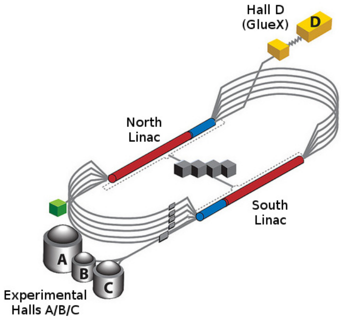
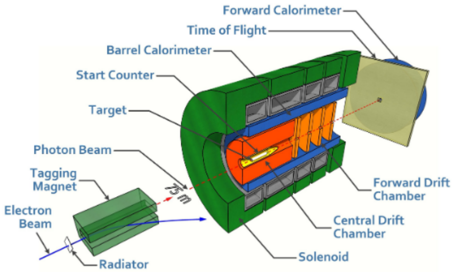
The GlueX Kinematic Fitter in $\sim$ 20 min

Daniel Lersch

May 23, 2022

The Gluonic Excitation Experiment - GlueX



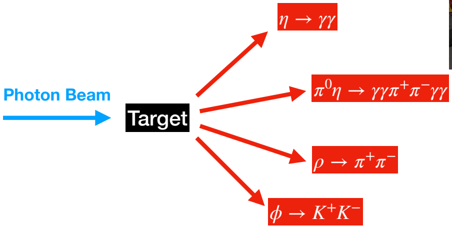


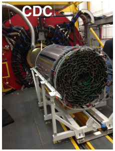
Experimental Hall D:

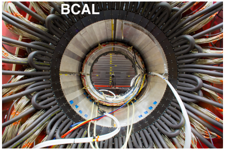
- Over 130 scientists from:
  - 30 Institutions
  - 10 Countries
- Experiments with polarized photon beam

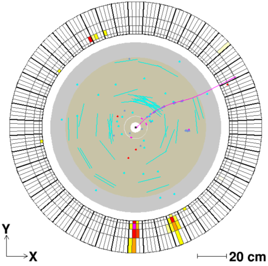
Phase	Run Period	Raw Data [PB]
GlueX-I	Spring 2017	0.9
	Spring 2018	1.9
	Fall 2018	1.1
GlueX-II	Spring 2020	2.8
	Summer 2020	1.7

Event Reconstruction





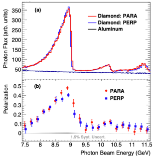


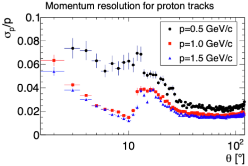


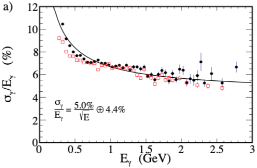
Charge q	Reconstructed via	Observables
>0, <0	FDC/CDC + mag. field	Momenta, track parameters, ddE/dx,...
0	BCAL/FCAL/CompCAL	Clusters, shower info, E,..

- **Goal:** Reconstruct entire reaction from final state particles
- Many final states particles
  - Overlapping topologies, combinatorics,... (See talk by Benedikt Zihlmann)
  - Poorly reconstructed events
    $\rightarrow$ Need an efficient filter
- **Kinematic fit algorithm:** Information from detector + reaction hypothesis

Event Reconstruction







Beam Photons
- Energy
- Position
- Resolution

Charged Tracks
- Trajectory
- Magnetic field
- Resolution

Neutral Showers
- Position
- Time
- Resolution

GlueX Kinematic Fit

Reaction Hypothesis: $\gamma p \rightarrow p \ldots$

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Constrained Least Squares Fitting

- Backbone of kinematic fit algorithm

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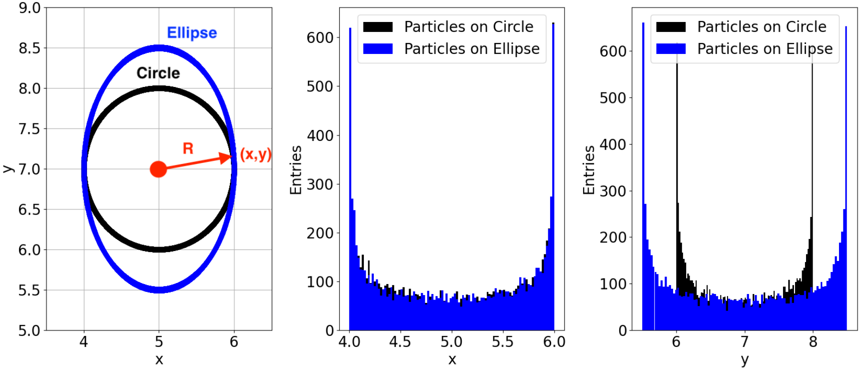
...each particle is characterized by: $\vec{v} = \begin{pmatrix} x \\ y \end{pmatrix}$

