2. a) $H_0: K'\tilde{\beta} - \tilde{m} = \begin{bmatrix} \beta_0 + 70\beta_1 + 10\beta_2 + 10\beta_3 \\ \beta_1 \\ \beta_2 - \beta_3 \end{bmatrix} - \begin{bmatrix} 80 \\ 4 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow \beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}, K' = \begin{bmatrix} 1 & 70 & 10 & 10 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}, m = \begin{bmatrix} 80 \\ 4 \\ 0 \end{bmatrix}$

b) • $(K'\hat{\beta} - m)'(K'(X'X)^{-1}K)^{-1}(K'\hat{\beta} - m) = 1800 \Rightarrow SSR$

• $Y'(I-H)Y = 7800 \Rightarrow SSE$

$F^* = \frac{MSR}{MSE}$

$= \frac{SSR/p}{SSE/n-p'} = \frac{1800/3}{7800/n-p'} = 1.538462$

$\Rightarrow$ F p-value $\Rightarrow$ $1 - pf(F^*, 3, 20)$

$= 0.2354379 \Rightarrow > \alpha = 0.05$

$\Rightarrow$ DO NOT REJECT H_0 //

3. a) $$X'X = \begin{bmatrix} n & \sum I_1 & \sum I_2 \\ \sum I_1 & \sum I_1^2 & \sum I_1 I_2 \\ \sum I_2 & \sum I_2 I_1 & \sum I_2^2 \end{bmatrix}$$

$$= \begin{bmatrix} n & nA & nB \\ nA & nA & 0 \\ nB & 0 & nB \end{bmatrix}$$

$$X'Y = \begin{bmatrix} \sum Y_i \\ \sum Y_i I_1 \\ \sum Y_i I_2 \end{bmatrix} = \begin{bmatrix} nA\bar{Y}_A + nB\bar{Y}_B + nC\bar{Y}_C \\ nA\bar{Y}_A \\ nB\bar{Y}_B \end{bmatrix}$$

3.b) <u>OPTION 1</u>

$$(X'X)\hat{\beta} = \begin{bmatrix} n & nA & nB \\ nA & nA & 0 \\ nB & 0 & nB \end{bmatrix} \begin{bmatrix} \hat{\beta_0} \\ \hat{\beta_1} \\ \hat{\beta_2} \end{bmatrix}$$

$$= \begin{bmatrix} n & nA & nB \\ nA & nA & 0 \\ nB & 0 & nB \end{bmatrix} \begin{bmatrix} \bar{Y}_C \\ \bar{Y}_A - \bar{Y}_C \\ \bar{Y}_B - \bar{Y}_C \end{bmatrix}$$

$$= \begin{bmatrix} (n_A + n_B + n_C)\cdot \bar{Y}_C + n_A\cdot(\bar{Y}_A - \bar{Y}_C) + n_B\cdot(\bar{Y}_B - \bar{Y}_C) \\ n_A\cdot\bar{Y}_C + n_A\cdot(\bar{Y}_A - \bar{Y}_C) \\ n_B\cdot\bar{Y}_C + n_B\cdot(\bar{Y}_B - \bar{Y}_C) \end{bmatrix}$$

$$= \begin{bmatrix} n_A\bar{Y}_A + n_B\bar{Y}_B + n_C\bar{Y}_C \\ n_A\bar{Y}_A \\ n_B\bar{Y}_B \end{bmatrix}$$

$$= X'Y \Rightarrow (X'X)\hat{\beta} = X'Y \Rightarrow \hat{\beta} = (X'X)^{-1}X'Y$$ as needed. $\blacksquare$

OPTION 2

- $S(\beta_0, \beta_1, \beta_2) = \sum_{i=1}^{n} (Y_i - \beta_0 - \beta_1 I_i - \beta_2 I_{2i})^2$
- $\Rightarrow \dfrac{\partial S}{\partial \beta_1} = -2\sum_{i=1}^{n} (Y_i - \beta_0 - \beta_1 I_{1i} - \beta_2 I_{2i}) I_1$

$$0 = \sum Y_i I_1 - \beta_0 \sum I_1 - \beta_1 \sum I_1^2 - \beta_2 \sum I_2 I_1$$

$$n_A \beta_1 = n_A \bar{Y}_A - n_A \beta_0 - 0 \Rightarrow \boxed{\beta_1 = \bar{Y}_A - \beta_0}$$

- $\Rightarrow \dfrac{\partial S}{\partial \beta_2} = -2\sum_{i=1} (Y_i - \beta_0 - \beta_1 I_{1i} - \beta_2 I_{2i}) I_2$

$$0 = \sum Y_i I_2 - \beta_0 \sum I_2 - \beta_1 \sum I_1 I_2 - \beta_2 \sum I_2^2$$

$$n B_2 = n_B \bar{Y}_B - n_B \beta_0 - 0 \Rightarrow \boxed{\beta_2 = \bar{Y}_B - \beta_0}$$

WTF β_0 from original eqtn.

