

Coordinates of central spectrometer point (unit vector):

$$x_0 = \sin(\theta_0) * \cos(\phi_0)$$

$$y_0 = \sin(\theta_0) * \sin(\phi_0)$$

$$z_0 = \cos(\theta_0)$$

$(\Delta x, \Delta y, \Delta z)$ coming from xptar and yptar:

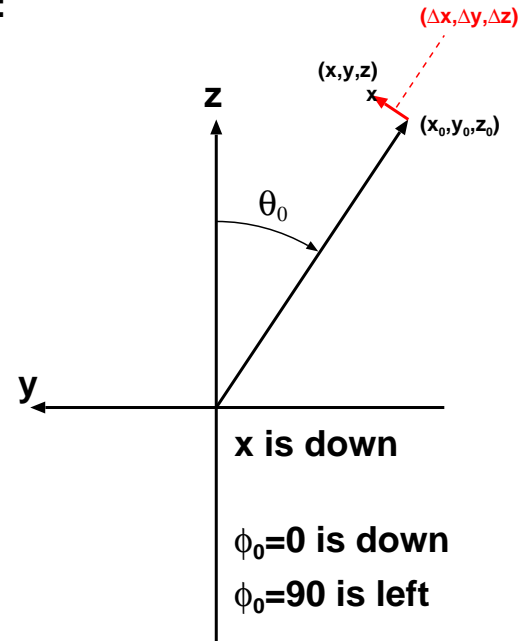
* $x_p = dx/dz = x_{ptar}$, $y_p = dy/dz = y_{ptar}$

* Assume spectrometers in plane ($\phi_0 = 90/270$)

$$\Delta x = x_p$$

$$\Delta y = y_p * \cos(\theta_0)$$

$$\Delta z = -y_p * \sin(\theta_0) * \sin(\phi_0)$$



(x, y, z) for the real event, in terms of central spectrometer and x_p/y_p (x_{ptar}/y_{ptar}).

$$x = x_0 + \Delta x = \sin(\theta_0) * \cos(\phi_0) + x_p = x_p \quad (\text{for in-plane spectrometers}).$$

$$y = y_0 + \Delta y = \sin(\theta_0) * \sin(\phi_0) + y_p * \cos(\theta_0)$$

$$z = z_0 + \Delta z = \cos(\theta_0) - y_p * \sin(\theta_0) * \sin(\phi_0)$$

To get the direction (unit vector) for the real event, need to divide by total length (which is easy to express in terms of the spectrometer unit vector and x_{ptar}/y_{ptar}).

$$d = \sqrt{x^2 + y^2 + z^2} = \sqrt{1 + x_p^2 + y_p^2}$$

$$\hat{x} = x / d, \quad \hat{y} = y / d, \quad \hat{z} = z / d$$

Next, the physics angles for the event (θ, ϕ) are calculated as follows:

$$\cos(\theta) = z / d = \frac{[\cos(\theta_0) - y_p * \sin(\theta_0) * \sin(\phi_0)]}{\sqrt{1 + x_p^2 + y_p^2}}$$

$$\tan(\phi) = y / x = \frac{\sin(\theta_0) * \sin(\phi_0) + y_p * \cos(\theta_0)}{\sin(\theta_0) * \cos(\phi_0) + x_p}$$

1) Note that denom. = dx if HMS&SOS in-plane
2) Must take care to get correct 'phase' when taking $\text{atan}(y/x)$

And finally, getting x_{ptar}/y_{ptar} from $\hat{x}, \hat{y}, \hat{z}$. Start by getting angle between the spectrometer unit vector (p_0) and the event unit vector (p).

$$\hat{p}_0 \cdot \hat{p} = \cos(d\theta) = dz / \sqrt{dx^2 + dy^2 + dz^2} = 1/d$$

$$\hat{x} = x / d = dx / d \implies dx = \hat{x} * d \implies dx/dz = (\hat{x} * d)/dz = \hat{x} / (dz/d) = \hat{x} * \cos(d\theta)$$

$$d^2 = 1 + x_p^2 + y_p^2 \implies dy/dz = y_p = \sqrt{d^2 - 1 - x_p^2}$$