...particles either live on a circle or on an ellipse



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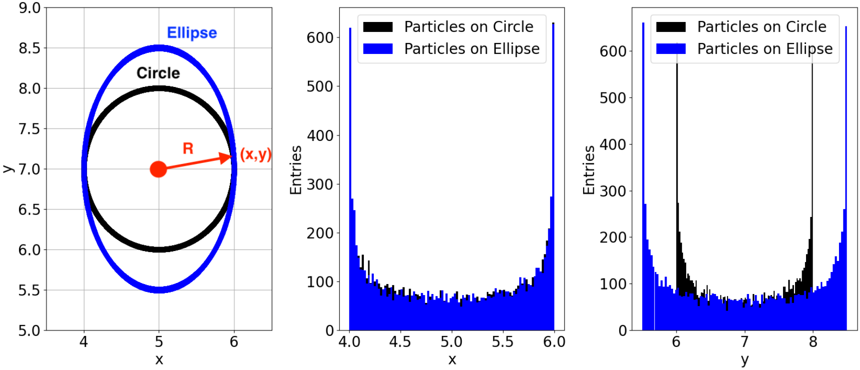
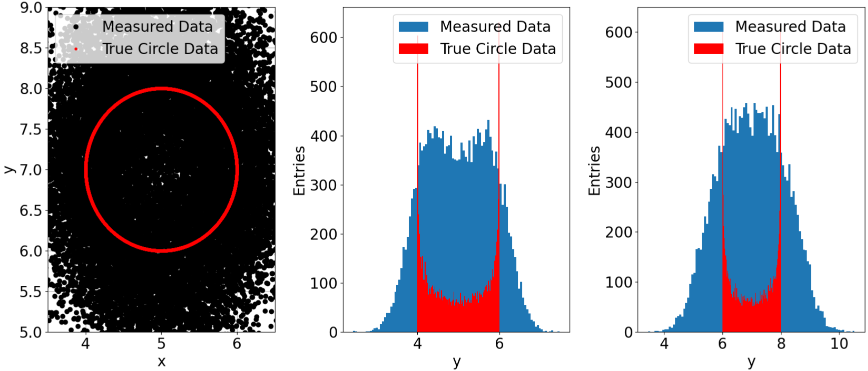


- Measure particles with imperfect apparatus
  $\rightarrow$ finite precision: $\sigma_x \approx 0.4$ and $\sigma \approx 0.6$

Measuring 2D Particles



- Measurement not very precise
  - Altered particle distributions
  - Can not distinguish between circular and elliptic particles
- **Approach(es):**
  1. Update / buy new apparatus $\rightarrow$ Requires money and time
  2. Recover true particle information via Least Squares Fit $\rightarrow$ Requires knowledge about experiment
- Your budget got cut $\rightarrow$ Go for option 2

Least Squares Fit

- Knowing the uncertainties on x an y, one may formulate:

$$\chi^2 = \left(\frac{x - x_{fit}}{\sigma_x}\right)^2 + \left(\frac{y - y_{fit}}{\sigma_y}\right)^2 \tag{1}$$

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- Just find new coordinates $\vec{v}_{fit} = (x_{fit}, y_{fit})$ which minimize Eq. 1 within the given uncertainties
- This does not help much $\rightarrow$ Just set: $x = x_{fit}$ and $y = y_{fit}$ $\rightarrow$ No improvement
- We need more information:
    - New coordinates need to reflect physics of the measured particles
    - Particles live on a circle: $(x_{fit} - x_c)^2 + (y_{fit} - y_c)^2 \stackrel{!}{=} R^2$

Least Squares Fit with Constraints

- Knowing the uncertainties on x an y AND the underlying physics, one may formulate:

$$\chi^2 = \left(\frac{x - x_{fit}}{\sigma_x}\right)^2 + \left(\frac{y - y_{fit}}{\sigma_y}\right)^2 + 2\lambda \cdot F(x_{fit}, y_{fit}) \tag{2}$$

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with: $F(x_{fit}, y_{fit}) = (x_{fit} - x_c)^2 + (y_{fit} - y_c)^2 - R^2$

$F(x_{fit}, y_{fit}) = 0$, if new particle coordinates are on a circle

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- Find new the new coordinates $\vec{v}_{fit} = (x_{fit}, y_{fit})$ by minimizing Eq. 2:

$$\frac{\partial \chi^2}{\partial \vec{v}_{fit}} = \left(\frac{\partial \chi^2}{\partial x_{fit}}, \frac{\partial \chi^2}{\partial y_{fit}}\right) = 0 \quad (3)$$

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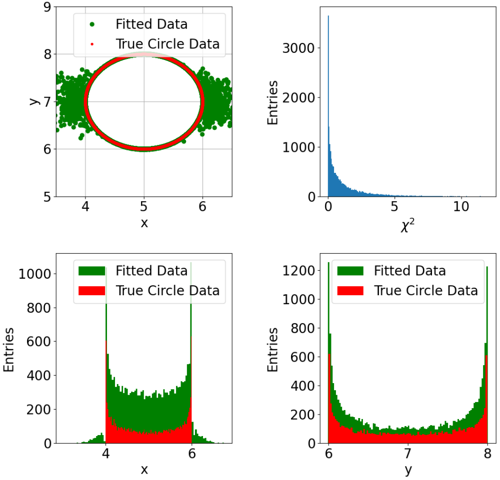
- The details of the (iterative) minimization routine can be found in the backup slides of this talk

Results from Least Squares Fit



- Fitted particle coordinates resemble a circle
- Recovered true information via constrained least squares fit