- $\Rightarrow \dfrac{\partial S}{\partial \beta_0} = -2\sum_{i=1} (Y_i - \beta_0 - \beta_1 I_{1i} - \beta_2 I_{2i})$

$$0 = \sum Y_i - \sum \beta_0 - \beta_1 \sum I_{1i} - \beta_2 \sum I_{2i}$$

$$n\beta_0 = \sum Y_i - n_A \beta_1 - n_B \beta_2$$

$$\Rightarrow n\beta_0 = \sum Y_i - n_A \beta_1 - n_B \beta_2$$

$$n\beta_0 = \sum Y_i - n_A(\bar{Y}_A - \beta_0) - n_B(\bar{Y}_B - \beta_0)$$

$$\Rightarrow (n - n_A - n_B)\beta_0 = \sum Y_i - n_A \bar{Y}_A - n_B \bar{Y}_B$$

sum of Y_i from A, B, C

$$n_C \beta_0 = n_C \bar{Y}_C$$

$$\beta_0 = \bar{Y}_C$$

$$\Rightarrow \beta_1 = \bar{Y}_A - \beta_0$$

$$\beta_1 = \bar{Y}_A - \bar{Y}_C$$

$$\Rightarrow \beta_2 = \bar{Y}_B - \beta_0$$

$$\beta_2 = \bar{Y}_B - \bar{Y}_C$$, as needed ∎

c)

- $S_A^2 = \frac{1}{n_A - 1} \cdot \sum_{i \in \{X = A\}}^{n_A} (Y_i - \bar{Y}_A)^2$
- $S_B^2 = \frac{1}{n_B - 1} \cdot \sum_{i \in \{X = B\}}^{n_B} (Y_i - \bar{Y}_B)^2$
- $S_C^2 = \frac{1}{n_C - 1} \cdot \sum_{i \in \{X = C\}}^{n_C} (Y_i - \bar{Y}_C)^2$
- $SSE = Y'Y - \hat{\beta}' X' Y$

- $y'y = \sum_{i=1}^{n} y_i^2 = \sum_{i \in \{x \in A\}}^{n_A} y_i^2 + \sum_{i \in \{x \in B\}}^{n_B} y_i^2 + \sum_{i \in \{x \in C\}}^{n_C} y_i^2$

- $\hat{\beta}' x' y = \begin{bmatrix} \hat{\beta}_0 & \hat{\beta}_1 & \hat{\beta}_2 \end{bmatrix} \begin{bmatrix} n_A \bar{Y}_A + n_B \bar{Y}_B + n_C \bar{Y}_C \\ n_A \bar{Y}_A \\ n_B \bar{Y}_B \end{bmatrix}$ from 3a)

$$= \overset{\bar{Y}_C}{\hat{\beta}_0} \left[n_A \bar{Y}_A + n_B \bar{Y}_B + n_C \bar{Y}_C \right] + \overset{\bar{Y}_A - \bar{Y}_C}{\hat{\beta}_1} \left[n_A \bar{Y}_A \right] + \overset{\bar{Y}_B - \bar{Y}_C}{\hat{\beta}_2} \left[n_B \bar{Y}_B \right]$$

$$= \cancel{n_A \bar{Y}_A \bar{Y}_C} + \cancel{n_B \bar{Y}_B \bar{Y}_C} + n_C \bar{Y}_C^2 + n_A \bar{Y}_A^2 - \cancel{n_A \bar{Y}_A \bar{Y}_C} + n_B \bar{Y}_B^2 - \cancel{n_B \bar{Y}_B \bar{Y}_C}$$

$$= n_C \bar{Y}_C^2 + n_A \bar{Y}_A^2 + n_B \bar{Y}_B^2$$

$$SSE = y'y - \hat{\beta}' x' y$$

$$\Rightarrow \quad = \sum_{i \in \{x \in A\}}^{n_A} y_i^2 + \sum_{i \in \{x \in B\}}^{n_B} y_i^2 + \sum_{i \in \{x \in C\}}^{n_C} y_i^2 - \left[n_C \bar{Y}_C^2 + n_A \bar{Y}_A^2 + n_B \bar{Y}_B^2 \right]$$

$$= \left[\sum_{i \in \{x \in A\}}^{n_A} y_i^2 - n_A \bar{Y}_A^2 \right] + \left[\sum_{i \in \{x \in B\}}^{n_B} y_i^2 - n_B \bar{Y}_B^2 \right] + \left[\sum_{i \in \{x \in C\}}^{n_C} y_i^2 - n_C \bar{Y}_C^2 \right]$$

$$= \sum_{i \in \{x = A\}}^{n_A} (y_i - \bar{Y}_A)^2 + \sum_{i \in \{x = B\}}^{n_B} (y_i - \bar{Y}_B)^2 + \sum_{i \in \{x = C\}}^{n_C} (y_i - \bar{Y}_C)^2$$

$$= (n_A - 1) \cdot s_A^2 + (n_B - 1) \cdot s_B^2 + (n_C - 1) \cdot s_C^2$$

$\blacksquare$