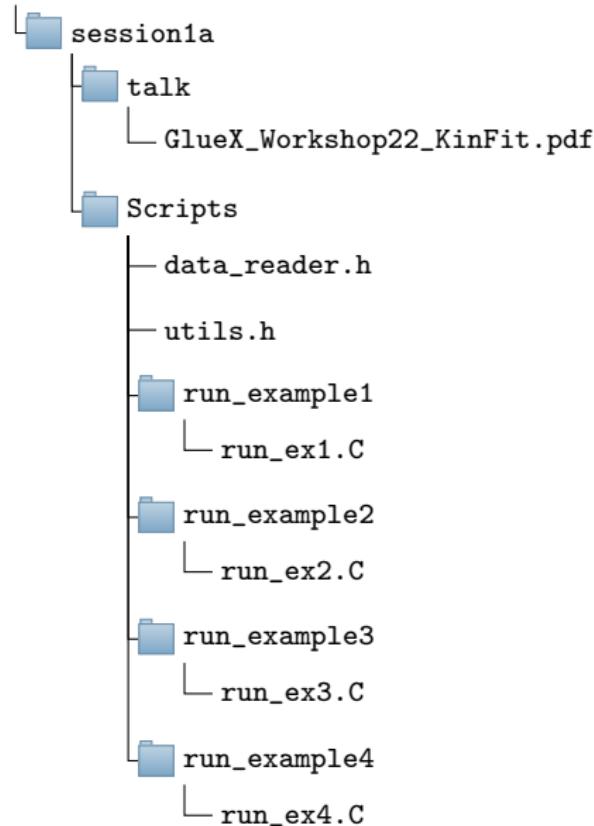
- ▶ Define fitter settings in config file for reaction filter plugin
- ▶ Use analysis-launch webpage
- ▶ Access information from kinematic fitter
- ▶ Use fitted momenta in analysis

Kinematic Fitting - DIY

- Run simple scripts to monitor the kinematic fit performance on $\gamma p \rightarrow \pi^+ \pi^- \pi^0 p$ final states
 - ▶ Similar to the exercises from 2021 Kinematic Fit Workshop
 - ▶ Flat trees from sub-section of 2017 data
- Clone GitHub repository: **JeffersonLab/gluex_workshops/tutorial_2022**
- Or check out on the ifarm:
group/halld/Software/gluex_workshops/tutorial_2022/session1a
- Follow instructions in the README.md file

Kinematic Fitting - DIY

/group/halld/Software/gluex_workshops/tutorial_2022



Backup: χ^2 -Minimization I

- **Note 1:** Most of the calculations and procedures shown here have been taken from:
 - ▶ "Note on kinematical fit", Nov. 20, 1995, A. Kupsc
 - ▶ "Least Squares Kinematic Fitting of Physics Reactions", April 4, 2016, Paul Mattione
- **Note 2:** For the sake of simplicity, unmeasured particles are not discussed in the calculations listed here
- **Goal:** Minimize:

$$\chi^2 = \epsilon^T \hat{V}^{-1} \epsilon + \lambda F \quad (6)$$

with: $\epsilon = \mathbf{y}_{fit} - \mathbf{y}_{meas}$

\mathbf{y}_{meas} / \mathbf{y}_{fit} : a 3N-dimensional vector, containing the measured / fitted momentum components of all N particles

- We achieve the goal by solving:

$$\frac{\partial \chi^2}{\partial \epsilon} = \frac{\partial \chi^2}{\partial \mathbf{y}_{fit}} = 0 \quad (7)$$

- This results, after some mathematical Harakiri, in the equation:

$$\hat{V}^{-1} \epsilon + \hat{D} \lambda = 0 \quad (8)$$

- with:

$$\hat{D} = \left. \frac{\partial F}{\partial \mathbf{y}} \right|_{\mathbf{y}=\mathbf{y}_{fit}}, \quad D_{hk} = \left. \frac{\partial F_h}{\partial y_k} \right|_{y_k=y_{fit,k}} \quad (9)$$