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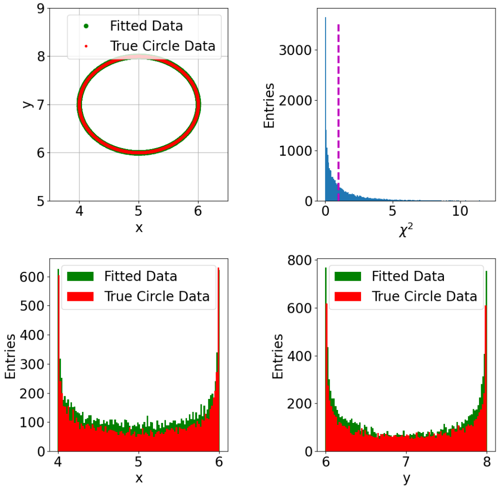

- Fitted particle coordinates resemble a circle
- Recovered true information via constrained least squares fit
- Further background removal via: $\chi^2 \leq 1.0$

From Circles to entire Physics Reactions

- Constrained least squares fitting is very successful, just needed:
  1. Uncertainties related to measurements
  2. Constraints, i.e. physics
- Use this technique to find particle coordinates for an entire physics reaction!

Particles	Coordinates	Uncertainties	Constraints
on a circle	(x, y)	σ_x,σ_y	$(x_{fit} - x_c)^2 + (y_{fit} - y_c)^2 - R^2 = 0$
from a reaction	(p, θ, ϕ, m)	$\sigma(p)$, $\sigma(\theta)$, $\sigma(\phi)$	Energy and momentum conservation
	(X, Y, Z)	$\sigma(X)$, $\sigma(Y)$, $\sigma(Y)$	Production vertex
	M	$\sigma(M)$	Mass of decaying particle

Kinematic Fitting at GlueX

- Master equation

$$\chi^2 = \boldsymbol{\epsilon}^T \hat{V}^{-1} \boldsymbol{\epsilon} + 2\boldsymbol{\lambda}^T \boldsymbol{F} \tag{4}$$

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- Residuals: $\boldsymbol{\epsilon} \equiv \begin{pmatrix} p_{fit,1} - p_{meas,1} \\ \theta_{fit,1} - \theta_{meas,1} \\ \vdots \\ \phi_{fit,N} - \phi_{meas,N} \end{pmatrix} \in \mathbb{R}^{3N}$

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- Uncertainties from experiment:

$$\hat{V} = \begin{pmatrix} \sigma^2(p_1) & \rho_{p_1,\theta_1}\sigma(p_1)\sigma(\theta_1) & \cdots & \rho_{p_1,\phi_{3N}}\sigma(p_1)\sigma(\phi_{3N}) \\ \rho_{p_1,\theta_1}\sigma(p_1)\sigma(\theta_1) & \sigma^2(\theta_1) & \cdots & \rho_{\theta_2,\phi_{3N}}\sigma(\theta_1)\sigma(\phi_{3N}) \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{p_1,\phi_{3N}}\sigma(p_1)\sigma(\phi_{3N}) & \rho_{\theta_2,\phi_{3N}}\sigma(\theta_1)\sigma(\phi_{3N}) & \cdots & \sigma^2(\phi_{3N}) \end{pmatrix}$$

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- Reaction specific constraints: $\boldsymbol{F}$

Kinematic Fitting at GlueX

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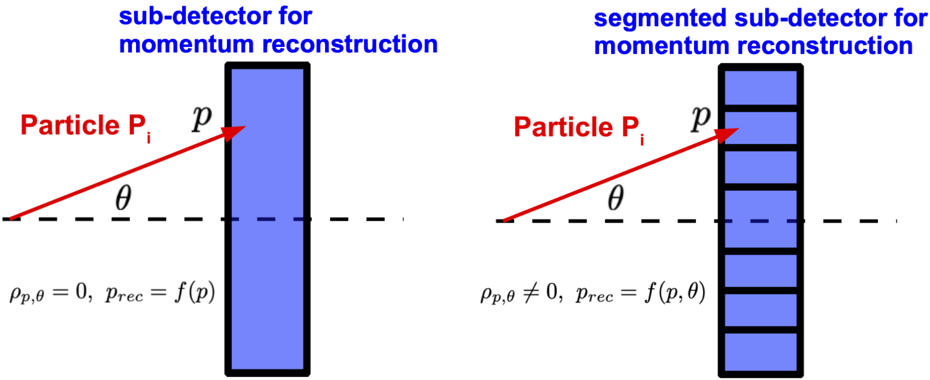
- Reaction specific constraints: $\boldsymbol{F}$
- Find ϵ by solving:

$$\frac{d\chi^2}{d\epsilon} = 0 \tag{5}$$

Know your Detector - Know your Covariance Matrix $\hat{V}$



- **Left:** Detector response is uniform with respect to $\theta \rightarrow$ Reconstructed momentum p_{rec} does not depend on
- **Right:** Detector response is not uniform with respect to $\theta \rightarrow$ Reconstructed momentum p_{rec} depends on θ

Covariance Matrix, Constraints and NDF

- Covariance matrix $\hat{V}$ is determined for every event and every particle

Particle Charge	Track Info	Dimension of $\hat{V}$
$q \neq 0$	P_x, P_y, P_z, X_x, X_y, X_z, t	7×7
$q = 0$	X_x, X_y, X_z, E, t	5×5

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- Basic constraints:

Fit Type	Constraint	Observable used
P4-Fit	$\sum_{i=1}^{N} P_{i,\mu} = 0,\ \mu = x, y, z, E$	$P_\mu,\ m$
P4-Vertex-Fit	$\sum_{i=1}^{N} P_{i,\mu} = 0,\ \sum_{i=1}^{N} X_{i,\mu} = 0\ ,\ \mu = x, y, z, (E)$	$P_\mu,\ X_\mu,\ m$
Vertex-Fit	$\sum_{i=1}^{N} X_{i,\mu} = 0,\ \mu = x, y, z$	X_μ

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- Number of degrees of freedom: NDF$= N_m - N + N_c$

  N: Number of all observables (momenta, angles,...)
  N_m: Number of measured observables
  N_c: Number of constraints

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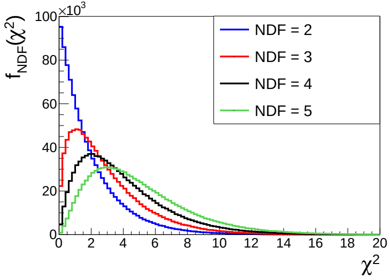
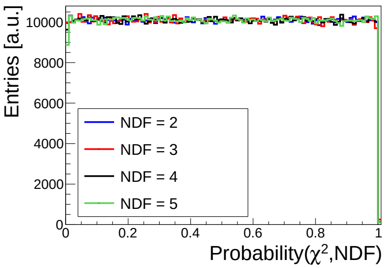
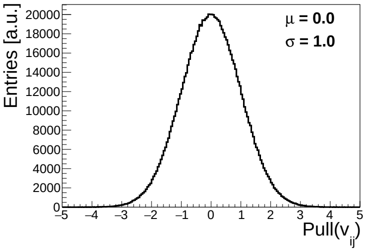
- **Example:** All observables measured + P4-Fit
  $\Rrightarrow$ NDF $= N_m - N + N_c = N - N + 4 = 4$

Performance Monitoring Tools

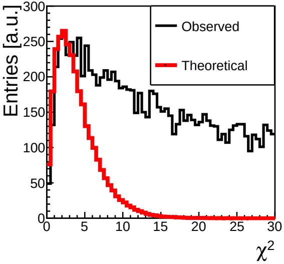


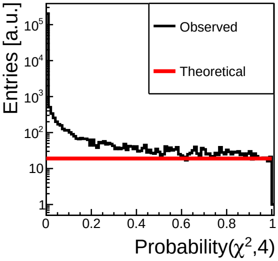


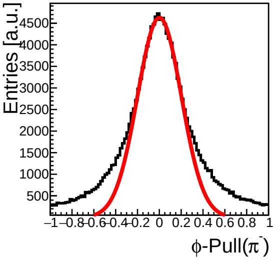


- Expected distributions for gaussian errors and independent measurements
- $f_{NDF}(x) = \frac{x^{(NDF/2-1)}}{2^{(NDF/2)}\Gamma(NDF/2)} \cdot e^{-x/2}$
- $\text{Probability}(\chi^2, NDF) = \frac{\int_{\chi^2}^{\infty} t^{NDF/2-1} e^{-t/2} dt}{\sqrt{2^{NDF}\Gamma(NDF/2)}}$
- $\text{Pull}(v_{ij}) = \frac{v_{meas,ij} - v_{fit,ij}}{\sqrt{|\sigma^2_{meas,ij} - \sigma^2_{fit,ij}|}}$

Impact of Error Estimation in $\hat{V}$



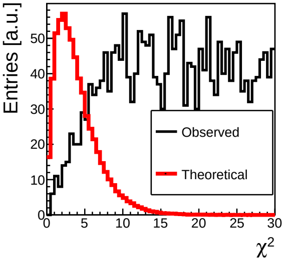


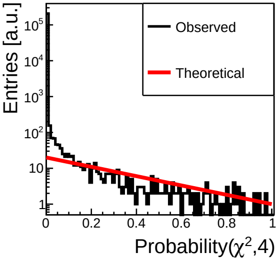


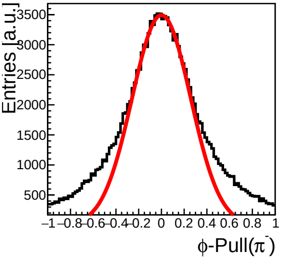
mean[ϕ-Pull(π^-)]	≈ 0
σ[ϕ-Pull(π^-)]	0.201

- Look at toy data set: $pp \to pp\eta[\eta \to \pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]]$
- Measured particle observables are correlated
- Realistic error estimation for $\hat{V}$

Impact of Error Estimation in $\hat{V}$



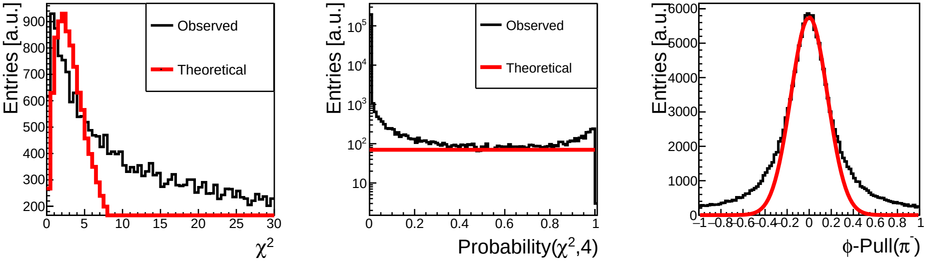




mean[ϕ-Pull(π^-)]	≈ 0
σ[ϕ-Pull(π^-)]	0.257

- Look at toy data set: $pp \to pp\eta[\eta \to \pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]]$
- Measured particle observables are correlated
- Underestimated errors by $\sim 25\%$