- Solving Eq. 8 \Leftrightarrow Find solutions for Eq. 7
- The calculations are continued on the next slide

Backup: χ^2 -Minimization II

- Eq. 8 is solved iteratively, by using a Taylor expansion around $\epsilon \approx 0$
- For the i -th iteration one finds:

$$0 = \hat{V}^{-1}\epsilon_{i+1} + \hat{D}_i^T \lambda_{i+1} \Leftrightarrow \quad (10)$$

$$\epsilon_{i+1} = -\hat{V}\hat{D}_i^T \lambda_{i+1} \quad (11)$$

- The corresponding constraints are Taylor expanded to:

$$0 = \mathbf{F} \simeq \mathbf{F}_i + \hat{D}_i(\mathbf{y}_{fit,i+1} - \mathbf{y}_{fit,i}) = \mathbf{F}_i + \hat{D}_i(\epsilon_{i+1} - \epsilon_i) \quad (12)$$

- Plugging Eq. 11 into Eq. 12 yields:

$$0 = \mathbf{F}_i - \underbrace{\hat{D}_i \hat{V} \hat{D}_i^T}_{\equiv \hat{S}} \lambda_{i+1} - \hat{D}_i \epsilon_i \Leftrightarrow \quad (13)$$

$$\lambda_{i+1} = \hat{S}^{-1}[\mathbf{F}_i - \hat{D}_i \epsilon_i] \quad (14)$$

- Eq. 14 can be plugged back into Eq. 11 and one obtains a solution for ϵ_{i+1} :

$$\epsilon_{i+1} = -\hat{V}\hat{D}_i^T \hat{S}^{-1}[\mathbf{F}_i - \hat{D}_i \epsilon_i] \quad (15)$$

- The next slide gives a short summary on the minimization procedure itself

Backup: χ^2 -Minimization III

Using the definitions and results from the last two slides, allows to formulate a rough outline for the minimization procedure:

1. Calculate $\epsilon_i = \mathbf{y}_{fit}(i) - \mathbf{y}_{meas}$, with: $\mathbf{y}_{fit}(i=0) = \mathbf{y}_{meas}$
2. Determine the derivative matrix:

$$\hat{D}_i = \left. \frac{\partial \mathbf{F}}{\partial \mathbf{y}} \right|_{\mathbf{y}=\mathbf{y}_{fit}(i)}, \text{ with elements: } D_{hk} = \left. \frac{\partial F_h}{\partial y_k} \right|_{y_k=y_{fit,k}(i)}$$

3. Use \hat{D}_i and \hat{V} to calculate the S-matrix: $\hat{S}_i = \hat{D}_i \hat{V} \hat{D}_i^T$
4. Evaluate the constraint functions at $\mathbf{y}_{fit}(i)$:

$$\mathbf{F}_i = \mathbf{F}[\mathbf{y}_{fit}(i)] \tag{16}$$

5. Calculate λ_{i+1} , according to Eq. 14 and thus, determine ϵ_{i+1} (see Eq. 11)
6. This allows for a new evaluation: $\mathbf{F}_{i+1} = \mathbf{F}[\mathbf{y}_{i+1}]$
7. Use λ_{i+1} , \hat{S} and \mathbf{F}_{i+1} to calculate the corresponding χ^2 :

$$\chi_{i+1}^2 = \lambda_{i+1}^T \hat{S}_i \lambda_{i+1} + 2 \lambda_{i+1}^T \mathbf{F}_{i+1} \tag{17}$$

8. Repeat steps 1. to 7. with the new $\mathbf{y}_{fit,i+1}$, until a convergence criterium is fulfilled (e.g. $|\chi_{i+1}^2 - \chi_i^2| \leq 0.001$)
9. Use the last reported values of ϵ_{i+1} for further analysis