Impact of Error Estimation in $\hat{V}$



mean[ϕ-Pull(π^-)]	≈ 0
σ[ϕ-Pull(π^-)]	0.165

- Look at toy data set: $pp \to pp\eta[\eta \to \pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]]$
- Measured particle observables are correlated
- Overestimated errors by $\sim 25\%$

In a Nutshell

$$\chi^2 = \underbrace{\boxed{\epsilon^T \hat{V}^{-1} \epsilon}}_{\text{Inputs from experiment}} + \underbrace{\boxed{2\lambda^T \boldsymbol{F}}}_{\text{Inputs from physics}} \stackrel{!}{=} 0$$

Kinematic Fit provides:

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Kinematic Fit provides:

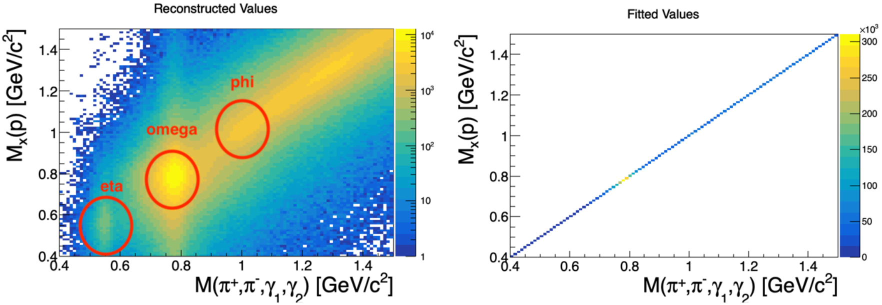
1. Fitted particle 4-momenta

2. Possibility to reject events which do not match the reaction hypothesis

- Kinematic fitter is a least squares fit with constraints
- Use probability / χ^2 and pull-distributions to monitor and diagnose algorithm

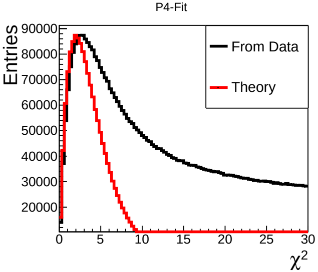
mean[Pull(v_{ij})]	σ[Pull(v_{ij})]	Scenario
0	1.0	everything is fine
0	< 1.0	$\sigma_{meas,ij}$ is overestimated
0	> 1.0	$\sigma_{meas,ij}$ is underestimated
$\neq 0$	$\in [0, 1]$	introduced bias
n.a.	n.a.	non-gaussian pulls $\rightarrow$ you are in trouble

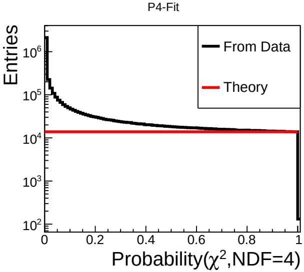
Analysis of $\gamma p \to \pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]$ with the GlueX Kinematic Fit - Mass Spectra



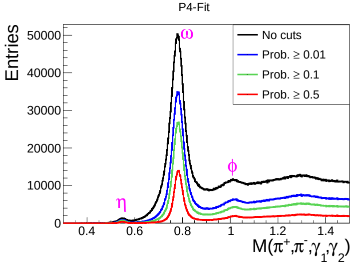
- Analyzed sub-sample of 2017 data
- Used a P4-Fit with no additional constraints
- **Missing Mass:** $M_x(p) \equiv \|P_{beam} + P_{target} - P_p\|$
- **Invariant Mass:** $M(\pi^+, \pi^-, \gamma_1, \gamma_2) \equiv \|P_{\pi^+} + P_{\pi^-} + P_{\gamma_1} + P_{\gamma_2}\|$
- **Ideally:** $M_x(p) = M(\pi^+, \pi^-, \gamma_1, \gamma_2)$ ($\rightarrow$ Fulfilled via kinematic fit)

Analysis of $\gamma p \to \pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]$ with the GlueX Kinematic Fit - Performance Plots

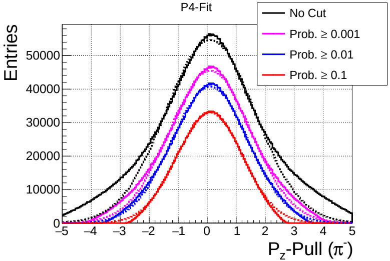




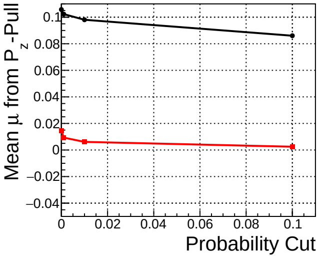
- Enhancement at small probabilities / large χ^2
  - Purely reconstructed events
  - Tracks with resolution outside of $\hat{V}$
  - Background reactions that do not match reaction hypothesis
- Check impact of different probability cuts

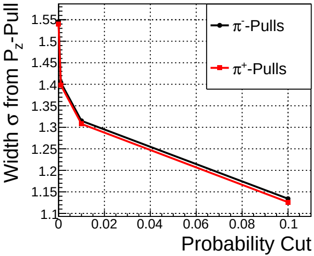


Analysis of $\gamma p \to \pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]$ with the GlueX Kinematic Fit - Pull Distributions



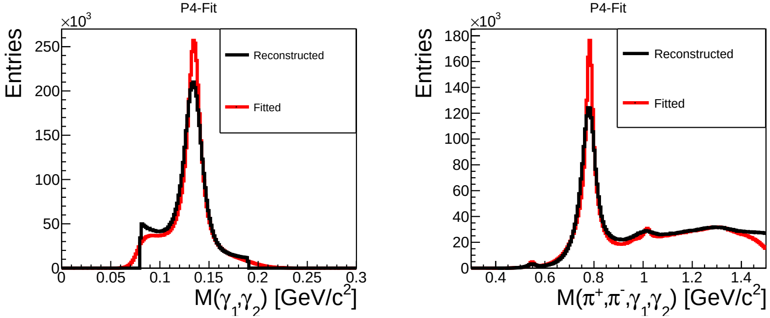
- Look at P_z-pulls for π^- and π^+
- Determine μ and σ for different probability cuts
- Fitter seems to "treat" oppositely charged pions differently
- Observe: $\lim_{\textbf{prob}\to 1} \mu = 0$ and: $\lim_{\textbf{prob}\to 1} \sigma = 1$ ($\to$ As it should)





Analysis of $\gamma p \to \pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]$ with the GlueX Kinematic Fit - Mass Constraints

- Kinematic fit helps to improve the resolution
  ($\to$ Combine information from all sub-detectors)
- Wish to reconstruct: $\eta \to \pi^+\pi^-\pi^0$
- Why not using mass constraints ?
  $M(\gamma_1, \gamma_2) = m(\pi^0)$ and $M(\pi^+, \pi^-, \gamma_1, \gamma_2) = m(\eta)$



Analysis of $\gamma p \rightarrow \pi^+\pi^-\pi^0[\pi^0 \rightarrow \gamma\gamma]$ with the GlueX Kinematic Fit - Mass Constraints

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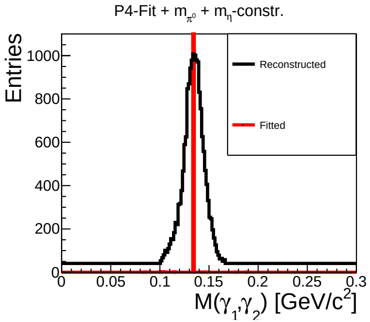
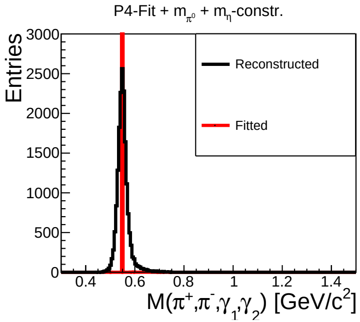
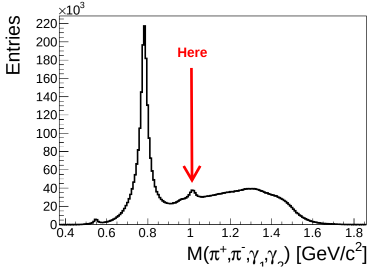
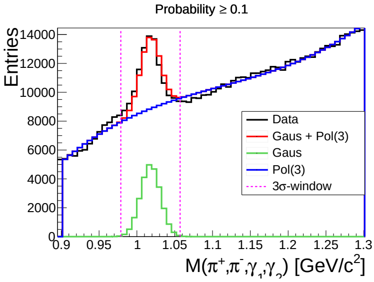
- Why not using mass constraints ?
  $M(\gamma_1, \gamma_2) = m(\pi^0)$ and $M(\pi^+, \pi^-, \gamma_1, \gamma_2) = m(\eta)$
- Additional mass constraints are helpful, but make it impossible to judge the involved background

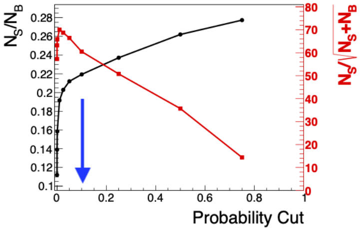




Analysis of $\gamma p \to \pi^+\pi^-\pi^0[\pi^0 \to \gamma\gamma]$ with the GlueX Kinematic Fit - Cut Optimization







- **Goal:** Find optimum probability cut to select $\phi \to \pi^+\pi^-\pi^0$
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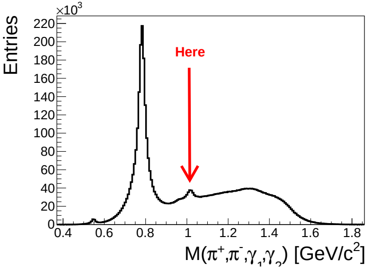

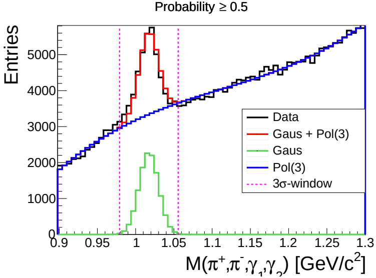

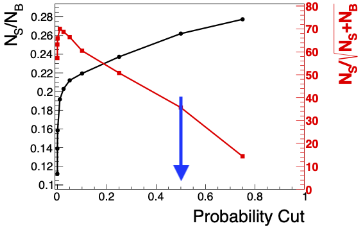

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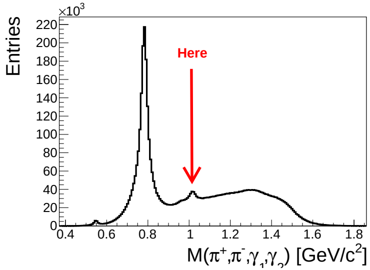

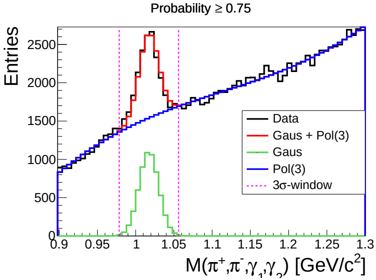

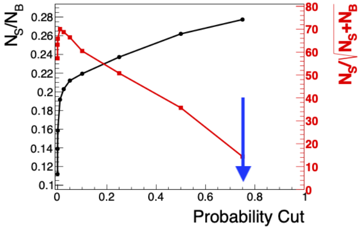

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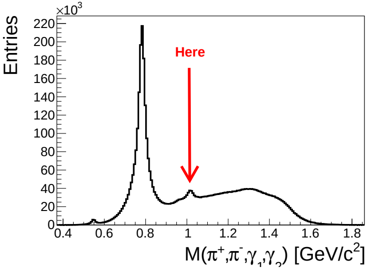

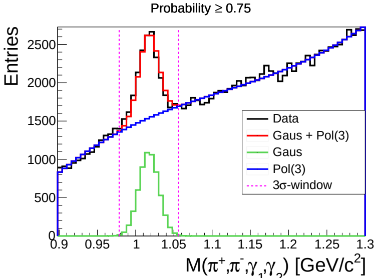

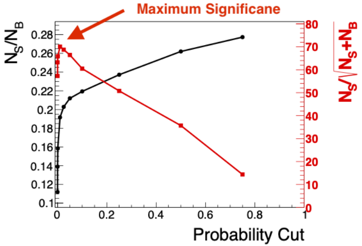

- **Goal:** Find optimum probability cut to select $\phi \rightarrow \pi^+\pi^-\pi^0$
- Vary probability cut and determine number of ϕ / background events
- Significance at maximum for 0.01
  $\Rightarrow$ Sweet spot between background rejection and signal loss

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    $\rightarrow$ Reality in many experiments
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  - The kinematic fit algorithm itself has no systematic error
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  - Try not to solve everything with mass constraints
- Use the kinematic fit in your own analysis
  (See following talks by: Justin Stevens, Beni Zihlman, Lawrence Ng)
  - Define fitter settings in config file for reaction filter plugin
  - Use analysis-launch webpage
  - Access information from kinematic fitter
  - Use fitted momenta in analysis

Kinematic Fitting - DIY

- Run simple scripts to monitor the kinematic fit performance on $\gamma p \to \pi^+\pi^-\pi^0 p$ final states
  - Similar to the exercises from 2021 Kinematic Fit Workshop
  - Flat trees from sub-section of 2017 data
- Clone GitHub repository: **JeffersonLab/gluex_workshops/tutorial_2022**
- Or check out on the ifarm: **group/halld/Software/gluex_workshops/tutorial_2022/session1a**
- Follow instructions in the README.md file

Kinematic Fitting - DIY

/group/halld/Software/gluex_workshops/tutorial_2022

```
└─ session1a
    ├─ talk
    │   └─ GlueX_Workshop22_KinFit.pdf
    └─ Scripts
        ├─ data_reader.h
        ├─ utils.h
        ├─ run_example1
        │   └─ run_ex1.C
        ├─ run_example2
        │   └─ run_ex2.C
        ├─ run_example3
        │   └─ run_ex3.C
        └─ run_example4
            └─ run_ex4.C
```

Backup: χ^2-Minimization I

- **Note 1:** Most of the calculations and procedures shown here have been taken from:
  - "Note on kinematical fit", *Nov. 20, 1995*, A. Kupsc
  - "Least Squares Kinematic Fitting of Physics Reactions", *April 4, 2016*, Paul Mattione
- **Note 2:** For the sake of simplicity, unmeasured particles are not discussed in the calculations listed here
- **Goal:** Minimize:

$$\chi^2 = \boldsymbol{\epsilon}^T \hat{V}^{-1} \boldsymbol{\epsilon} + \boldsymbol{\lambda F} \tag{6}$$

  with: $\boldsymbol{\epsilon} = \boldsymbol{y}_{fit} - \boldsymbol{y}_{meas}$

  $\boldsymbol{y}_{meas}$ / $\boldsymbol{y}_{fit}$: a 3N-dimensional vector, containing the measured / fitted momentum components of all N particles
- We achieve the goal by solving:

$$\frac{\partial \chi^2}{\partial \boldsymbol{\epsilon}} = \frac{\partial \chi^2}{\partial \boldsymbol{y}_{fit}} = 0 \tag{7}$$

- This results, after some mathematical Harakiri, in the equation:

$$\hat{V}^{-1} \boldsymbol{\epsilon} + \hat{D} \boldsymbol{\lambda} = 0 \tag{8}$$

- with:

$$\hat{D} = \left. \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{y}} \right|_{\boldsymbol{y} = \boldsymbol{y}_{fit}}, \quad D_{hk} = \left. \frac{\partial F_h}{\partial y_k} \right|_{y_k = y_{fit,k}} \tag{9}$$

- Solving Eq. 8 $\Leftrightarrow$ Find solutions for Eq. 7
- The calculations are continued on the next slide

Backup: χ^2-Minimization II

- Eq. 8 is solved iteratively, by using a Taylor expansion around $\boldsymbol{\epsilon} \approx 0$
- For the i-th iteration one finds:

$$\begin{aligned} 0 &= \hat{V}^{-1}\boldsymbol{\epsilon}_{i+1} + \hat{D}_i^T \boldsymbol{\lambda}_{i+1} \Leftrightarrow & (10) \\ \boldsymbol{\epsilon}_{i+1} &= -\hat{V}\hat{D}_i^T \boldsymbol{\lambda}_{i+1} & (11) \end{aligned}$$

- The corresponding constraints are Taylor expanded to:

$$0 = \boldsymbol{F} \simeq \boldsymbol{F}_i + \hat{D}_i(\boldsymbol{y}_{fit,i+1} - \boldsymbol{y}_{fit,i}) = \boldsymbol{F}_i + \hat{D}_i(\boldsymbol{\epsilon}_{i+1} - \boldsymbol{\epsilon}_i) \quad (12)$$

- Plugging Eq. 11 into Eq. 12 yields:

$$\begin{aligned} 0 &= \boldsymbol{F}_i - \underbrace{\hat{D}_i \hat{V} \hat{D}_i^T}_{\equiv \hat{S}} \boldsymbol{\lambda}_{i+1} - \hat{D}_i \boldsymbol{\epsilon}_i \Leftrightarrow & (13) \\ \boldsymbol{\lambda}_{i+1} &= \hat{S}^{-1}[\boldsymbol{F}_i - \hat{D}_i \boldsymbol{\epsilon}_i] & (14) \end{aligned}$$

- Eq. 14 can be plugged back into Eq. 11 and one obtains a solution for $\boldsymbol{\epsilon}_{i+1}$:

$$\boldsymbol{\epsilon}_{i+1} = -\hat{V}\hat{D}_i^T \hat{S}^{-1}[\boldsymbol{F}_i - \hat{D}_i \boldsymbol{\epsilon}_i] \quad (15)$$

- The next slide gives a short summary on the minimization procedure itself

Backup: χ^2-Minimization III

Using the definitions and results from the last two slides, allows to formulate a rough outline for the minimization procedure:

1. Calculate $\boldsymbol{\epsilon}_i = \boldsymbol{y}_{fit}(i) - \boldsymbol{y}_{meas}$, with: $\boldsymbol{y}_{fit}(i=0) = \boldsymbol{y}_{meas}$
2. Determine the derivative matrix:

$$\hat{D}_i = \left.\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{y}}\right|_{\boldsymbol{y}=\boldsymbol{y}_{fit}(\boldsymbol{i})}, \text{ with elements: } D_{hk} = \left.\frac{\partial F_h}{\partial y_k}\right|_{y_k = y_{fit,k}(i)}$$

3. Use $\hat{D}_i$ and $\hat{V}$ to calculate the S-matrix: $\hat{S}_i = \hat{D}_i \hat{V} \hat{D}_i^T$
4. Evaluate the constraint functions at $\boldsymbol{y}_{fit}(i)$:

$$\boldsymbol{F}_i = \boldsymbol{F}[\boldsymbol{y}_{fit}(i)] \tag{16}$$

5. Calculate $\boldsymbol{\lambda}_{i+1}$, according to Eq. 14 and thus, determine $\boldsymbol{\epsilon}_{i+1}$ (see Eq. 11)
6. This allows for a new evaluation: $\boldsymbol{F}_{i+1} = \boldsymbol{F}[\boldsymbol{y}_{i+1}]$
7. Use $\boldsymbol{\lambda}_{i+1}$, $\hat{S}$ and $\boldsymbol{F}_{i+1}$ to calculate the corresponding χ^2:

$$\chi^2_{i+1} = \boldsymbol{\lambda}^T_{i+1} \hat{S}_i \boldsymbol{\lambda}_{i+1} + 2\boldsymbol{\lambda}^T_{i+1} \boldsymbol{F}_{i+1} \tag{17}$$

8. Repeat steps 1. to 7. with the new $\boldsymbol{y}_{fit,i+1}$, until a convergence criterium is fulfilled (e.g. $|\chi^2_{i+1} - \chi^2_i| \leq 0.001$)
9. Use the last reported values of $\boldsymbol{\epsilon}_{i+1}$ for further analysis